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Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

Course Number	ELE532
Course Title	Signals and Systems I
Semester/Year	Fall 2021

Instructor	Dr. Javad Alirezaie

ASSIGNMENT No.	**2**

Assignment Title	System Properties and Convolution

Submission Date	October 29, 2021
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Problem A:

A.1

```
%Rendel Abrasia, Reza Aablue
%500942743, 500966944
%Section 4

% Problem A.1

% Setting component values.
R = [1e4, 1e4, 1e4];
C = [1e-6, 1e-6];
% Determining the coefficients for characteristic equation.
A1 = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determining characteristic equation's roots.
lambda = roots(A1);

% The poly command takes the matrix of the roots and return the original
% polynomial equation.
poly(lambda);
```

Command Window

```
>> Lab2PartA
>> lambda

lambda =

 -261.8034
  -38.1966
```

A.2

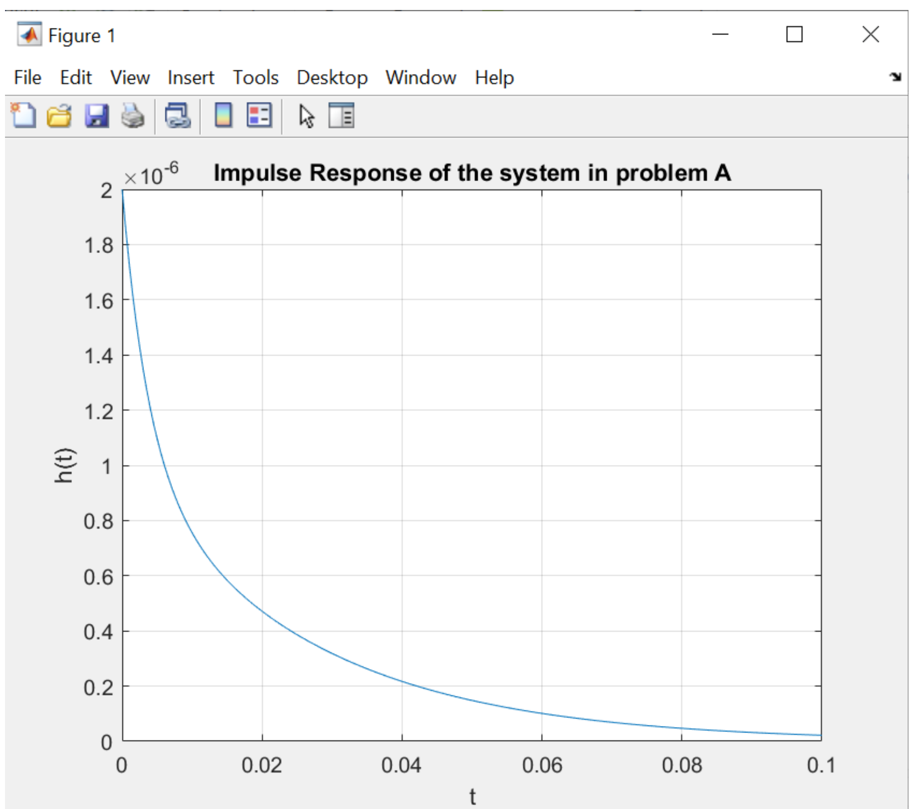
```
% Problem A.2

% Set up time interval and step size, along with u(t) step function.
t = (0:0.0005:0.1);
u = @(t) 1.0* (t>=0);

h = @(t) (C(1).* exp(lambda(1).* t) + C(2).* exp(lambda(2).*t)).*(u(t));
% Case 1 of the 2nd order DE solution applies for our h(t) function.

plot(t,h(t)) % Plots h(t) function.

% Setting up x and y-labels and title for graph.
xlabel('t');
ylabel('h(t)');
title('Impulse Response of the system in problem A');
grid;
```



A.3

```
%Rendel Abrasia, Reza Aablue
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%Section 4

% Problem A.3

function [lambda] = ProblemA3(R,C)
% CH2MP2.m : Chapter 2, MATLAB Program 2
% Function M-file finds characteristic roots of op-amp circuit.
% INPUTS: R = length-3 vector of resistances
% C = length-2 vector of capacitances
% OUTPUTS: lambda = characteristic roots
% Determine coefficients for characteristic equation:
A = [1, (1/R(1)+1/R(2)+1/R(3))/C(2), 1/(R(1)*R(2)*C(1)*C(2))];
% Determine characteristic roots:
lambda = roots(A);

% To test this function, just use this command in the command window with your desired "R" and
% "C" values to get the eigenvalues:
% ie. lambda = ProblemA3([1e4, 1e4, 1e4],[1e-9, 1e-6])
```

Command Window

```
>> Lab2PartA
>> lambda = ProblemA3([1e4, 1e4, 1e4],[1e-9, 1e-6])

lambda =

   1.0e+03 *

  -0.1500 + 3.1587i
  -0.1500 - 3.1587i

fx >>
```

Problem B

B.1

```
%Rendel Abrasia, Reza Aablue
%500942743, 500966944
%Section 4

% Problem B.1
figure(1) % Create figure window and make visible on screen
u = @(t) 1.0*(t>=0); % u(t) step function.
x = @(t) 1.5*sin(pi*t).*(u(t)-u(t-1)); % Assigned x(t) function.
h = @(t) 1.5*(u(t)-u(t-1.5))-u(t-2)+u(t-2.5); % h(t).
dtau = 0.005; tau = -1:dtau:4; % Time intervals and step size for tau.
ti = 0; tvec = -.25:.1:2.25; % Time intervals and step size for tvec.
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```

B.2

```
% Problem B.2
% x(t) = u(t) - u(t-2)
% h(t) = (t+1)(u(t+1)-u(t))
% y(t) = x(t)*h(t) (Convolution)

figure(2) % Create figure window and make visible on screen
u = @(t) 1.0*(t>=0); % u(t) step function.
x = @(t) u(t)-u(t-2); % Assigned x(t) function (Figure 2.4-28 (a)).
h = @(t) (t+1).*(u(t+1)-u(t)); % h(t) = (t+1)(u(t+1)-u(t)).
dtau = 0.005; tau = -1:dtau:4; % Time intervals and step size for tau.
ti = 0; tvec = -1.0:.1:2.25; % Time intervals and step size for tvec.
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
```

```
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```

B.3

```
% Problem B.3 - part 1 (Assume A=1, B=2)
% x1(t) = x(t) = u(t-4)-u(t-6)
% x2(t) = h(t) = (2)(u(t+5)-u(t+4))
% y(t) = x(t)*h(t)

figure(3) % Create figure window and make visible on screen
A = 1.0; B = 2.0; % Assumptions for A and B values.
u = @(t) 1.0*(t>=0); % u(t) step function.
x = @(t) A*(u(t-4)-u(t-6)); % Assigned x(t) function (Figure 2.4-28 (a)).
h = @(t) B*(u(t+5)-u(t+4)); % h(t) = (t+1)(u(t+1)-u(t)).
dtau = 0.005; tau = -6:dtau:2.5; % Time intervals and step size for tau.
ti = 0; tvec = -5.0:.1:5; % Time intervals and step size for tvec.
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
```

