G.C.E. (Advanced Level) Examination - August 2011

Combined Mathematics II

Three hours

Instructions :

This question paper consists of two parts;

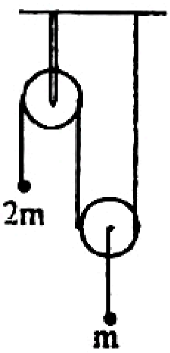
Part A (Questions 1 - 10) and Part B (Questions 11 - 17)

PART - A

- Answer all equstions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.

(01) A particle P is projected vertically upwards from a point O in space with velocity $2u$. At the same instant, another particle Q is projected vertically downwards from the same point O with the velocity u. Both particles move under gravity. Draw the velocity-time graphs for the motions of the particles P and Q in the same figure and show that the speed of the particle Q when the particle P reaches its maximum height, is $3u$.

(02) One end of a light inextensible string which passes ove a smooth fixed pulley carries a particle of mass $2m$. The string passes under a smooth light pulley which carries a particle of mass m. The other end of the string is attached to a ceiling as shown inthe figure. The system moves freely under gravity. Show that the tension of the string is $\frac{2}{3}mg$.



(03) The total mass of a cyclist and his bicycle is M kg. When he rides directly up a straight road inclined at an angle α to the horizontal, at a constant speed of $V\ ms^{-1}$ against a resistance to motion of RN, he works at a constant rate of HW. Show that $H = (R + Mg\sin\alpha)V$.

(04) A thin light elastic spring of natural length l and modulus of elasticity λ rests on a smooth horizontal table. One of its ends is fasten to a fixed point onthe table. A particle of mass m is attached to the other end. The spring is stretched along the table and released. Show that the particle performs a simple harmonic motion with periodic time $2\pi\sqrt{\frac{ml}{\lambda}}$

(05) Let $-2p + 5q$, $7p - q$, and $p + 3q$ be the position vectors of three points A, B and C respectively, with respect to a fixed origin O, where p and q are two non-parallel vectors. Show that the points A, B and C are collinear and find the ratio in which C divides AB.

(06) A weight W is suspended by two light inextensible strings of lenths a and b from two points at the same horizontal level which are at a distance $\sqrt{a^2+b^2}$ apart. Show that the tensions in the strings are $\frac{Wa}{\sqrt{a^2+b^2}}$ and $\frac{Wb}{\sqrt{a^2+b^2}}$.

(07) Let A and B be two exhaustive events in a sample space Ω (that is $A \cup B = \Omega$)

If $P(A) = \frac{2}{5}$ and $P(A \cap B) = \frac{1}{3}$, find

(i) P(B) (ii) P(A|B) (iii) P(A' | B'), where A' and B' are the complementary events of A and B respectively.

(08) Two friends sttempt independently to solve a problem; their probabilities of success being $\frac{1}{3}$ and $\frac{1}{4}$.

Find the probability that (i) both of them, (ii) none of them, will succeed in solving the problem.

(09) The daily expenditure of 1000 families is given in the following table:

Daily expenditure in rupees	Number of families
400 - 600	50
600 - 800	x
800 - 1000	500
1000-1200	y
1200 - 1400	50

If the median of the distribution is 900 Rupees, find the frequencies x and y, and show that the mean of the distribution is also 900 Rupees.

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(10) Over the past 15 months, the number of order received for a certain product has an average of 24 orders per month. The best three months has an average of 35 orders per month. There were 11, 14, 16 and 22 orders for the product in the lowest four months.

Find

(i) the average of the number of orders received in the remaining 8 months,

(ii) the first quartile of the number of orders of the 15 months.

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PART - B

- Answer five questions only. Write your answers on the sheets provided.
- At the end of the time allotted, tie the answers of the two parts together so that **Part A** is on top of **Part B** before handing them over to the supervisor.
- You are permitted to remove only **Part B** of the question paper from the Examination Hall.

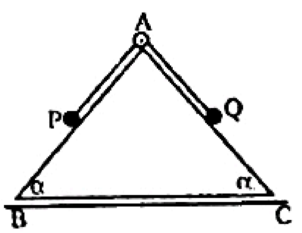
(11)

(a) The top-most points A, B and C of three lamp-posts lie in a horizontal plane at the vertices of an equilanteral triangle of side a. A wind blows in the direction of $\overline{AC}$ at a steady speed u. A bird, whose speed relative to the wind is v (> u), files from A to B alond AB and then from B to C along BC.

Draw the velocity triangles of relative velocities for both parts of the journey **in the same figure.**

Hence, show that the total time taken for the journey from A to C through B is $\dfrac{4a}{u+\sqrt{4v^2-3u^2}}$.

