RYERSON UNIVERSITY

Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

Course Number	ELE632
Course Title	Signals and Systems II
Semester/Year	Winter 2022

Instructor	Dr. Dimitri Androutsos

ASSIGNMENT No.	**2**

Assignment Title	Time-Domain Analysis of Discrete-Time Systems - Part 2

Submission Date	February 20, 2022
Due Date	February 20, 2022

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Problem A

1i) $y[n] + \frac{1}{6} y[n-1] - \frac{1}{6} y[n-2] = \frac{1}{3} x[n]$

$y[n+2] + \frac{1}{6} y[n+1] - \frac{1}{6} y[n] = \frac{1}{3} x[n+2]$

$\left(E^2 + \frac{1}{6}E - \frac{1}{6}\right) y[n] = \frac{1}{3} E^2 x[n]$

$\lambda^2 + \frac{1}{6}\lambda - \frac{1}{6} = 0$

$6\lambda^2 + \lambda - 1 = 0$

$\lambda = \frac{-1 \pm 5}{12}$

$\lambda_1 = 0.33$

$\lambda_2 = -0.5$

$y_0[n] = C_1 (0.33)^n + C_2 (-0.5)^n$

$h[n] = u[n]\left(C_1 (0.33)^n + C_2 (-0.5)^n\right)$

$h[n+2] + \frac{1}{6} h[n+1] - \frac{1}{6} h[n] = \frac{1}{3} \delta[n+2]$

$\underline{n = -2}$

$h[0] = \frac{1}{3} \delta[0] = \frac{1}{3}$

$\boxed{h[0] = \frac{1}{3}}$

$\underline{n = -1}$

$h[1] + \frac{1}{6} h[0] = \frac{1}{3} \cancelto{0}{\delta[1]}$

$h[1] + \frac{1}{18} = 0$

$\boxed{h[1] = -1/18}$

$\underline{h[0] = 1/3}$

$C_1 + C_2 = \frac{1}{3}$

$C_1 = \frac{1}{3} - C_2$

$\underline{h[1] = -1/18}$

$6\left(\frac{C_1}{3} - \frac{C_2}{2}\right) = -\frac{1}{18} (6)$

$2C_1 - 3C_2 = -\frac{1}{3}$

$\frac{2}{3} - 2C_2 - 3C_2 = -\frac{1}{3}$

$-5C_2 = -1$

$\boxed{C_2 = 1/5}$

$\boxed{C_1 = \frac{2}{15}}$

$\therefore h[n] = u[n]\left(\frac{2}{15}(0.33)^n + \frac{1}{5}(-0.5)^n\right).$

1ii) $y[n] + \frac{1}{4} y[n-2] = x[n]$

$y[n+2] + \frac{1}{4} y[n] = x[n+2]$

$\left(E^2 + \frac{1}{4}\right) y[n] = E^2 x[n]$

$\lambda^2 + \frac{1}{4} = 0 \qquad \lambda = \pm \frac{j}{2}$

$\lambda_1 = 0.5\, e^{j\pi/2}$

$\lambda_2 = 0.5\, e^{-j\pi/2}$

$y_0[n] = C_1 \left(\frac{1}{2}\right)^n e^{jn\pi/2} + C_2 \left(\frac{1}{2}\right)^n e^{-jn\pi/2}$

$h[n] = u[n]\left(C_1 \left(\frac{1}{2}\right)^n e^{jn\pi/2} + C_2 \left(\frac{1}{2}\right)^n e^{-jn\pi/2}\right)$

$h[n+2] + \frac{1}{4} h[n] = \delta[n+2]$

$\underline{n = -2}$

$h[0] + \frac{1}{4} \cancel{h[-2]}^{\,0} = \cancel{\delta[0]}^{\,1}$

$\boxed{h[0] = 1}$

$\underline{n = -1}$

$h[1] + \frac{1}{4} \cancel{h[-1]}^{\,0} = \cancel{\delta[1]}^{\,0}$

$\boxed{h[1] = 0}$

$y_0[n] = C \left(\frac{1}{2}\right)^n \cos\left(\frac{n\pi}{2}\right)$

$h[n] = u[n]\left(C \left(\frac{1}{2}\right)^n \cos\left(\frac{n\pi}{2}\right)\right)$

