Generalized Far-Field Distance of Antennas and the Concept of Classical Photons

Arthur D. Yaghjian, *Life Fellow, IEEE*

***Abstract*— A generalized far-field (Rayleigh) distance is derived for a frequency-domain or time-domain antenna in terms of the radius of the significant reactive power of the antenna, where this radius is given in terms of the maximum value *N* of the degree number *n* of the spherical waves needed to accurately represent the far-field of the antenna. The maximum possible gain of the antenna is given in terms of the radius of the significant reactive power, which enables supergain to be defined in terms of the physical radius of the antenna. Although the energy in pulses from finite-energy, finite-extent sources, including the energy in "electromagnetic missiles," must eventually decay, it is shown that wavelength-size wavepackets with a well-defined center frequency can remain localized in free space for a limited amount of travel time and distance. These quasi-monochromatic "classical photons" are used, along with Planck's constant, to determine the electromagnetic energy density below which quantum scattering theory, rather than the classical Maxwell equations, may be required to determine electromagnetic scattering.**

***Index Terms*— Antennas, classical photons, far-field distance, quantum scattering, supergain.**

I. Introduction

THE approximate distance at which the $e^{-i\omega t}$ time-harmonic (single frequency ω) fields of an antenna form a far-field angular pattern with e^{ikr}/r radial dependence ($k = \omega/c = 2\pi/\lambda$, with c the speed of light and λ the wavelength), sometimes referred to as the Rayleigh distance, is commonly given as $2D_0^2/\lambda$, where D_0 is the spherical circumscribing diameter of the source region of the antenna, and the radial distance r is measured from the center of the circumscribing sphere [1, sec. 7.7]. However, this is not a generally valid formula, not only because it does not apply if $D_0 \ll \lambda$ but, more importantly, because the sources of any given antenna with nonzero D_0 can be replaced by sources in a smaller region of space that have practically the same fields outside the circumscribing diameter D_0 of the original sources [2], [3]. Thus, this article begins with a derivation using spherical-wave representations to obtain a generalized far-field distance for arbitrary single-frequency antennas with sources confined to a volume of finite extent.

For time-domain antennas, the sources, in principle, need not be bandlimited and, unlike single-frequency far-fields, the time-domain far-fields can be pulses that travel to an infinite radial distance with slower than $1/r$ decay and power decay slower than $1/r^2$ to produce "electromagnetic missiles" if and only if the frequency spectrum of part of the source current decays as $1/|\omega|^{3/2}$ or slower as $|\omega| \to \infty$ [4], [5, sec. 5.4]. Moreover, finite-energy sources within a finite radius of space cannot produce *nondecaying* electromagnetic missiles or any other *nondecaying-energy* wavepacket that remains localized[1] for an indefinitely long time or indefinitely large travel distance.

Nonetheless, proof is given that for limited times and travel distances, classical localized electromagnetic wavepackets can be produced in free space that approximate some of the properties of photons in that the wavepackets have a well-defined center frequency and a nearly constant energy concentrated in a cubic volume approximately one center-frequency wavelength on a side.

This article concludes by using this classical model of the photon, along with Planck's constant, to determine the smallest average energy density that assures a monochromatic electromagnetic field can be treated classically with Maxwell's equations rather than with quantum scattering theory—a result that previously had been obtained only from quantum electrodynamics.

II. Far-Field Distance of Antennas

Consider single-frequency antennas with sources located within a circumscribing sphere of finite radius $a_0 = D_0/2$, so that there is free space for the values of $r > a_0$, where r is the radial distance from the center of the circumscribing sphere. In this infinite free-space region defined by $r > a_0$, assume that the time-harmonic electric and magnetic fields satisfy the homogeneous vector wave equation along with the outgoing radiation condition as $r \to \infty$, and thus, they can be represented in a complete set of outgoing vector spherical waves [6, ch. 7], [7, sec. 9.7], [8, th. 16]. For a scalar (acoustic) field that satisfies the homogeneous Helmholtz equation, the complete set of spherical waves reduces to

$$E(r,\theta,\phi) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n} c_{nm} h_n^{(1)}(kr) P_n^m(\cos\theta) e^{im\phi}, \quad r > a_0 \tag{1}$$

where the $P_n^m(\cos\theta)$ are associated Legendre polynomials of degree n and order m and the $h_n^{(1)}(kr)$ are the nth-order

[1] The term "localized" is used to mean that the energy of the wavepacket is approximately constant and resides predominantly in a volume of fixed finite dimensions.

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The author resides in Concord, MA 01742 USA (e-mail: a.yaghjian@comcast.net).

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spherical Hankel functions of the first kind. The (r, θ, ϕ) are the usual spherical coordinates. The constant spherical wave coefficients c_{nm} define the fields of the antenna for $r > a_0$. We have assumed a scalar field $E(r, \theta, \phi)$, because it simplifies the algebra, and the results obtained for the far-field distance are the same as for the full vector electromagnetic fields. The n summation in (1) begins at $n = 1$ rather than $n = 0$, because electromagnetic antennas, unlike acoustic radiators, have no monopoles. For $r \leq a_0$, the double summation in (1) may or may not converge, depending on the asymptotic values of the coefficients c_{nm}.

