G.C.E. (Advanced Level) Examination - August 2001
Combined Mathematics - II
Three hours

- Answer six questions only

(01) (a) A stationary police car observes a van travelling past it, with constant velocity 72 km h^{-1}. Ten seconds later the police car starts in pursuit of the van. Moving with constant acceleration f ms^{-2} for a distance of 200m, the police car reaches a velocity of 90 km h^{-1} which it maintains until overtaking the van. Sketch velocity-time graphs for the two vehicles on the same diagram.

Calculate the acceleration f of the police car in the first 200m of its journey, and the total time taken by it to overtake the van, calculated from the instant of their first encounter.

(b) A motor-boat whose speed is u km h^{-1} is to intercept a ship which moves with constant velocity v (< u) km h^{-1} in the North-West direction. Initially, the ship is located at a distance d km north of the motor-boat. Draw a velocity triangle and find the direction in which the motor-boat should move in order to intercept the ship. Show that the interception takes place after a time

$$\frac{\sqrt{2}\,d\left[\sqrt{2u^2-v^2}+v\right]}{2\left(u^2-v^2\right)} \text{ hours.}$$

(02) (a) A particle of mass m slides down a smooth face, of inclination α to the horizontal, of a wedge of mass M which is free to move on a smooth horizontal table. Show that the acceleration of the wedge is $\dfrac{mg\sin\alpha\cos\alpha}{M+m\sin^2\alpha}$, and find the reaction between the particle and the wedge.

(b) The greatest speed of a car, of mass 1000kg, moving on a straight horizontal road, against a resistance of 400 N, is 144 km h^{-1}. Show that the power of its engine is 16 kw.

The car pulls a trailer of mass 600 kg on the same road against an sdditional resistance of 200 N. If the engine is working at the same power as before, find the tension in the tow-rope in newtons, when the speed is 24 km h^{-1}.

(03) (a) The components of the initial velocity of a particle, projected under gravity from a point O are u, v in the directions of horizontal and upward vertical axes Ox, Oy respectively. Show that when it has moved a horizontal distance x it attains a vertical height

$$y=\left(\frac{v}{u}\right)x-\left(\frac{g}{2u^2}\right)x^2$$

The particle just clears a vertical wall of height $\frac{a}{2}$ at a horizontal distance a from O and has a range 4a on the horizontal plane through O. Determine u, v and show that the direction of projection makes an angle $\tan^{-1}\left(\frac{2}{3}\right)$ with the horizontal.

(b) Two smooth spheres A, B of equal radii, are moving **in opposite directions** on a smooth horizontal table, so as to collide directly. Their masses are 2m, 3m and their speeds are 7u, 3u respectively. The coefficient of restitution between the spheres is e. Show that the impulse of the collision is of magnitude 12mu (1+e)

If the smaller sphere, A is brought to rest by the impact determine the value of e, and show that $\frac{1}{15}$ of the original kinetic energy is retained in the system.

(04) (a) The figure 1 depicts a narrow smooth tube bent in the form of a circle of centre O and radius a, fixed in a vertical plane.

Inside the tube are two particles P, Q of masses m, 3m respectively connected by a light inextensible taut string of length πa. Initially, the system is released from rest, when the particles are at the opposite ends of the horizontal diameter of the tube and the string occupies the upper-half of the tube.

If OP has turned through an angle θ, at time t after release, **using the principle of conservation of energy** show that

$$a\dot{\theta}^2 = g\sin\theta\left(0 \le \theta \le \frac{\pi}{2}\right)$$

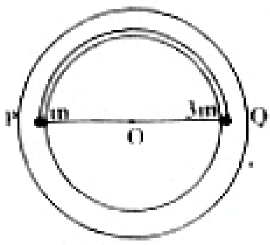

Figure 1

Find the force exerted by the tube on the particle P, at this instant.

(b) A particle of mass m hangs in equalibrium, being attached to one end of a light elastic string, of natural lenght l, whose other end is tied to a fixed point O. If the particle is at a point C whose displacement below O is $2l$ show that the modulus of elasticity of the string is mg.

The particle is now projected vertically downwards from C, with initial speed $\sqrt{gl}$. At time t, its **downward displacement from** O is x. Show that $\ddot{x} + \frac{g}{l}(x - 2l) = 0$, and identify the centre and the period of the simple harmonic motion of the particle.

Obtain the maximum and the minimum values of x.

(05) Forces P, 7P, 8P, 7P, 3P newtons act along the sides AB, CB, CD, ED, FE respectively, of a regular hexagon ABCDEF of side a metres, in the directions indicated by the order of the letters. Taking i and j to be unit vectors in the direction of $\overrightarrow{AB}$ and $\overrightarrow{AE}$ respectively, express each force in terms of i, j and P.