$\underline{h[0] = 1}$

$C(1)(1) = 1$

$\boxed{C = 1}$

$\underline{h[1] = 0}$

$C\left(\frac{1}{2}\right)(0) = 0$

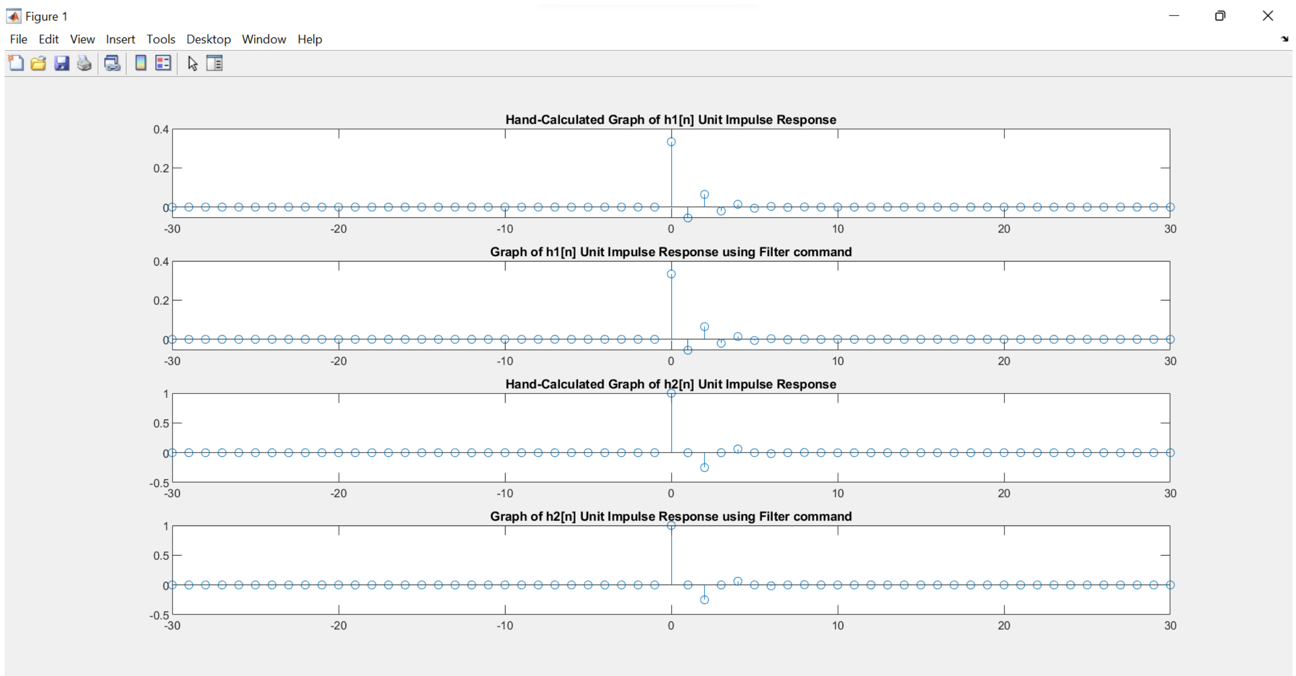
$\underline{0 = 0}$

$\therefore h[n] = \left(\left(\frac{1}{2}\right)^n \cos\left(\frac{n\pi}{2}\right)\right) u[n].$

```
% Reza Aablue
% 500966944
% Section 05

%Problem A
n = [-30:30];
u = @(n) (n>=0) * 1.0 .* (mod(n,1)==0);

% Calculating h1[n] and h2[n] by hand - detailed work attached in report.
h1Handcalc = @(n) ((1/5).*(-0.5).^n + (2/15).*(1/3).^n).*u(n);
h2Handcalc = @(n) ((1/2).^n.*cos(n*pi/2).*u(n));

% Calculating h1[n] and h2[n[ using filter command.
impulse = @(n) (n==0) * 1.0 .* (mod(n,1)==0);

B = 1/3; % Coefficients from x[n] side.
A = [1 1/6 -1/6]; % Coefficients from y[n] side.

h1Filter = filter (B,A,impulse(n)); % h1[n].

D = 1; % Coefficients from x[n] side.
C = [1 0 1/4]; % Coefficients from y[n] side.

```

```
h2Filter = filter (D,C,impulse(n)); % h2[n].

figure (1);

subplot (4,1,1);
stem (n,h1Handcalc(n));
title("Hand-Calculated Graph of h1[n] Unit Impulse Response");

subplot (4,1,2);
stem (n,h1Filter);
title("Graph of h1[n] Unit Impulse Response using Filter command");

subplot (4,1,3);
stem (n,h2Handcalc(n));
title("Hand-Calculated Graph of h2[n] Unit Impulse Response");

subplot (4,1,4);
stem (n,h2Filter);
title("Graph of h2[n] Unit Impulse Response using Filter command");

```

```
% Verify value of h[3] in both cases.
threshold = 1e-10; % Used to compare the difference between the
% hand-calculated and simulated graphs.

h1Check = all (h1Handcalc(n)-h1Filter <= threshold);
h2Check = all (h2Handcalc(n)-h2Filter <= threshold);
```



Command Window

```
>> Lab2ProblemA
>> h1Check

h1Check =

  logical

   1

>> h2Check

h2Check =

  logical

   1
```

A.3) Both of h1Check and h2Check correspond to 1, or "true". Therefore, h[3] is the same in both methods.

Problem B

```
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% Section 05

%Problem B
B = [2]; % Coefficients on x[n] side.
A = [1 -0.3 -0.1]; %Coefficients on y[n] side.
Init = [1 2]; % Initial conditions.
xInitConditions = filtic (B,A,Init); % System's initial conditions.
ZIResponse = filter (B,A,zeros(1,20),xInitConditions); % Zero-input response of system.

figure (1);
stem (ZIResponse);
title ("Zero-Input Response of the System in Problem B");
```

