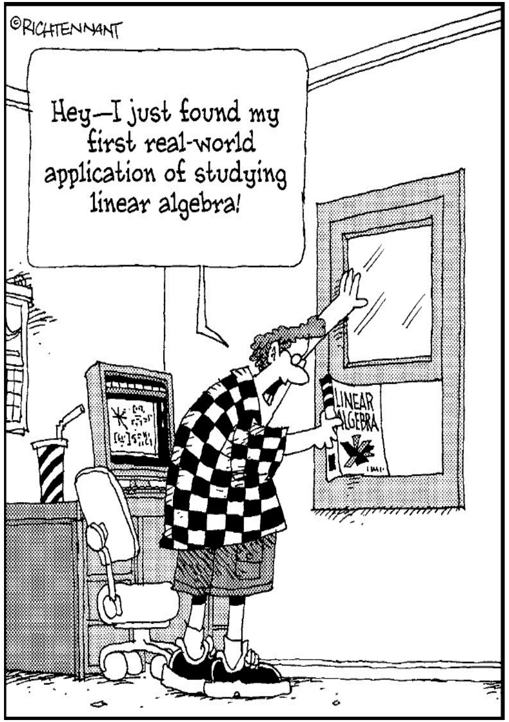
Taller de Matemática Computacional TUDAI

2019 Exactas - UNICEN

Algebra lineal

Escena 1

Algebra lineal



Ejemplo

- 4 VENDEDORES, 5 PRODUCTOS (A,D,W,T,H)
- A ($110),D($200),W($600),T($60),H($100)
- ¿Cuánto vendió cada uno?

Ejemplo

- 4 VENDEDORES, 5 PRODUCTOS (A,D,W,T,H)
- A ($110),D($200),W($600),T($60),H($100)
- ¿Cuánto vendió cada uno?

$$
\begin{array}{r c}
 & \begin{array}{ccccc} \mathrm{A} & \mathrm{D} & \mathrm{W} & \mathrm{T} & \mathrm{H} \end{array} \\
\begin{array}{r} \text{Lucas} \\ \text{Pablo} \\ \text{Nacho} \\ \text{Diego} \end{array} &
\begin{bmatrix} 15 & 10 & 5 & 9 & 1 \\ 10 & 9 & 4 & 9 & 2 \\ 20 & 0 & 0 & 23 & 1 \\ 15 & 6 & 10 & 6 & 5 \end{bmatrix}
\end{array}
\times
\begin{bmatrix} 110 \\ 200 \\ 600 \\ 60 \\ 100 \end{bmatrix}
=
\begin{bmatrix} 7290 \\ 6040 \\ 3680 \\ 9710 \end{bmatrix}
$$

