නව නිර්දේශය / புதிய பாடத்திட்டம் / New Syllabus

NEW

ශ්‍රී ලංකා විභාග දෙපාර්තමේන්තුව
இலங்கைப் பரீட்சைத் திணைக்களம்
Department of Examinations, Sri Lanka

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය, 2019 අගෝස්තු
கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
General Certificate of Education (Adv. Level) Examination, August 2019

සංයුක්ත ගණිතය I
இணைந்த கணிதம் I
Combined Mathematics I

10 E I

05.08.2019 / 0830 - 1140

පැය තුනයි
மூன்று மணித்தியாலம்
Three hours

අමතර කියවීම් කාලය - මිනිත්තු 10 යි
மேலதிக வாசிப்பு நேரம் - 10 நிமிடங்கள்
Additional Reading Time - 10 minutes

Use **additional reading time** to go through the question paper, select the questions and decide on the questions that you give priority in answering.

Index Number							

Instructions:

* *This question paper consists of two parts;*
  Part A (Questions 1 - 10) and **Part B** (Questions 11 - 17).
* **Part A:**
  *Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.*
* **Part B:**
  *Answer **five** questions only. Write your answers on the sheets provided.*
* *At the end of the time allotted, tie the answer scripts of the two parts together so that* **Part A** *is on top of* **Part B** *and hand them over to the supervisor.*
* *You are permitted to remove **only** **Part B** of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics I		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Total

In Numbers	
In Words	

Code Numbers

Marking Examiner		
Checked by:	1	
	2	
Supervised by:		

[see page two

Part A

1. Using the **Principle of Mathematical Induction**, prove that $\sum_{r=1}^{n}(2r-1) = n^2$ for all $n \in \mathbb{Z}^+$.

2. Sketch the graphs of $y=|4x-3|$ and $y=3-2|x|$ in the same diagram.
Hence or otherwise, find all real values of x satisfying the inequality $|2x-3|+|x|<3$.

[see page three

3. Sketch, in an Argand diagram, the locus of the points that represent complex numbers z satisfying $\operatorname{Arg}(z-2-2i) = -\frac{3\pi}{4}$.

Hence or otherwise, find the minimum value of $|i\bar{z}+1|$ such that $\operatorname{Arg}(z-2-2i) = -\frac{3\pi}{4}$.

4. Show that the coefficient of x^6 in the binomial expansion of $\left(x^3 + \frac{1}{x^2}\right)^7$ is 35.

Show also that there **does not exist** a term independent of x in the above binomial expansion.

[see page four

5. Show that $\lim_{x \to 3} \dfrac{\sqrt{x-2}-1}{\sin(\pi(x-3))} = \dfrac{1}{2\pi}$.

6. The region enclosed by the curves $y = \sqrt{\dfrac{x+1}{x^2+1}}$, $x=0$, $x=1$ and $y=0$ is rotated about the x-axis through 2π radians. Show that the volume of the solid thus generated is $\dfrac{\pi}{4}(\pi + \ln 4)$.

7. Let C be the parabola parametrically given by $x = at^2$ and $y = 2at$ for $t \in \mathbb{R}$, where $a \neq 0$. Show that the equation of the normal line to the parabola C at the point $\left(at^2, 2at\right)$ is given by $y + tx = 2at + at^3$.

The normal line at the point $P \equiv (4a, 4a)$ on the parabola C meets this parabola again at a point $Q \equiv (aT^2, 2aT)$. Show that $T = -3$.

8. Let l_1 and l_2 be the straight lines given by $x + y = 4$ and $4x + 3y = 10$, respectively. Two distinct points P and Q are on the line l_1 such that the perpendicular distance from each of these points to the line l_2 is 1 unit. Find the coordinates of P and Q.

[see page six

9. Show that the point $A \equiv (-7, 9)$ lies outside the circle $S \equiv x^2 + y^2 - 4x + 6y - 12 = 0$.

Find the coordinates of the point on the circle $S = 0$ nearest to the point A.

10. Let $t = \tan\frac{\theta}{2}$ for $\theta \neq (2n+1)\pi$, where $n \in \mathbb{Z}$. Show that $\cos\theta = \frac{1-t^2}{1+t^2}$.

Deduce that $\tan\frac{\pi}{12} = 2 - \sqrt{3}$.

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[see page seven

නව නිර්දේශය/புதிய பாடத்திட்டம்/New Syllabus

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கல்விப் பொதுத் தராதரப் பத்திர (உயர் தர)ப் பரீட்சை, 2019 ஓகஸ்ற்
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Part B

※ Answer five questions only.

11. (*a*) Let $p\in\mathbb{R}$ and $0<p\leq 1$. Show that 1 is not a root of the equation $p^2x^2+2x+p=0$.

