G.C.E. (Advanced Level) Examination - August 2015
Combined Mathematics - II
Three hours

Part A

- **Answer all questions**

(01) Two particles A and B, of masses m and 2m respectively are attached to the two ends, of a light inexitensible string of length $2l$, which passes over a fixed small light smooth pulley C. The system is helt with each particle at a depth l below C, and released from rest in this position. Using the principle of conservation of energy. Show that the speed V of each particle after moving a distance $x(< l)$ is given by $v^2 = \frac{2gx}{3}$, Hence or otherwise, find the acceler ation of the system.

(02) A straight narrow smooth tube OA, of length l, with both ends open, is fixed with the upper end O at a height h ($> l$) above the horizantal ground, making an angle $\pi/3$ with the downward vertical. A particle, gently placed inside the tube at O slides down along the tube. Next the particle leaves the tube at the end A and strikes the ground at a point B at a horisontal distance $\sqrt{3}l$ from O. Show that (i) the speed of the particle at A is $\sqrt{gl}$, and (ii) $h = 3l/2$.

(03) A particle P of mass m, moving on a smooth horizontal table with velocity u, collides directly with another particle Q of mass m which lies at rest on the path of P. If the coefficient of restitution between the two particles is e ($0 < e < 1$) obtain expressions for the sum and the difference of the velocities of P and Q after impact in terms of u and e Hence, or otherwise, show that ratio of the kinetic energy retained in the system after collision to the criginal kinetic energy is $(1 + e^2) : 2$.

(04) A lorry of mass M metric tons moves at constant velocity $u\,ms^{-1}$ along a straight level road, with the engine working at power H KW. Next, the lorry with the engine working at power 2H kw moves up along a straight road inclined to the horizontal at an angle α, and the resistance to the motion is the same as the resistance in the horizontal motion. Show that the maximum speed of the lorry. in this case, is

$$\frac{2Hu}{H + Mgu\sin\alpha}\ ms^{-1}$$

(05) In the usual notation, the position vecters of two points A and B with respect to an origin O, are $\lambda\underline{i} + \mu\underline{j}$ and $\mu\underline{i} - \lambda\underline{j}$ respectively, where λ and μ are real numbers such that $0 < \lambda < \mu$ show that $A\hat{O}B$ is a right angle. Let C be the mid point of the line segment AB. If the vector $\overrightarrow{OC}$ is of magnitude 2 and it makes an angle $\frac{\pi}{6}$ with the unit vector i, find the values of λ and μ.

(06) A uniform thin heavy rod rests with one end on a rough horizontal floor and the other end against a smooth vertical wall. The rod lies in a vertical plane perpendicular to the wall, making an a cute angle θ with the wall. Show that for the rod to be in equilibrium in this position, the coefficient of friction μ between the rod and the floor must satisfy $\mu \geqslant \frac{1}{2}\tan\theta$

(07) Let A, B and C be three independent events in a sample space s. In the usual notation, express the probability $P(A \cup B \cup C)$ in terms of probabilities P(A), P(B) and P(C). Given further that $P(A) = 1/4$, $P(B) = 1/2$ and $P(A \cup B \cup C) = 3/4$, find the probability P(C).

(08) A box contains 7 eletric bulbs which appear to be identical. Out of these bulbs 2 are known to be defective and the rest are usable. The bulbs are tested one after the other until the 2 defective bulbs are identified. Find the probability that the two defective bulbs will be identified ofter testing

(i) just two bulbs

(ii) juts three bulbs.

(09) A set S of seven **whole numbers** is aranged in the **ascending order** as follows:

$S = (1, 2, 4, x, y, 11, 13)$

If y is the mean of the numbers, determine the values of x and y. Show that the variance of the numbers is $\frac{120}{7}$

(10) When a dice with faces marked 1, 2, 3, 4, 5, 6 is tossed 50 times. The frequency distribution of the respective numbers appearing on the upper face of the dice is shown below.

Number	1	2	3	4	5	6
Frequency	α	9	γ	11	8	7

Given that the mean of the frequency distribution is 3.66 determine the values of α and γ, and find the mode and the median.