The Hankel function $h_n^{(1)}(kr)$ rapidly becomes extremely large as n gets larger than kr, namely, as [9, eqs. (10.19.2) and (10.47.5)]

$$h_n^{(1)}(kr) \overset{n > kr}{\sim} -\sqrt{\frac{2}{e}}\,\frac{i}{kr}\left[\frac{2n+1}{ekr}\right]^n \tag{2}$$

where $e = 2.718, \ldots$ is Euler's number. For vector electric and magnetic fields $(\mathbf{E}, \mathbf{H})$ of antennas (as well as the scalar fields of acoustic radiators) having spherical waves with n significantly larger than kr, this implies a near-field region of the nth spherical waves ($m = \pm 1, \pm 2, \ldots, \pm n$) of extremely high electric and magnetic fields whose complex Poynting vector is predominantly imaginary (reactive) [10]. The integration of the real part of the complex Poynting vector dotted into the normal of a surface enclosing the sources of an antenna determines the power being radiated by the antenna and, thus, its input radiation resistance. The same integration of the imaginary part of the complex Poynting vector contributes to the antenna's input reactance [11]. Specifically, we have

$$\frac{1}{2}\int_S (\mathbf{E}\times\mathbf{H}^*)\cdot\hat{\mathbf{n}}\,dS = P_{\text{rad}} - \frac{i\omega}{2}\int_{V_{\text{out}}} \left(\mu_0|\mathbf{H}|^2 - \epsilon_0|\mathbf{E}|^2\right)dV \tag{3}$$

where $\hat{\mathbf{n}}$ is the outward unit normal to the surface S enclosing the sources of the antenna, V_{out} is the volume outside S, μ_0 and ϵ_0 are the permeability and permittivity of free space, and * denotes the complex conjugate. The real part of (3) (P_{rad}) is the average radiated power that determines the input radiation resistance of the antenna. The imaginary part of (3) is the reactive power outside S that contributes to the input reactance of the antenna.[2]

Although $\mathbf{E}$ or $\mathbf{H}$ by itself (without its spatial derivatives) does not divide in the near field into radiative and reactive parts, unit-area vector radiative and reactive powers of the fields can be defined at every point in free space in terms of the complex Poynting vector, namely, as the real and imaginary parts, respectively, of

$$\frac{1}{2}\mathbf{E}\times\mathbf{H}^* = \frac{i}{2\omega\mu_0}\mathbf{E}\times(\nabla\times\mathbf{E}^*) = \frac{i}{2\omega\epsilon_0}(\nabla\times\mathbf{H})\times\mathbf{H}^*. \tag{4}$$

Arguably, one should refer to the imaginary part of (4) as the "reactance power" rather than the "reactive power" so as not to confuse it with the antenna's stored or Q-energy that can be written as the sum of the electric and magnetic energies of the quasistatic fields of electrically small antennas [12] or, in general, as the energy in the total fields with the radiated fields subtracted [11], [13]. Nevertheless, we will conform to the IEEE nomenclature for reactive power and refer to the imaginary part of (4) as reactive power [14]. In the far-field, the reactive power goes to zero leaving only the radiative power. The region close to the antenna, where the reactive power can dominate over the radiative power, is commonly called the reactive near-field zone. For most antennas, the near-field zone where the unit-area reactive power can dominate over the unit-area radiative power is confined to a distance of a few $\lambda/(2\pi)$ from the physical circumscribing radius a_0 of the sources of the antenna. The region between the reactive near-field zone and the far-field is commonly called the radiative near-field zone [15].

Although most antennas have negligible reactive power beyond a wavelength from their circumscribing spheres, we keep open the possibility of highly reactive antennas (such as superdirective antennas [16] and electrically small dielectric resonator antennas with higher order multipole resonances) where the extent of the reactive power can be significantly larger [2]. Theoretically, the radius a of the reactive near-field zone can have an arbitrarily large finite value compared with the physical radius a_0 of the sphere circumscribing the sources.

A. Truncation of Spherical Wave Summation

Beyond some finite value of n in the spherical wave expansion (1), the magnitudes of the spherical wave coefficients $|c_{nm}|$ will decay extremely rapidly with increasing n. To prove this, we use (1) to obtain the following summation for the scalar-field complex power integrated over any sphere of radius r in free space (see Footnote 2):

$$\frac{1}{2}\int_S iE\,\nabla E^*\cdot\hat{\mathbf{n}}\,dS = \frac{ikr^2}{2}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}|c_{nm}|^2 h_n^{(1)}(kr)h_n^{(1)\prime *}(kr), \quad r > a_0 \tag{5}$$

where the normalization of the spherical harmonics has been chosen in accordance with Jackson [7, p. 108] to make the orthonormality integrals equal to unity. The $'$ denotes the derivative with respect to the argument of the spherical Hankel function. (The electromagnetic case using Poynting's vector in (3) goes through in the same way by considering the electric and magnetic multipole field summations separately.)

The asymptotic expansion of the spherical Hankel-function product in (5) for order n larger than the argument kr can be found from (2) to be

$$h_n^{(1)}(kr)h_n^{(1)\prime *}(kr) \overset{n > kr}{\sim} -\frac{2(n+1)}{e(kr)^3}\left[\frac{2n+1}{ekr}\right]^{2n}. \tag{6}$$

Insertion from (6) into (5) reveals that, in order for the summation in (5) to converge for any given value of $kr > ka_0$, the coefficients $|c_{nm}|^2$ have to rapidly decrease in value, namely, faster than $O[((2n+1)/ekr)^{-(2n+1)}]$ for $n > kr$. In particular, there is a finite $\mathcal{N}$ as $kr \to ka_0$ ($r > a_0$), such

[2] The scalar (acoustic) complex power integral corresponding to the complex Poynting vector integral on the left-hand side of (3) is proportional to $(1/2)\int_S iE\,\nabla E^*\cdot\hat{\mathbf{n}}\,dS$ [5, sec. 2.4.8].

that the $|c_{nm}|^2$ in (5) become extremely small for $n > \mathcal{N}$ and all $m = \pm 1, \pm 2, \ldots, \pm \mathcal{N}$; specifically,

$$|c_{nm}|^2 \overset{n>\mathcal{N}}{\sim} o\left[\left(\frac{2n+1}{eka_0^+}\right)^{-(2n+1)}\right] \tag{7}$$

where $o[\ ]$ is the usual mathematical notation for "order of growth less than the function in the bracket" and a_0^+ denotes a value approaching a_0 but always larger than a_0.