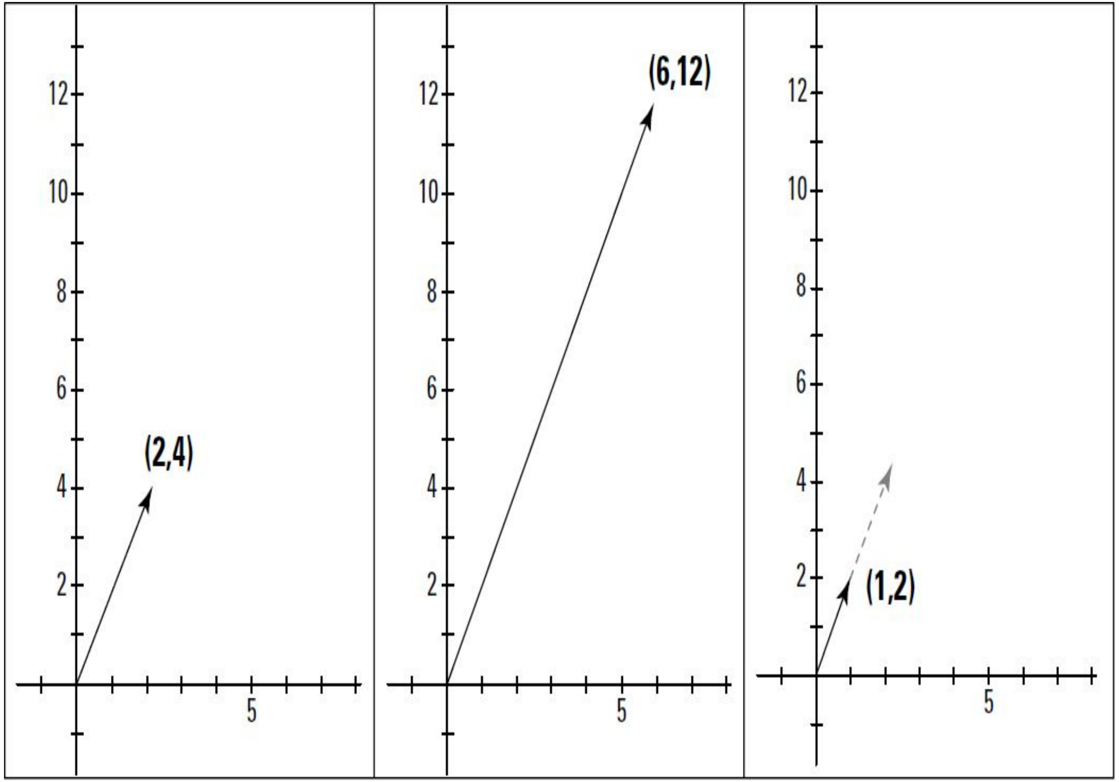
Vectores

Un vector es una tupla de **n** números.

El conjunto de todos los vectores reales de dimensión **n** se expresa como $\mathbb{R}^{\mathbf{n}}$

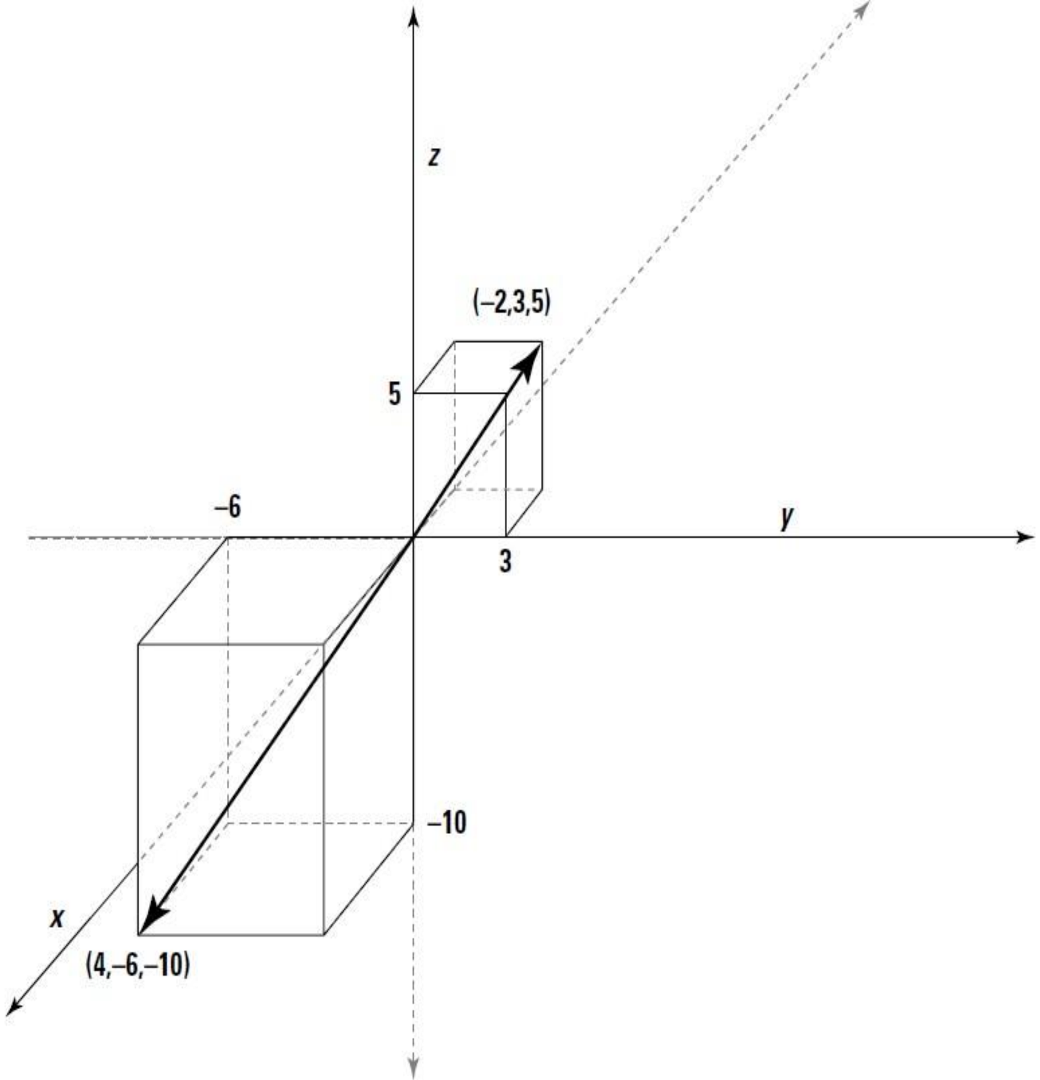
$v = (v_1, v_2, v_3, ..., v_{\mathbf{n}}) \in R^{\mathbf{n}}$