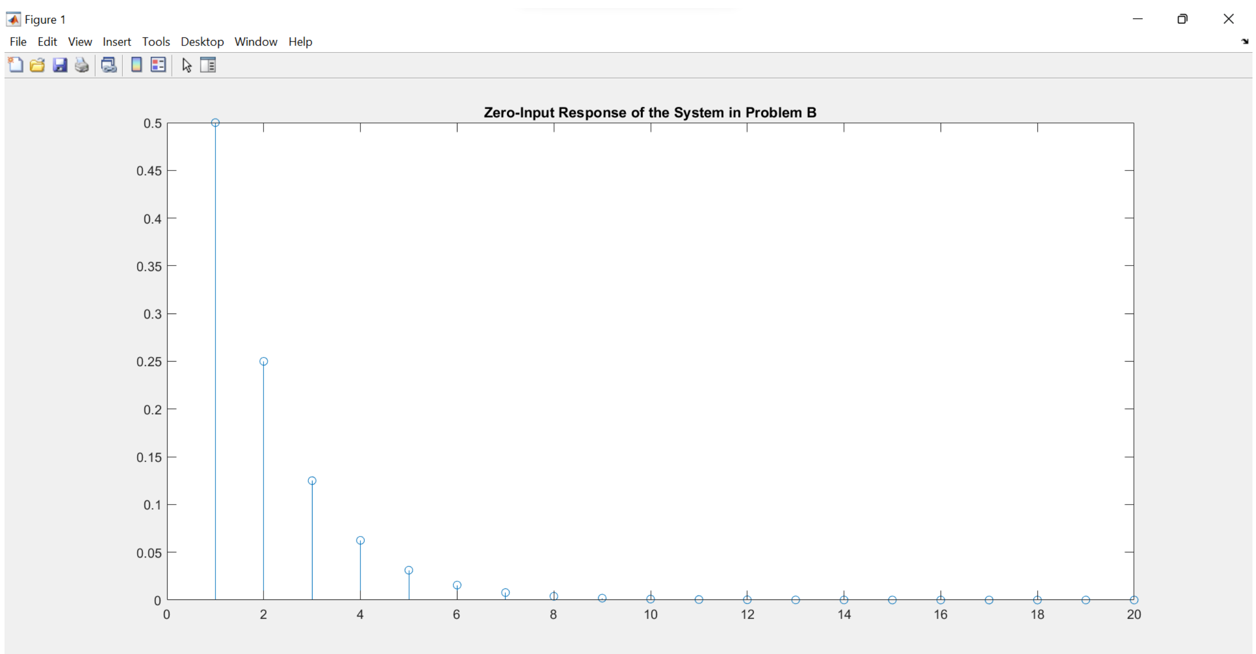

Problem C

```
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% Section 05

%Problem C
n = [-10:20];
u = @(n) (n>=0) * 1.0 .* (mod(n,1)==0);
xofn = @(n) 2*cos ((n*pi)/3).*(u(n)-u(n-10));

B = [2]; % Coefficients on x[n] side. (Difference eq'n of part B)
