Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

Course Number	ELE532
Course Title	Signals and Systems I
Semester/Year	Fall 2021

Instructor	Dr. Javad Alirezaie

ASSIGNMENT No.	**3**

Assignment Title	Fourier Series Analysis using MATLAB

Submission Date	November 12, 2021
Due Date	November 21, 2021

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Problem A.1

Problem A.1)

$$x_1(t) = \cos\left(\frac{3\pi}{10}t\right) + \frac{1}{2}\cos\left(\frac{\pi}{10}t\right)$$

$$x_1(t) = \frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}$$

Finding ω_0

$$\left.\begin{aligned}\omega_{01} &= \frac{3\pi}{10}\\ \omega_{02} &= \frac{\pi}{10}\end{aligned}\right\} \left.\begin{aligned}\frac{3\pi}{10}T_0 &= k_1\cdot 2\pi\\ \frac{\pi}{10}T_0 &= k_2\cdot 2\pi\end{aligned}\right\} \begin{aligned}T_0 &= \frac{20}{3}k_1\\ T_0 &= 20k_2\end{aligned} \Rightarrow T_0 = 20 \text{ sec.}$$

$$\omega_0 = 2\pi/T_0 \qquad \boxed{\omega_0 = \pi/10 \text{ rad/s}}$$

$\therefore$ From $x_1(t)$, $D_3 = \frac{1}{2}$, $D_{-3} = \frac{1}{2}$, $D_1 = \frac{1}{4}$ and $D_{-1} = \frac{1}{4}$.

$$D_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j\omega_0 n t}\, dt$$

$$D_n = \frac{1}{20}\int_{-10}^{10}\left(\frac{1}{2}e^{j\frac{3\pi}{10}t} + \frac{1}{2}e^{-j\frac{3\pi}{10}t} + \frac{1}{4}e^{j\frac{\pi}{10}t} + \frac{1}{4}e^{-j\frac{\pi}{10}t}\right)e^{-j\frac{\pi}{10}nt}\, dt$$

$$D_n = \frac{1}{(2)20}\int_{-10}^{10}\left(e^{j\frac{\pi}{10}(3-n)t} + e^{-j\frac{\pi}{10}(3+n)t} + \frac{1}{2}e^{j\frac{\pi}{10}(1-n)t} + \frac{1}{2}e^{-j\frac{\pi}{10}(1+n)t}\right)dt$$

$$D_n = \frac{1}{20}\left[\frac{e^{j\frac{\pi}{10}(3-n)t}}{2j\frac{\pi}{10}(3-n)} + \frac{e^{-j\frac{\pi}{10}(3+n)t}}{-2j(\frac{\pi}{10})(3+n)} + \frac{e^{j\frac{\pi}{10}(1-n)t}}{2(2)j\frac{\pi}{10}(1-n)} + \frac{e^{-j\frac{\pi}{10}(1+n)t}}{2(2)(-j)\frac{\pi}{10}(n+1)}\right]_{-10}^{10}$$

$$D_n = \frac{1}{20}\left(\frac{e^{j\pi(3-n)} - e^{-j\pi(3-n)}}{2j\frac{\pi}{10}(3-n)} + \frac{e^{-j\pi(3+n)} - e^{j\pi(3+n)}}{-2j\frac{\pi}{10}(3+n)} + \frac{e^{j\pi(1-n)} - e^{-j\pi(1-n)}}{2(2)j\frac{\pi}{10}(1-n)} + \frac{e^{-j\pi(1+n)} - e^{j\pi(1+n)}}{-2(2)j\frac{\pi}{10}(n+1)}\right)$$

$$D_n = \frac{1}{20}\left(\frac{10}{1}\operatorname{sinc}(\pi(3-n)) + 10\operatorname{sinc}(\pi(3+n)) + 5\operatorname{sinc}(\pi(1-n)) + 5\operatorname{sinc}(\pi(1+n))\right)$$

$$\boxed{D_n = \frac{1}{2}\left(\operatorname{sinc}(\pi(3-n)) + \operatorname{sinc}(\pi(3+n))\right) + \frac{1}{4}\left(\operatorname{sinc}(\pi(1-n)) + \operatorname{sinc}(\pi(1+n))\right)}$$

Problem A.2

Problem A.2)

$x_2(t)$: $T_0 = 20$ sec. $\quad \omega_0 = 2\pi/T_0 \quad \omega_0 = \pi/10$ rad/s

$$D_n = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-j\omega_0 n t}\, dt$$

$$D_n = \frac{1}{20}\int_{-5}^{5} e^{-j\frac{\pi}{10}nt}\, dt = \frac{1}{20}\left[\frac{e^{-j\frac{\pi}{10}nt}}{-j\frac{\pi}{10}n}\right]_{-5}^{5}$$

$$D_n = \frac{1}{20}\left(\frac{e^{-j\frac{\pi}{2}n} - e^{j\frac{\pi}{2}n}}{-j\frac{\pi}{10}n}\right)$$

$$D_n = \frac{1}{2n\pi}\left(\frac{e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}}{j}\right) \qquad \boxed{D_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)}$$

$x_3(t)$: $T_0 = 40$ sec. $\quad \omega_0 = 2\pi/T_0 \quad \omega_0 = \frac{\pi}{20}$ rad/s

$$D_n = \frac{1}{40}\int_{-5}^{5} e^{-j\frac{\pi}{20}nt}\, dt = \frac{1}{40}\left[\frac{e^{-j\frac{\pi}{20}nt}}{-j\frac{\pi}{20}n}\right]_{-5}^{5}$$

$$D_n = \frac{1}{40}\left(\frac{20}{n\pi}\right)\left(\frac{e^{-j\frac{\pi}{4}n} - e^{j\frac{\pi}{4}n}}{-j}\right) = \frac{1}{2\pi n}\left(\frac{e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}}{j}\right)$$

$$\boxed{D_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{4}\right)}$$

Problem A.3

```
%Rendel Abrasia, Reza Aablue
%500942743, 500966944
%Section 4

%Problem A.3
function [D]=Dn(d,n)

D1 = [0.5,0,-0.5*1i,0.5*1i,-0.5];
D2 = (1/(n.*pi)*sin((n.*pi)/2));
D3 = (1/(n.*pi)*sin((n.*pi)/4));

if (d==1)
    D=D1;
end

if (d==2)
    D=D2;
end

if (d==3)
    D=D3;
end

end
```