Vectores

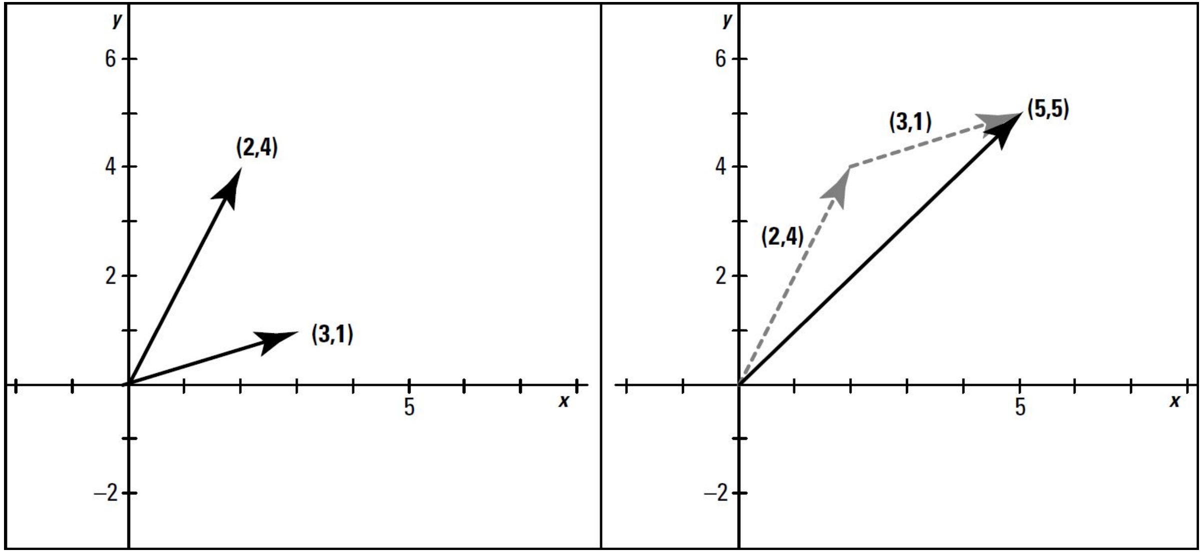


$$k\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} kv_1 \\ kv_2 \\ kv_3 \\ \vdots \\ kv_n \end{bmatrix}$$

Vectores



Vectores



$$\mathbf{u} + \mathbf{v} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_n \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \\ \vdots \\ u_n + v_n \end{bmatrix}$$