Let α and β be the roots of this equation. Show that α and β are both real.
Write down $\alpha+\beta$ and $\alpha\beta$ in terms of p, and show that

$$\frac{1}{(\alpha-1)}\cdot\frac{1}{(\beta-1)}=\frac{p^2}{p^2+p+2}.$$

Show also that the quadratic equation whose roots are $\dfrac{\alpha}{\alpha-1}$ and $\dfrac{\beta}{\beta-1}$ is given by $(p^2+p+2)x^2-2(p+1)x+p=0$ and that both of these roots are positive.

(*b*) Let c and d be two **non-zero** real numbers and let $f(x)=x^3+2x^2-dx+cd$. It is given that $(x-c)$ is a factor of $f(x)$ and that the remainder when $f(x)$ is divided by $(x-d)$ is cd. Find the values of c and d.

For these values of c and d, find the remainder when $f(x)$ is divided by $(x+2)^2$.

12. (*a*) Let P_1 and P_2 be the two sets given by $\{A,B,C,D,E,1,2,3,4\}$ and $\{F,G,H,I,J,5,6,7,8\}$ respectively. It is required to form a password consisting of 6 elements taken from $P_1\cup P_2$ of which 3 are different letters and 3 are different digits. In each of the following cases, find the number of different such passwords that can be formed:

(i) all 6 elements are chosen only from P_1,

(ii) 3 elements are chosen from P_1 and the other 3 elements from P_2.

(*b*) Let $U_r=\dfrac{1}{r(r+1)(r+3)(r+4)}$ and $V_r=\dfrac{1}{r(r+1)(r+2)}$ for $r\in\mathbb{Z}^+$.

Show that $V_r-V_{r+2}=6U_r$ for $r\in\mathbb{Z}^+$.

Hence, show that $\displaystyle\sum_{r=1}^{n}U_r=\frac{5}{144}-\frac{(2n+5)}{6(n+1)(n+2)(n+3)(n+4)}$ for $n\in\mathbb{Z}^+$.

Let $W_r=U_{2r-1}+U_{2r}$ for $r\in\mathbb{Z}^+$.

Deduce that $\displaystyle\sum_{r=1}^{n}W_r=\frac{5}{144}-\frac{(4n+5)}{24(n+1)(n+2)(2n+1)(2n+3)}$ for $n\in\mathbb{Z}^+$.

Hence, show that the infinite series $\displaystyle\sum_{r=1}^{\infty}W_r$ is convergent and find its sum.

[see page eight

13. (*a*) Let $\mathbf{A} = \begin{pmatrix} a & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -a & 4 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} b & -2 \\ -1 & b+1 \end{pmatrix}$ be matrices such that $\mathbf{AB}^{\mathrm{T}} = \mathbf{C}$, where $a, b \in \mathbb{R}$.

Show that $a = 2$ and $b = 1$.

Show also that, $\mathbf{C}^{-1}$ **does not** exist.

Let $\mathbf{P} = \frac{1}{2}(\mathbf{C} - 2\mathbf{I})$. Write down $\mathbf{P}^{-1}$ and find the matrix $\mathbf{Q}$ such that $2\mathbf{P}(\mathbf{Q}+3\mathbf{I}) = \mathbf{P} - \mathbf{I}$, where $\mathbf{I}$ is the identity matrix of order 2.

(*b*) Let $z, z_1, z_2 \in \mathbb{C}$.

Show that (i) $\operatorname{Re} z \leq |z|$, and

(ii) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ for $z_2 \neq 0$.

Deduce that $\operatorname{Re}\left(\frac{z_1}{z_1+z_2}\right) \leq \frac{|z_1|}{|z_1+z_2|}$ for $z_1 + z_2 \neq 0$.

Verify that $\operatorname{Re}\left(\frac{z_1}{z_1+z_2}\right) + \operatorname{Re}\left(\frac{z_2}{z_1+z_2}\right) = 1$ for $z_1 + z_2 \neq 0$,

and show that $|z_1 + z_2| \leq |z_1| + |z_2|$ for $z_1, z_2 \in \mathbb{C}$.

(*c*) Let $\omega = \frac{1}{2}\left(1 - \sqrt{3}\,i\right)$.

Express $1+\omega$ in the form $r(\cos\theta + i\sin\theta)$; where $r(>0)$ and $\theta\left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$ are constants to be determined.

Using De Moivre's theorem, show that $(1+\omega)^{10} + (1+\bar{\omega})^{10} = 243$.

14. (*a*) Let $f(x) = \dfrac{9(x^2 - 4x - 1)}{(x-3)^3}$ for $x \neq 3$.