With $|c_{nm}|$ decreasing so rapidly for $n > \mathcal{N}$, where $\mathcal{N}$ is a finite integer, the far-field can be written from (1) as follows:

$$E(r,\theta,\phi) \overset{kr\to\infty}{\sim} \frac{e^{ikr}}{ikr} \sum_{n=1}^{\mathcal{N}} \sum_{m=-n}^{n} i^{-n} c_{nm} P_n^m(\cos\theta) e^{im\phi} \tag{8}$$

where use has been made of the asymptotic form of the spherical Hankel functions as $kr \to \infty$ [6, p. 406]. To determine the minimum value of $\mathcal{N}$ allowable in (8), assume we know or can estimate the highest value N of n required to accurately expand the far-field of the antenna. For example, assume it is known that the narrowest far-field lobes of the antenna have a null-to-null beamwidth of $\alpha_{\min}$. Then, because the spherical harmonics $P_n^m(\cos\theta)e^{im\phi}$ contain the n and $|m| \le n$ sinusoidal functions $\substack{\sin \\ \cos} n\theta$ and $\substack{\sin \\ \cos} m\phi$, respectively, the highest value of n required to expand the far-field (which, incidentally, is an analytic function of complex θ and ϕ [5, sec. 3.2.4]) will be about $N \approx \pi/\alpha_{\min}$. In any case, the required far-field N is assumed known (determined from a given, computed, or measured far-field) and, thus, can replace $\mathcal{N}$ in (8). That is,

$$E(r,\theta,\phi) \overset{kr\to\infty}{\sim} \frac{e^{ikr}}{ikr} \sum_{n=1}^{N} \sum_{m=-n}^{n} i^{-n} c_{nm} P_n^m(\cos\theta) e^{im\phi} \tag{9}$$

where *it is assumed that we have a reasonably good estimate of the maximum value N of n for the spherical waves to accurately represent the far-fields of the antenna, such that $|c_{nm}|$ rapidly decays for $n > N$.*[3] Below, it is shown that the extent of the reactive power and the far-field distance of the antenna will be proportional to $(N+1/2)$ and N^2, respectively.

Next, replace the asymptotic form of the spherical Hankel function in (9) with the original spherical Hankel function in (1). Because we now know from the far-fields that the $|c_{nm}|$ rapidly becomes very small for $n > N$, the resulting equation will hold for all kr until kr becomes less than about N, and the spherical Hankel functions grow extremely large, so that the reactive power begins to dominate; see (2) and (6). Letting this value of r, below which the spherical Hankel functions become extremely large and the reactive power starts to dominate, be denoted by a, such that $ka \approx N$ or $a \approx N/k$, we have

$$E(r,\theta,\phi) \approx \sum_{n=1}^{N} \sum_{m=-n}^{n} c_{nm} h_n^{(1)}(kr) P_n^m(\cos\theta) e^{im\phi}, \quad r > a \tag{10}$$

where a is the radius less than which the reactive power of the finite summation of spherical waves starts to dominate the radiative power; specifically,

$$a \approx (N + 1/2)/k, \quad N \ge 1 \tag{11}$$

where the $1/2$ in (11) is included to ensure that for electrically small dipoles ($N = 1$), the significant reactive fields extend a little further than $\lambda/(2\pi)$. For ordinary (nonsuper-reactive) antennas, such as reflector or horn antennas, with source radii a_0 (for example, the radius of the sphere that circumscribes the aperture of a horn or reflector antenna), one finds from [17, sec. 3.4.1] and [18, sec. 2.2.3] that truncation of the spherical wave summation index n to

$$N_0 \approx \mathrm{Int}\Big[ka_0 + [\ln(1/\varepsilon)]^{\frac{2}{3}}(ka_0)^{\frac{1}{3}} + 1\Big] \tag{12}$$

gives a relative error in energy density outside the reactive zone of about ε, where Int[ ] denotes the nearest integer.

Note that if the antenna of source radius a_0 is oversized in the sense that N in (10) is $< ka_0$, *then a can be less than the radius a_0 of the sources of the antenna*. In other words, the original sources extending to the radius a_0 can be replaced by sources in a smaller radius, such that a in (10) can be less than a_0.[4] For the usual case of $a > a_0$, the truncated summation in (10) is generally not accurate in the reactive zone $a_0 < r < a$. That is, the truncated summation gives reduced reactive power in this annulus, because the spherical Hankel functions become so large in this annulus that some of the neglected c_{nm} for $n > N$ can still give significant reactive power. For $r > a$, the fields are radiative and, thus, can be expanded in the same number of spherical waves as the far-field. Equations (11) and (12) show that far-fields expandable with a minimum degree number N of spherical waves with respect to an origin O can be produced by ordinary (nonsuper-reactive) sources within a radius a_0 from O equal to or greater than about $a - 3\lambda/(4\pi)$, where a is given in terms of N by (11).

1) Maximum Gain and Supergain: As an aside, we mention that Harrington [19, sec. 6–13] has proven that for an antenna with $n = N$ electric and magnetic vector spherical waves, the maximum possible gain of the antenna is

$$G = N(N+2), \quad N \ge 1. \tag{13}$$

For electric or magnetic spherical waves alone, the maximum possible gain is half the value in (13). Thus, the gain of an antenna with its sources contained within a finite radius a_0 can, in principle, be made arbitrarily large, provided that spherical

[3]The maximum degree number N of spherical waves required to expand the far-fields of an antenna depends on the choice of the origin of the coordinate system in which the far-fields are expanded. Thus, if we want to determine approximately the smallest radius of the significant reactive power, the center of the circumscribing sphere of source radius a_0 must be close to the origin that requires the smallest value of the degree number N of spherical waves to expand the far-fields. If it is not, so that another origin requires a smaller value of N to expand the far-fields, this origin can be chosen, and a_0 will then be the radius of the smallest sphere that is centered at this origin and encloses the sources of the antenna.