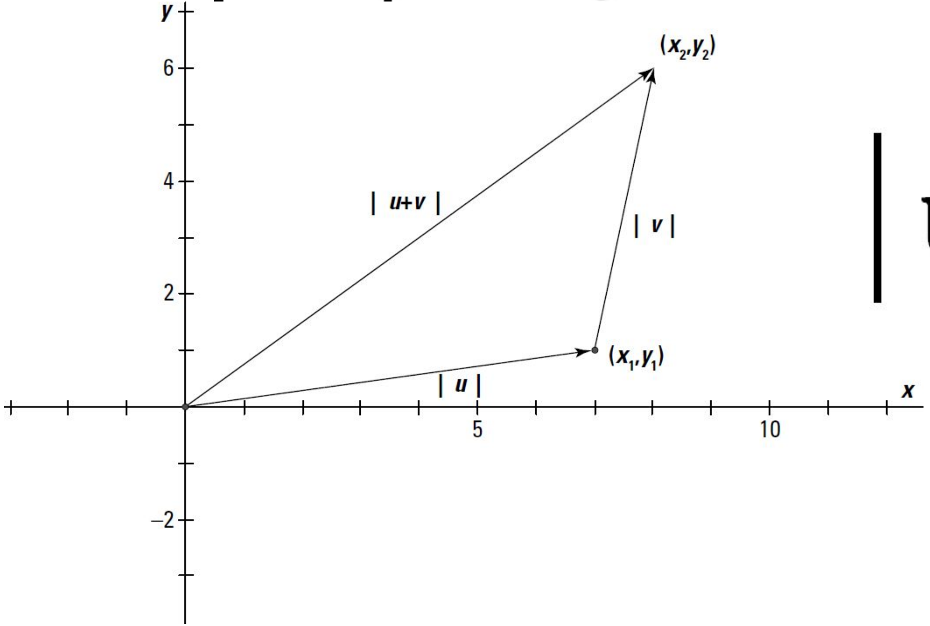
$$\mathbf{u} - \mathbf{v} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_n \end{bmatrix} - \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} u_1 - v_1 \\ u_2 - v_2 \\ u_3 - v_3 \\ \vdots \\ u_n - v_n \end{bmatrix}$$

Vectores

$$\left|\mathbf{v}\right| = \sqrt{v_1{}^2 + v_2{}^2 + \cdots + v_n{}^2}$$

Vectores

$$\left| \mathbf{v} \right| = \sqrt{v_1^{\ 2} + v_2^{\ 2} + \cdots + v_n^{\ 2}}$$



$$\left| \mathbf{u} + \mathbf{v} \right| \leq \left| \mathbf{u} \right| + \left| \mathbf{v} \right|$$