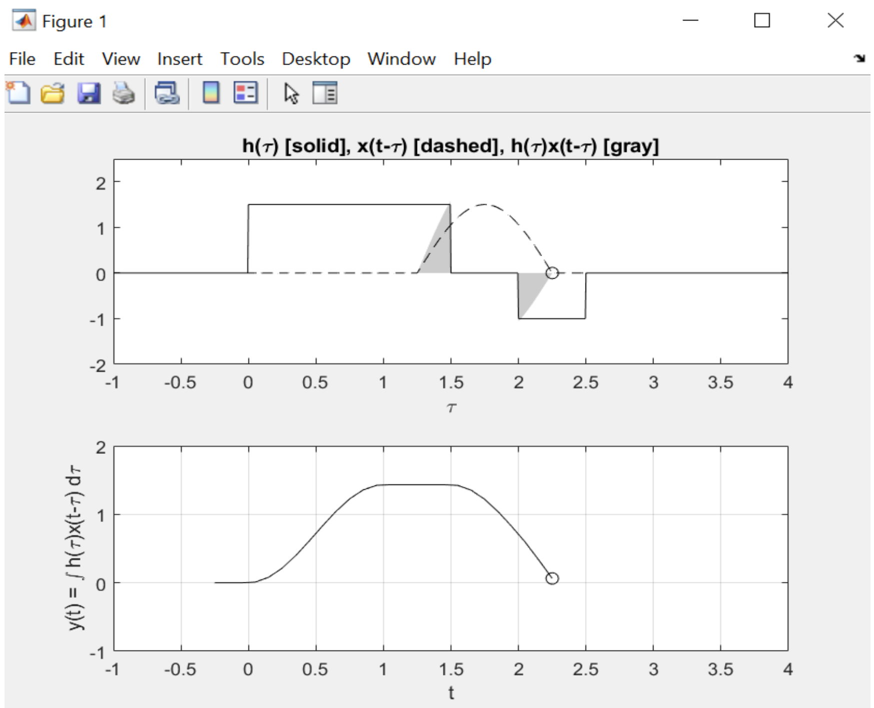
```
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```

```
% Problem B.3 - part 2 (Assume A=1, B=2)
% x1(t) = x(t) = u(t-3)-u(t-5)
% x2(t) = h(t) = (2)(u(t+5)-u(t+3))
% y(t) = x(t)*h(t)

figure(4) % Create figure window and make visible on screen
A = 1.0; B = 2.0; % Assumptions for A and B values.
u = @(t) 1.0*(t>=0); % u(t) step function.
x = @(t) A*(u(t-3)-u(t-5)); % Assigned x(t) function (Figure 2.4-28 (a)).
h = @(t) B*(u(t+5)-u(t+3)); % h(t) = (t+1)(u(t+1)-u(t)).
dtau = 0.005; tau = -6:dtau:2.5; % Time intervals and step size for tau.
ti = 0; tvec = -5.0:.1:5; % Time intervals and step size for tvec.
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
```

```
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 4.0]); grid;
drawnow;
end

```

```
% Problem B.3 - part 3
% x1(t) = x(t) = e^t(u(t+2)-u(t))
% x2(t) = h(t) = e^(-2t)(u(t)-u(t-1))
% y(t) = x(t)*h(t)

figure(5) % Create figure window and make visible on screen
u = @(t) 1.0*(t>=0); % u(t) step function.
x = @(t) exp(t).*(u(t+2)-u(t)); % Assigned x(t) function (Figure 2.4-28 (a)).
h = @(t) exp(-2*t).*(u(t)-u(t-1)); % h(t) = (t+1)(u(t+1)-u(t)).
dtau = 0.005; tau = -6:dtau:2.5; % Time intervals and step size for tau.
ti = 0; tvec = -5.0:.1:5; % Time intervals and step size for tvec.
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
```

```
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```



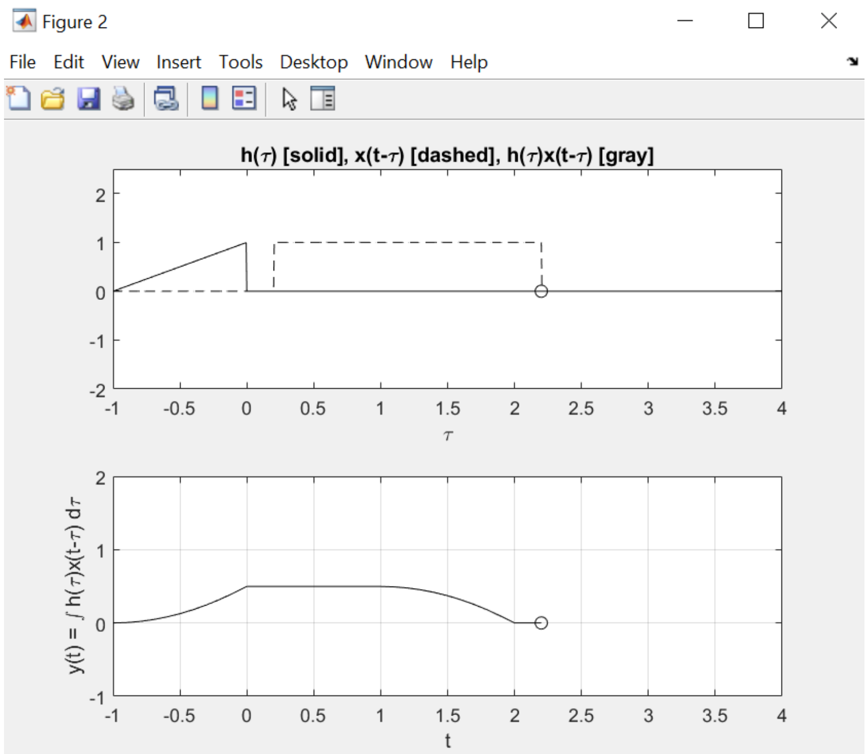


Figure 2
File Edit View Insert Tools Desktop Window Help
h(τ) [solid], x(t-τ) [dashed], h(τ)x(t-τ) [gray]
τ
y(t) = ∫ h(τ)x(t-τ) dτ
t