[4]For example, consider the hypothetical antenna with equivalent electric and magnetic surface currents of an electric or magnetic dipole ($N = 1$) on a sphere of radius $a_0 = 100\lambda$. The fields in the region $r > a_0$ are exactly the fields of the dipole yet the same fields in the region $r > a_0$ could be produced by an infinitesimal dipole at $r = 0$ with the radius a of the reactive fields given from (11) as $a = 3\lambda/(4\pi) \ll a_0 = 100\lambda$.

waves are allowed with arbitrarily large $n = N$. The result in (11) implies that such $G = N(N+2)$ high-gain antennas will also have high reactive power extending to a distance $a \approx N/k$, so that as the gain approaches an infinite value, high reactive power will extend to an infinite distance ($a \to \infty$) from the center of the antenna whose sources are located within the finite radius $r = a_0$. (The reactive power and quality factor Q of this antenna will also approach ∞ as $N \to \infty$.)

The maximum gain formula in (13) can be written in terms of ka, the electrical size of the reactive power, by substituting $N = ka - 1/2$ from (11) into (13) to give

$$G \approx (ka - 1/2)(ka + 3/2) = (ka)^2 + ka - 3/4, \quad ka \geq 3/2 \tag{14}$$

which implies that $G \geq 3$, where $G = 3$ is the maximum gain of electric plus magnetic dipoles ($N = 1, ka = 3/2$).

Ordinary (nonsuper-reactive) antennas can be defined from (11) and (12) as having large reactive fields extending no further than a radius equal to about $a = a_0 + 3\lambda/(4\pi)$. Consequently, the maximum gain G_0 of ordinary electric plus magnetic multipole antennas is given from (13) as follows:

$$G_0 = (ka_0 + 1)(ka_0 + 3) = (ka_0)^2 + 4ka_0 + 3, ka_0 \geq 0. \tag{15}$$

Thus, it may seem reasonable to define a supergain antenna as having a gain

$$G > G_0 = (ka_0)^2 + 4ka_0 + 3, ka_0 \geq 0. \tag{16}$$

However, it is found in engineering practice that it is very challenging to obtain gains greater than that of a maximum-gain Huygens source (elementary electric and magnetic dipoles with a gain of 3) for an antenna with electrical size $ka_0 \lesssim 1$ [16]. Therefore, it is more realistic to define supergain as follows:

$$G > \begin{cases} 3 + ka_0, & ka_0 \leq 1 \\ (ka_0)^2 + ka_0 + 2, & ka_0 \geq 1 \end{cases} \tag{17}$$

which is close to the supergain criterion, $G > (ka_0)^2 + 3$, proposed by Kildal and Best [20]. For electrically large antennas ($ka_0 \gg 1$), the supergain criterion in (17) gives

$$G \gtrsim (ka_0)^2. \tag{18}$$

Predicted and measured supergains of about $G = 5$ (7 dB) were first obtained by Yaghjian et al. [16], [21], [22], and later by Lim and Ling [23], for two-element supergain endfire arrays that fit inside free-space spheres with electrical sizes ka_0 between 0.5 and 1.0.

B. Frequency-Domain Far-Fields

Performing the summation over m, the spherical wave expansion in (10) can be recast as a single summation

$$E(r,\theta,\phi) \approx \sum_{n=1}^{N} c_n(\theta,\phi) h_n^{(1)}(kr), \quad r > a. \tag{19}$$

As a means of finding the radial distance beyond which the E-field of the antenna in (19) exhibits far-field behavior, that is, varies predominantly as $F(\theta,\phi)e^{ikr}/r$, express $h_n^{(1)}(kr)$ in a large argument asymptotic expansion, namely [6, sec. 7.4]

$$h_n^{(1)}(kr) \overset{kr\to\infty}{\sim} \frac{e^{ikr}}{i^{n+1}kr} \left[1 - \frac{n(n^2-1)(n+2)}{8(kr)^2} + \frac{in(n+1)}{2kr} + O\left(\frac{1}{(kr)^3}\right)\right]. \tag{20}$$

The terms that are higher order than 1 in the square brackets of (20) are negligible if

$$\left[1 - \frac{n(n^2-1)(n+2)}{8(kr)^2} + \frac{in(n+1)}{2kr}\right] \approx 1 \tag{21}$$

or $n^2/(2kr) \ll 1$. If this inequality holds for the largest value of n equal to $N \geq 1$, it will hold, for $n < N$ to an increasingly better approximation, the smaller n than N. Thus, it follows that the inequality $N^2/(2kr) \ll 1$, or for a phase error of approximately $\pi/8$ in the square brackets of (21), the inequality

$$\frac{N^2}{2kr} \lesssim \frac{\pi}{8} \tag{22}$$

which corresponds to the phase error usually chosen to define the conventional Rayleigh distance [1, sec. 7.7], will assure that all the terms higher order than 1 in the square brackets of (20) can be ignored. Since $N \approx ka$, we have

$$r \gtrsim \frac{8a^2}{\lambda} = \frac{2D^2}{\lambda} = R \tag{23}$$

with $D = 2a$. Then, the far-field is given by

$$E(r,\theta,\phi) \approx F(\theta,\phi)\frac{e^{ikr}}{r}, \quad r \gtrsim R \tag{24}$$

where the complex far-field pattern is found from (19) as follows:

$$F(\theta,\phi) \approx \frac{1}{k}\sum_{n=1}^{N}(-i)^{(n+1)}c_n(\theta,\phi). \tag{25}$$

In a null of the far-field pattern ($F(\theta,\phi) = 0$), the fields decay as $O(1/r^2)$. A criterion similar to (22) was obtained in [18, sec. 2.2.3] but for N_0 given in (12) rather than for a general N that determines the far-fields.