Problem A.4

```
%Rendel Abrasia, Reza Aablue
%500942743, 500966944
%Section 4

%Problem A.4 (x1(t)) - part 1
n = (-5:5);
Dna1 = (1./2).*((1./(n.*pi)).*sin((3-n).*pi)+(1./(n.*pi)).*sin((3+n).*pi)+(1./(2.*n.*pi)).*sin((1+n).*pi)+(1./(2.*n.*pi)).*sin((1-n).*pi));
figure (1);

subplot (1,2,1);
stem (n, abs(Dna1), '.k');
xlabel('n');
ylabel('|Dn| for x1(t)');

subplot (1,2,2);
stem (n, angle(Dna1), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x1(t)');
```

```
%Problem A.4 (x2(t)) - part 1
n = (-5:5);
Dna2 = (1./(n.*pi).*sin((n.*pi)./2));

figure (2);

subplot (1,2,1);
stem (n, abs(Dna2), '.k');
xlabel('n');
ylabel('|Dn| for x2(t)');

subplot (1,2,2);
stem (n, angle(Dna2), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x2(t)');

%Problem A.4 (x3(t)) - part 1
n = (-5:5);
Dna3 = (1./(n.*pi).*sin((n.*pi)./4));

figure (3);

subplot (1,2,1);
stem (n, abs(Dna3), '.k');
xlabel('n');
ylabel('|Dn| for x3(t)');

subplot (1,2,2);
stem (n, angle(Dna3), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x3(t)');
```

```
%Problem A.4 (x1(t)) - part 2
n = (-20:20);
Dna4 = (1./2).*((1./(n.*pi)).*sin((3-n).*pi)+(1./(n.*pi)).*sin((3+n).*pi)+(1./(2.*n.*pi)).*sin((1+n).*pi)+(1./(2.*n.*pi)).*sin((1-n).*pi));
figure (4);

subplot (1,2,1);
stem (n, abs(Dna4), '.k');
xlabel('n');
ylabel('|Dn| for x1(t)');

subplot (1,2,2);
stem (n, angle(Dna4), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x1(t)');

%Problem A.4 (x2(t)) - part 2
n = (-20:20);
Dna5 = (1./(n.*pi).*sin((n.*pi)./2));

figure (5);

subplot (1,2,1);
stem (n, abs(Dna5), '.k');
xlabel('n');
ylabel('|Dn| for x2(t)');

subplot (1,2,2);
stem (n, angle(Dna5), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x2(t)');
```

```
%Problem A.4 (x3(t)) - part 2
n = (-20:20);
Dna6 = (1./(n.*pi).*sin((n.*pi)./4));

figure (6);

subplot (1,2,1);
stem (n, abs(Dna6), '.k');
xlabel('n');
ylabel('|Dn| for x3(t)');

subplot (1,2,2);
stem (n, angle(Dna6), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x3(t)');

%Problem A.4 (x1(t)) - part 3
n = (-50:50);
Dna7 = (1./2).*((1./(n.*pi)).*sin((3-n).*pi)+(1./(n.*pi)).*sin((3+n).*pi)+(1./(2.*n.*pi)).*sin((1+n).*pi)+(1./(2.*n.*pi)).*sin((1-n).*pi));
figure (7);

subplot (1,2,1);
stem (n, abs(Dna7), '.k');
xlabel('n');
ylabel('|Dn| for x1(t)');

subplot (1,2,2);
stem (n, angle(Dna7), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x1(t)');
```

```
%Problem A.4 (x2(t)) - part 3
n = (-50:50);
Dna8 = (1./(n.*pi).*sin((n.*pi)./2));

figure (8);

subplot (1,2,1);
stem (n, abs(Dna8), '.k');
xlabel('n');
ylabel('|Dn| for x2(t)');

subplot (1,2,2);
stem (n, angle(Dna8), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x2(t)');

%Problem A.4 (x3(t)) - part 3
n = (-50:50);
Dna9 = (1./(n.*pi).*sin((n.*pi)./4));

figure (9);

subplot (1,2,1);
stem (n, abs(Dna9), '.k');
xlabel('n');
ylabel('|Dn| for x3(t)');

subplot (1,2,2);
stem (n, angle(Dna9), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x3(t)');
```

```
%Problem A.4 (x1(t)) - part 4
n = (-500:500);
Dna10 = (1./2).*((1./(n.*pi)).*sin((3-n).*pi)+(1./(n.*pi)).*sin((3+n).*pi)+(1./(2.*n.*pi)).*sin((1+n).*pi)+(1./(2.*n.*pi)).*sin((1-n).*pi));
figure (10);

subplot (1,2,1);
stem (n, abs(Dna10), '.k');
xlabel('n');
ylabel('|Dn| for x1(t)');

subplot (1,2,2);
stem (n, angle(Dna10), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x1(t)');

%Problem A.4 (x2(t)) - part 4
n = (-500:500);
Dna11 = (1./(n.*pi).*sin((n.*pi)./2));

figure (11);

subplot (1,2,1);
stem (n, abs(Dna11), '.k');
xlabel('n');
ylabel('|Dn| for x2(t)');

subplot (1,2,2);
stem (n, angle(Dna11), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x2(t)');
```

```
%Problem A.4 (x3(t)) - part 4
n = (-500:500);
Dna12 = (1./(n.*pi).*sin((n.*pi)./4));

figure (12);

subplot (1,2,1);
stem (n, abs(Dna12), '.k');
xlabel('n');
ylabel('|Dn| for x3(t)');

subplot (1,2,2);
stem (n, angle(Dna12), '.k');
xlabel('n');
ylabel('Phase of Dn [Radians] for x3(t)');
```

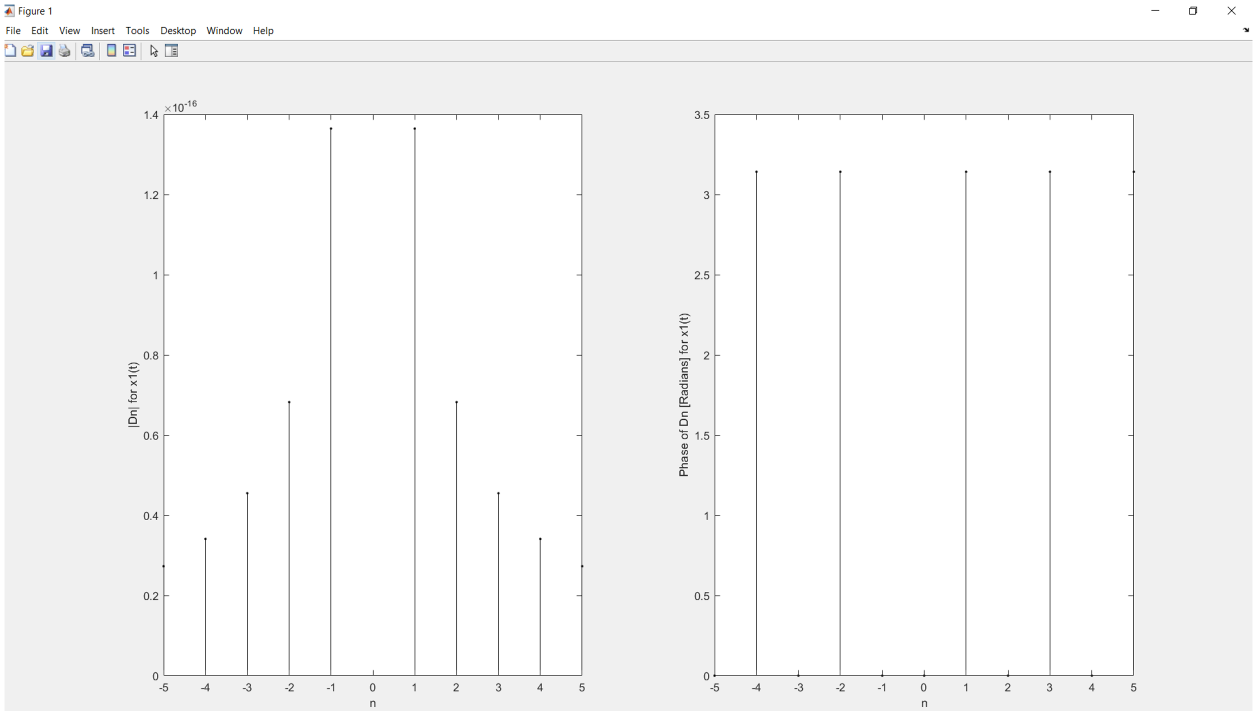
Figure 1
File Edit View Insert Tools Desktop Window Help
|Dn| for x1(t)
Phase of Dn [Radians] for x1(t)
n
n