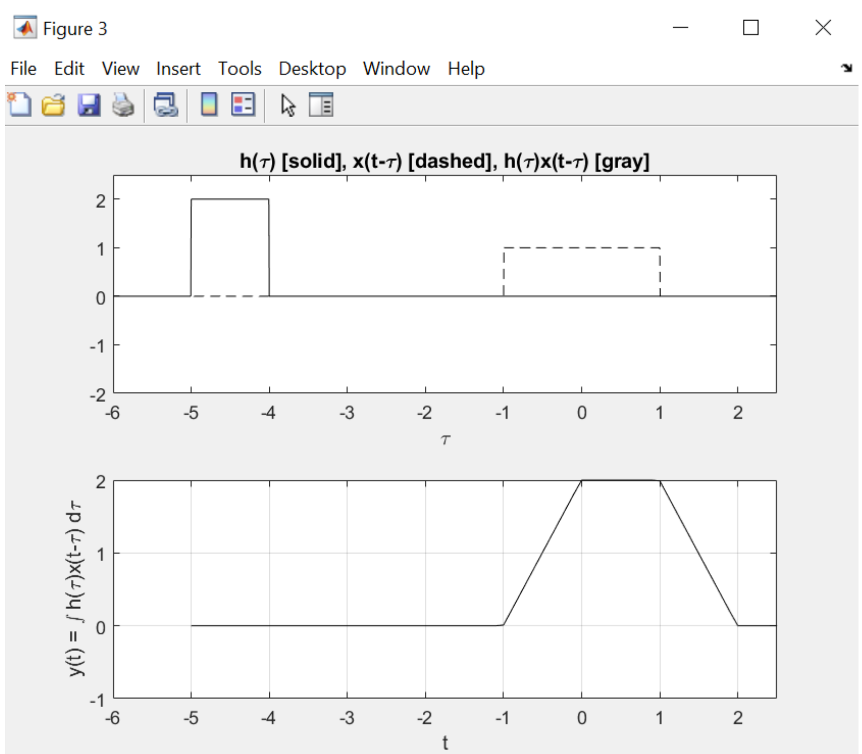


Figure 3
File Edit View Insert Tools Desktop Window Help
h(τ) [solid], x(t-τ) [dashed], h(τ)x(t-τ) [gray]
τ
y(t) = ∫ h(τ)x(t-τ) dτ
t

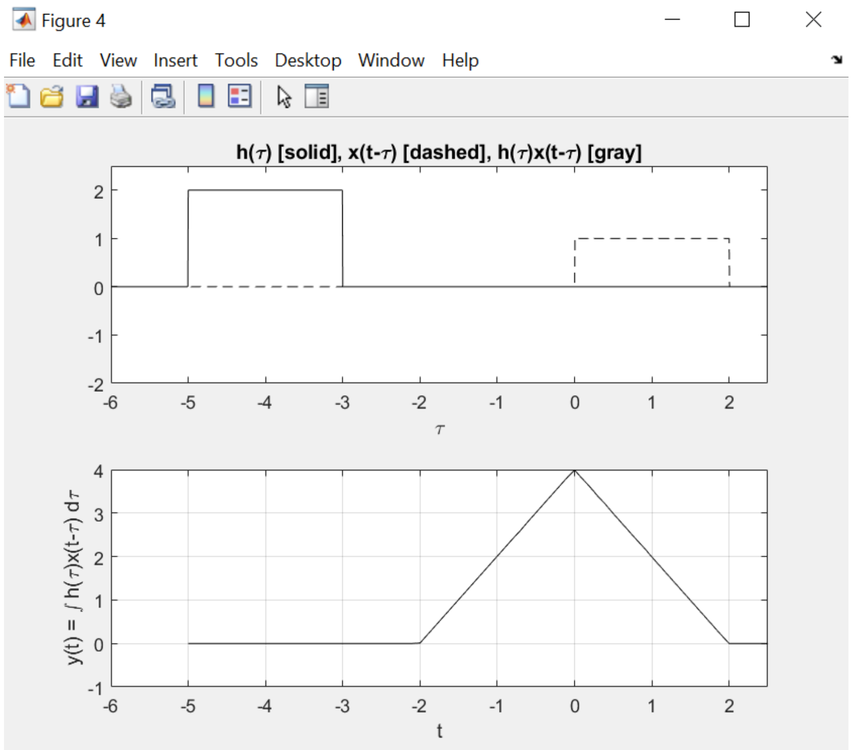


Figure 4
File Edit View Insert Tools Desktop Window Help
h(τ) [solid], x(t-τ) [dashed], h(τ)x(t-τ) [gray]
τ
y(t) = ∫ h(τ)x(t-τ) dτ
t

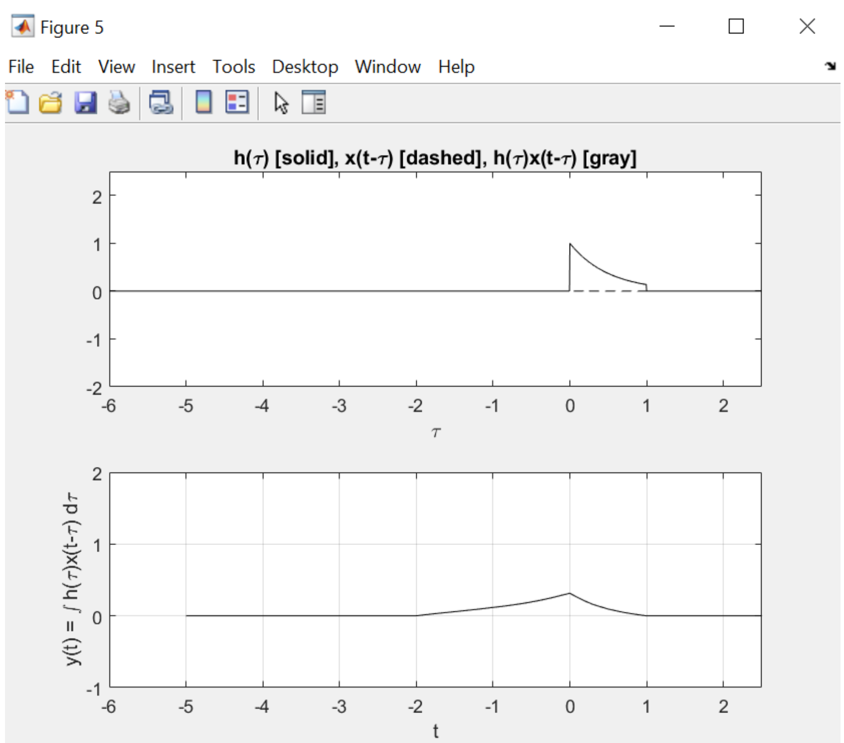


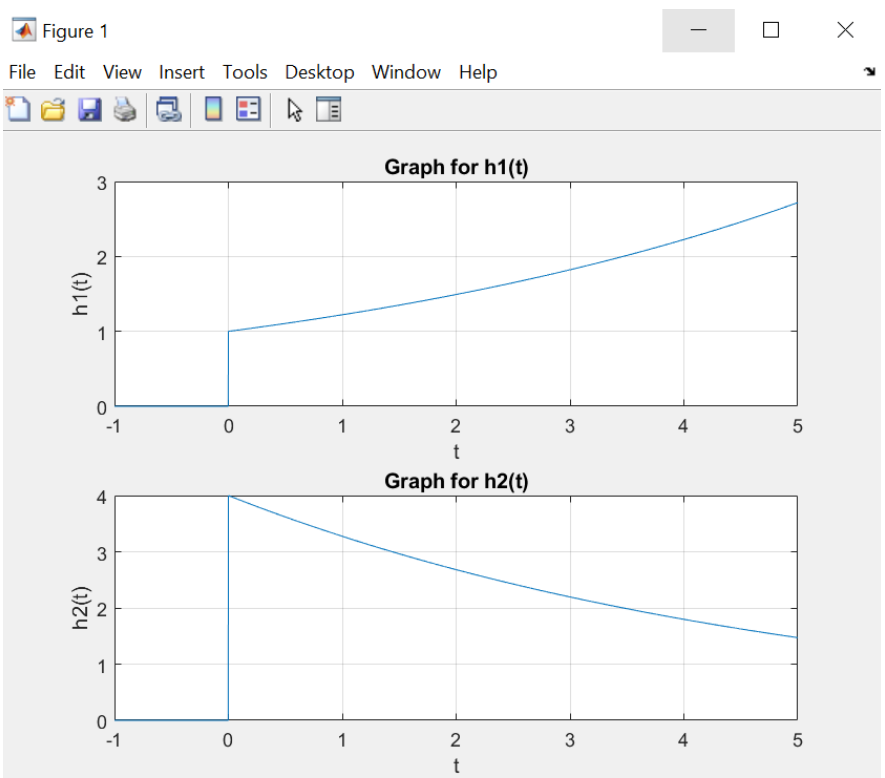
Figure 5
File Edit View Insert Tools Desktop Window Help
h(τ) [solid], x(t-τ) [dashed], h(τ)x(t-τ) [gray]
τ
y(t) = ∫ h(τ)x(t-τ) dτ
t

