RYERSON UNIVERSITY

Faculty of Engineering, Architecture and Science

Department of Electrical and Computer Engineering

Course Number	ELE632
Course Title	Signals and Systems II
Semester/Year	Winter 2022

Instructor	Dr. Dimitri Androutsos

ASSIGNMENT No.	**3**

Assignment Title	Discrete-Time Fourier Series

Submission Date	March 13, 2022
Due Date	March 13, 2022

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By: Reza Aablue
Date: 13/03/2022

Problem A.1

[redacted] $x[n] = 4\cos(2.4n\pi) + 2\sin(3.2n\pi)$ $\qquad \Omega_o = \frac{2\pi}{N_o}$

$\Omega_{01} = 2.4\pi = \frac{12\pi}{5} \times \frac{2\pi}{N_{01}}$ $\qquad \Omega_{02} = 3.2\pi = \frac{16\pi}{5} \times \frac{2\pi}{N_2}$

$12\cancel{\pi}(N_1) = 10\cancel{\pi}$ $\qquad 16\cancel{\pi}(N_2) = 10\cancel{\pi}$

$\underline{N_1 = \frac{5}{6}}$ $\qquad N_2 = \frac{10}{16}$ $\quad \underline{N_2 = \frac{5}{8}}$

$$\frac{N_1}{N_2} = \frac{5}{6} \cdot \left(\frac{5}{8}\right)^{-1} \Rightarrow \frac{\cancel{5}}{6} \cdot \frac{8}{\cancel{5}} \Rightarrow \frac{8}{6} \Rightarrow \frac{4}{3} \in \mathbb{Q} \text{ (rational number)}$$

$\therefore$ Since N_1/N_2 is rational, $x[n]$ is periodic.

$N_o = \text{lcm}(N_1, N_2)$

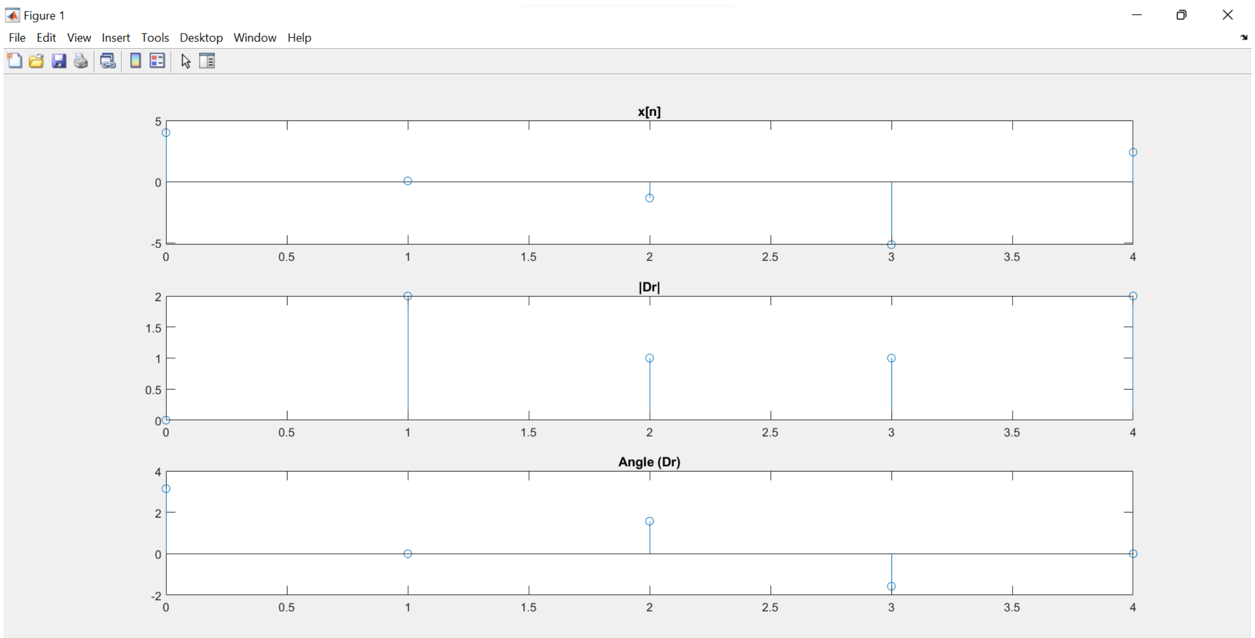
$N_1 = 5^1 6^{-1}$, $N_2 = 5^1 8^{-1}$ $\qquad N_o = 5^1 \cdot \cancel{6}^0 \cdot \cancel{8}^0 \Rightarrow \boxed{N_o = 5}$