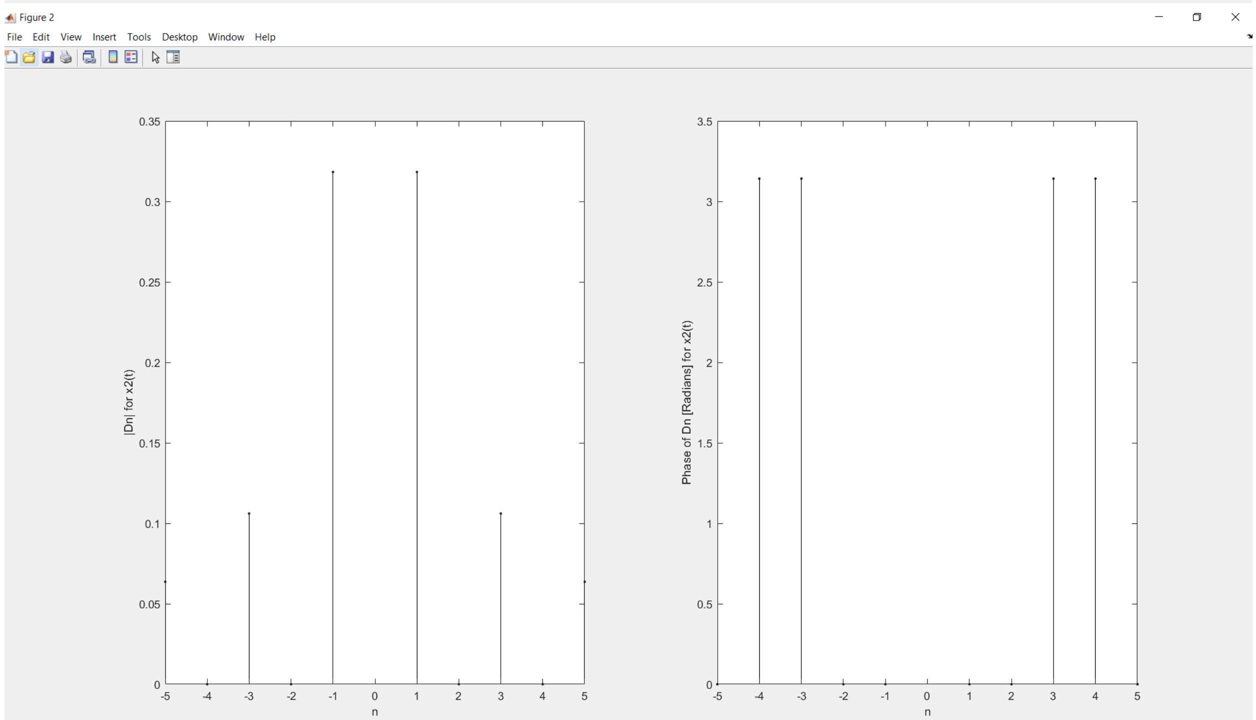
Figure 2
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|Dn| for x2(t)
Phase of Dn [Radians] for x2(t)
n
n

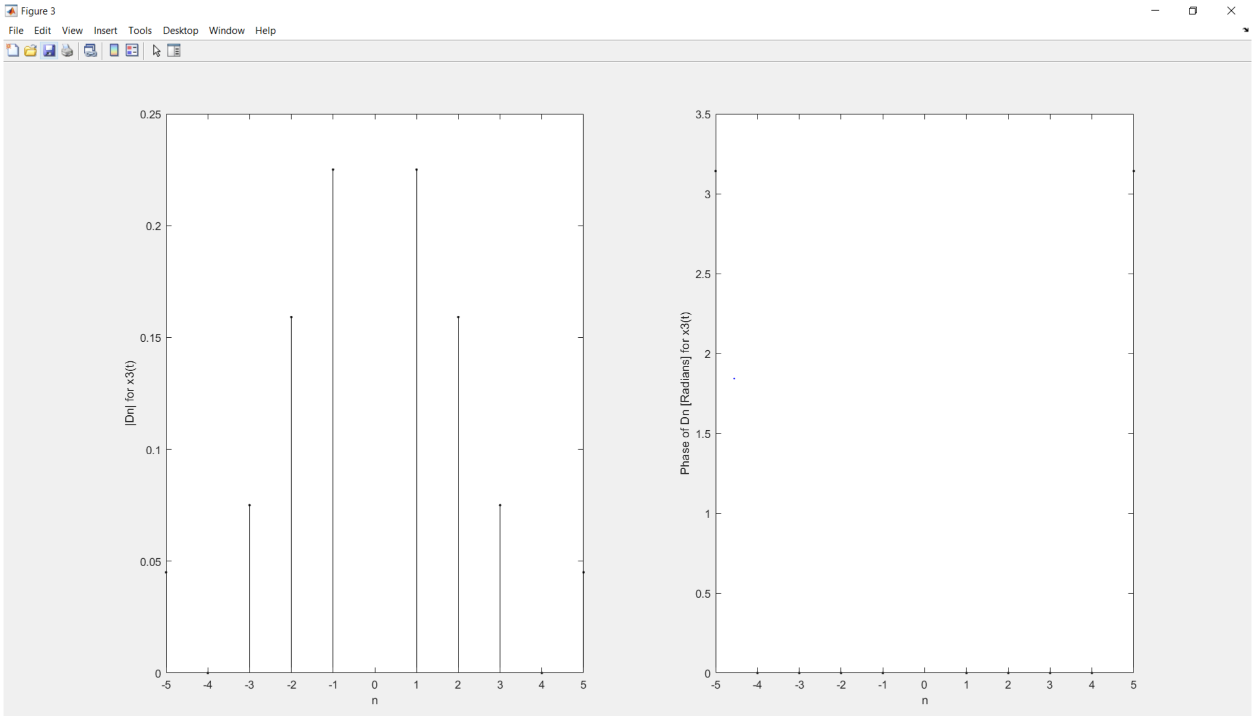
Figure 3
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|Dn| for x3(t)
n
Phase of Dn [Radians] for x3(t)
n

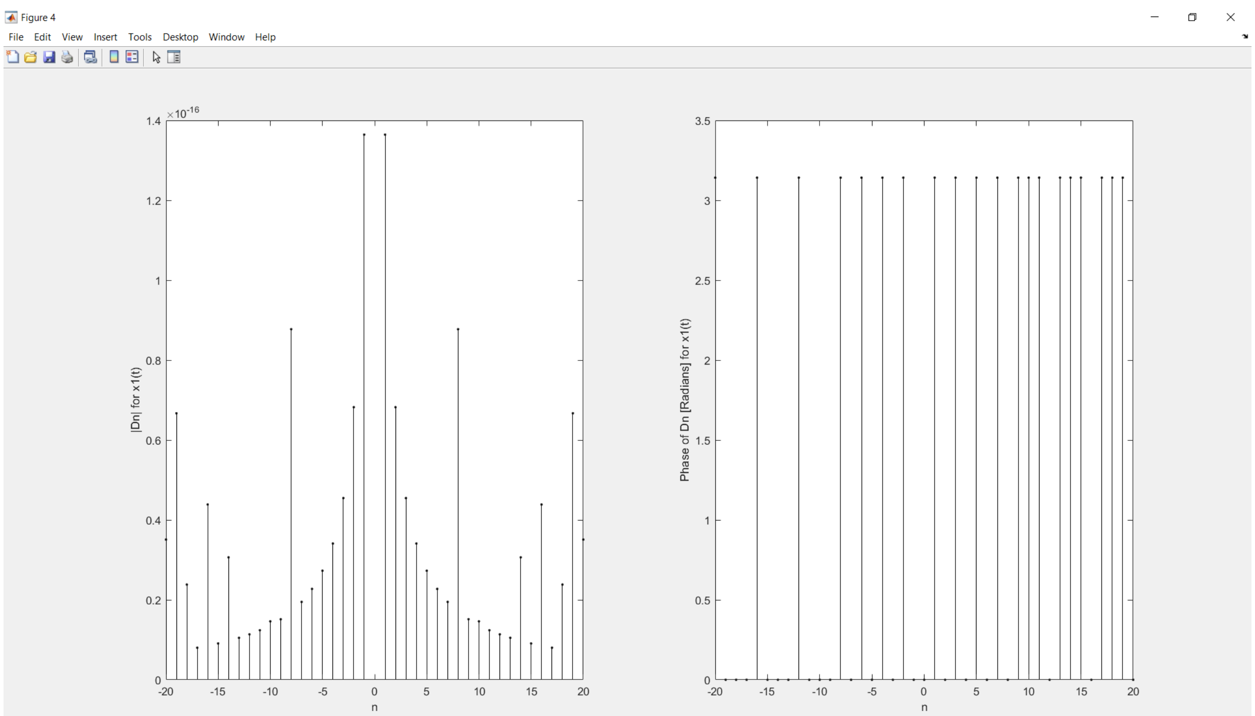
Figure 4
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|Dn| for x1(t)
n
Phase of Dn [Radians] for x1(t)
n