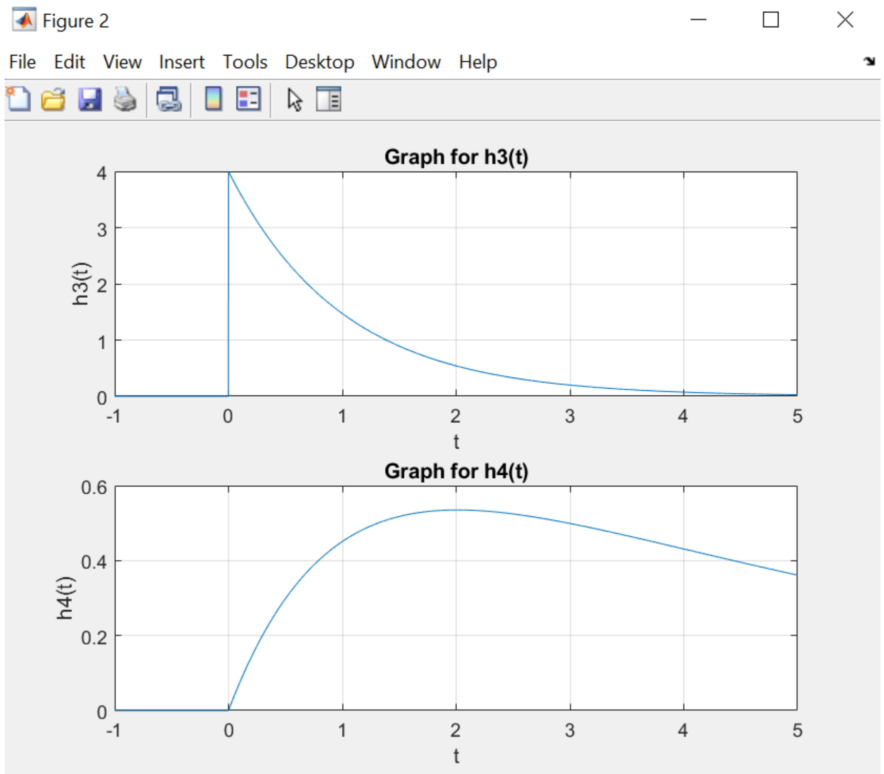
Problem C

C.1

```
%Rendel Abrasia, Reza Aablue
%500942743, 500966944
%Section 4

% Problem C.1

% Set up time interval and step size, along with u(t) step function.
t = (-1:0.001:5);
u = @(t) 1.0* (t>=0);

h1 = @(t) (exp((1/5) .* t)).*(u(t)); % Equation for h1(t) function.

h2 = @(t) 4 .* (exp((-1/5) .* t)).*(u(t)); % Equation for h2(t) function.

h3 = @(t) 4 .* (exp((-1) .* t)).*(u(t)); % Equation for h3(t) function.

h4 = @(t) (exp((-1/5) .* t) - exp((-1).*t)).*(u(t)); % Equation for h4(t) function.

figure (1);

subplot(211);
plot(t,h1(t));
xlabel('t');
ylabel('h1(t)');
title('Graph for h1(t)');
grid;

subplot(212);
plot(t,h2(t));
xlabel('t');
ylabel('h2(t)');
title('Graph for h2(t)');
grid;
```

```
figure (2);

subplot(211);
plot(t,h3(t));
xlabel('t');
ylabel('h3(t)');
title('Graph for h3(t)');
grid;

subplot(212);
plot(t,h4(t));
xlabel('t');
ylabel('h4(t)');
title('Graph for h4(t)');
grid;
```





C.2

Determine the characteristics values (eigenvalues) of systems S1-S4

For h1(t), the eigenvalue is 1/5.

For h2(t), the eigenvalue is -1/5.

For h3(t), the eigenvalue is -1.

For h4(t), the eigenvalues are -1/5 and -1.

C.3

```matlab
% Problem C.3 - Part 1
% h1(t) = e^(t/5)u(t)

figure(1) % Create figure window and make visible on screen
u = @(t) 1.0*(t>=0); % u(t) step function.
x = @(t) sin(5*t).*(u(t)-u(t-3)); % Assigned x(t) function (Figure 2.4-28 (a)).
h = @(t) exp(t/5).*(u(t)-u(t-20)); % h(t) = (t+1)(u(t+1)-u(t)).
dtau = 0.005; tau = 0:dtau:20; % Time intervals and step size for tau.
ti = 0; tvec = 0:.1:20; % Time intervals and step size for tvec.
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```

```matlab
% Problem C.3 - Part 2
% h2(t) = 4e^(-t/5)u(t)

figure(2) % Create figure window and make visible on screen
u = @(t) 1.0*(t>=0); % u(t) step function.
x = @(t) sin(5*t).*(u(t)-u(t-3)); % Assigned x(t) function (Figure 2.4-28 (a)).
h = @(t) 4*exp(-t/5).*(u(t)-u(t-20)); % h(t) = (t+1)(u(t+1)-u(t)).
dtau = 0.005; tau = 0:dtau:20; % Time intervals and step size for tau.
ti = 0; tvec = 0:.1:20; % Time intervals and step size for tvec.
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```

```
% Problem C.3 - Part 3
% h3(t) = 4e^(-t)u(t)

figure(3) % Create figure window and make visible on screen
u = @(t) 1.0*(t>=0); % u(t) step function.
x = @(t) sin(5*t).*(u(t)-u(t-3)); % Assigned x(t) function (Figure 2.4-28 (a)).
h = @(t) 4*exp(-t).*(u(t)-u(t-20)); % h(t) = (t+1)(u(t+1)-u(t)).
dtau = 0.005; tau = 0:dtau:20; % Time intervals and step size for tau.
ti = 0; tvec = 0:.1:20; % Time intervals and step size for tvec.
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```