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Part B

- **Answer five questions only**

(11).(a) Two particles P and Q are simultaneously projected vertically upwards with speeds u and $\frac{u}{\sqrt{2}}$ respectively, from two points on a fixed horizontal floor. There is a fixed smooth horizontal ceiling at a height $\frac{u^2}{4g}$ from the floor. The coefficient of restitution between the ceiling and the particle P which strikes it is $\frac{1}{\sqrt{2}}$, and the two particles move upwards and downwards only under gravity.

(i) Find the speed of the particle P just before it strikes the ceiling and the time T_1 up to the instant of collision.

Show that the particle P returns to its point of projection with speed $\frac{u\sqrt{3}}{2}$

(ii) Show that the particle Q just reaches the ceiling, and find the time T_2 up to that instant.

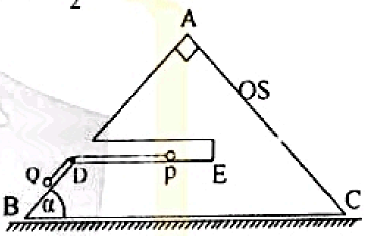
(iii) Sketch, on the same diagram, the velocity-time graphs for the motions of the two particles P and Q from the instant of projection until they return to the respective points of projection.

(iv) Using the velocity-time graphs show that, at the instant when P strikes the ceiling, Q is at a vertical distance $\frac{u^2}{2g}\left(\sqrt{2}-1\right)^2$ below the ceiling.

(b) A ship S sails due North with uniform speed u. Its straight line path is at a perpendicular distance P Eastward from a port P. At a certain instant when the direction of $\overrightarrow{PS}$ makes an angle 45° South of East, two supply boats B_1 and B_2, each moving with uniform speed $v\left(\frac{u}{\sqrt{2}} < v < u\right)$, start from port P at the same instant in two different directions so as to intercept the ship S. These boats reach the ship S at times T_1 and T_2 ($> T_1$), respectively. Given further that $\frac{v}{u} = \sqrt{\frac{2}{3}}$, sketch the two relative velocity triangles for the motions of the boats B_1 and B_2 relative to the ship S, on the same diagram and find the actual directions of motion of boats B_1 and B_2 as they move from the port P to the ship S.

Show further that $T_2 - T_1 = \frac{2\sqrt{3}P}{u}$

(12).(a) The triangle ABC in the given figure represents a vertical cross section through the centre of gravity of a uniform smooth wedge of mass M. There is a thin smooth groove DE parallel to BC, within the wedge. The lines AB and AC are the lines of greatest slope of the respective faces, $A\hat{B}C = \alpha$ and $B\hat{A}C = \frac{\pi}{2}$

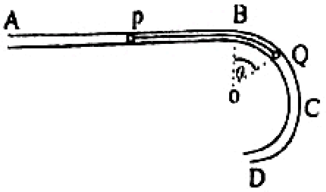


The wedge is placed with the face containing BC on a fixed smooth horizontal table. Two particles P and Q each of mass m, placed on DE and DB respectively, are connected by a light inextensible string which passes over a small smooth light pulley at the point D. A particle S of mass $\frac{m}{2}$ is placed at a point on AC, and the system is released from rest in this position, with the string connecting P and Q taut.

Write down the equations of motion for particle P along ED, for particle Q along DB and for particle S along AC. Further, write down the equation of motion for the whole system along BC. **Hence** show that the acceleration of the wedge is $\frac{mg\sin\alpha}{2M + 3m - 2m\cos\alpha}$, in the direction of $\overrightarrow{BC}$.

(b) A narrow smooth tube $ABCD$ is bent into the form indicated in the figure below. The portion AB of the tube is straight. The portion BCD is of semicircular shape with radius a, centre O and the diameter BD perpendicular to AB. The tube is fixed in a vertical plane with AB horizontal and uppermost. Inside the tube there is a particle P of mass m and a particle Q of mass $3m$ connected by a light inextensible string of length $l\left(>\frac{\pi a}{2}\right)$.