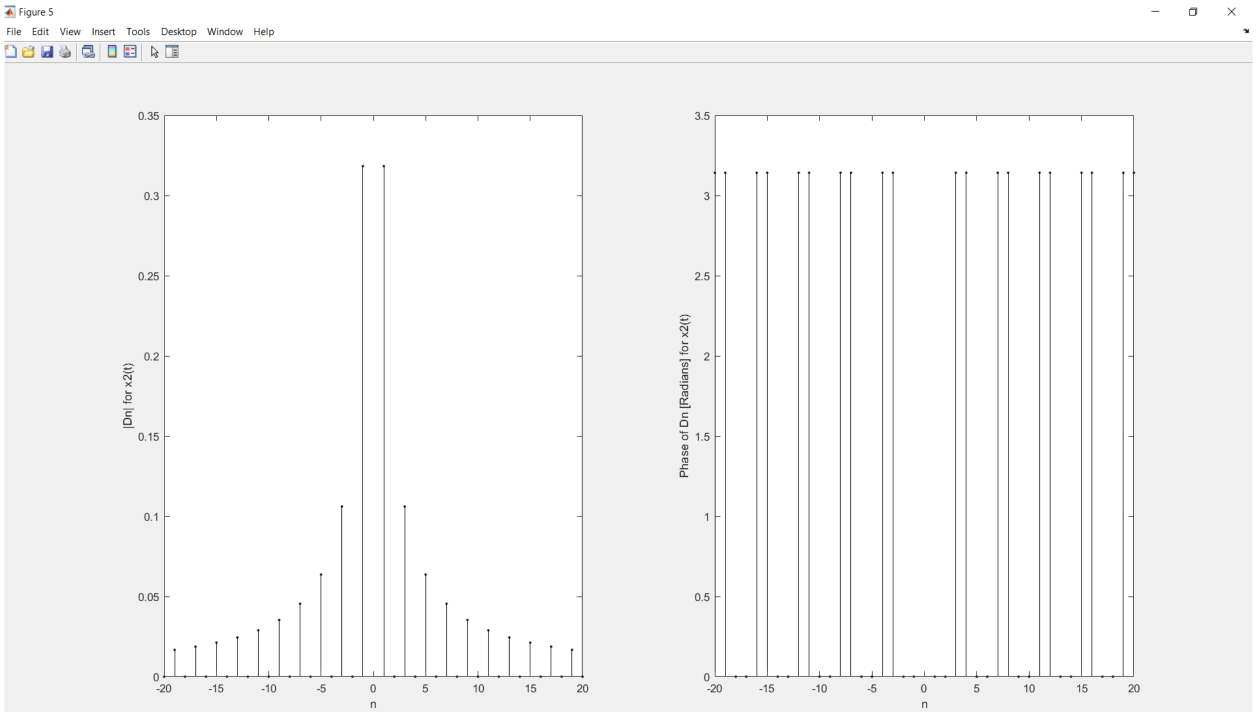
Figure 5
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|Dn| for x2(t)
Phase of Dn [Radians] for x2(t)
n
n

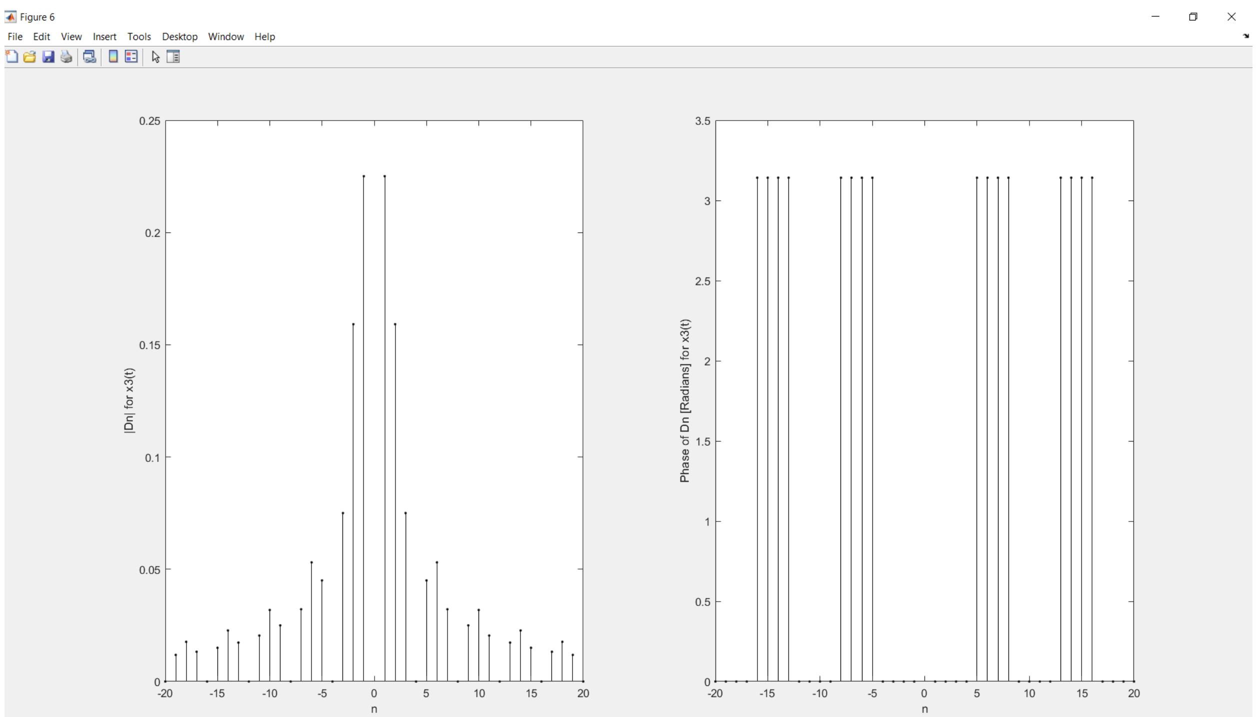
Figure 6
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|Dn| for x3(t)
Phase of Dn [Radians] for x3(t)
n
n

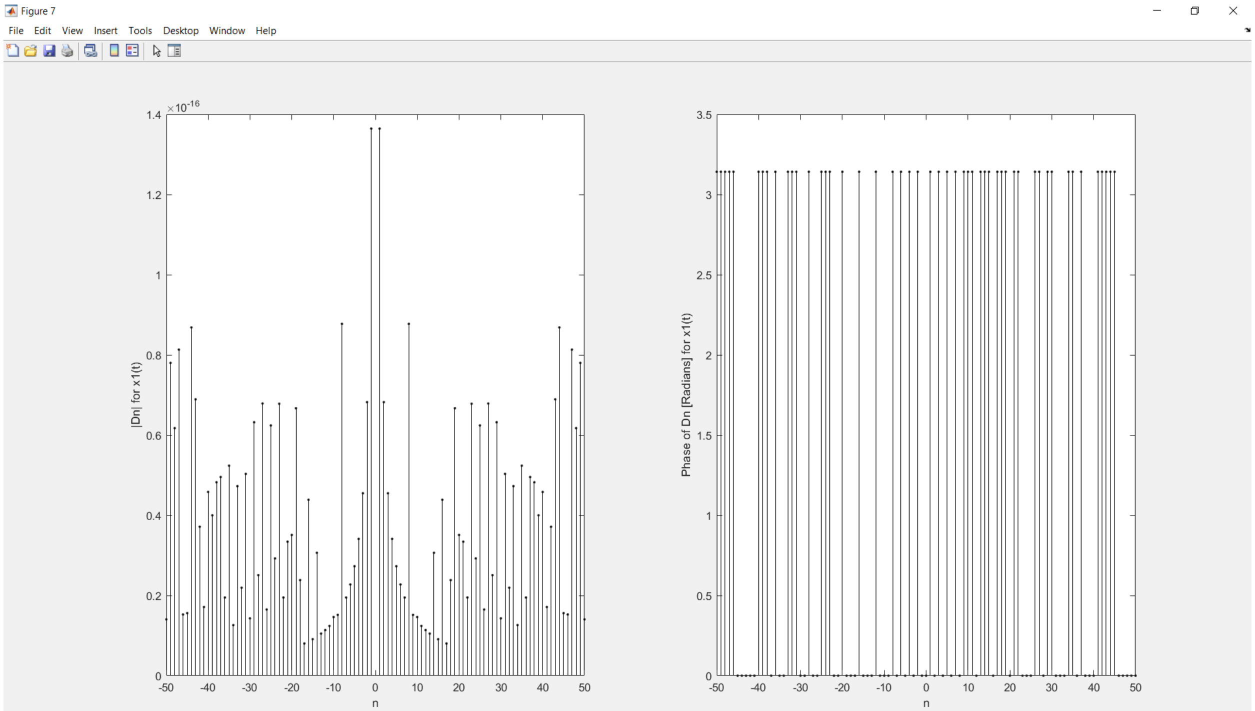
Figure 7
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×10^-16
|Dn| for x1(t)
Phase of Dn [Radians] for x1(t)
n
n

