G.C.E. (Advanced Level) Examination - April 2004
Combined Mathematics - II
Three hours

- Answer six questions only.
- In this question paper, g denotes the acceleration due to gravity.

(01) (*a*) A particle projected horizontally with speed $\sqrt{2}\,gT$, from a height h above the ground, where T is a constant, moves under gravity.

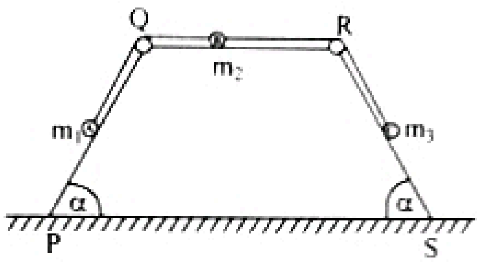
Draw separate speed-time graphs for the horizontal and vertical components of the velocity of the particle.

If the particle is at a distance $\frac{3}{2}gT^2$ from the point of projection, when it falls to the ground, show that, using the speed-time graphs, the time taken by the particle to reach the ground is T, and $h = \frac{1}{2}gT^2$.

(b) A motor car of width w moves uniformly along a straight road, parallel to the pavement almost touching it. A pedestrian on the edge of the pavement at a distance l ahead of the car begins to walk uniformly to cross the road. If v is the speed of the car and u is the speed of the pedestrian relative to the road, show that the pedestrian can cross the road safely in front of the car if $u > v \sin \alpha$, where $\alpha = \tan^{-1}\left(\frac{w}{l}\right)$.

If $u = v \sin \alpha$, show that the pedestrian can cross the road just in front of the car, by walking relative to the road in a direction making an angle $\frac{\pi}{2} - \alpha$ with the direction of the motion of the car relative to the road.

(02) The figure represents a vertical cross section of a smooth block of mass M with PQ and RS inclined at an angle α to the horizontal, and QR and PS being horizontal. A smooth light inelastic string passes over two small smooth pulleys at Q and R. Two small smooth particles A and C of masses m_1 and m_3 respectively are attached to the ends of the string. A third small smooth particle B of mass m_2 is attached to the string in between Q and R. The block is free to move on a smooth horozonatal plane. **Write down** equations to determine the acceleration of the block relative to the plane, the accelerations of the particles relative to the block, and the tensions in the portions AB and BC of the string.



If mass of the particle B is negligible, show that the tensions in the two portions AB and BC of the string are the same.

If, further the mass of the block is also negligible, show that each of the magnitudes of the reactions of the block on A and C is equal to $\frac{2m_1m_3}{m_1 + m_3}\, g \cos \alpha$.

(03) (a) A particle is projected from a point O on the ground, with speed u at an angle α to the horozontal, under gravity. At the same instant, a vertical screen a perpendiculat distance d form O and at right angles to the vertical plane of motion of the particle, is made to move away from the particle, in a horizontal direction with uniform speed v

If the particle strikes the screen at a height h above the ground, show that $u \cos \alpha > v$, and $gd^2 - 2u \sin \alpha (u \cos \alpha - v) d + 2h (u \cos \alpha - v)^2 = 0$

Deduce that the particle cannot strike the screen, if $d > \frac{2u}{g} \sin\alpha(u \cos \alpha - v)$

(b) A small smooth particle P of mass m is free to move under gravity in a **thin** smooth circular tube of radius r and centre O, fixed in a vertical plane. The particle is projected horizonatlly from the lowest point of the tube with speed $\sqrt{3gr}$.

Explaim why the law of conservation of energy can be applied for the motion of the particle.

If v is the speed of the particle when OP makes an angle θ with the downward vertical, show that $v^2 = gr(1 + 2\cos\theta)$.

Hence show that the reaction of the tube on the particle changes its direction when $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$ and find the speed of the particle at that point.

(04) (a) A small smooth particle, A and a small smooth elastic particle B of mass m are attached to the two ends of an inelastic string of length l and are at rest on a smooth horizontal plane. The system now moves with speed u in the direction AB with the string taut. The particle B, after some time, collides with a small smooth elastic particle C of mass M which is at rest on the plane. If e is the coefficeient of restitution between the particles B and C, show that the particle B moves with speed $\frac{m - eM}{m + M}u$ after colliding with particle C, and the particle A collides with the particle B after a time $\frac{(m + M)l}{M(1 + e)u}$ from the moment of collision between B and C.

(b) One end of a light elastic string of natural lenght l, passing through a small smooth ring of mass m, is attached to a point O, of a ceiling. A particle P of mass M attached to the other end of the string hangs in equalibrium, with the ring being held at rest at the point O. if $2Mg$ is the modulus of elasticity of the string, show that the extension of the string in the equilibrium position is $\frac{l}{2}$.

The ring, now released from rest at O, slides verticaly downward along the string, under gravity, collides and coalesces with P. Show that the composite body consisting of the ring and the particle, wil begin to move vertically downward with velocity $\frac{m}{M + m}\sqrt{3gl}$.

Write down the equation of motion for the composite body. when x is the extension of the string, and show that the composite body performs simple harmonic motion with frequency $\sqrt{\frac{2Mg}{(M + m)l}}$.