$\Omega_o = \frac{2\pi}{N_o} = \frac{2\pi}{5}$ $\quad \boxed{\Omega_o = 0.4\pi}$

Problem A.2

```
% Problem A.2
No = 5;
Wo = 0.4*pi;
n=(0:4); % Range from 0 to (No-1).

x = @(n) (4.*cos(2.4.*pi.*n) + 2.*sin(3.2.*pi.*n));

for r=0:4
    xr(r+1) = sum (x(n).*exp(-1i*r*(0.4*pi)*n))/5;
end

r=n;

figure(1);

subplot (3,1,1);
a=(0:4);
y=x(a);
```

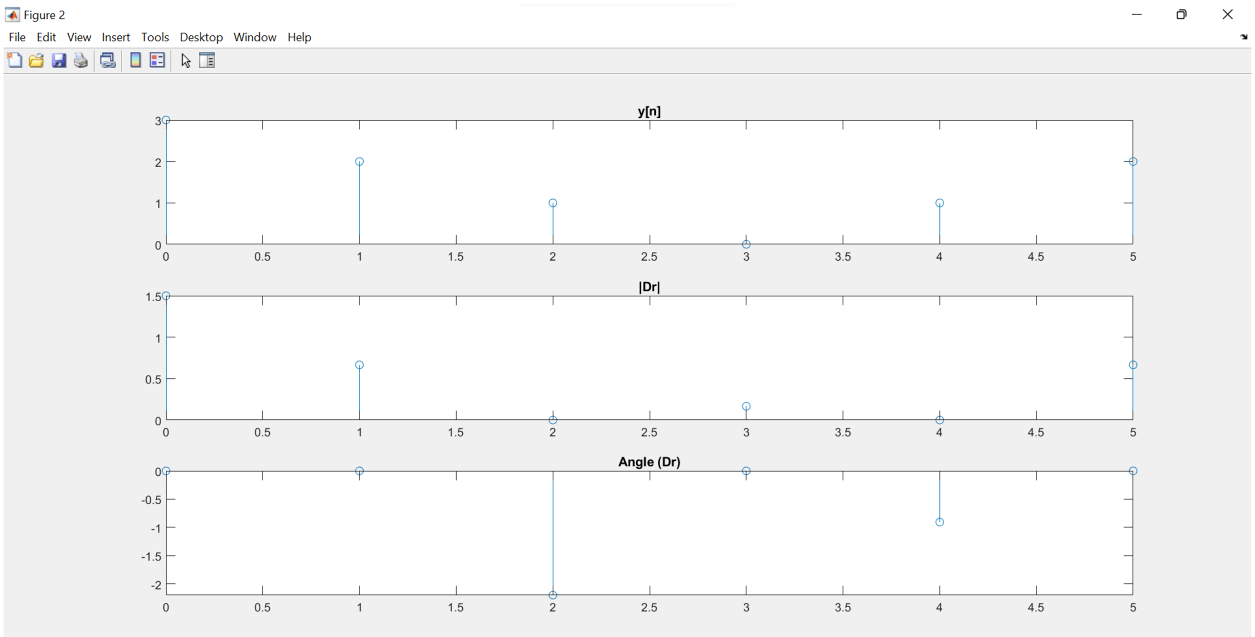
```
stem (a,y);
title ('x[n]');

subplot (3,1,2);
stem (r,abs(xr));
title ('Dr');

subplot (3,1,3);
stem (r,angle(xr));
title ('Angle (Dr)');
```