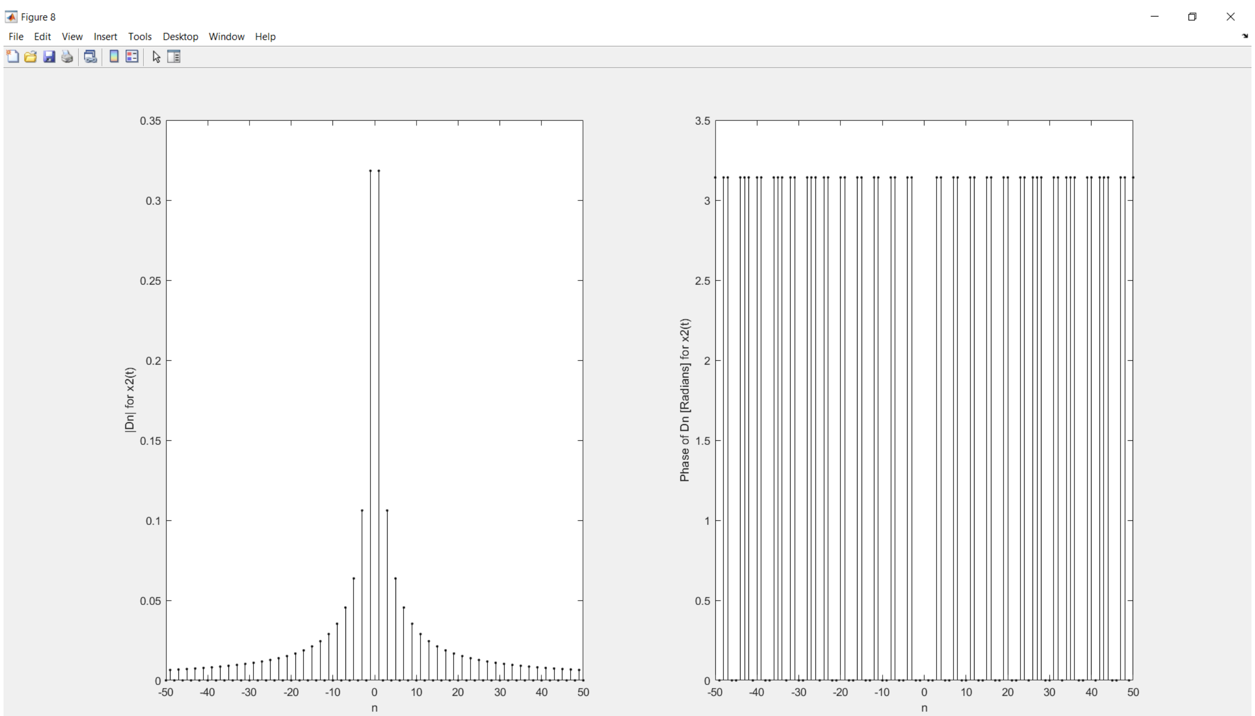
Figure 8
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|Dn| for x2(t)
Phase of Dn [Radians] for x2(t)
n
n

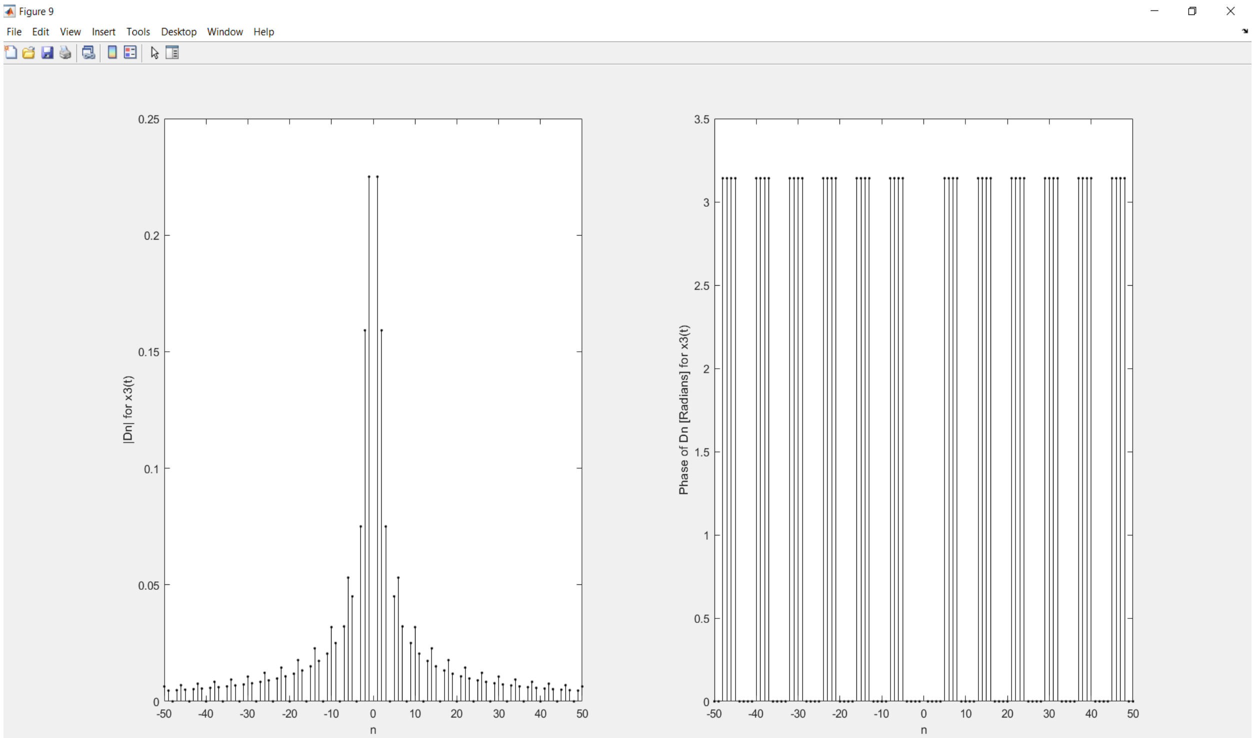
Figure 9
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|Dn| for x3(t)
n
Phase of Dn [Radians] for x3(t)
n

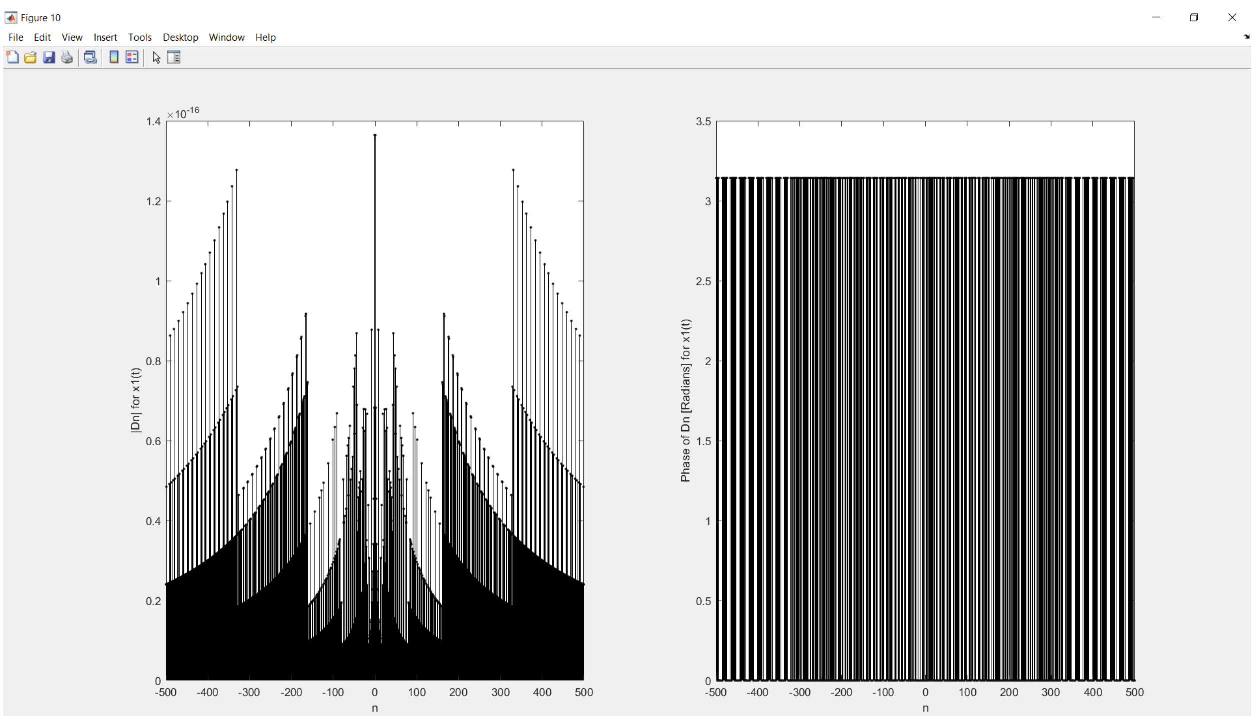
Figure 10
File Edit View Insert Tools Desktop Window Help
|Dn| for x1(t)
n
Phase of Dn [Radians] for x1(t)
n