The distance

$$R = \frac{8a^2}{\lambda} = \frac{2D^2}{\lambda} \approx \frac{N^2\lambda}{5} \tag{26}$$

is a generalized "Rayleigh distance" that applies to all single-frequency (narrow band in practice) antennas with N-degree significant spherical waves in their far-fields and, thus, significant reactive power extending to the radius a given in (11). *It is noteworthy that in regard to determining its far-field (generalized Rayleigh) distance, the effective radius of an antenna is simply the radius to which the significant reactive power of the antenna extends.* This curious, quite remarkable result was apparently first obtained and noted in the context of maximum effective area of resonant antennas and maximum possible total scattering cross section of resonant scatterers [2].

For unbounded aperture radiators, such as nondiffracting (Bessel, Airy, and so on) beams [24], [25], [26], [6, secs. 6.6–6.7], neither N nor the radiated energy is bounded, that is,

$(a_0, a) \to \infty$, and the Rayleigh distance approaches infinity, such that the $1/r$ far-field decay is never reached. *However, if these beam apertures are truncated to a finite radius, their far-field distance is still given by (26) with $N = N_0$ in (12).*

C. Time-Domain Far-Fields

The expressions derived so far have been for a single frequency. If this frequency dependence is made explicit, the far-field expression in (24) becomes

$$E_\omega(r,\theta,\phi) \approx F_\omega(\theta,\phi)\frac{e^{ikr}}{r}, \quad r \gtrsim R_\omega = 8\ a_\omega^2/\lambda = 2D_\omega^2/\lambda \tag{27}$$

where for antennas with finite source radii a_0, it is assumed that $N_\omega = ka_\omega$ is bounded, and thus, a_ω is bounded for $\omega \neq 0$. Taking the Fourier transform of (27) gives the time-domain far-field

$$E(r,\theta,\phi,t) \approx \frac{F(\theta,\phi,t-r/c)}{r} \tag{28}$$

assuming the existence of the infinite integral

$$F(\theta,\phi,t) = \int_{-\infty}^{+\infty} F_\omega(\theta,\phi)e^{-i\omega t}d\omega. \tag{29}$$

For all known sources of fields (available generators of voltages and currents), the frequency spectrum is effectively bandlimited; that is, the generator produces signals that are finite and that rapidly go below the noise levels for $|\omega| > \omega_g$. Also, actual antennas are finite in size and radiate negligible power below some nonzero frequency, $|\omega| < \omega_{\rm a}$. Thus, the Fourier transform in (29) can be rewritten with finite limits as follows:

$$F(\theta,\phi,t) \approx \int_{-\omega_g}^{+\omega_g} F_\omega(\theta,\phi)e^{-i\omega t}d\omega, \quad r \gtrsim R_{\max} \tag{30}$$

where $R_{\max} = \max[8a_\omega^2/\lambda]$ with $\omega_{\rm a} \le |\omega| \le \omega_g$, and thus, $R_{\max}$ is bounded for N_ω bounded. Consequently, the time-domain far-field pattern $F(\theta,\phi,t)$ exists, such that the radiated time-domain far-fields decay as $1/r$ for $r > R_{\max}$. *In other words, all effectively bandlimited sources with their significant reactive power located in a volume of finite extent (N_ω bounded)—such as all known sources—have far-fields that cannot decay slower than $1/r$.*

In theory, one can consider far-field frequency-domain patterns that are not effectively bandlimited, such that in a certain direction (θ_0,ϕ_0), they decay slower with frequency than $1/|\omega|$ as $|\omega| \to \infty$ [5, p. 239], so that the value of the time-domain far-field in that direction for some time t is infinite

$$F(\theta_0,\phi_0,t) = \int_{-\infty}^{+\infty} F_\omega(\theta_0,\phi_0)e^{-i\omega t}d\omega = \infty. \tag{31}$$

That is, nonbandlimited time-domain far-fields can decay slower than $1/r$ for some time t and in some (θ_0,ϕ_0) direction. Indeed, it is required [5, sec. 5.4] that antennas have time-domain current densities $\mathbf{J}(\mathbf{r}',t)$ that have an infinite time derivative at some t and frequency-domain dependence at some $\mathbf{r}'$ that decays slower than $1/\omega^2$ as $|\omega| \to \infty$ for slower than $1/r$ decaying far-fields in some direction. Moreover, it is also shown [5, sec. 5.4] that "electromagnetic missiles" [4], pulses with radiated far-field energy decaying slower than $1/r^2$, require that the frequency-domain current density at some $\mathbf{r}'$ decays as $1/|\omega|^{3/2}$ or slower as $|\omega| \to \infty$. For example, surface current density with unit-step time dependence on a circular disk of radius a_0 radiates a boresight pulse to infinite r with diminishing pulselength equal to $a_0^2/(2r)$ and total *energy* in this electromagnetic missile that decays as $1/r$. The transverse dimensions of the significant energy in the pulse are on the order of a_0, and the frequency spectrum of the far-field pattern in the boresight direction approaches a nonzero constant as $|\omega| \to \infty$ [5, sec. 5.4.3]. Nevertheless, no sources with such slow frequency decay as $|\omega| \to \infty$ have ever been found, and electromagnetic missiles or other slower than $1/r^2$ energy decaying far-field pulses appear to be academic, impractical curiosities. Of course, the far-field (Rayleigh) distance becomes larger with increasing effective bandwidth ω_g of the generator and increasing reactive-power radius $a_g = N_g/k_g$ of the antenna.

III. Classical Photons

In theory, a pulse can be sent to infinity through a finite area. For example, a uniform current source with delta-function $\delta(t)$ time dependence on a circular disk radiates two nondecaying delta-function pulses to an indefinitely large distance along the boresight axis normal to the center of the disk. Moreover, the general expressions for the impulse-response fields show that the transverse area of the delta-function pulses is equal to the area of the disk, and thus, this electromagnetic missile carries an infinite amount of energy, since $\int_{-\infty}^{+\infty}\delta^2(t)dt = \infty$ [5, secs. 5.4.3–5.4.4].

