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1.) Periksa apakah $\vec{v} = (4, -10, 8)$ merupakan kombinasi linier dari $\{\vec{a} = (1, 4, 2), \vec{b} = (2, -1, 0), \vec{c} = (3, 4, 1)\}$

$\vec{v} = x\vec{a} + y\vec{b} + z\vec{c}$, x, y, z suatu skalar

$$\begin{bmatrix} 4 \\ -10 \\ 8 \end{bmatrix} = x\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + y\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + z\begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\downarrow$$

$$4 = x + 2y + 3z$$
$$-10 = 4x + (-y) + 4z$$
$$8 = 2x + z$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 4 & -1 & 4 & -10 \\ 2 & 0 & 1 & 8 \end{array}\right] E_{21(-4)} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -9 & -8 & -26 \\ 2 & 0 & 1 & 8 \end{array}\right]$$

$$E_{31(-2)}\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -9 & -8 & -26 \\ 0 & -4 & -5 & 0 \end{array}\right] E_{32(-\frac{4}{9})}\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -9 & -8 & -26 \\ 0 & 0 & -\frac{18}{9} & \frac{104}{9} \end{array}\right]$$

$\Rightarrow -\frac{18}{9}z = \frac{104}{9}$ $\Rightarrow -9y - 8z = -26$ $\Rightarrow x + 2y + 3z = 4$

$z = -8$ $-9y - 8(-8) = -26$ $x + 2(10) + 3(-8) = 4$

$-9y + 64 = -26$ $x + 20 + (-24) = 4$

$y = 10$ $x = 8$

$\therefore \vec{v} = 8\vec{a} + 10\vec{b} + (-8)\vec{c}$, Sehingga v kombinasi linier dari $\vec{a}, \vec{b}, \vec{c}$.

2.) Periksa apakah $\vec{v} = (2, 1, 8)$ merupakan kombinasi linier dari $\{\vec{a} = (1, 0, 2), \vec{b} = (2, -1, 0), \vec{c} = (3, -1, 1)\}$

$\vec{v} = x\vec{a} + y\vec{b} + z\vec{c}$, x, y, z suatu skalar

$$\begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix} = x\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + y\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + z\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

$$\downarrow$$

$$2 = x + 2y + 3z$$
$$1 = -y + z$$
$$8 = 2x + z$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 1 & 1 \\ 2 & 0 & 1 & 8 \end{array}\right] E_{31(-2)} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 1 & 1 \\ 0 & -4 & -5 & 4 \end{array}\right]$$

$$E_{32(-4)}\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & -9 & 0 \end{array}\right]$$

$\Rightarrow -9z = 0$ $\Rightarrow -y + z = 1$ $\Rightarrow x + 2y + 3z = 2$

$z = 0$ $-y + 0 = 1$ $x + 2(-1) + 3(0) = 2$

$-y = 1$ $x - 2 + 0 = 2$

$y = -1$ $x = 4$

$\therefore \vec{v} = 4\vec{a} + (-1)\vec{b} + 0\vec{c}$, Sehingga v kombinasi linier dari $\vec{a}, \vec{b}, \vec{c}$

3.) Periksa apakah $\vec{v} = (4, 2, 4)$ merupakan kombinasi linier dari $\{\vec{a} = (1, 0, 2), \vec{b} = (2, -1, 0), \vec{c} = (2, -1, 1)\}$

$\vec{v} = x\vec{a} + y\vec{b} + z\vec{c}$, x, y, z suatu skalar

$$\begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix} = x\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + y\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + z\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\downarrow$$

$$4 = x + 2y + 2z$$
$$2 = -y - z$$
$$4 = 2x + z$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 4 \\ 0 & -1 & -1 & 2 \\ 2 & 0 & 1 & 4 \end{array}\right] E_{31(-2)} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 4 \\ 0 & -1 & -1 & 2 \\ 0 & -4 & -3 & -4 \end{array}\right] E_{32(-4)} \left[\begin{array}{ccc|c} 1 & 2 & 2 & 4 \\ 0 & -1 & -1 & 2 \\ 0 & 0 & 1 & -12 \end{array}\right]$$