```
% Problem C.3 - Part 4
% h3(t) = 4(e^(-t/5) - e^(-t))u(t)

figure(4) % Create figure window and make visible on screen
u = @(t) 1.0*(t>=0); % u(t) step function.
x = @(t) sin(5*t).*(u(t)-u(t-3)); % Assigned x(t) function (Figure 2.4-28 (a)).
h = @(t) 4*(exp(-t/5) - exp(-t)).*(u(t)-u(t-20)); % h(t) = (t+1)(u(t+1)-u(t)).
dtau = 0.005; tau = 0:dtau:20; % Time intervals and step size for tau.
ti = 0; tvec = 0:.1:20; % Time intervals and step size for tvec.
y = NaN*zeros(1,length(tvec)); % Pre-allocate memory
for t = tvec,
ti = ti+1; % Time index
xh = x(t-tau).*h(tau); lxh = length(xh);
y(ti) = sum(xh.*dtau); % Trapezoidal approximation of convolution integral
subplot(2,1,1),plot(tau,h(tau),'k-',tau,x(t-tau),'k--',t,0,'ok');
axis([tau(1) tau(end) -2.0 2.5]);
patch([tau(1:end-1);tau(1:end-1);tau(2:end);tau(2:end)],...
[zeros(1,lxh-1);xh(1:end-1);xh(2:end);zeros(1,lxh-1)],...
[.8 .8 .8],'edgecolor','none');
xlabel('\tau'); title('h(\tau) [solid], x(t-\tau) [dashed], h(\tau)x(t-\tau) [gray]');
c = get(gca,'children'); set(gca,'children',[c(2);c(3);c(4);c(1)]);
subplot(2,1,2),plot(tvec,y,'k',tvec(ti),y(ti),'ok');
xlabel('t'); ylabel('y(t) = \int h(\tau)x(t-\tau) d\tau');
axis([tau(1) tau(end) -1.0 2.0]); grid;
drawnow;
end
```

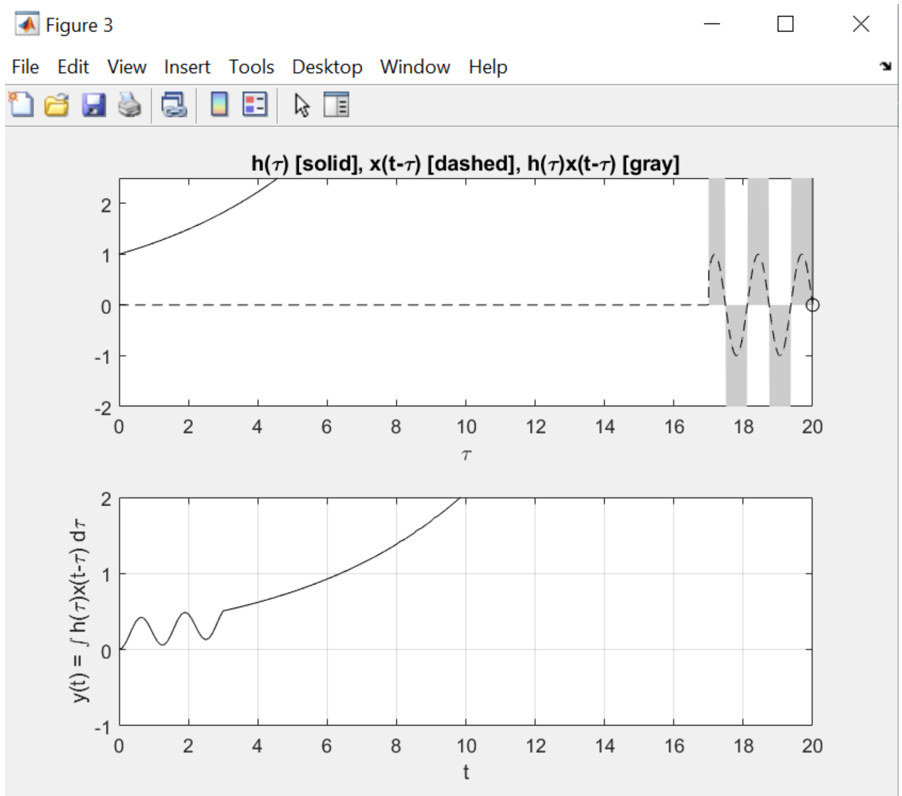


Figure 3
File Edit View Insert Tools Desktop Window Help
h(τ) [solid], x(t-τ) [dashed], h(τ)x(t-τ) [gray]
τ
y(t) = ∫ h(τ)x(t-τ) dτ
t

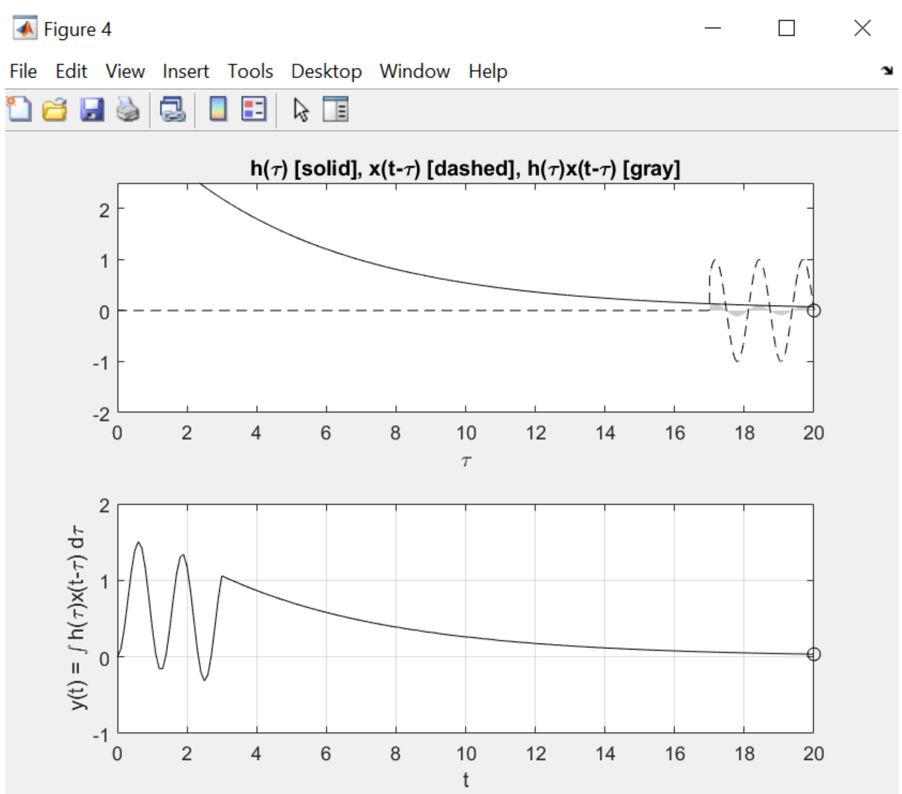


Figure 4
File Edit View Insert Tools Desktop Window Help
h(τ) [solid], x(t-τ) [dashed], h(τ)x(t-τ) [gray]
τ
y(t) = ∫ h(τ)x(t-τ) dτ
t

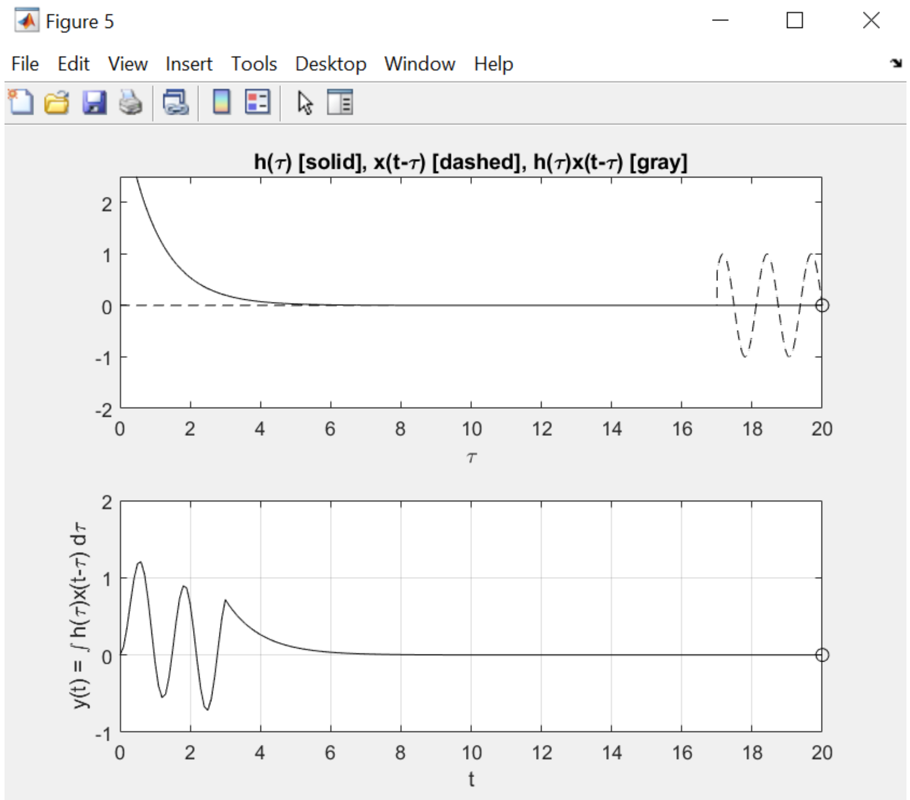


Figure 5
File Edit View Insert Tools Desktop Window Help
h(τ) [solid], x(t-τ) [dashed], h(τ)x(t-τ) [gray]
τ
y(t) = ∫ h(τ)x(t-τ) dτ
t

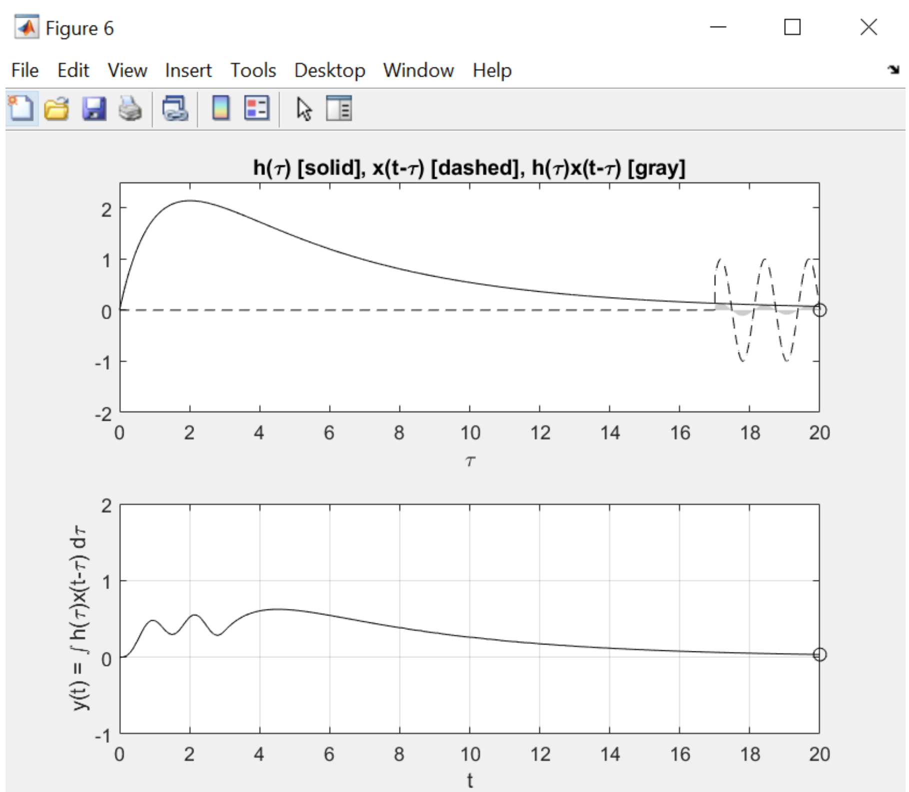


Figure 6
File Edit View Insert Tools Desktop Window Help
h(τ) [solid], x(t-τ) [dashed], h(τ)x(t-τ) [gray]
τ
y(t) = ∫ h(τ)x(t-τ) dτ
t

Is there any relationship between the output of systems S2, S3, S4? Explain.