A = [1 -0.3 -0.1]; %Coefficients on y[n] side. (Difference eq'n of part B)

ZSResponse = filter (B, A, xofn (n)); % Zero-state response.

figure(1);
stem(ZSResponse);
title ("Zero-State Response of the input in Problem C with the system of Problem B");
```
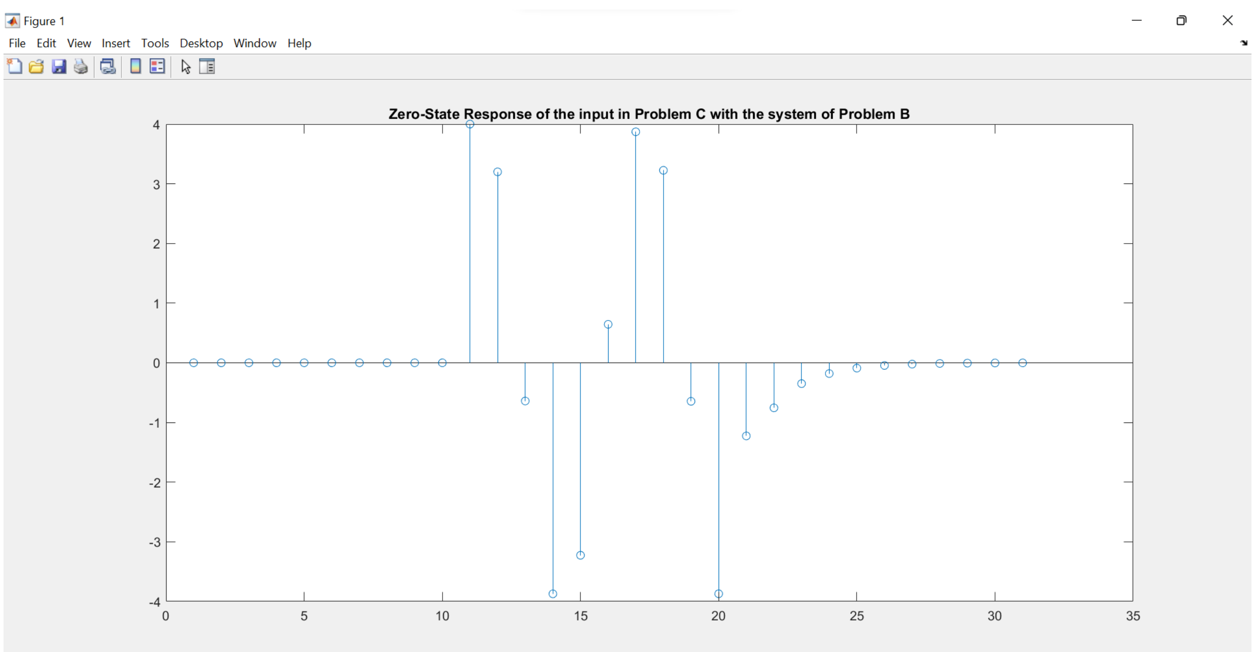
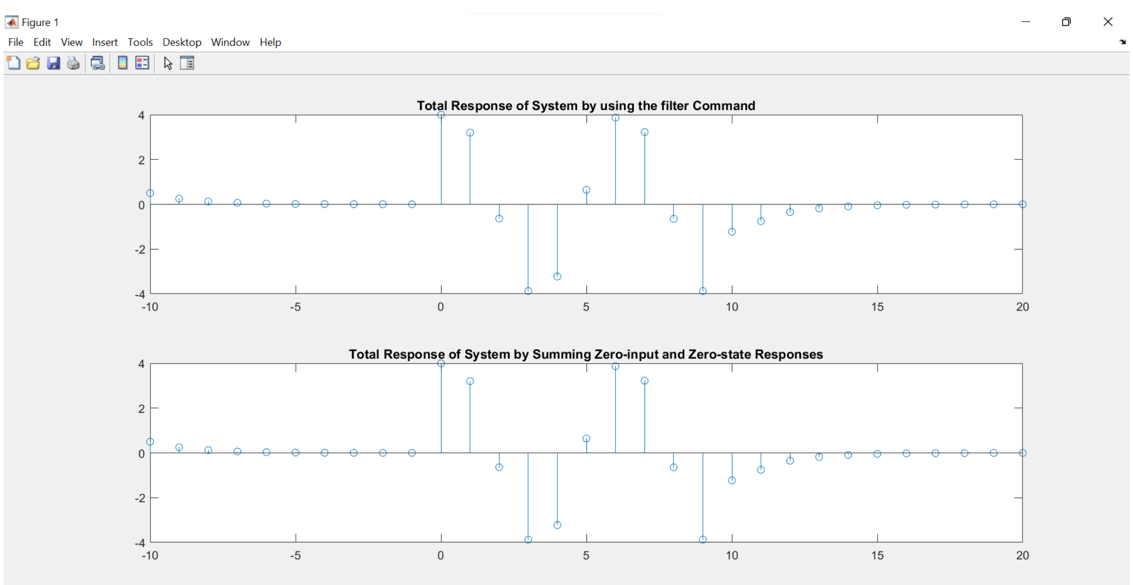