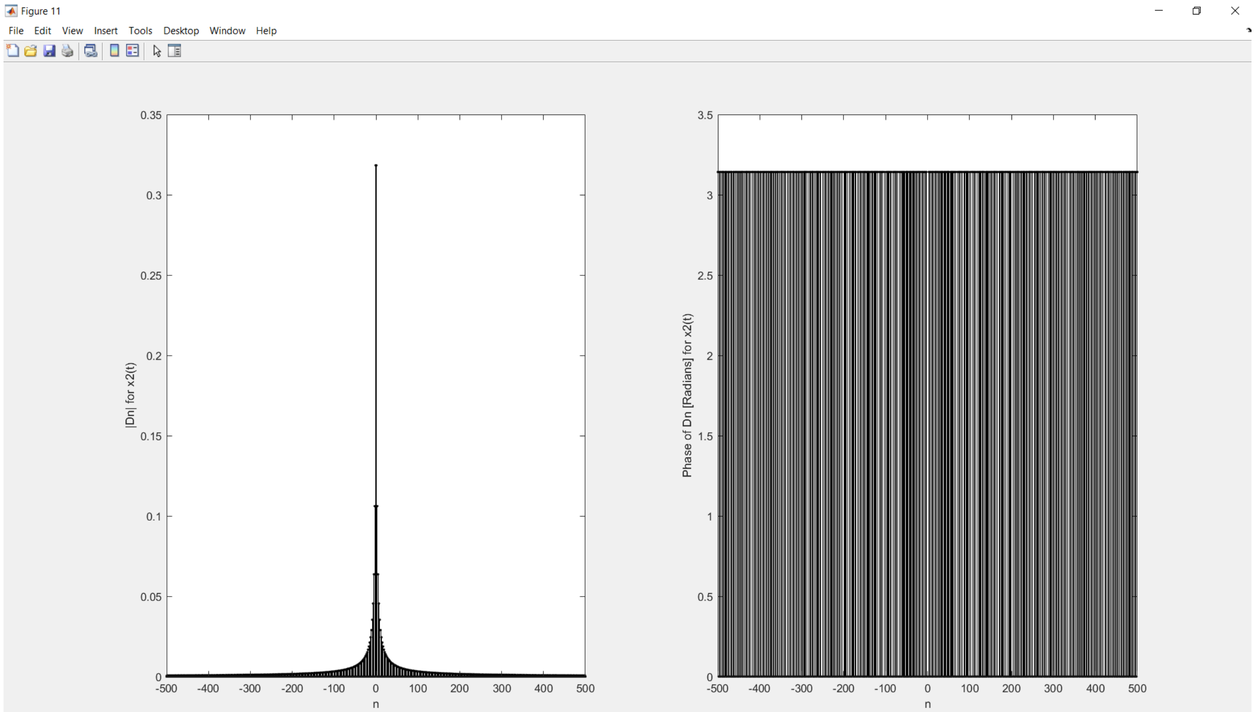
Figure 11
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|Dn| for x2(t)
n
Phase of Dn [Radians] for x2(t)
n

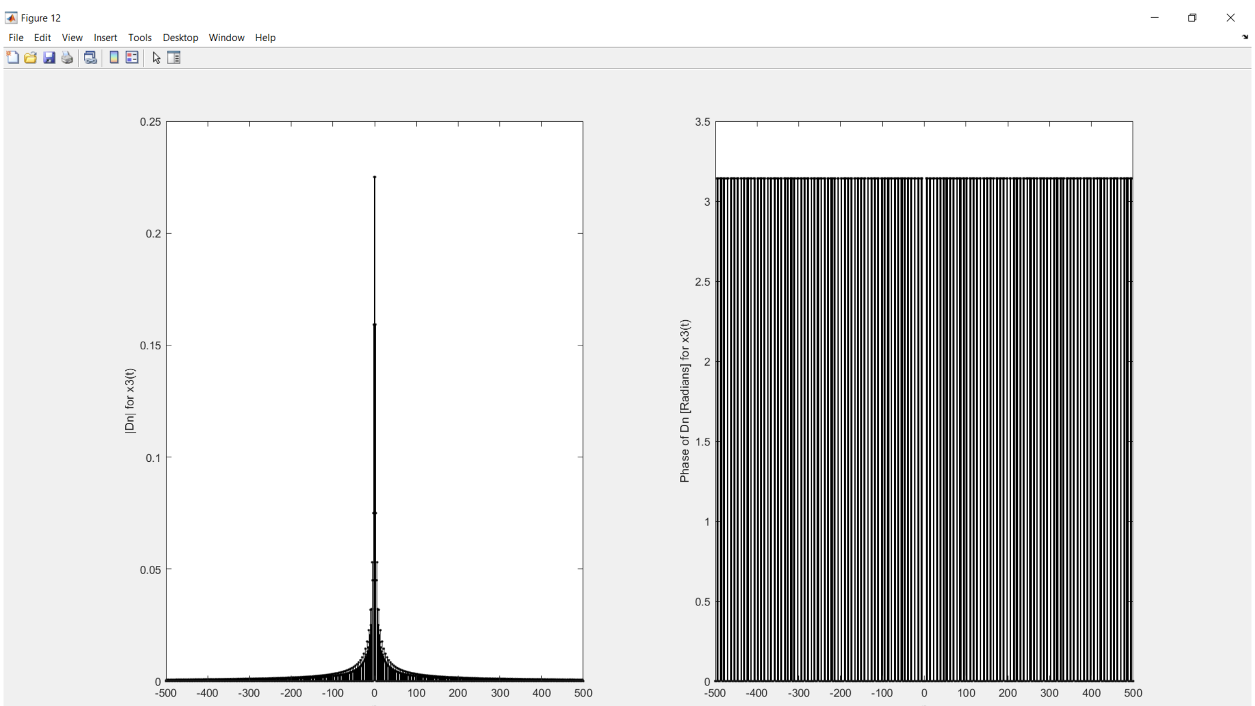


Figure 12
File Edit View Insert Tools Desktop Window Help
|Dn| for x3(t)
n
Phase of Dn [Radians] for x3(t)
n