(b) A small smooth pulley is fixed at the vertex A of the triangular vertical cross-section ABC of a smooth wedge of mass 2m through its centre of mass. The face through BC is placed on a fixed smooth horizontal table. It is given that AB and AC are lines of greatest slope of the relevant faces and $A\hat{B}C = A\hat{C}B = \alpha$. Two smooth particles P and Q of masses m and λm $(\lambda > 1)$ respectively, are attached to the ends of a light inextensible string. The string passes over the pulley and the particles P and Q are placed on AB and AC respectively, with the string taut as shown in the figure.



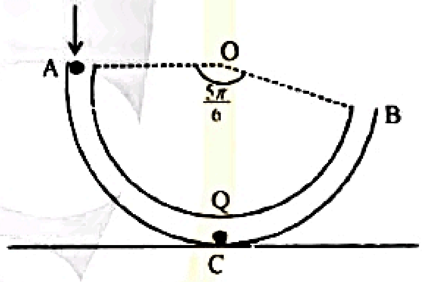
The system is released from rest.
Obtain the equations of motion for the particles P and Q along BA and AC respectively, and for the system horizontally.

Show that the magnitude of the acceleration of each of the particles P and Q relative to the wedge is

$$\frac{(\lambda-1)(\lambda+3)g\sin\alpha}{(\lambda+1)\left[(\lambda+3)-(\lambda+1)\cos^2\alpha\right]}.$$

When the particle Q reaches C, the string is suddenly broken. Assuming that P has not reached the pulley, write down the magnitude of the acceleration of the particle P relative to the wedge just after the string is broken.

(12)



A thin smooth tube ACB in the shape of a circular arc of radius a that subtends an angle $\frac{5\pi}{6}$ at its centre O is fixed in a vertical plane with OA horizontal and the lowest point C of the tube touching a fixed horizontal floor as shown in the figure. A smooth particle P of mass m is projected verically downwards into the tube at the end A with speed $\sqrt{2ga}$.

Show that the speed of the particle P, when OP makes an anlge $\theta\left(0 \le \theta \frac{\pi}{2}\right)$ with OA is $\sqrt{2ga(1+\sin\theta)}$ and the magnitude of the reaction on the particle P from the tube is $mg(2 + 3\sin\theta)$. The particle P, when it reaches the point C, strikes another smooth particle Q of mass m which is at test inside the tube at C. The coefficient of restitution between the particles P and Q is $\frac{1}{2}$.

Find the speed of the particle P just before the collision and show that the speeds of the particles P and Q just after the collision are $\frac{1}{2}\sqrt{ga}$ and $\frac{3}{2}\sqrt{ga}$ respectively.

Show further that the psrticle P never leaves the tube and that the particle Q reaches the point B with speed $\frac{1}{2}\sqrt{5ga}$.

Find the maximum height from the floor reached by the particle Q after it leaves the tube.

(13) A particle P of mass m is attached to one end of a light elatic string of natural length l. The other end of the string is attached to a fixed point O at a height $4l$ from a horizontal floor. When the particle P hangs in equilibrium, the extension of the string is l.

Show that the modulus of elasticity of the string is mg.

The particle P is now held at O and projected vertically downwards with a velocity $\sqrt{gl}$. Find the velocity of the particle P when it has fallen a distance l.

Write down the equation of motion for the particle P, when the lenght of the string is $2l + x$, where $-l \le x \le 2l$, and show that $\ddot{x} + \frac{g}{l}x = 0$, in the usual notation.

Assuming that the above equation gives $\dot{x}^2 = \frac{g}{l}\left(c^2 - x^2\right)$, where c (>0) is a constant, find c.

Show that the particle P comes to instantaneous rest when it reaches the floor and that the time taken from O to reach the floor is $\frac{1}{3}\left(3\sqrt{3} - 3 + 2\pi\right)\sqrt{\frac{l}{g}}$.

(14) (a) Define the dot product $a.b$ of two vectors a and b.

Assuming $(a + b).(c + d) = a.c + b.c + a.d + b.d$ for any four vectors a, b, c and d show that

$|a+b|^2 = |a|^2 + 2(a.b) + |b|^2$.

Write down a similar expression for $|a - b|^2$.

Show that, if $|a+b|^2 = |a-b|^2$ then $a.b = 0$.

Hence, show that if the diagonals of a parallelogram are equal, then it is a rectangle.

(b) The points A, B, C, D, E and F are the vertices of a regular hexagon of side $2a$ metres taken in the anti-clockwise sense. Forces of magnitude P, $2P$, $3P$, $4P$, $5P$, L, M and N newtons act along AB, CA, FC, DF, EB, BC, FA and FE respectively, in the sense indicated by the order of the letters.