Problem A.3

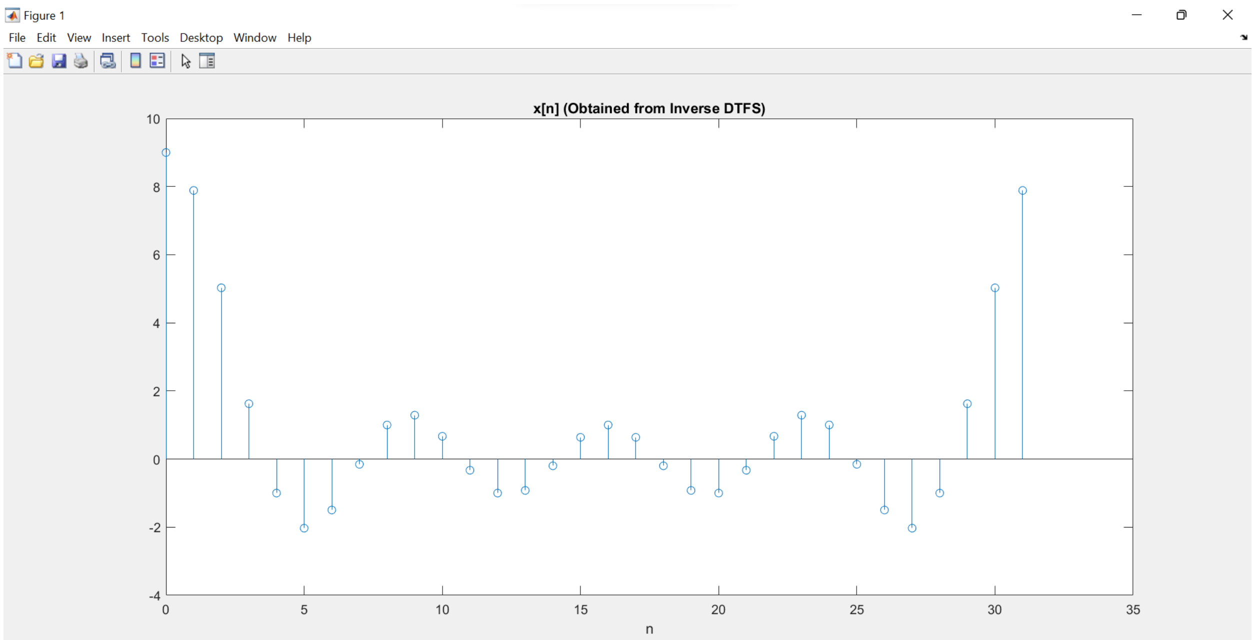
```
% Problem A.3
No1 = 6;
Wo1 = pi/3;
n1 = (0:5);

yofn = [3 2 1 0 1 2]; % All values for y[n] in one period.

for s=0:5
    ys(s+1)=sum(yofn.*exp(-1i*s*(pi/3)*n1))/6;
end

s=n1;

figure(2);
b=(0:5);

subplot (3,1,1);
stem (b,yofn);
title ('y[n]');

subplot (3,1,2);
```

```
stem (s,abs(ys));
title ('|Dr|');

subplot (3,1,3);
stem (s,angle(ys));
title ('Angle (Dr)');
```



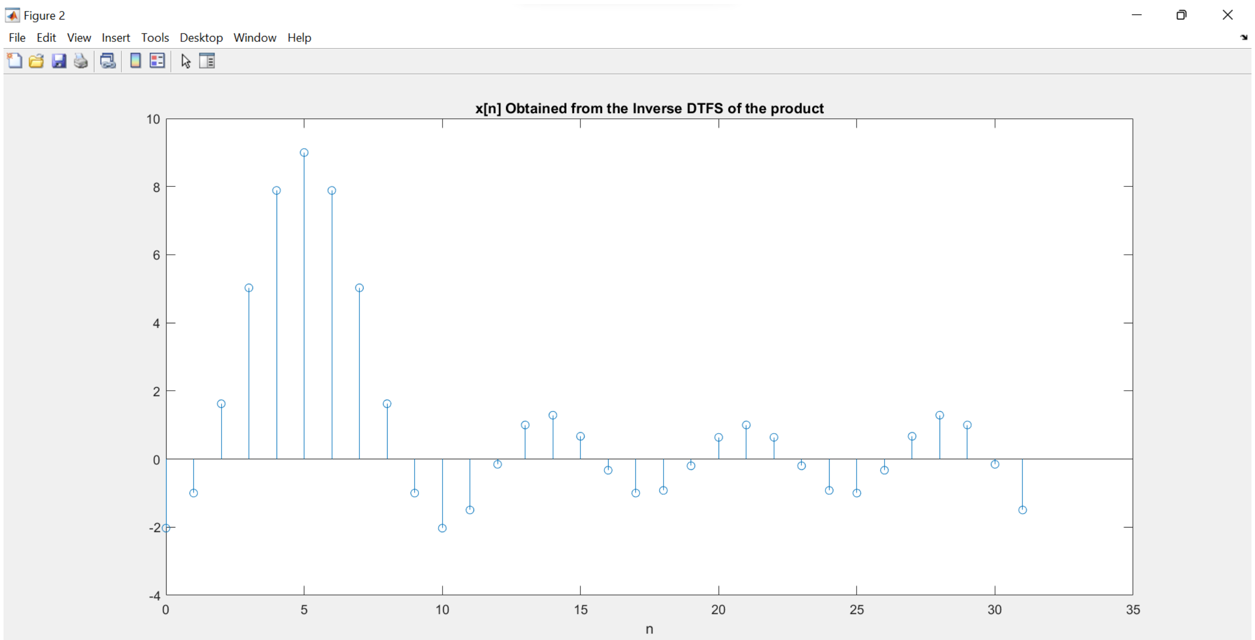
Problem B.1

```
% Problem B.1
No = 32;
Wo = pi/16;
n = (0:31);

Xofr = [ones(1,5) zeros(1,23) ones(1,4)]; % Plotting all points from r=0 up to r=31.

for r = 0:31
    xn(r+1) = sum (Xofr.*exp(1i*n*(pi/16)*r));
end

figure (1);
stem (n, xn);
xlabel ('n');
title ('x[n] (Obtained from Inverse DTFS)');
```