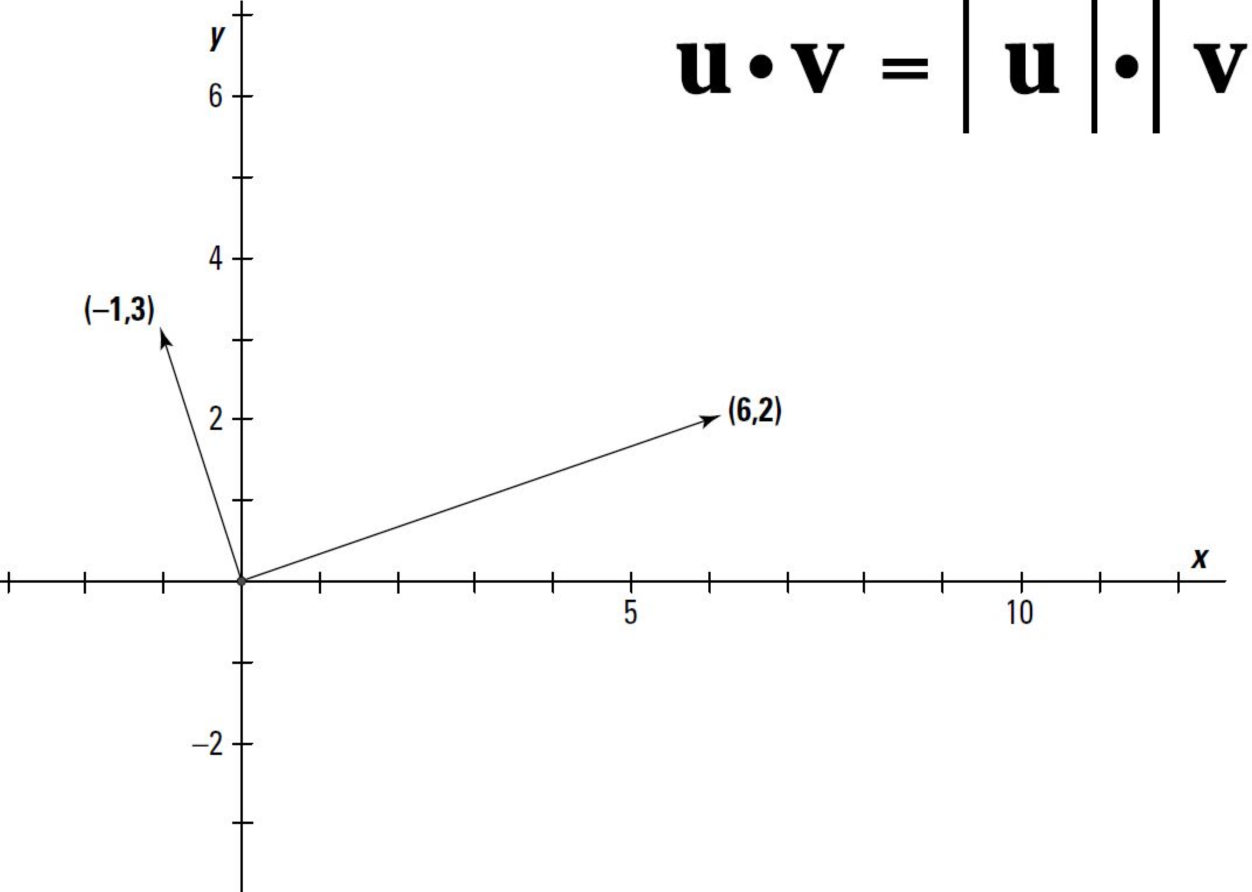
Vectores

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}, \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

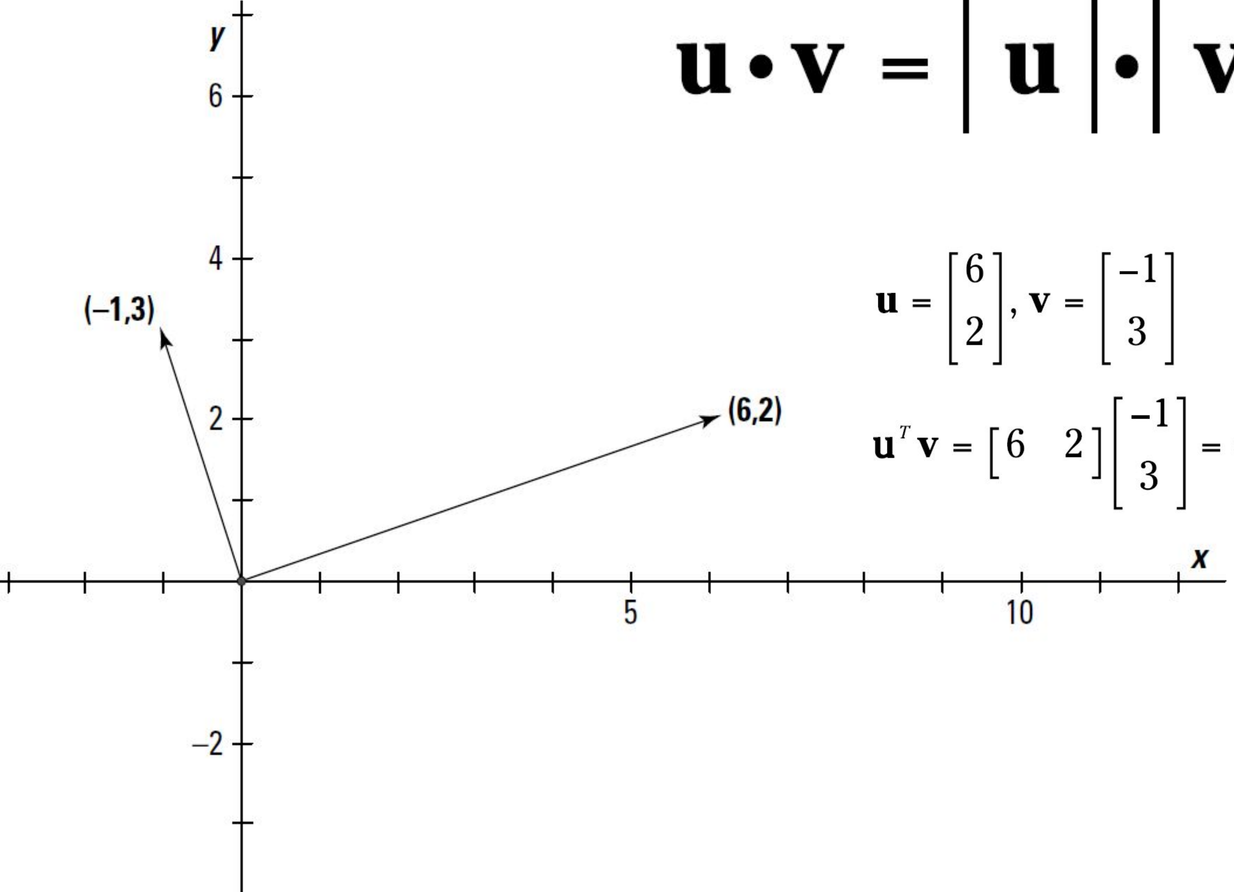
$$\mathbf{u}^T \mathbf{v} = \underbrace{\begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}}_{1 \times N} \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}}_{N \times 1} = \underbrace{u_1 v_1 + u_2 v_2 + \cdots + u_n v_n}_{1 \times 1}$$