Problem D

```matlab
% Reza Aablue
% 500966944
% Section 05

%Problem D
n = [-10:20];
u = @(n) (n>=0) * 1.0 .* (mod(n,1)==0);
xofn = @(n) 2*cos ((n*pi)/3).*(u(n)-u(n-10));

B = [2]; % Coefficients on x[n] side. (Difference eq'n of part B)
A = [1 -0.3 -0.1]; %Coefficients on y[n] side. (Difference eq'n of part B)
Init = [1 2]; % Initial conditions.

xInitConditions = filtic (B, A, Init); % Find initial conditions of the system.

ZIResponse = filter (B, A, zeros(1,31), xInitConditions); % Zero-input response of system.
ZSResponse = filter (B, A, xofn (n)); % Zero-state response.

totalResponseMethodOne = filter (B,A, xofn(n), xInitConditions);

totalResponseMethodTwo = ZIResponse + ZSResponse; % Finding total response through summing up
                                                  % zero-input and zero-state responses.
```

```matlab
figure (1);

subplot (2,1,1);
stem (n, totalResponseMethodOne);
title ("Total Response of System by using the filter Command");

subplot (2,1,2);
stem (n, totalResponseMethodTwo);
title ("Total Response of System by Summing Zero-input and Zero-state Responses");
```



D.2) It is observed that whether the filter command is used or the zero-input and zero-state responses are added, the total response of the system is the exact same, as shown in the plots of problem D.