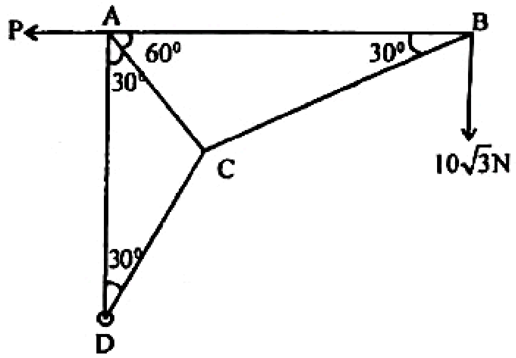
If the systemis in equilibrium, find L, M and N in terms of P.

(15) (a) Two uniform rods AB and BC are equal in length. The weight of AB is $2w$ and the weight of BC is w. The rods are smoothly hinged at B and the midpoints of the rods are connected by a light inelastic string.. The system stands in equilibrium in a vertical plane with A and C on a smooth horizontal table.

If $A\hat{B}C = 2\theta$, show that the tension of the string is $\frac{3}{2}w\tan\theta$.

Find the magnitude of the reaction at B and the angle it makes with the horizontal.

(b) Five light rods AB, BC, CD, DA and AC are smoothly jointed at their ends to form a framework as shown in the figure.



$A\hat{B}C = A\hat{D}C = D\hat{A}C = 30^0$ and $B\hat{A}C = 60^0$ .The framework is smoothly hinged at D and carries a weight of $10\sqrt{3}$ newtons at B. The framework is held in a vertical plane, with AB horizontal, by a horizontal force of P newtons at A.

(i) Find the value of P.
(ii) Find the magnitude and the direction of the reaction at D.
(iii) Using Bow's notation, draw a stress diagram for the framework and find the stresses in all the rods, distinguishing between tension and thrusts.

(16) Show that the centre of mass of a uniform solid hemisphere of radius a is on its axis of symmetry at a distance $\frac{3}{8}a$ from the base of the hemisphere.

The inner and outer radii of a uniform solid hemispherical shell are a and b (> a). Show that the distance of its centre of mass from the centre along the axis of symmetry is $\frac{3(a+b)(a^2+b^2)}{8(a^2+ab+b^2)}$.

This hemispherical shell rests in equilibrium so that its curved surface is in contact with a rough horizontal ground and equally rough vertical wall.
Show that if the equilibrium is limiting, the inclination of the base to the horizontal is

$\sin^{-1}\left\{\frac{8\mu b(1+\mu)(a^2+ab+b^2)}{3(1+\mu^2)(a+b)(a^2+b^2)}\right\}$, where μ is the coefficient of friction between the shell and the rough surfaces.

(17) (a) Nimal, Sunil and Piyal play a game with a biased coin which has probability p of landing a head. Nimal, Sunil and Piyal in that order toss the coin in turns. The first person who gets a tail will win the game.
Find the probability that Nimal wins the game in his
(i) second turn,
(ii) third turn.
Hence, find the probability that Nimal wins the game eventually.
Deduce that, if the coin is more likely to land tails than heads, Nimal has more than 50% chance of winning the game.

(b) The mean and the standard deviation of a set of observations $\{x_1, x_2, \ldots, x_n\}$ are $\bar{x}$ and S_x respectively. Suppose that a linear transformation $y_i = a + bx_i$, where a and b are constants, transforms the original data set $\{x_1, x_2, \ldots, x_n\}$ to the set $\{y_1, y_2, \ldots, y_n\}$.
Show that $\bar{y} = a + b\bar{x}$ and $S_y^2 = b^2 S_x^2$, where $\bar{y}$ and S_y are the mean and the standard deviation of the set $\{y_1, y_2, \ldots, y_n\}$.

(i) Find the mean and the standard deviation of the set of observations {1, 2, 3, 4, 5, 6, 7}.
Hence, find
(a) the mean and the standard deviation of the set of obsetvations
{2.01, 3.02, 4.03, 5.04, 6.05, 7.06, 8.07}
(b) seven values whose mean is 5 and the standard deviation is 6.

(ii) Salt is packed in bags which the manufacturer claims contain 25kg each. The following information is given for 80 such bags whose actual weights are not known:

$\sum_{i=1}^{80}(x_i - 25) = 27.2$ and $\sum_{i=1}^{80}(x_i - 25)^2 = 85.1$

where x_i(i = 1, 2,..., 80) denotes the actual weight of the i^{th} bag. Using an appropriate linear transformation or otherwise, find the mean and the variance of the actual weights of the eighty of the eighty bags.