Problem B.2

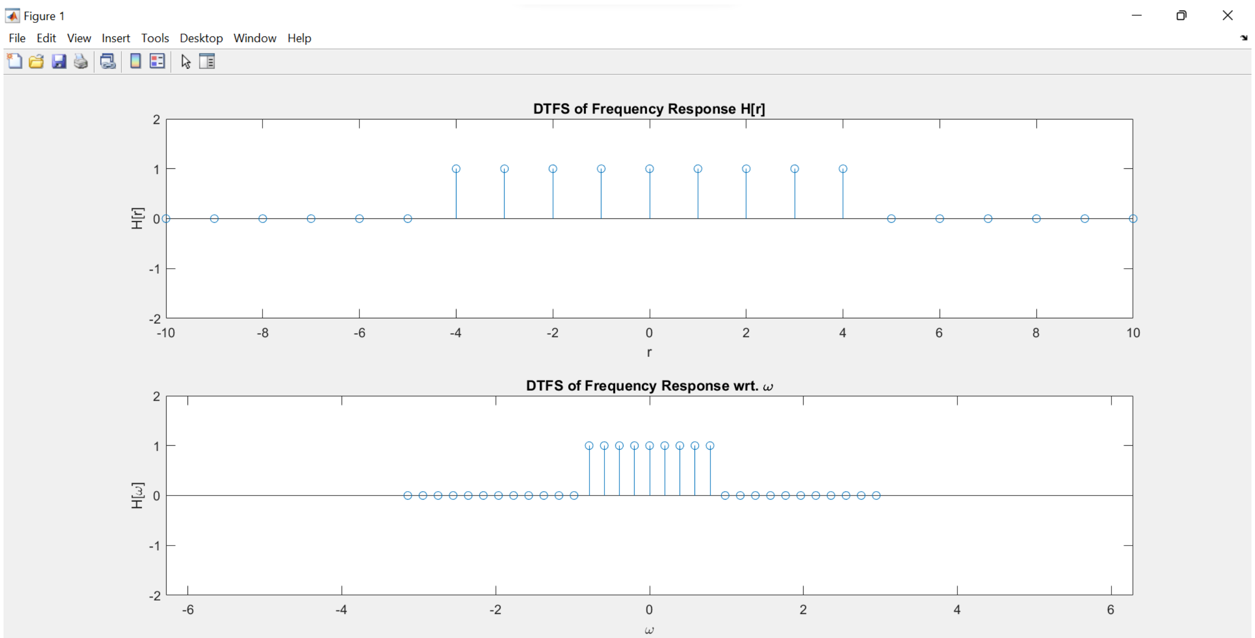
```
% Problem B.2
% No = 32, n = (0:31), and Wo = pi/16.
XofrProduct = [ones(1,5) zeros(1,23) ones(1,4)].*exp(-1i*5*(pi/16)*n);
for r = 0:31
    xn(r+1) = sum (XofrProduct.*exp(1i*n*(pi/16)*r));
end
r=n;

figure (2);
stem (r,xn);
xlabel ('n');
title ('x[n] Obtained from the Inverse DTFS of the product');
```



It is observed that the x[n] graph of part 3 obtained from the inverse DTFS is part 1's x[n] graph shifted by 5 units to the left.