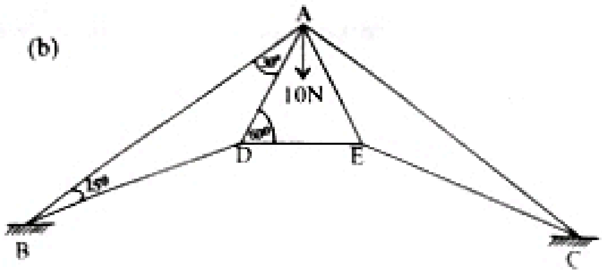
(05) (a) Forces of magnitudes 5, 6, 1 and 2 newtons act along the sides AB, CB, CD and AD in the directions indicated by the order of letters, respectively, of a swuare $ABCD$ of side 1m. Find the magnitude, direction and the line of action of the resutant force.

Another force of magnitude $4\sqrt{2}$ newtons acting along BD, in the direction from B to D, is added to the system. Show that the system reduces to a couple of magnitude 1Nm.

(b) A ladder whose centre of gravity is at a distance b from the foot, stands on a rough horizontal ground and leans in equilibrium against a rough cylindrical pipe of radius r, fixed on the ground. The ladder projects beyond the point of contact with the pipe and is perpendicular to the axis of the pipe. Let λ be the angle of friction at both points where friction acts and 2α ($b < r\cot\alpha$) be the inclination of the ladder to the horizontal. A load of weight equal to that of the ladder is suspended from a point at a distance x measured along the ladder from its foot. The ladder is in limiting equilibrium at both points where friction acts. Show that $(b + x)\sin^2\alpha\cos 2\alpha = r\sin\lambda\cos\lambda$.

(06) (a) AB, BC and CD are three uniform rods of equal weight and length, smoothly hinged at B *and* C. The ends A and D are hinged to fixed smooth horizontal pins at the same level. The system hangs in equilibrium. If AB and CD are inclined at the same angle α to the horizontal, and β is the inclination of the reaction at A on AB to the horizontal, show that $\tan\alpha = \frac{2}{3}\tan\beta$.

(b)

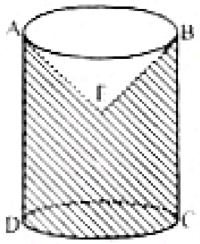


The light framework shown in the diagram lies in a vertical plane, and is symmetrical about the vertical line through A. The framwork rests in equilibrium with supports at B and C at the same horizontal level. The angles $D\hat{B}A$, $D\hat{A}B$ and $A\hat{D}E$ are 15°, 30° and 60° respectively. If a load of 10N hangs from the point A, draw a stress diagram using Bow's notation, and hence show that the rod CE has a tension of magnitude $\frac{5\sqrt{3}}{2}$ cosec 15° N.

Determine the stress in each of the other rods indicating whether it is a tension or a thrust.

(07) Show that the centre of gravity of a uniform solid right circular cone of height h is at a distance $\frac{1}{4}h$, on its axis, from the base.

The figure shows what remains of a uniform solid right circular cylinder $ABCD$ of height H and base radius R, after a solid right circular cone EAB of heighty h and base radius R is scooped out. Find the distance of the centre of gravity of the resulting body S, from AB.



Hence, show that, if the centre of gravity of S is at E, then $h = (2 - \sqrt{2})H$.

The body S is placed on a rough plane making an angle $\alpha\left(< \frac{\pi}{2}\right)$ with the horizontal, the base DC being on the plane. The plane is rough enough to prevent S from slipping. Assuming that the centre of gravity of S is at E, show that S will not topple if $R \tan\alpha > (\sqrt{2} - 1)H$.

(08) (a) The probability that a randomly selected item is faulty is P_1. The probability that a defect is detected in a faulty item is P_2. Find the probability that the defect is detected in an item selected at random.(You may assume that the probability of detection a defect in a good item is 0).

Suppose that three such items are selected at random. Determine the probability that

(i) no defects are detected among the three items.

(ii) the defects are detected in two items.

(iii) the defects are detected at least in two items.

(b) Two forecasters X and Y predict weather independently of each other. The probability that the forecaster X predicts the weather correctly is α and the probability that the forecaster Y predicts the weather correctly is β. For a given day the forecaster X predicted fair weather and the forecater Y predicted bad weather. Find the probability thar the forecaster X is correct.

(09) (a) The following table gives the information in respect of the monthly wages of 100 employees in a certain factory:

Monthly Wage (in Rupees)	Number of employees
6000	35
10 000	30
15 000	25
20 000	10

Find the mean, the median and the mode of this wage distribution.

Which of these values will be altered if 4 employees wotk overtime and each increases his montly wage by 3750 Rupees? Justify your answers.

(b) The weights of 200 men were meaured to the neatest kilogram. The results obtained are shown in the following table

Weight (kg)	45-54	55-64	65-74	75-84	85-94	95-104
Frequencey	24	50	58	35	21	12

(i) Identify the modal class and compute the mode of the distribution.

(ii) Identify the median class and compute the median of the distribution.

(iii) Evaluate the mean and the standard deviation of the distribution.