However, it can be proven that a current source of finite energy in a finite region of space cannot radiate an electromagnetic missile with *nondecaying* energy to an infinitely large distance from the source [4], [5, sec. 5.4.4]. That this result must hold can be explained by the following simple argument. Because we showed that a source pulse of finite effective bandwidth ω_g cannot generate an electromagnetic missile, any electromagnetic missile must maintain itself on the energy in the increasingly higher frequencies. If the total energy radiated by the sources is finite, the energy in the frequency spectrum at frequencies higher than any finite value must approach zero, and the energy in the electromagnetic missile must decay, albeit slower than $1/r^2$. *Thus, it can be unequivocally stated that classical finite-energy sources in a finite region of space cannot produce fixed-energy wavepackets that remain localized. Finite-energy sources of finite extent cannot produce nondecaying electromagnetic missiles or any other nondecaying-energy wavepacket that remains localized for an indefinitely long time or indefinitely large travel distance.* This result implies that there are no free-space analogs to localized soliton pulses that can exist in nonlinear material, pulses that travel indefinitely without decay or change of shape [27]. Also, there are no free-space analogs to lossless waveguide or surface-wave pulses that disperse with travel time and distance but maintain a nondecaying total energy content.

However, for limited times and travel distances, classical electromagnetic wavepackets can be excited in free space that mimic photons in the limited sense that they can have a well-defined center frequency and a nearly constant energy concentrated within a small volume (as small as about a cubic center-frequency wavelength). The proof of this result follows.

Again, confining our attention to a scalar space-time field $E(\mathbf{r},t)$ that satisfies the homogeneous wave equation, $\nabla^2 E - \partial^2 E/(c\partial t)^2 = 0$, expand the spatial dependence in a 3-D Fourier transform in $\mathbf{k}$ space and the time dependence in an "analytic Fourier transform" over positive frequencies ω [5, ch. 5], [28] to get

$$E(\mathbf{r},t) = 2\Re e \int_{\mathbf{k}^\pm} \mathcal{E}(\mathbf{k}) e^{i[\mathbf{k}\cdot\mathbf{r} - \omega(\mathbf{k})t]} d^3\mathbf{k} \tag{32}$$

where the wave equation demands that $\omega(\mathbf{k}) = c(k_x^2 + k_y^2 + k_z^2)^{1/2}$ so as to collapse the integral over ω to one value for each $\mathbf{k}$. The $d^3\mathbf{k}$ denotes $dk_x dk_y dk_z$, and the integration limits $\mathbf{k}^\pm$ are $(-\infty,-\infty,-\infty) < (k_x,k_y,k_z) < (+\infty,+\infty,+\infty)$.

A wavepacket has its $\mathbf{k}$ spectrum concentrated about a central propagation vector $\mathbf{k}_0$ that can be chosen in the z-direction, that is, $\mathbf{k}_0 = k_0\hat{\mathbf{z}}$, such that $\mathcal{E}(\mathbf{k})$ is negligible unless $|\mathbf{k} - k_0\hat{\mathbf{z}}|/k_0 = |\Delta\mathbf{k}|/k_0 \ll 1$. Expanding $\omega(\mathbf{k})$ in a power series about $\mathbf{k}_0 = k_0\hat{\mathbf{z}}$ gives

$$\omega(\mathbf{k}) = \omega(\mathbf{k}_0) + \nabla_k\omega(\mathbf{k}_0)\cdot\Delta\mathbf{k}[1 + O(|\Delta\mathbf{k}|/k_0)] \tag{33}$$

with the subscript k on ∇_k denoting the gradient with respect to (k_x,k_y,k_z). Then, $E(\mathbf{r},t)$ in (32) can be approximated as follows:

$$E(\mathbf{r},t) \approx 2\Re e\left[e^{i(k_0z-\omega_0t)}\int_{\Delta\mathbf{k}}\mathcal{E}(\mathbf{k}_0+\Delta\mathbf{k}) \cdot e^{i\Delta\mathbf{k}\cdot[\mathbf{r}-\nabla_k\omega(\mathbf{k}_0)t]}d^3\Delta\mathbf{k}\right] \tag{34}$$

or simply

$$E(\mathbf{r},t) \approx \Re e\left\{g[\mathbf{r}-\nabla_k\omega(\mathbf{k}_0)t]e^{i(k_0z-\omega_0t)}\right\} \tag{35}$$

where $\omega_0 = \omega(\mathbf{k}_0) = ck_0$ and $g(\mathbf{r})$ is an envelope function that varies slowly over a distance equal to the wavelength $\lambda_0 = 2\pi/k_0$. The magnitude of the "group velocity," $\nabla_k\omega(\mathbf{k}_0) = c\hat{\mathbf{z}}$, is equal to c, the free-space speed of light, as it must, since the wavepacket is traveling in free space. Thus, (35) further simplifies to

$$E(\mathbf{r},t) \approx \Re e\left[g(\mathbf{r}-ct\hat{\mathbf{z}})e^{i(k_0z-\omega_0t)}\right]. \tag{36}$$

Choosing the envelope to be an imaginary function $g = -ig_0$, the wavepacket in (36) becomes in rectangular coordinates

$$E(\mathbf{r},t) = E(x,y,z-ct) \approx g_0(x,y,z-ct)\sin(k_0z-\omega_0t) \tag{37}$$

a quasi-monochromatic wavepacket traveling in the z-direction in free space with both group and phase speeds equal to c.[5]

[5] $E(\mathbf{r},t)$ in (37) cannot satisfy the wave equation $\nabla^2E - \partial^2E/(c\partial t)^2$ for g_0 bounded as $\rho = \sqrt{x^2+y^2} \to 0$ or ∞ except for g_0 independent of (x,y) [29, p. 302]. This seeming inconsistency is explained by the neglect of the $O(|\Delta\mathbf{k}|/k_0)$ terms in (33). These terms cannot be neglected in the evaluation of ∇^2E even though they can contribute negligibly to E itself.