Problem E

```
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% 500966944
% Section 05

%Problem E
n = [-10:20];
impulse = @(n) (n==0) * 1.0 .* (mod(n,1)==0);

B = [2]; % Coefficients on x[n] side. (Difference eq'n of part B)
A = [1 -0.3 -0.1]; %Coefficients on y[n] side. (Difference eq'n of part B)
ZIResponse = filter (B, A, impulse(n)); % Zero-input response.

u = @(n) (n>=0) * 1.0 .* (mod(n,1)==0);
xofn = @(n) 2*cos ((n*pi)/3).*(u(n)-u(n-10)); % Input from Problem C.
ZSResponse = filter (B, A, xofn (n)); % Zero-state response.

convolution = conv (xofn(n), ZIResponse);
```
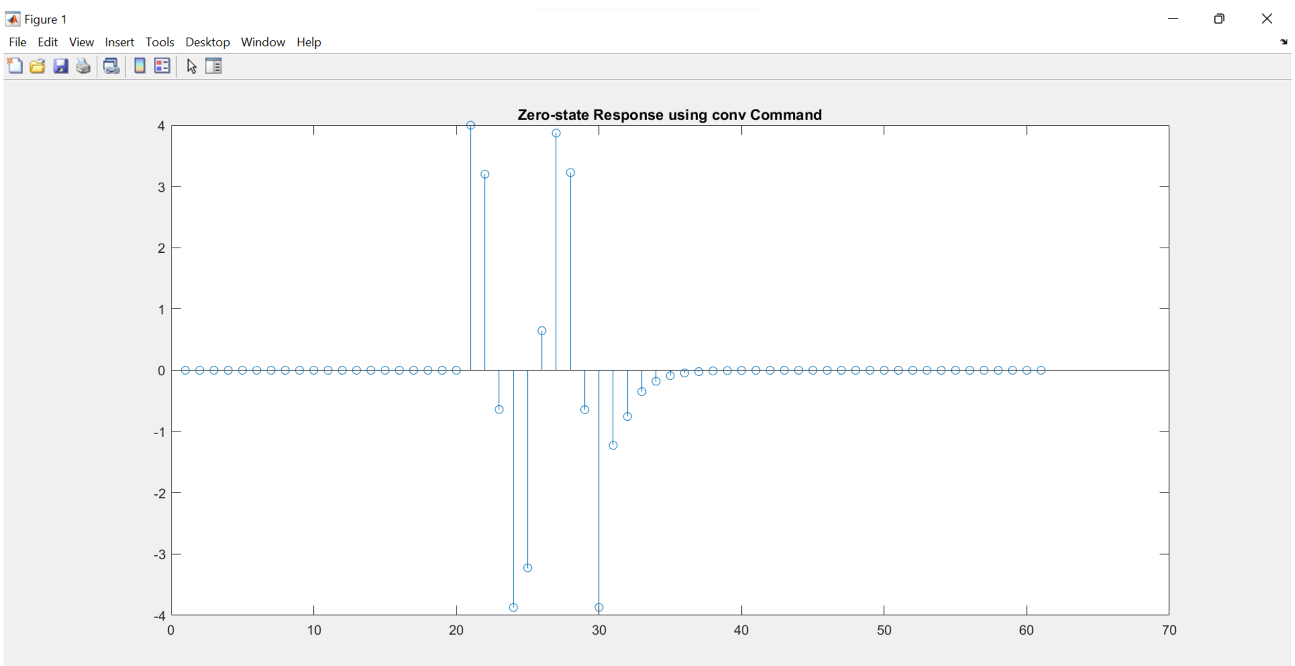

```
figure (1);
stem (convolution);
title ("Zero-state Response using conv Command");
```



E.2) The Zero-state responses of problems C and E are the exact same.

E.3) The system is asymptotically stable, as, with a bounded input, the output (being bounded) converges to zero.