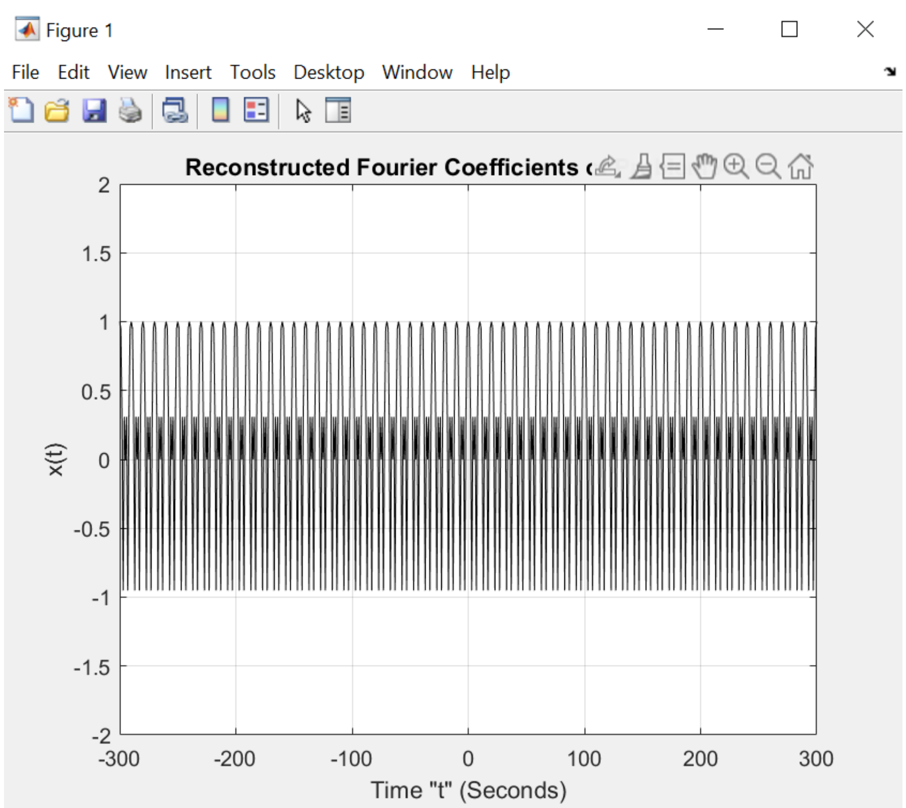
Problem A.5

```
%Rendel Abrasia, Reza Aablue
%500942743, 500966944
%Section 4

%Problem A.5
n = (-500:500);
t = (-300:1:300);
w = pi*(0.1);
x = zeros(size(t));

for i = 1:length(n)-400
    x(i)=exp(1i*n(i)*w*t(i));
end

figure (1);
plot (t,x,'k')
axis ([-300 300 -2 2]);
xlabel('Time "t" (Seconds)');
ylabel('x(t)');
title ('Reconstructed Fourier Coefficients of Problem A.5');
grid;
```



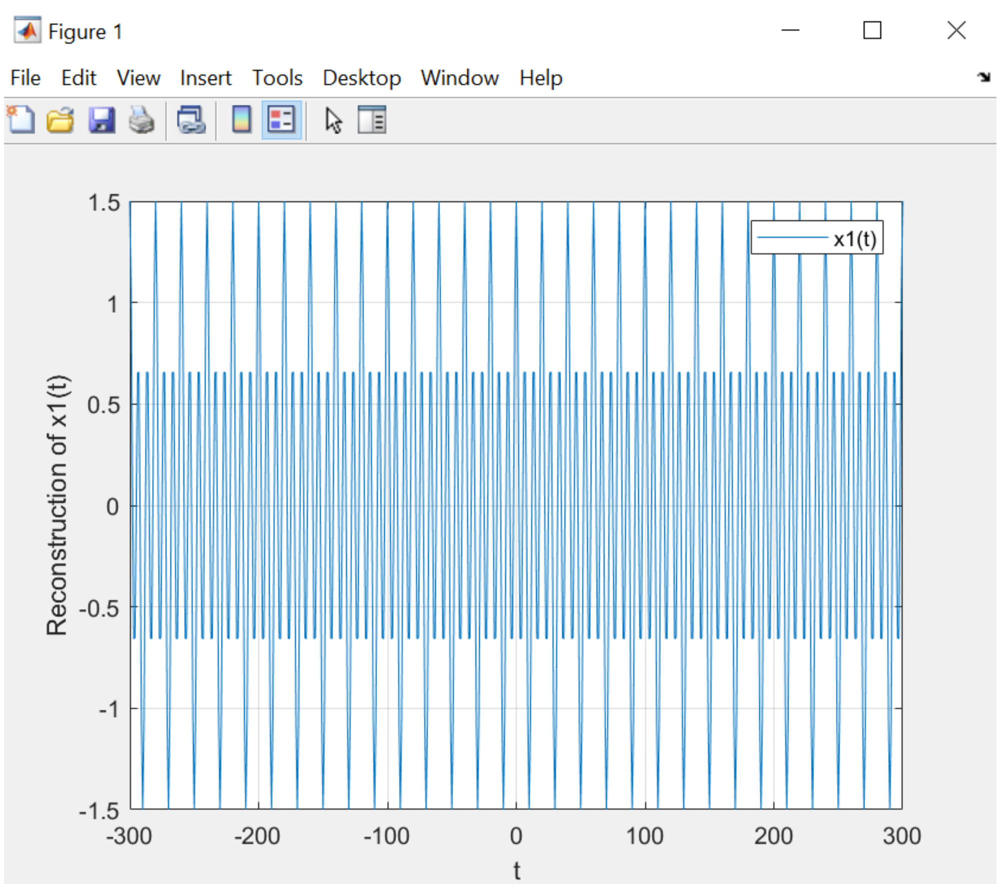
Problem A.6 - Part 1

```
%Rendel Abrasia, Reza Aablue
%500942743, 500966944
%Section 4

%Problem A.6 - part 1 (x1(t))
clf;
n = 0;
D_n = [-500 : 500]
x = 0;
w = pi/10
t = [-300:1:300]

for n = -500 : 1 : 500
    if(abs(n)==1)
        D_n(n+500+1) = 0.25;
    elseif(abs(n) == 3)
        D_n(n+500+1) = 0.5;
    else
       D_n(n+500+1) = 0;
end
end
 s = 300 + 1;
 x = zeros(1,601);
 for t = -300 : 1 : 300
    for n = -500 : 1 : 500
        m = n+500+1;
        x(t+s) = x(t+s) + real(D_n(m).*exp(n.*1i*w*t));
    end
 end

t=[-300:1:300];
plot(t,real(x));
ylabel('Reconstruction of x1(t)');xlabel('t');
legend('x1(t)');
grid;
```



Problem A.6 - Part 2

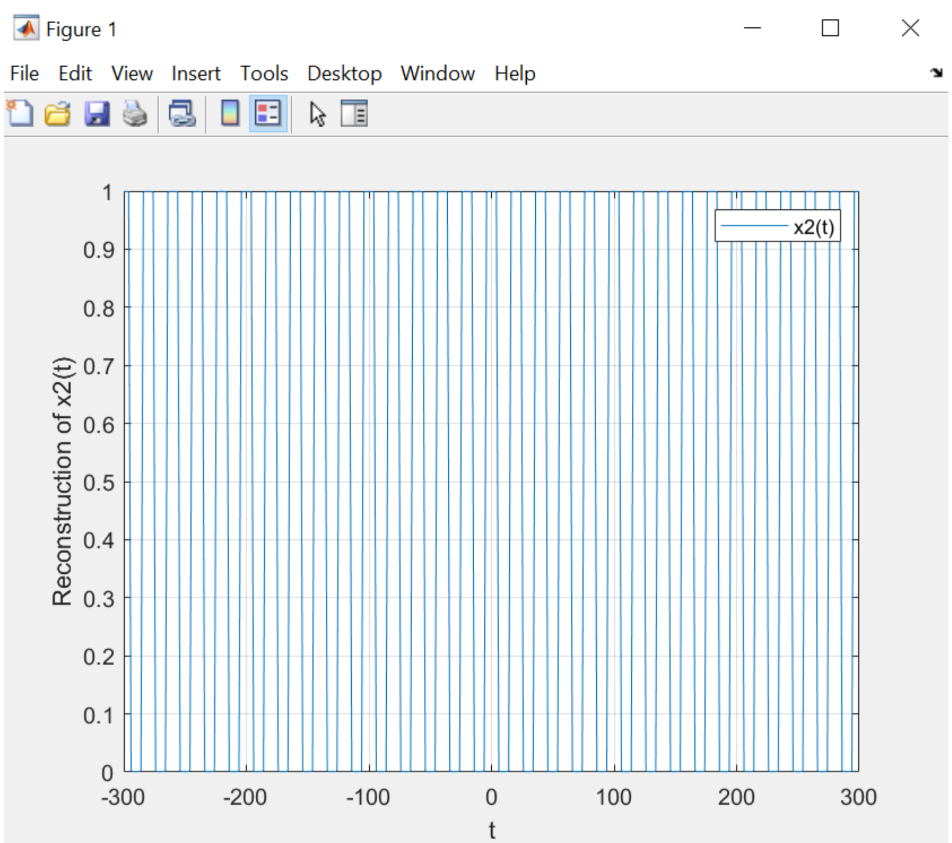
```matlab
%Rendel Abrasia, Reza Aablue
%500942743, 500966944
%Section 4

%Problem A.6 - part 2 (x2(t))
clf;
n = 0;
D_n = [-500:500]
for n = -500 : 1 : 500
    i = n+500+1;
    if n == 0
        D_n(i) = 0.5;
    else
        D_n(i) = (1/(pi.*n)).*sin(0.5.*pi*n);
end
end

w = pi/10;
s = 300 + 1;
x = zeros(1,601);

