

Introduction

SUMSRI is a program designed to prepare participants for the rigor and pace of graduate school. We feel that this preparation will allow the participants to successfully complete and compete in graduate school. Because of the small number of African Americans, Latinos, and Native Americans with doctoral degrees in the mathematical sciences, we are particularly interested in these undergraduate students. The Institute aids participants by intervening in their learning development at a crucial stage. The main goals are:

- Address the shortage of underrepresented minorities and women mathematicians by producing minority and women research mathematicians.
- Provide the students with a research environment and improve their research abilities.
- Improve the student's ability to work in groups and give them a long term support group.
- Provide role models.
- Improve the students' technical writing skills.
- Give the participants an opportunity to give a talk and to write a technical research paper.
- Familiarize them about graduate school and inform them about available financial aid for graduate school.
- Make the students aware of career opportunities in the mathematical sciences.
- Prepare the students for the GRE.

In the summer of 2001, the Institute operated for seven weeks. During these seven weeks, students participated in a research seminar in graph theory, statistics or algebra. Workshops in technical writing and GRE preparation were held. Also, eight colloquium talks and two short courses on algebra and real analysis were given.

Fifteen participants were accepted into the Institute. These students were from colleges across the country. All of them had completed college level introductory mathematics and/or statistics courses and at least one proof based mathematics course. The average GPA in the mathematical science courses for the participants was approximately 3.87. Each applicant wrote a brief essay on why they wished to participate in the Institute. Two recommendations from faculty members at the student's home institution were received for each applicant. Three of the participants were African American, one was Hispanic, ten were Caucasian, and one was Asian American. Of the fifteen participants, twelve were female.

Seminar instructors suggested topics that would challenge students to work in teams, draw on their critical thinking and research skills, familiarize them with current literature on the topic, set parameters of the research, and utilize computer modeling programs.

In the statistics seminar, Jennifer Everson, Melissa Hildt, Jason Popovic and Sarah Zimmermann determined the desirability of living in any state by using a set of several different variables. The multivariate statistical methods of factor analysis and discriminant analysis lend themselves to this issue. They used factor analysis to reduce a large number of variables to a smaller set of common factors which describe state

desirability. They then used discriminant analysis to classify states according to their desirability level based on a set of measured variables.

The graph theory seminar researched various possibilities in the "Lights Out" game. Jillian Martens, Lisa McGee, Christian Roldan Santos and Lisa Sparrgrove investigated the properties of a mod 3 variation of the game "Lights Out" played on an arbitrary graph or digraph, following work done by Craft, Miller and Pritikin. They prove that there are $2n-1$ unwinnable configuration for trees with n vertices, and show a procedure to derive and recognize these configurations. For each graph, they exhibit a nontrivial initial configuration X , which is unwinnable. They characterize which initial configurations are winnable on complete graphs, complete bipartite graphs and acyclic digraphs, certain tournaments, and some other graphs.

Khalilah Beal looked at variations of the original Lights Out game for patterns of winnability. Varieties of move shapes and grid sizes are considered, and general formulas for winnability are obtained. When H is an $m \times n$ grid with a move shape p , there is created a puzzle space (H,p) . (H,p) is a vector space of patterns, called even-dominating sets, for the graph. Both linear algebra (mod 2) and ad hoc methods are used in generating all even-dominating sets.

The algebra seminar looked at several aspects of the Josephus Dilemma. Ledah Casbern and Tuyet-Linh Phan considered the Josephus Problem from a new perspective. $J(n,k)$ represents the position of the survivor when n people are eliminated with a skip factor of k . They demonstrate that there exists an explicit formula for $J(n,k)$ when n is fixed. They show that the set of all cycles generated by the orders of elimination (for a fixed n) is a group if and only if $1 < n < 5$.

Sarah Breede and Christy Finch considered a variation of the classical Josephus Dilemma. In a circle of n men when they eliminate every third person, the order of elimination defines an $(n-3)$ -Josephus permutation. They discover and prove fifteen remarkable patterns in the fixed points of these permutations.

Sarah Burke and Robert Davis also considered another aspect of the Josephus Problem. Instead of killing all but one person, they considered killing exactly half of the group, which is distinct from the other half. For example, suppose there are n bad guys and n good guys and all the bad guys are killed. Sarah and Robert wrote a program that will find a skip factor that will achieve this goal for any arbitrary ordering of bad guys and good guys.

Final presentations of the research projects were given in the final week using overhead projector slides and Power Point.

It is hoped that many of these students will attend national meetings such as NAM Fest, MathFest, and the Joint Winter Meetings of the AMS-MAA in order to present the results of their work.

By providing this intensive research program, we endeavored to encourage those who attended SUMSRI to pursue advanced degrees. On student evaluations, 9 students stated that, before attending SUMSRI, they were thinking about pursuing a graduate education. After participating, twelve were considering graduate school. In the brief period that has followed the summer Institute, the program has made a difference in their lives by encouraging them to participate in national meetings, doing poster presentations, forming supportive groups for one another, and finding role models in the instructors and colloquium speakers.

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