Problem F

F.1) For the impulse response to be h[n], the coefficient for “A”, or y[n] side, is 1, and the coefficients for “B”, or x[n] side, are equal to the N-numbered sum of 1/N.

```
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% Section 05

%Problem F
n = [0:50];
impulse = @(n) (n==0) * 1.0 .* (mod(n,1)==0);
xofn = @(n) cos((n*pi)/5) + impulse (n-30) - impulse (n-35);

figure (1);

subplot(3,1,1);
[a, b] = params (4); % Filter with N=4 case.
stem(n,filter(b, a, xofn (n)));
title ("Filter with N = 4");

subplot(3,1,2);
[a, b] = params (8); % Filter with N=8 case.
stem(n,filter(b, a, xofn (n)));
title ("Filter with N = 8");
```
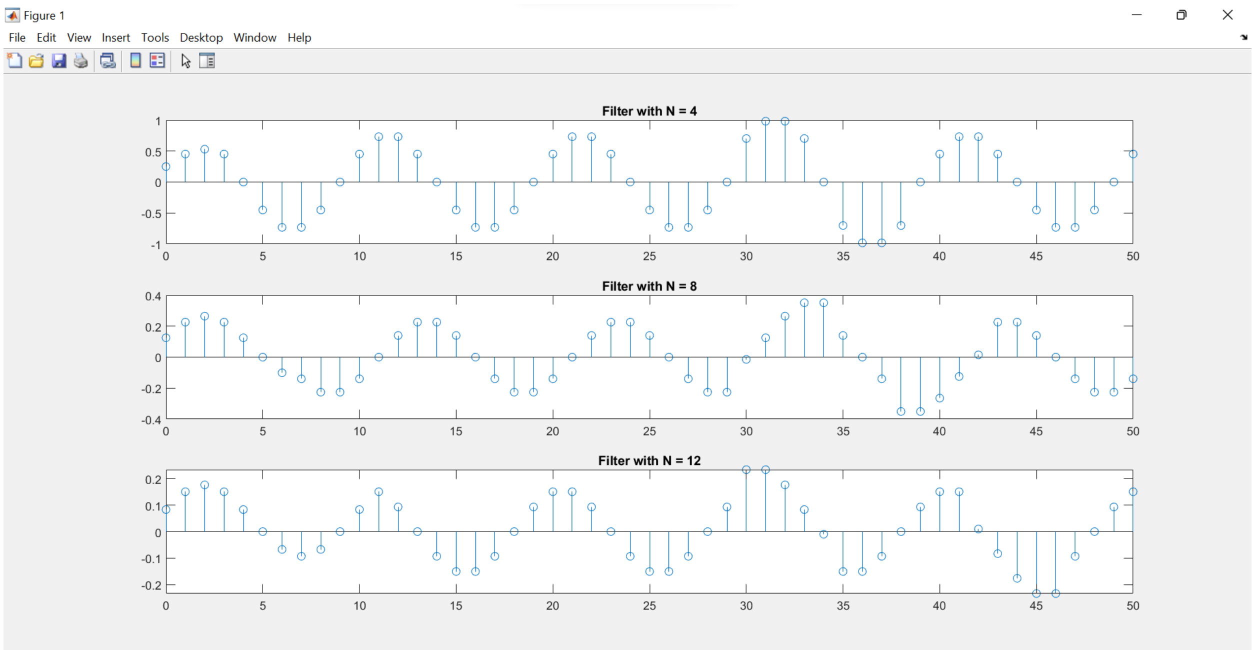

```
subplot(3,1,3);
[a, b] = params (12); % Filter with N=12 case.
stem(n,filter(b, a, xofn (n)));
title ("Filter with N = 12");

function [a,b] = params (N)
    a = 1;
    b = ones(N,1)/N;
end
```



F.3) From the properties of a cosine signal, it is known that its average value is zero. So, in the case of the three filters, as N increases, the filtered signal approaches zero, causing the effect of the impulse on the size of the output to decrease.