The wavepacket in (37) has been derived under the assumption that the frequency spectrum of the field is negligible for $|\Delta\mathbf{k}|/k_0 \ll 1$. Numerical evaluation of $E(x,y,z-ct)$ for frequency spectra with different bandwidths shows that this criterion can be relaxed to $|\Delta\mathbf{k}|/k_0 \lesssim 1/2$ for a wavepacket with its energy density predominantly confined to as small a volume as a cubic center-frequency wavelength while propagating with little distortion for a few wavelengths; see also [30]. For example, a frequency spectrum that falls off from $\mathbf{k} = \mathbf{k}_0$ as a Gaussian function can produce a Gaussian wavepacket in time and space given by

$$E(x,y,z-ct) \approx \mathcal{E}_0\sin(k_0z-\omega_0t)e^{-\frac{(k_0z-\omega_0t)^2}{15}}e^{-\frac{k_0^2\rho^2}{15}} \tag{38}$$

where $\rho^2 = x^2+y^2$. At any instant of time t, this wavepacket has most of its energy, which is proportional to $|E|^2$, confined to about a cubic center-frequency wavelength. This minimum-size wavepacket with a dominant center frequency is about the best approximation obtainable for a classical photon propagating with little distortion for a few wavelengths in free space. It can be produced, for example, by the focused fields of a parabolic reflector. By adjusting the value of the electric-field constant to $\mathcal{E}_0 \approx \sqrt{k_0^3\hbar\omega_0/(\pi^2\epsilon_0)}$, the energy contained in the wavepacket can be made to equal the energy $\hbar\omega_0$ in a photon of frequency ω_0, where $\hbar = h/(2\pi)$ is the reduced Planck's constant. Of course, these "classical photons" are not as narrowband as actual photons. Neither do they have a probabilistic character, nor are they required to have the discrete photon energy $\hbar\omega_0$, since $\hbar$ is a quantum fundamental constant that is foreign to Maxwell's classical equations.

It is noteworthy that the energy in a vector electromagnetic plane-wave pulse corresponding to the scalar pulse in (38) is $W = \int(\epsilon_0|\mathbf{E}|^2 + \mu_0|\mathbf{H}|^2)dt/2 = \int\epsilon_0|\mathbf{E}|^2dt$, and the momentum carried by the pulse is $\mathbf{G} = \int\mathbf{E}\times\mathbf{H}dt/c^2 = \hat{\mathbf{z}}\int\epsilon_0|\mathbf{E}|^2dt$. If one assigns a mass m to the pulse, such that $\mathbf{G} = mc\hat{\mathbf{z}}$, then one sees that $W = mc^2$; that is, the Einstein mass-energy relation is inherently contained in Maxwell's equations for a propagating pulse.

A. Quantum Scattering

If the minimum-size (cubic-wavelength) quasi-monochromatic wavepacket exemplified in (38) is the closest that we can get to a classical representation of a photon, its cubic-wavelength volume possibly can predict the approximate number of photons per unit volume required for a quantum electrodynamic field to be treated classically. Less than this number of photons per unit volume, quantum scattering by individual photons may be required to accurately predict the scattering of incident electromagnetic fields [31, sec. 8.6.5], [32].

Consider the average photon density n_{ph} (number of photons per unit volume) in plane wave fields (E_0,H_0) with frequency ω_0. Specifically, equate the average energy density of the photons to the average energy density in the classical plane-wave fields, to get

$$n_{ph}\hbar\omega_0 = \frac{1}{4}\left(\epsilon_0E_0^2 + \mu_0H_0^2\right) = \frac{1}{2}\epsilon_0E_0^2 = I_0/c \tag{39a}$$

$$n_{ph} = \epsilon_0 E_0^2/(2\hbar\omega_0) \tag{39b}$$

where $I_0 = c\epsilon_0 E_0^2/2$ is the intensity of the electromagnetic plane wave. The average cubic-grid distance between the photons is $d_{ph} = 1/(n_{ph})^{1/3}$. If this average photon separation distance is too large, then classical electromagnetic field analysis needed, for example, to determine the scattering of a single electron in a plane wave (Thomson scattering), may not suffice, and a quantum approach to scattering of individual photons bouncing off the electron (Compton scattering) may be required.

The critical separation distance, beyond which classical field theory may fail and quantum theory may be required, can be determined from our above classical model of a photon as the smallest possible quasi-monochromatic electromagnetic wavepacket that can propagate in free space. The linear dimensions of this minimum-size free-space classical quasi-monochromatic wavepacket with well-defined center frequency ω_0 were found above and in [30] to be approximately one wavelength λ_0. Thus, these classical photons will create an approximately uniform classical electromagnetic field in free space if $d_{ph} \lesssim \lambda_0/4$, such that there is appreciable overlap of the wavepackets. Consequently, it seems reasonable to assume that the inequality $d_{ph} \gtrsim \lambda_0/4$ gives the criterion for leaving the homogeneous continuum regime of the classical Maxwellian electromagnetic fields and for encountering significant quantum scattering effects produced by the individual incident photons.[6] This inequality can be re-expressed as follows:

$$n_{ph} \lesssim \frac{64}{\lambda_0^3} \tag{40}$$

or from (39b)

$$\frac{\epsilon_0 E_0^2}{2}\left(\frac{\lambda_0}{4}\right)^3 = \frac{I_0}{c}\left(\frac{\lambda_0}{4}\right)^3 \lesssim \hbar\omega_0 \tag{41}$$