Initially, the string is taut, lying along AB, with the particle Q at the point B. The particle Q is slighty displaced from this position, and in time t, radius OQ turns through an acute angle θ.



Applying the principle of conservation of energy, show that $\left(\frac{d\theta}{dt}\right)^2 = \frac{3g}{2a}(1-\cos\theta)$

Hence, or otherwise, Show that the acceleration of the particle P is $\frac{3g}{4}\sin\theta$.

Find the reaction from the tube on the particle Q and the tension in the string, at time t.

(13) One end of a light elastic string of natural length a and modulus of elasticity $2mg$ is tied to a fixed point A. The string passes over a small smooth peg B which is fixed above the level of A, and a particle P of mass m is attached to the other end of the string. The distance AB is a, and the angle made by BA with the downward vertical is $\frac{\pi}{4}$. Initially, the particle P is placed just below the peg B and projected vertically downwards with speed $u = \sqrt{\frac{5ga}{8}}$.

Let x be the extension of the string at time t. Show that the equation for the simple harmonic motion of the particle P can be expressed in the form $\ddot{X} + \omega^2 X = 0$, where $X = x - \frac{a}{2}$ and $\omega^2 = \frac{2g}{a}$

Assuming a solution for this equation of motion in the form $\dot{X}^2 = \omega^2(A^2 - X^2)$, show that the simple harmonic motion is of amplitude $A = \frac{3a}{4}$, and find the lowest point E reached by the particle.

Show that the speed of the particle as it passes the centre C of the simple harmonic motion is $\frac{3u}{\sqrt{5}}$.

By considering the corresponding circular motion, or otherwise, show that the time taken by the particle P to pass C in its downward motion is $\sqrt{\frac{a}{2g}}\left\{\frac{\pi}{2} - \cos^{-1}\left(\frac{2}{3}\right)\right\}$.

Further, find the time taken by the particle P to reach its lowest position E, and the maximum magnitude of the force exerted by the string on the peg.

(14) With respect to the origin O in the xy-plane, the position vectors of points A, B and C, in the usual notation, are $i + j$, $2i + 3j$ and $4i + 2j$, respectively. Find the position vector of the point P on BC such that $\overrightarrow{BP} = \frac{1}{3}\overrightarrow{BC}$. The vertex D of a trapezium $ABCD$ is taken such that the side BC is parallel to AD and PD is perpendicular to AC. Show that the position vector of D is $\frac{11}{3}i - \frac{1}{3}j$

A system consists of four forces in the xy-plane with distance measured in metres and force in newtons, is given as follows:

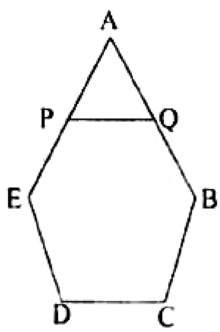
Coordinated of point of action	components of force in ox, oy directions
$B(2, 3)$	$F_1 = (2, 4)$
$C(4, 2)$	$F_2 = (3, 1)$
$L(0, 1)$	$F_3 = (6, 12)$
$M(0, 6)$	$F_4 = (9, 3)$

(i) Show that the moments of the two forces F_1 and F_2 about the origin O and the point $A(1, 1)$ are zero, and **hence** show that the moment G of the system consisting of four forces F_1, F_2, F_3 and F_4 about the origin is of magnitude $60\ Nm$, in the clockwise sense.

(ii) Find the components (X, Y) of the resultant $\mathbf{R}$ of the system. **Hence** find the point where the line of action of $\mathbf{R}$ meets the y-axis.