<u>Problem C.1</u>

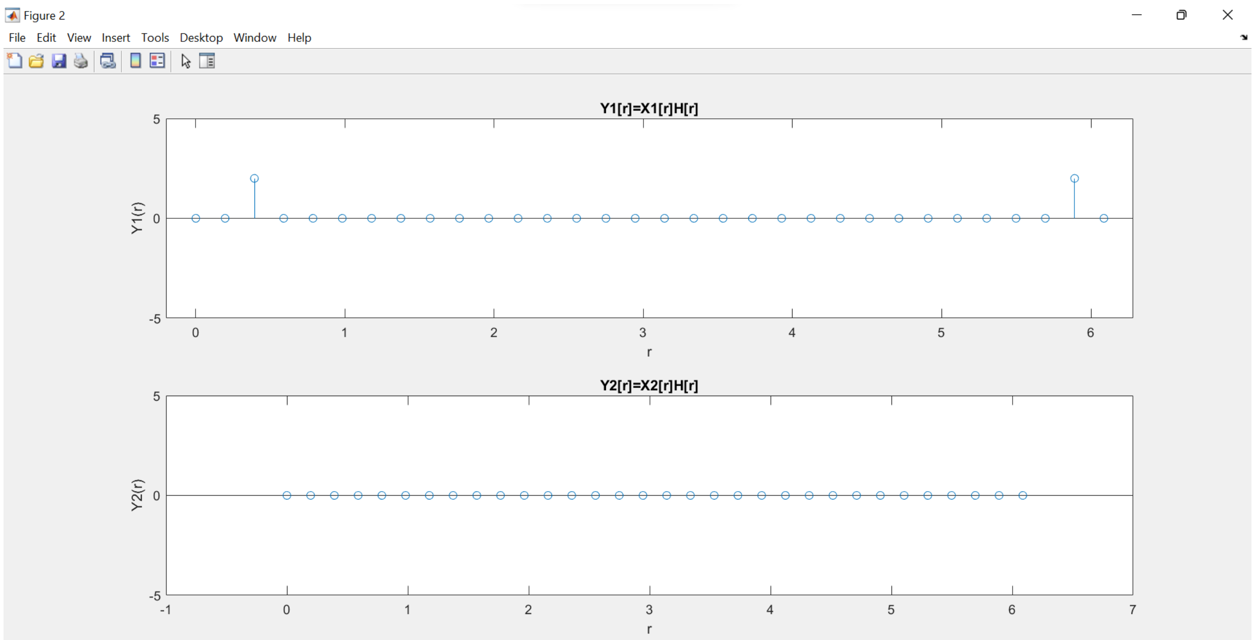
```
% Problem C.1
u = @(n) (n>=0) * 1.0 .* (mod(n,1)==0); % Unit step function.
H = @(n) u(n+4)-u(n-5); % H[r] frequency response.
No = 32;
Wo = (2*pi/No);
n = (-16:15);

figure (1);
subplot(2,1,1);
stem (n,H(n));
xlabel('r');
ylabel('H[r]');
axis ([-10 10 -2 2]);
title ('DTFS of Frequency Response H[r]');

subplot(2,1,2);
stem (n.*Wo, H(n));
xlabel('\omega');
ylabel('H[\omega]');
axis ([-2*pi 2*pi -2 2]);
title ('DTFS of Frequency Response wrt. \omega');
```



Problem C.2 and C.3

```
% Problem C.2 and C.3
% No = 32 and Wo = (2*pi/No).
n1 = (0:31);
H1 = [ones(1,5) zeros(1,23) ones(1,4)]; % Frequency response H[r].
x1 = @(n1) 4.0*cos(n*pi/8); % x1[n] function.
x2 = @(n1) 4.0*cos(n*pi/2); % x2[n] function.

for r = 0:31
    X1(r+1) = sum (x1(n1).*exp(-1i.*r.*Wo.*n1))/No;
    X2(r+1) = sum (x2(n1).*exp(-1i.*r.*Wo.*n1))/No;
end

figure (2);

subplot(2,1,1);
stem (n1.*Wo, H1.*abs(X1));
axis ([-0.2 2*pi -5 5]);
xlabel ('r');
ylabel ('Y1(r)');
title ('Y1[r]=X1[r]H[r]');
```

```
subplot(2,1,2);
stem (n1.*Wo, H1.*abs(X2));
axis ([-1 7 -5 5]);
xlabel ('r');
ylabel ('Y2(r)');
title ('Y2[r]=X2[r]H[r]');
```



Problem C.4) In part 1, H[r] is a lowpass filter that allows signal components (in the range of $-\pi/4$ to $\pi/4$) to be contained and the rest to be filtered out. As both input signals, x1[n] and x2[n], are cosine sinusoidal signals, they both produce impulses at the positive and negative locations of ω_0 (fundamental frequency). This property is known from the DTFT of the cosine function. In part 2, x1[n]'s DTFT impulse components are within the filter's range and are contained in the Y1[r] graph, but for x2[n], as its DTFT impulse components are at $\omega_0 = \pm\ \pi/2$, they are not contained and filtered out (resulting in Y2[n] being zero at all points).