Show that the given system is equivalent to a single resultant force, $R = 2p(i + \sqrt{3}j)$, parallel to $\overrightarrow{BC}$

What is the magnitude of **R**?

Show further that the line of action of the resultant passes through the common point of DE and AF (both produced)

If the system is equivalent to a force **R** acting through the vertex A together with a couple, find the moment of this couple, in magnitude and sense.

(06) (a) A smooth peg is fixed at a point P at distance a from a smooth vertical wall. A uniform rod AB of length 6a and weight W is in equilibrium resting on the peg with the end A in contact with the wall.

Taking θ to be the angle made by the rod AB with the horizontal draw a triangle of forces, representing forces acting on the rod. Find the reaction at P, in terms of W and θ

Show that $3\cos^3\theta = 1$

(b) A framework consisting of four light rods AB, BC, CD, BD is shown in figure 2. It is freely hinged to a vertical wall at A and D. A load 500N is hung from the joint C, and BC is horizontal. Draw a stress diagram for the framework using Bow's notation, and hence find the stresses in all the rods, distinguishing between tensions and thrusts.

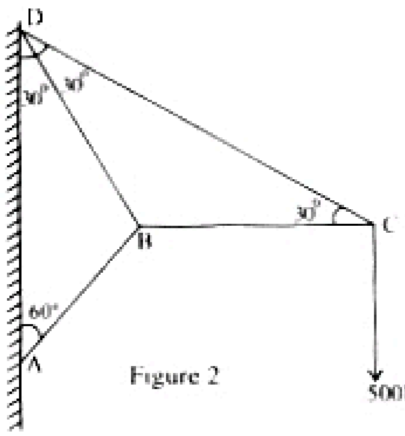

Figure 2

(07) Show that the position of the centre of mass of a uniform solid right circular cone of height h is on its axis at a distance $\frac{3h}{4}$ from the vertex.

Such a cone, of semi-vertical angle 15°, rests with its base on a rough horizontal floor. It is tilted to one side by a light inextensible string attached to its vertex. The string pulls downwrads making an angle 45° with the horozontal, in a vertical plane through the axis of the cone. The edge of the cone is about to slip on the floor.

when the vertex is vertically above the point of contact of the edge and the floor. Write down sufficient equations to determine the tension T in the string, the normal reaction and the frictional force. Hence show that

(i) $T = \frac{3\sqrt{2}W}{16}$;

(ii) the value of the coefficient of friction, is $\frac{3}{19}$

(08) (a) Define the probabilities $P(A \cup B)$, $P(A \cap B)$ and $P(A \mid B)$ in relation to two random events A and B.

Two random events A, B have probabilities $P(A) = 0.6$, $P(B) = 0.2$ and $P(A|B) = 0.1$. For the events A and B, calculate the probability of

(i) both events occurring,
(ii) exactly one of the events occurring and
(iii) neither of the events occurring.

(b) One of three coins is biassed so that the probability of obtaining a head when it is tossed once is p. The other two coins are unbiassed. One of the three coins is chosen at random and tossed twice. Draw a tree-diagram to show the possible outcomes. If the probability of obtaining heads on both tosses is $\frac{17}{54}$, find the value of p.

For this value of p, given that heads were, in fact, obtained on both tosses, find the probability that the chosen coin is biassed.

(09) Define the mean $\bar{x}$ of a grouped distribution.

By means of the coding $y = \frac{x-a}{c}$, where a is the assumed mean and c is a positive constant, show that $\bar{x} = a + c\bar{y}$

Starting from the definition $\sigma^2 = \frac{\sum f\left(x - \bar{x}\right)^2}{\sum f}$ for the variance, and using the same coding as above, derive the formula $\sigma = c\sqrt{\frac{\sum fy^2}{\sum f} - \bar{y}^2}$, for the standard deviation.

The following age-class distribution gives the estimated total population of Sri Lanka for the year 2003, in millions.

Age Class (Years)	Frequency (Number of people in millions)
0 and more, less than 10	4.2
10and more, less than 20	3.9
20 and more, less than 30	3.4
30 and more, less than 40	3.2
40 and more, less than 50	2.8
50 and more, less than 60	2.8
60 and more, less than 70	2.5
70 and more, less than 80	1.6
80 and more, less than 90	0.6
Total population	25.0

[Note : The width of each class is 10 years. The number of people older than 90 is negligible.]

Taking $a = 45$years and $c = 10$ years, the width of each class, and using the above coding calculate y, fy and fy^2 for each class.

Hence estimate the mean age and the standard deviation of the population in years, each correct to one decimal place.