Due to the linearity of the three LTI systems (S2, S3 and S4), the convolutions of S2 and S3 are the exact same shape. As for S4, it is defined as the difference between S2 and S3, and its convolution is fairly similar to S2 and S3. In essence, all three LTI systems among S2, S3 and S4 look fairly similar to each other, with S2 and S3(t) having the same convolution graph, and S4(t) having a similar shape to S2 and S3(t).

Problem D

D.1

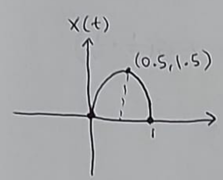
B.1) $x(t) = 1.5 \sin(\pi t)\,(u(t) - u(t-1))$

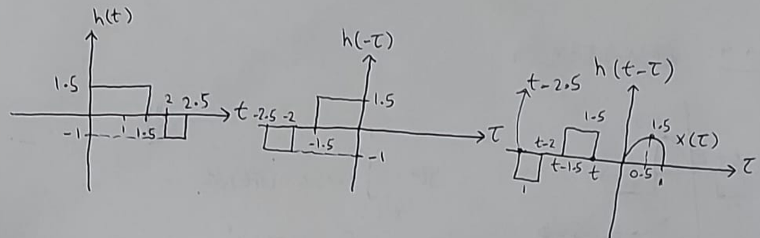
$h(t) = 1.5\,(u(t) - u(t-1.5)) - u(t-2) + u(t-2.5)$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$$

or

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$

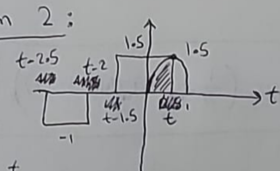




Region 1: $t < 0$

$$\boxed{y(t) = 0, \ \text{(No overlap)}}$$

Region 2:

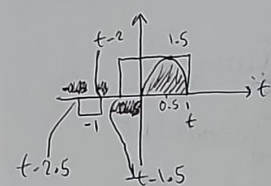


$(0 \le t \le 1)$

$$y(t) = \int_0^t (1.5)(1.5 \sin(\pi t))\,dt = 2.25 \left[-\frac{\cos(\pi t)}{\pi} \right]_0^t$$

$$\boxed{y(t) = 2.25 \left(\frac{1 - \cos(\pi t)}{\pi} \right)}$$

Region 3:

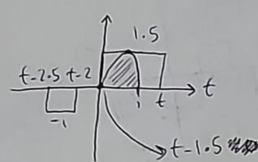


$$y(t)=\int_0^1 (1.5)(1.5\sin(\pi t))\,dt$$

$$y(t)=2.25\left[-\frac{\cos(\pi t)}{\pi}\right]_0^1 = 2.25\left(\frac{1}{\pi}+\frac{1}{\pi}\right)$$

$$\boxed{y(t) \doteq 1.432}$$

Region 4:

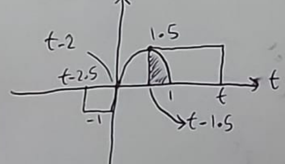


$$y(t)=\int_{t-1.5}^{1} 2.25\sin(\pi t)\,dt$$

$$y(t)=2.25\left[-\frac{\cos(\pi t)}{\pi}\right]_{t-1.5}^{1}$$

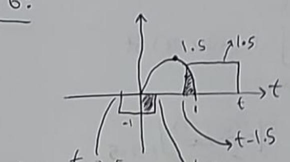
$$\boxed{y(t)=\frac{2.25}{\pi}\left(1+\cos(\pi(t-1.5))\right)}$$

Region 5:



$$\boxed{y(t)=2.25\left(\frac{1}{\pi}+\cos(\pi(t-1.5))\right)}$$

Region 6:



$$y(t) = \int_{0}^{t-2} -1.5\sin(\pi t)\,dt + \int_{t-1.5}^{1} (1.5)(1.5\sin(\pi t))\,dt$$

$$y(t) = -1.5\left[-\frac{\cos(\pi t)}{\pi}\right]_{0}^{t-2} + 2.25\left[-\frac{\cos(\pi t)}{\pi}\right]_{t-1.5}^{1}$$

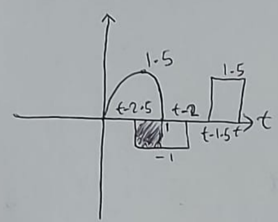
$$\boxed{y(t) = -1.5\left(\frac{-\cos(\pi(t-2))+1}{\pi}\right) + 2.25\left(\frac{1+\cos(\pi(t-1.5))}{\pi}\right)}$$