(iii) The system of forces is replaced by a single force acting at the point $(0, -4)$ and a couple of moment G_1. Find the value of G_1 and show that the line of action of the single force passes through the point $D\left(\frac{11}{3}, -\frac{1}{3}\right)$

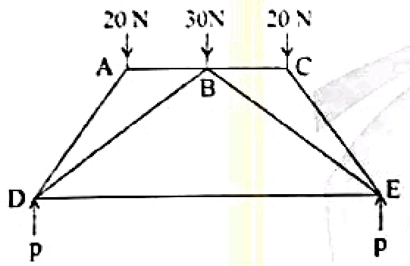
(15).(a) Five uniform heavy rods AB, BC, CD, DE and EA are smoothly jointed at their ends to form a framework in the shape of a pentagon $ABCDE$, as shown in the figure. The rods BC, CD and DE are each of length l and weight W. The rods AB and EA are each of length $2l$ and weight 2w. The two ends P and Q of a light rod PQ of length l are smoothly hinged to the mid-points of AE and AB respectively. The framework, freely suspended from the joint A is in equilibrium in a vertical plane



Write down equations sufficient to determine the horizontal and vertical components (X, Y) of the reaction at the joint B and the thrust T in the light rod PQ. **Hence** find the reaction on the rod AB at the joint B, and show that $T = \frac{7w}{\sqrt{3}}$.

(b) A symmetrical framework of seven rigid light rods freely jointed at their ends is shown in the figure. Rods AB, BC and DE are horizontal. $A\hat{D}E = C\hat{E}D = 45°$ and $B\hat{D}E = B\hat{E}D = 30°$. The framework is loaded as indicated in the figure at the joints A, B and C and is supported by equal vertical forces P at the joints D and E. Find the value of P.

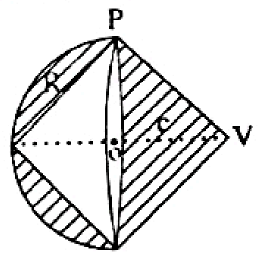
Draw stress diagrams for the joints A and D on the same figure, using Bow's notation. **Hence** find the stresses in the rods AD, AB, DE and DB, and state whether they are tensions or thrusts.



(16) Using **integration**, find the positions of the centres of mass of a uniform solid cone of base radius a and height h, and a uniform solid hemisphere of radius a.

Let R denote the solid body obtained by removing a right circular cone C, of base radius a and height a from a uniform solid hemisphere of mass M, radius a and centre O. Find the mass of the solid body R, in terms of M, and the position of its centre of mass.

The solid cone C is next fixed to the solid body R so as to form a composite body S, as shown in the figure. The circular edge of the base of C is rigidly attached to the rim of R, so that the centre O of the rim is coincident with centre of the base of C.



Show that the centre of gravity G of the composite body S is on its axis of symmetry at a distance $\frac{a}{8}$ from the common centre O of the bases.

(a) The composite body S is freely suspended from a point P of the edge.

(i) Find the inclination of the axis of symmetry OV to the horizontal, where V is the vertex of C.

(ii) Find, in terms of M, the mass m of the particle that should be attached to the vertex V so as to make the axis of symmetry horizontal.

(b) The composite body S, with the mass m attached at V is removed from the point of suspension and kept in equilibrium, with the hemispherical surface placed on a fixed smooth horizontal plane. Find the range of values of the angle between the axis OV and the upward vertical.

17. (a) A man makes a risky journey along a definite route, by using only one of the three modes of transport: motor cycle, bicycle or on foot.

If the probabilities of the man using these modes of transport are P, $2P$ and $3P$ respectively, find the value of P.

If the probabilities of occurring an accident when he uses these modes of transport are $\frac{1}{5}, \frac{1}{10}$ and $\frac{1}{20}$ respectively, calculate the probability of occurring an accident in a single journey.

If an accident is known to have happened, to the man during the journey, calculate the probability that the man was travelling (i) by motor cycle, (ii) by bicycle, (iii) on foot.

Which mode of transport was the safest? Justify your answer.

(b) A group of 100 technical college students measured the length of a certain portion of a main road, and their measurements are given in the following frequency table.

Length (metres) x	99.8	99.9	100.0	100.1	100.2	100.3	100.4
Frequency f	5	7	12	33	25	15	3

By means of the transformation $y = \frac{x - \bar{x}_a}{d}$, for an assumed mean $\bar{x}_a = 100.1$ and d = 0.1, extend the above table to include the corresponding values of y and y^2. Find the mean of y and hence show that the mean of x is 100.123.

Taking that $\sqrt{1.917} \approx 1.385$, calculate, approximately the standard deviation of the frequency distribution, correct to three decimal places.