Zipf-Mandelbrot:
$f(k;q,z,n) = \frac{C}{(k+q)^z} \quad \text{avec}\, C^{-1} = \sum_{i=1}^{n} \frac{1}{(i+q)^z}$

- loglik $= \sum_{k=1}^{n} \log(C) - z\log(k+q)$
- negloglik $= \sum_{k=1}^{n} -\log(C) + z\log(k+q)$
  $= \sum_{k=1}^{n} \log(C^{-1}) + z\log(k+q) = \sum_{k=1}^{n} \log\left(\sum_{i=1}^{n} \frac{1}{(i+q)^z}\right) + z\log(k+q)$
  $= \sum_{k=1}^{n} \log\left(\sum_{i=1}^{n} \exp(-z\log(i+q))\right) + z\log(k+q).$

Gradient:

$\frac{\partial \text{negloglik}}{\partial q} = \sum_{k=1}^{n} \frac{\sum_{i=1}^{n} \frac{-z}{(i+q)^{z+1}}}{\sum_{j=1}^{n} \frac{1}{(j+q)^z}} + \frac{z}{k+q}$

$\frac{\partial \text{negloglik}}{\partial z} = \sum_{k=1}^{n} \frac{\sum_{i=1}^{n} -\log(i+q)\exp(-z\log(i+q))}{\sum_{j=1}^{n} \exp(-z\log(j+q))} + \log(k+q)$