which says that quantum scattering theory, rather than classical electromagnetic-wave theory, may be required if the classical electromagnetic-field energy density in a quarter wavelength cubed becomes less than about the photon energy $\hbar\omega_0$. Conversely, if

$$\frac{\epsilon_0 E_0^2 \lambda_0^3}{2} = \frac{I_0}{c}\lambda_0^3 \gtrsim 64\hbar\omega_0 \quad \left(n_{ph} \gtrsim \frac{64}{\lambda_0^3}\right) \tag{42}$$

then the ensemble of photons can be treated as a classical Maxwellian electromagnetic field. For a given energy density, classical field behavior occurs for frequencies $\omega_0^4 \lesssim 2c^3\epsilon_0 E_0^2/\hbar = 4c^2 I_0/\hbar$.[7] It is especially noteworthy that the inequality in (42) agrees well with the condition obtained from quantum electrodynamics by Berestetskii, Lifshitz, and Pitaevskii for the "averaged [quantum-electrodynamic] field to be quasi-classical" [32, eq. (5.2)], namely, $\epsilon_0 E_0^2 \lambda_0^3 \gg 2\pi^2\hbar\omega_0$ (in SI units).[8]

IV. Conclusion

A generalized far-field (Rayleigh) distance derived for an arbitrary frequency-domain antenna with sources in a finite region of space reveals that the effective size of such an antenna is determined by the radius a of its significant reactive power, which, in turn, is determined by the highest value N of the degree number n of spherical waves needed to accurately represent the far-fields of the antenna. Moreover, ordinary (nonsuper-reactive) sources within a radius $a_0 \gtrsim a - 3\lambda/(4\pi)$ can produce these N-degree spherical-wave far-fields. The radius a of the significant reactive power determines the maximum possible gain of the antenna and allows supergain to be defined in terms of the physical radius a_0 of the antenna. It is also proven that time-domain antennas having effectively bandlimited sources with their significant reactive power located in a volume of finite extent (such as all known existing sources) cannot have far-fields that decay slower than $1/r$. Although finite-extent sources that are not effectively bandlimited can produce "electromagnetic-missile" pulses whose far-field energy decays slower than $1/r^2$, finite-energy, finite-extent sources cannot produce *nondecaying* electromagnetic missiles or any other nondecaying-energy wavepacket that remains localized for an indefinitely long travel time or distance. However, it is shown that for limited times and travel distances, classical electromagnetic wavepackets can be excited in free space that mimic photons in the sense that they can have a well-defined center frequency and a nearly constant energy concentrated within a small volume (as small as about a cubic center-frequency wavelength). This classical quasi-monochromatic model of the photon is then used, along with the photon quantum energy $\hbar\omega_0$, to derive an inequality, previously obtained only from quantum electrodynamics, that determines the minimum average energy density in a monochromatic electromagnetic field that is sufficient for the field to be treated classically with Maxwell's equations rather than quantum electrodynamically with photon scattering theory.

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[6] We are assuming that each of the photons can be replaced by the minimum-size classical wavepackets. This assumption is a plausibility argument that is finally justified by the results obtained agreeing with the rigorous derivation from quantum electrodynamics.

[7] For example, the irradiance of visible sunlight at the surface of the Earth under clear skies is about 1.5 Watts/m^2 per nanometer of wavelength bandwidth [33]. Therefore, if the molecules of the cone cells in the retina of the human eye that detect one of the three primary colors (red, green, or blue) have a bandwidth of roughly 100 nanometers, then $I_0 \lesssim 150$ Watts/m^2 in daylight. Even with the maximum value of $I_0 = 150$ Watts/m^2, (42) predicts $\lambda_0 \lesssim 70\,000$ nanometers, so that the incident light (center wavelength between 300 and 700 nanometers) is received by these molecules as individual photons separated by a hundred wavelengths or more rather than as a classical continuous field.

[8] It should be pointed out that even if (42) is strongly violated, such that (41) strongly holds, averaging over long enough time can still produce results predicted by the classical Maxwell equations; for example, double-slit diffraction patterns made by light so weak that the detected photons are separated in time by many periods of the light [34, vol. III, ch. 1].

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Arthur D. Yaghjian (Life Fellow, IEEE) received the B.S., M.S., and Ph.D. degrees in electrical engineering from Brown University, Providence, RI, USA, in 1964, 1966, and 1969, respectively, and the Honorary Doctorate degree from the Technical University of Denmark, Lyngby, Denmark, in 2020.

After teaching for a year, he joined the Research Staff of the National Institute of Standards and Technology (NIST), Boulder, CO, USA, in 1971, and transferred in 1983 to the Air Force Research Laboratories, Bedford, MA, USA, until 1996. Since then, he has been an Independent Researcher in electromagnetics. His early research at NIST helped pioneer the development of probe-corrected near-field antenna measurements for accurately characterizing modern antennas in both the frequency and time domains. He has derived the definitive microscopic and macroscopic force and energy expressions for both diamagnetic and paramagnetic media. He has contributed significantly to the determination and fundamental understanding of the classical equations of motion of accelerated charged particles. In the area of high-frequency diffraction, he obtained convenient expressions for incremental length diffraction coefficients that are currently used to predict bistatic scattering and reflector antenna performance in commercial high-frequency computer codes. His fundamental characterization of antennas, including the determination of the upper bounds on the bandwidth of complex antennas, has had a major impact on the research and development of modern electrically small antennas. He holds the patent on supergain electrically small antennas. He has authored two books, one in the *IEEE Press Series on Electromagnetic Wave Theory*.

Dr. Yaghjian has been an IEEE-APS Distinguished Lecturer. He has received the IEEE Electromagnetics Award, the IEEE-APS Distinguished Achievement Award, and four IEEE Schelkunoff Prize Paper Awards.