Vectores

$$\mathbf{u} \cdot \mathbf{v} = \left| \mathbf{u} \right| \cdot \left| \mathbf{v} \right| \cos\theta$$



Vectores



$$\mathbf{u} \cdot \mathbf{v} = \left| \mathbf{u} \right| \cdot \left| \mathbf{v} \right| \cos\theta$$

$$\mathbf{u} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$\mathbf{u}^T \mathbf{v} = \begin{bmatrix} 6 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = 6(-1) + 2(3) = -6 + 6 = 0$$

Matrices

Una matriz es arreglo bidimensional de **n**x**m** números.

El conjunto de todos los vectores de dimensiones **n**x**m** se expresa como $\mathbb{R}^{\mathbf{nxm}}$

$$a = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \dots & a_{1,n} \\ a_{2,1} & a_{2,2} & a_{2,3} & \dots & a_{1,n} \\ \dots & & & & \dots \\ a_{m,1} & a_{m,2} & a_{m,3} & \dots & a_{m,n} \end{pmatrix} \in R^{\mathbf{nxm}}$$

Matrices

$$A + B = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \cdots & a_{1n} + b_{1n} \\ a_{21} + b_{21} & a_{22} + b_{22} & \cdots & a_{2n} + b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} + b_{m1} & a_{m2} + b_{m2} & \cdots & a_{mn} + b_{mn} \end{bmatrix}$$

$$A - B = \begin{bmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \cdots & a_{1n} - b_{1n} \\ a_{21} - b_{21} & a_{22} - b_{22} & \cdots & a_{2n} - b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} - b_{m1} & a_{m2} - b_{m2} & \cdots & a_{mn} - b_{mn} \end{bmatrix}$$

Matrices

$$kA = k\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} & \cdots & ka_{1n} \\ ka_{21} & ka_{22} & \cdots & ka_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ka_{m1} & ka_{m2} & \cdots & ka_{mn} \end{bmatrix}$$

Matrices

$$A * B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + \cdots + a_{1m}b_{m1} & a_{11}b_{12} + a_{12}b_{22} + \cdots a_{1m}b_{m2} & \cdots \\ a_{21}b_{11} + a_{22}b_{21} + \cdots + a_{2m}b_{m1} & a_{21}b_{12} + a_{22}b_{22} + \cdots a_{2m}b_{m2} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

Matrices

Matriz identidad

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrices

Triangular Superior

$$A = \begin{bmatrix} 4 & 3 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

Triangular Inferior

$$B = \begin{bmatrix} 7 & 0 & 0 & 0 \\ 7 & 8 & 0 & 0 \\ 5 & 2 & 3 & 0 \\ 5 & -4 & -9 & 1 \end{bmatrix}$$

Diagonal

$$C = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Matrices - Propiedades

- Conmutativa respecto de la suma: $A + B = B + A$
- Asociativa respecto de la suma: $(A + B) + C = A + (B + C)$
- Distributiva:
  - $k (A + B) = kA + kB$
  - $(k + l) A = kA + lA$
  - $A * (B + C) = A * B + A * C$
- <u>No conmutativa respecto al producto</u>: $A * B != B * A$

Matrices - Inversa

$M * M^{-1} = I$

$M * A * M^{-1} = A$

$$\mathbf{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\mathbf{M}^{-1} = \begin{bmatrix} \dfrac{d}{ad - bc} & \dfrac{-b}{ad - bc} \\ \dfrac{-c}{ad - bc} & \dfrac{a}{ad - bc} \end{bmatrix}$$