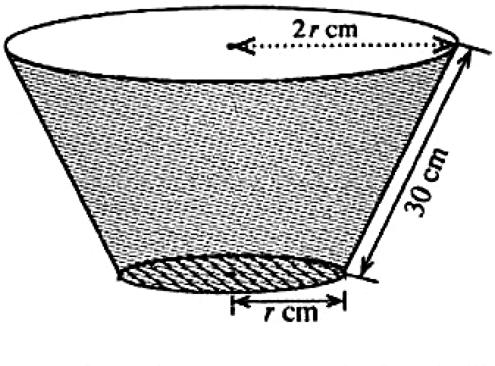
Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = -\dfrac{9(x+3)(x-5)}{(x-3)^4}$ for $x \neq 3$.

Sketch the graph of $y=f(x)$ indicating the asymptotes, y-intercept and the turning points.

It is **given** that $f''(x) = \dfrac{18(x^2 - 33)}{(x-3)^5}$ for $x \neq 3$.

Find the x-coordinates of the points of inflection of the graph of $y=f(x)$.

(*b*) The adjoining figure shows a basin in the form of a frustum of a right circular cone with a bottom. The slant length of the basin is 30 cm and the radius of the upper circular edge is twice the radius of the bottom. Let the radius of the bottom be r cm.



Show that the volume V cm³ of the basin is given by $V = \frac{7}{3}\pi r^2\sqrt{900 - r^2}$ for $0 < r < 30$.

Find the value of r such that volume of the basin is maximum.

[see page nine

15. (a) Using the substitution $x=2\sin^2\theta+3$ for $0 \leq \theta \leq \frac{\pi}{4}$, evaluate $\int_3^4 \sqrt{\frac{x-3}{5-x}}\,dx$.

(b) Using partial fractions, find $\int \frac{1}{(x-1)(x-2)}\,dx$.

Let $f(t) = \int_3^t \frac{1}{(x-1)(x-2)}\,dx$ for $t>2$.

Deduce that $f(t)=\ln(t-2)-\ln(t-1)+\ln 2$ for $t>2$.

Using integration by parts, find $\int \ln(x-k)\,dx$, where k is a real constant.
Hence, find $\int f(t)\,dt$.

(c) Using the formula $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$, where a and b are constants,

show that $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+e^x}\,dx = \int_{-\pi}^{\pi} \frac{e^x\cos^2 x}{1+e^x}\,dx$.

Hence, find the value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+e^x}\,dx$.

16. Write down the coordinates of the point of intersection A of the straight lines $12x-5y-7=0$ and $y=1$.

Let l be the bisector of the acute angle formed by these lines. Find the equation of the straight line l.

Let P be a point on l. Show that the coordinates of P can be written as $(3\lambda+1,\ 2\lambda+1)$, where $\lambda\in\mathbb{R}$.

Let $B\equiv(6,0)$. Show that the equation of the circle with the points B and P as ends of a diameter can be written as $S+\lambda U=0$, where $S\equiv x^2+y^2-7x-y+6$ and $U\equiv -3x-2y+18$.

Deduce that $S=0$ is the equation of the circle with AB as a diameter.

Show that $U=0$ is the equation of the straight line through B, perpendicular to l.

Find the coordinates of the fixed point which is distinct from B, and lying on the circles with the equation $S+\lambda U=0$ for all $\lambda\in\mathbb{R}$.

Find the value of λ such that the circle given by $S=0$ is orthogonal to the circle given by $S+\lambda U=0$.

[see page ten

17. (a) Write down $\sin(A+B)$ in terms of $\sin A$, $\cos A$, $\sin B$ and $\cos B$, and obtain a similar expression for $\sin(A-B)$.

Deduce that

$$2\sin A\ \cos B = \sin(A+B) + \sin(A-B) \text{ and}$$

$$2\cos A\ \sin B = \sin(A+B) - \sin(A-B).$$

Hence, solve $2\sin 3\theta \cos 2\theta = \sin 7\theta$ for $0 < \theta < \frac{\pi}{2}$.

(b) In a triangle ABC, the point D lies on AC such that $BD = DC$ and $AD = BC$. Let $B\hat{A}C = \alpha$ and $A\hat{C}B = \beta$. Using the Sine Rule for suitable triangles, show that $2\sin\alpha\cos\beta = \sin(\alpha + 2\beta)$.

If $\alpha : \beta = 3 : 2$, using the last result in (a) above, show that $\alpha = \frac{\pi}{6}$.

(c) Solve $2\tan^{-1}x + \tan^{-1}(x+1) = \frac{\pi}{2}$. **Hence**, show that $\cos\left(\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)\right) = \frac{3}{\sqrt{10}}$.

* * *