Randomized Response Experiment: Do Math Students Cheat on Tests Using Graphing Calculators?

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Questioning people directly about sensitive issues, such as cheating, is an important but difficult task. Because people may feel uncomfortable responding truthfully to a sensitive question, an alternative method must be employed. For sensitive issues, such as cheating on a significant other, sexual preference, or other similar topics, the randomized response method gives a better approximation of the proportion of people who belong to a particular class. This method allows for complete anonymity when responding to the statements because the experimenters do not know to which statement the participant responded. The standard and unrelated question randomized response methods are used to estimate the number of mathematics students who use graphing calculators to cheat on exams.

Procedure

The same setup and room was used for both experiments. The room contained two tables on top of which rested a deck of cards, a pen, slips of paper, and written instructions. The participants were informed that if they encountered any problems or did not understand the directions, they could leave the room to question the experimenters. Each participant entered the room and visited both tables to complete the experiment. The participants selected a card and responded to the corresponding statement by writing the answer on a slip of paper, without indicating the statement to which they responded. The participant then left the room and the experimenters entered the room to collect the data and reset the tables.

Standard Randomized Response Method

In the standard randomized response experiment, two statements were presented to the participants. They were the following:

1. I have cheated on a test using a graphing calculator.
2. I have not cheated on a test using a graphing calculator.

A standard deck of cards was used to determine to which statement the participant would respond. If participants chose a heart, then they responded to statement two, otherwise the participants responded to statement one. Thus, the probability of responding to statement one is ¾ and the probability of responding to statement two is ¼. There were 16 total participants[1] in the experiment, denoted by n in the calculations.

[1] The participants are assumed to be representative of student mathematics majors.

Calculations

The experiment resulted in 10 participants responding yes and 6 participants responding no. However, these results do not indicate which question was answered. Because participants were in a comfortable atmosphere it was assumed that the responses were honest. Thus the data can be analyzed to estimate the proportion of those who have cheated on a test using a graphing calculator. This proportion is denoted by π_A whereas the estimated value of it is denoted by $\hat{\pi}_A$.

To calculate the proportion of those who answered yes (m) out of the total number of participants asked (n), the following equation is used:

$$\hat{\phi} = \frac{m}{n} = \frac{10}{16} = 0.625$$

In order to find the true value of ϕ, the following equation is used:

$$\phi = \pi_A p + (1 - \pi_A)(1 - p)$$

where π_A = P(those who responded yes given that they responded to statement one). Solving for π_A the following equation can be used to estimate π_A.

$$\hat{\pi}_A = \frac{\hat{\phi} - (1 - p)}{(2p - 1)} = \frac{0.625 - \left(1 - \frac{3}{4}\right)}{\left(2\left(\frac{3}{4}\right) - 1\right)} = 0.75$$

Thus, the estimated number of mathematicians who have cheated using a graphing calculator is about 75 percent.

In order to calculate the confidence interval for π_A, the estimated variance of $\hat{\pi}_A$ must first be determined using the following equation:

$$\hat{V}(\hat{\pi}_A) = \frac{\hat{\pi}_A(1 - \hat{\pi}_A)}{n} + \frac{p(1 - p)}{n(2p - 1)^2} = \frac{0.75(1 - 0.75)}{16} + \frac{\frac{3}{4}\left(1 - \frac{3}{4}\right)}{16\left(2\left(\frac{3}{4}\right) - 1\right)^2} = 0.059$$

The final step in calculating the 95 percent confidence interval can be accomplished by using the following equation where z is the normal percentile[2] equal to 1.96.

$$\hat{\pi}_A \pm z\sqrt{\hat{V}(\hat{\pi}_A)} = 0.75 \pm 1.96\sqrt{0.059} = (0.276,\ \ 1.224)$$ [3]

Therefore, there is 95 percent confidence[4] that the actual proportion of mathematicians who cheat is within this interval. It is somewhat surprising that this interval is so large.

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[2] Normal approximation is used even though n is not large.

[3] Although 1.224 does not make sense (because it really can not be more than 1.00), this number is used in order to compare the confidence intervals.

This can be attributed to the small sample size. In order to reduce the variance, thus decreasing the size of the confidence interval, another method called unrelated-question-randomized response is used.

Unrelated-Question Randomized Response Method

This experiment is similar to the first, except that the second statement to which the participants responded to is not related to the first.

The statements that the participants responded to were the following:

1. I have cheated on a test using a graphing calculator.
2. I was born in the month of December.

Again, a standard deck of cards was used in making the probabilities of responding to statements one and two, ¾ and ¼, respectively.

Calculations:

This method yielded 7 participants who responded yes and 9 who responded no. π_A again represents the proportion of those who have cheated. Let π_μ be equal to the proportion of those born in the month of December. This is a known value of 1/12. Let $\hat{\phi} = \frac{m}{n}$ = the number of participants who responded yes (m) out of the total number of participants (n). In order to find the true value of ϕ, the equation for ϕ can be used as in the first method. The following equation is then used to estimate π_A where p= ¾:

$$\hat{\pi}_A = \frac{\hat{\phi} - (1-p)\pi_\mu}{p} .= \frac{\frac{7}{16} - (\frac{1}{4})(\frac{1}{12})}{\frac{3}{4}} = 0.556$$

The estimate of π_A indicates that about 56 percent of the population of mathematicians have cheated on a test using their graphing calculators.

In order to find the confidence interval for a more detailed analysis of the data, the variance must be calculated by the following:

$$\hat{v}(\hat{\pi}_A) = \frac{\hat{\phi} - (1-\hat{\phi})}{np^2} . = \frac{\hat{\phi}(1-\hat{\phi})}{np^2} = \frac{(\frac{7}{16})(\frac{9}{16})}{16(\frac{3}{4})^2} = 0.027$$

The following equation is used to calculate the 95% confidence interval for π_A:

$$\hat{\pi}_A \pm z\sqrt{\hat{V}(\hat{\pi}_A)} \;= 0.556 \pm 1.96\sqrt{0.027} = (0.231,\quad 0.880)$$

The z in this equation is the normal percentile. Thus there is 95 percent confidence that the true value of π_A lies in this interval.

[4] Note that the confidence is in the process and not the interval.

Conclusion

The unrelated-question experiment had an estimated proportion of 0.56. This means that 56 percent of those sampled have cheated on tests using graphing calculators. The confidence interval constructed was (0.231, 0.880). This interval is smaller than in the standard method because of the smaller variance. With 95 percent confidence it can be concluded that the percentage of all mathematics students who cheat on tests using a graphing calculator is between 23 percent and 88 percent.

The calculations completed in the above sections must be interpreted in order to determine the significance of the experiment performed. In the standard method, the estimate of the population proportion (π_A) of math students who have cheated on tests using graphing calculators was equal to 0.75. In the sample we tested, approximately 75 percent of the participants have cheated on tests using graphing calculators. This estimate can be used to determine the proportion of all math students who have cheated on tests using graphing calculators by constructing a confidence interval for the population proportion. This confidence interval is (0.276, 1.224). With 95 percent confidence it can be concluded that the population proportion falls in the calculated interval.

Both of the preceding confidence intervals are large because the sample size was small (16 participants). As the number of participants increases, the confidence intervals will decrease. To acquire a shorter interval, which would increase how precisely the population proportion could be estimated, a much larger sample would be needed.