Region 7:

$$y(t) = \int_{t-2.5}^{t-2} -1.5\sin(\pi t)\,dt$$

$$\boxed{y(t) = -1.5\left(\frac{-\cos(\pi(t-2))+\cos(\pi(t-2.5))}{\pi}\right)}$$

Region 8:

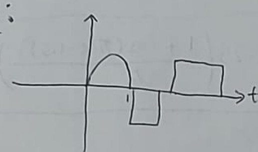


$$y(t) = \int_{t-2.5}^{1} -1.5\sin(\pi t)\,dt$$

$$y(t) = -1.5\left[-\frac{\cos(\pi t)}{\pi}\right]_{t-2.5}^{1}$$

$$\boxed{y(t) = -1.5\left(\frac{1+\cos(\pi(t-2.5))}{\pi}\right)}$$

Region 9:

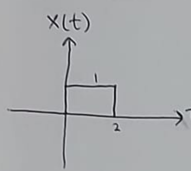


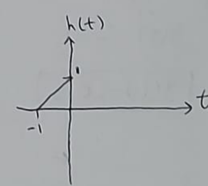
$$\boxed{y(t) = 0 \text{ , no overlap}}$$

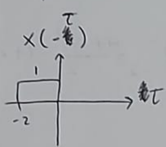
B.2) $x(t) = u(t) - u(t-2)$

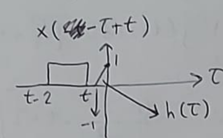
$h(t) = (t+1)(u(t+1) - u(t))$

$y(t) = x(t) * h(t)$





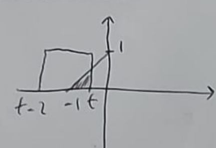




Region 1: $t \leq -1$

$\boxed{y(t) = 0 \text{ (no overlap)}}$

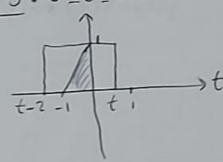
Region 2: $-1 \leq t \leq 0$



$$y(t) = \int_{-1}^{t} (t+1)\,dt = \left[t^2/2 + t\right]_{-1}^{t} = \frac{t^2}{2} + t - \frac{1}{2} + 1$$

$$\boxed{y(t) = \frac{t^2}{2} + t + \frac{1}{2}}$$

Region 3: $0 \le t \le 1$

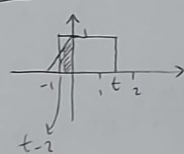


$$y(t) = \int_{-1}^{0} (t+1)\,dt = \left[\frac{t^2}{2} + t\right]_{-1}^{0}$$

$$y(t) = \frac{1}{2} - 1$$

$$\boxed{y(t) = -0.5}$$

Region 4: $1 \le t \le 2$



$$y(t) = \int_{t-2}^{0} (t+1)\,dt = \left[\frac{t^2}{2} + t\right]_{t-2}^{0} = \frac{-(t-2)^2}{2} - t + 2$$

$$\boxed{y(t) = -\frac{(t-2)^2}{2} - t + 2}$$

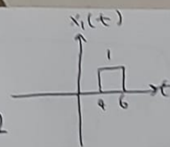
Region 5: $t > 2$

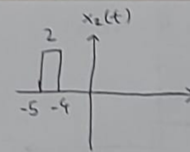
$\boxed{y(t) = 0}$, no overlap

B.3 - Part 1) $A=1,\ B=2$

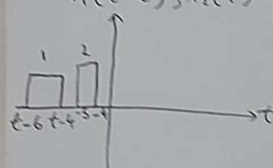
$x_1(t) = u(t-4) - u(t-6)$

$x_2(t) = (u(t+5) - u(t+4)) \cdot 2$





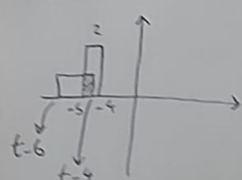
$x_1(t-\tau),\ x_2(t)$



Region 1: $(t-4) \le -5 \Rightarrow t \le -1$

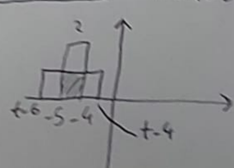
$\boxed{y(t) = 0 \text{ (no overlap)}}$

Region 2: $-5 \le t \le 4$



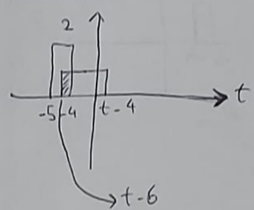