```

```matlab
for t = -300 : 1 : 300
    for n = -500 : 1 : 500
    m = n+500+1;
    x(t+s) = x(t+s) + real(D_n(m).*exp(n.*1i*w*t));
    end
end
t = [ -300 : 1 : 300];
plot(t,real(x));
ylabel('Reconstruction of x2(t)');xlabel('t');
legend('x2(t)');
grid;
```



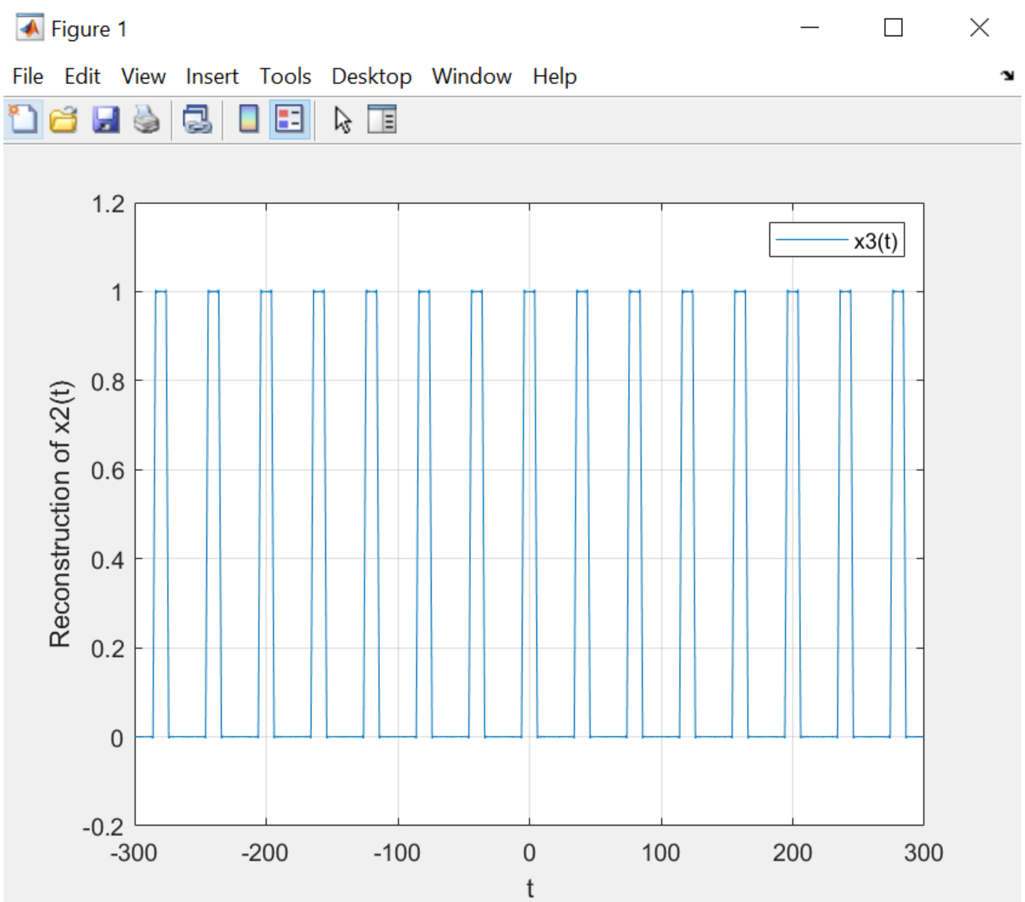
Problem A.6 - Part 3

```
%Rendel Abrasia, Reza Aablue
%500942743, 500966944
%Section 4

%Problem A.6 - part 3 (x3(t))
clf;
n = 0;
D_n = [-500:500]
for n = -500 : 1 : 500
    i = n+500+1;
    if n == 0;
        D_n(i) = 0.25;
    else
        D_n(i) = (1/(pi.*n)).*sin(0.25.*pi*n);
end
end

w = pi/20;
s = 300 + 1;
x = zeros(1,601);
for t = -300 : 1 : 300
    for n = -500 : 1 : 500
    m = n+500+1;
    x(t+s) = x(t+s) + real(D_n(m).*exp(n.*1i*w*t));
end
end

t = [ -300 : 1 : 300];
plot(t,real(x));
ylabel('Reconstruction of x2(t)');xlabel('t');
legend('x3(t)');
grid;
```



Problem B.1

Problem B.1)

$$x_1(t) = \cos\Big(\underbrace{\tfrac{3\pi}{10}}_{\omega_1} t\Big) + \frac{1}{2}\cos\Big(\underbrace{\tfrac{\pi}{10}}_{\omega_2} t\Big)$$

$$\omega_1 = \frac{3\pi}{10} \text{ rad/s} > \omega_2 = \frac{\pi}{10} \text{ rad/s}$$

$$\therefore \boxed{\omega_0 = \omega_2 = \pi/10 \simeq 0.314 \text{ rad/s.}}$$

$x_2(t)$

$T_0 = 20$ seconds

$$f_0 = \frac{1}{T_0} = \frac{1}{20} \text{ Hz}$$

$$\omega_0 = 2\pi f_0 = \cancel{2}\pi\Big(\frac{1}{\cancel{20}_{10}}\Big)$$

$$\therefore \boxed{\omega_0 = \frac{\pi}{10} \simeq 0.314 \text{ rad/s}}$$

$x_3(t)$

$T_0 = 40$ seconds

$$f_0 = \frac{1}{T_0} = \frac{1}{40} \text{ Hz}$$

$$\omega_0 = 2\pi f_0 = \cancel{2}\pi\Big(\frac{1}{\cancel{40}_{20}}\Big)$$

$$\therefore \boxed{\omega_0 = \pi/20 \simeq 0.157 \text{ rad/s}}$$

Problem B.2) The main difference between the Fourier coefficients of $x_1(t)$and $x_2(t)$is that $x_1(t)$ consists of the sinc function, whereas $x_2(t)$relies on the sine function. Another key difference is that $x_1(t)$has four distinct Fourier Series coefficients that contains the sinc function, whereas $x_2(t)$has an infinite number of Fourier Series coefficients containing the sine function.

Problem B.3) The signal $x_3(t)$has a smaller fundamental period and frequency relative to $x_2(t)$. To be precise, $x_2(t)$has double the fundamental period (20 seconds vs. 40 seconds) and fundamental frequency ($\frac{\pi}{10}$rads/second vs. $\frac{\pi}{20}$rads/second) over $x_3(t)$.

Problem B.4) For $x_2(t)$, the Fourier coefficient $D_o$is the DC value of the signal, which is 1. As for $x_4(t)$, the value for $D_o$would be 0.5 as that represents the magnitude or the DC value of the signal.

Problem B.5) Since the $x_1(t)$signal has a finite number of Fourier Series coefficients (4 "D_n" coefficients), the re-constructed results wouldn't be any different from the original if more coefficients were to be used for re-constructing the signal. However, for $x_2(t)$and $x_3(t)$ signals, increasing the Fourier Series coefficients results in a more accurate reconstruction of the signal since there are an infinite number of Fourier Series coefficients for these two signals.

Problem B.6) In the case of $x_1(t)$, only 4 Fourier coefficients are required to perfectly reconstruct the signal, as the signal has only 4 Fourier coefficients as shown in part A.1. However, for $x_2(t)$and $x_3(t)$, since there is an infinite number of Fourier coefficients for both of them, reconstructing them perfectly is not possible as one cannot combine an infinite number of signals. With that being said, the more coefficients used for reconstruction, the closer one can get to perfect reconstruction of $x_2(t)$and $x_3(t)$.

Problem B.7) This method is not a viable approach as the fundamental frequency of the signal is not known. Regardless of the number of Fourier Series coefficients, the fundamental frequency is essential in accurate reconstruction of the original signal.