$$\boxed{y(t) = \int_{-5}^{t-4} 1\, dt = t-4+5 \Rightarrow t+1}$$

Region 3: $-4 \le t \le 0$



$$\boxed{y(t) = \int_{-5}^{-4} 1\, dt \Rightarrow 1}$$

Region 4: $1 \le t \le 2$



$$\boxed{y(t) = \int_{t-6}^{-4} 1\,dt = -4-t+6 \Rightarrow 2-t}$$

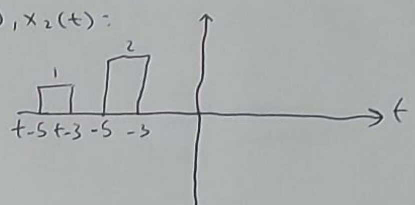
Region 5: $t > 2$

$\boxed{y(t) = 0 \text{ , no overlap}}$

B.3 - part 2) $A=1,\ B=2$

$x_1(t) = u(t-3) - u(t-5)\ ,\ x_2(t) = 2\left(u(t+5) - u(t+3)\right)$

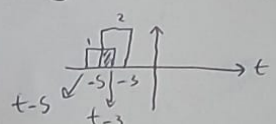
$x_1(t-\tau),\ x_2(t)$:



Region 1: $(t-3) < -5 \Rightarrow t < -2$

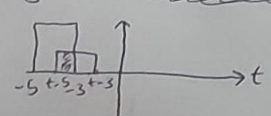
$y(t) = 0$ (no overlap

Region 2: $-5 < (t-3) < -3 \Rightarrow -2 < t < 0$



$$y(t) = \int_{-5}^{t-3} 1\, d\tau \Rightarrow t+2$$

Region 3: $-3 < (t-3) < 0 \Rightarrow 0 < t < 3$



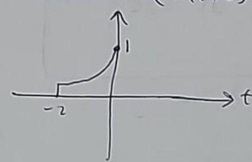
$$y(t) = \int_{t-5}^{-3} 1\, d\tau \Rightarrow -t+2$$

Region 4: $(t-3) > 0 \Rightarrow t > 3$

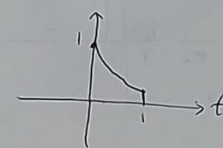
$y(t) = 0$ (no overlap)

B.3 - Part 3)

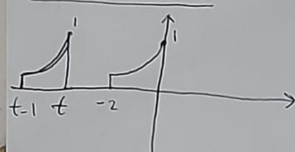
$x_1(t) = e^{t}\left(u(t+2) - u(t)\right)$



$x_2(t) = e^{-2t}\left(u(t) - u(t-1)\right)$



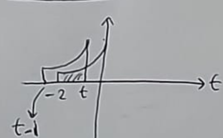
$\underline{x_2(t-\tau), x_1(t)}$



Region 1: $t \leq -2$

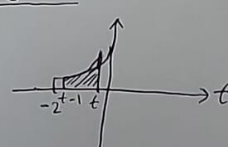
$\boxed{y(t) = 0 \text{ (no overlap)}}$

Region 2: $-2 \leq t \leq 0$

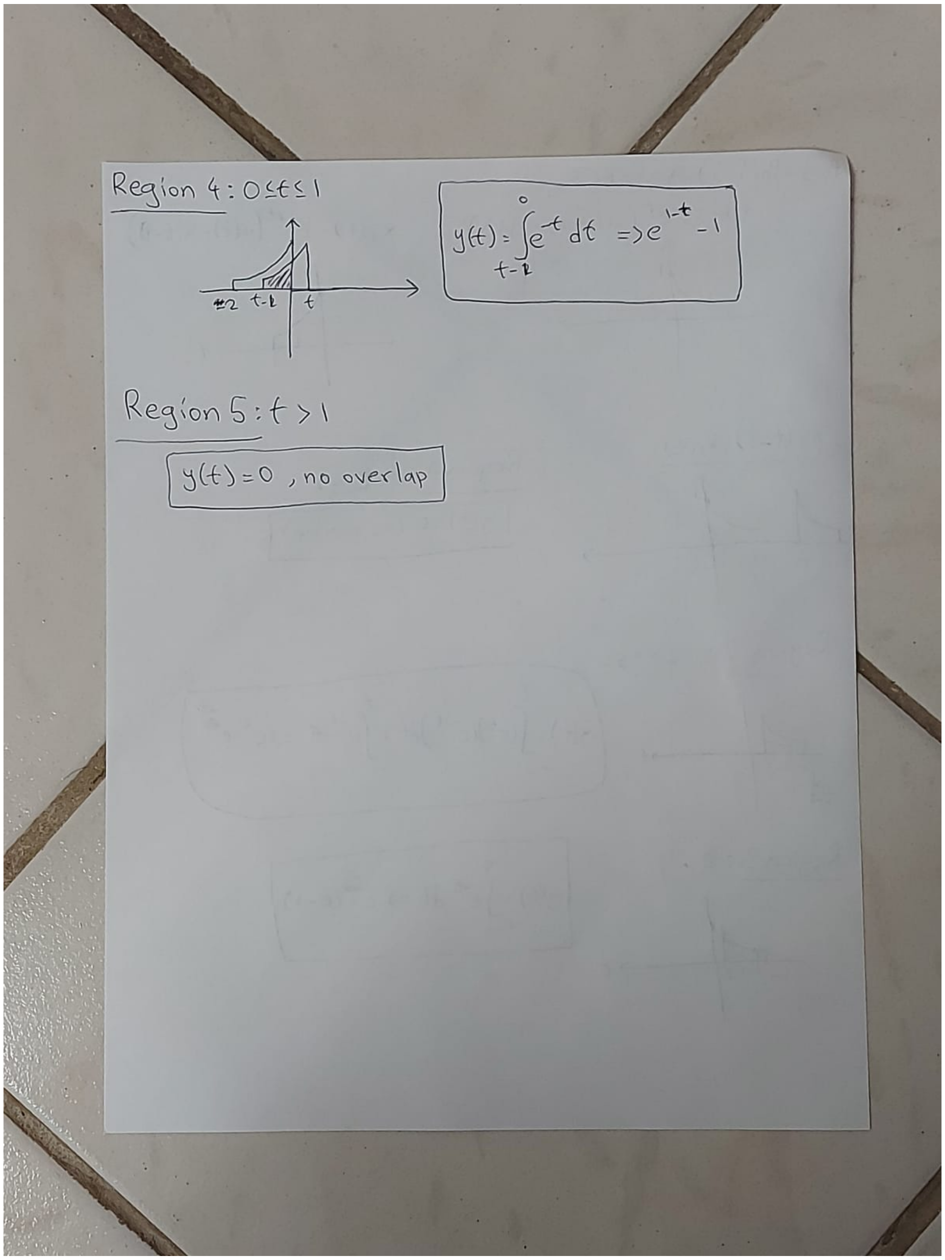


$$\boxed{y(t) = \int_{-2}^{t} (e^{t})(e^{-2t})\,dt = \int_{-2}^{t} e^{-t}\,dt \Rightarrow e^{2} - e^{-t}}$$

Region 3:



$$\boxed{y(t) = \int_{t-1}^{t} e^{-t}\,dt \Rightarrow e^{-t}(e-1)}$$



Overall, all convolution calculations performed by hand are very accurate when compared to the MATLAB results and the same results are obtained in both cases.

D.2 What can you say about the width/duration of the signal resulting from the convolution of two signals?

It is observed that the width/duration of the signal resulting from the convolution of the two signals is equal to the sum of the duration of the two functions.