$\Rightarrow z = -12$ $\Rightarrow -y - z = 2$ $\Rightarrow x + 2y + 2z = 4$

$-y - (-12) = 2$ $x + 2(10) + 2(-12) = 4$

$-y + 12 = 2$ $x + 20 - 24 = 4$

$-y = -10$ $x = 8$

$y = 10$

$\therefore \vec{v} = 8\vec{a} + 10\vec{b} - 12\vec{c}$, Sehingga v kombinasi linier dari $\vec{a}, \vec{b}, \vec{c}$

4.) Periksa apakah $\vec{v} = (2, -5, 3)$ merupakan kombinasi linier dari $\{\vec{a} = (1, -3, 2), \vec{b} = (2, -4, 1), \vec{c} = (1, -5, 1)\}$

$\vec{v} = x\vec{a} + y\vec{b} + z\vec{c}$, x, y, z suatu skalar

$$\begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} = x\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} + y\begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix} + z\begin{bmatrix} 1 \\ -5 \\ 1 \end{bmatrix}$$

$$\downarrow$$

$$2 = x + 2y + z$$
$$-5 = -3x - 4y - 5z$$
$$3 = 2x + y + z$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ -3 & -4 & -5 & -5 \\ 2 & 1 & 1 & 3 \end{array}\right] E_{21(3)} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 1 \\ 2 & 1 & 1 & 3 \end{array}\right] E_{31(-2)} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 1 \\ 0 & -3 & -1 & -1 \end{array}\right]$$

$$E_{32(\frac{3}{2})}\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & 2 & -2 & 1 \\ 0 & 0 & -4 & \frac{1}{2} \end{array}\right]$$

$\Rightarrow -4z = \frac{1}{2}$ $\Rightarrow 2y - 2z = 1$ $\Rightarrow x + 2y + z = 2$

$-z = \frac{1}{2}$ $2y - 2(-\frac{1}{8}) = 1$ $x + \frac{6}{8} + (-\frac{1}{8}) = 2$

$z = -\frac{1}{8}$ $2y + \frac{1}{4} = 1$ $x + \frac{5}{8} = 2$

$2y = \frac{3}{4}$ $x = \frac{11}{8}$

$y = \frac{3}{8}$

$\therefore \vec{v} = \frac{11}{8}\vec{a} + \frac{3}{8}\vec{b} + (-\frac{1}{8})\vec{c}$, Sehingga v kombinasi linier dari $\vec{a}, \vec{b}, \vec{c}$.

5) Untuk harga k yang manakah $\vec{a} = (2, 0, k)$ merupakan kombinasi linier dari $\vec{b} = (1, 2, 1)$ dan $\vec{c} = (1, -2, 0)$

$\vec{a} = u\vec{b} + v\vec{c}$

$$\begin{bmatrix} 2 \\ 0 \\ k \end{bmatrix} = u\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + v\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

$2 = u + v$
$0 = 2u - 2v$
$k = u$

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & -2 & 0 \\ 1 & 0 & k \end{array}\right] E_{21(-2)} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -4 & -4 \\ 1 & 0 & k \end{array}\right] E_{31(-1)} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -4 & -4 \\ 0 & -1 & k-2 \end{array}\right]$$

$$E_{32(-\frac{1}{4})} \left[\begin{array}{cc|c} 1 & 1 & 2 \\ 0 & -4 & -4 \\ 0 & 0 & k-1 \end{array}\right]$$

$\Rightarrow 0z = k - 1$
$0 = k - 1$ atau
$k = 1$

$\Rightarrow -4y = -4$
$\boxed{y = 1}$

$\Rightarrow u + y = 2$
$u + 1 = 2$
$\boxed{u = 1}$

$\Rightarrow k = u$
$k = 1$

$\therefore$ harga k adalah 1

6) Periksa apakah $\vec{v} = (3, 7, -4)$ sebagai kombinasi linier dari $\vec{a} = (1, 2, 3)$, $\vec{b} = (2, 3, 7)$, $\vec{c} = (3, 5, 6)$

$\vec{v} = u\vec{a} + v\vec{b} + z\vec{c}$

$$\begin{bmatrix} 3 \\ 7 \\ -4 \end{bmatrix} = u\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + v\begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix} + z\begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}$$

$3 = u + 2v + 3z$
$7 = 2u + 3v + 5z$
$-4 = 3u + 7v + 6z$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 2 & 3 & 5 & 7 \\ 3 & 7 & 6 & -4 \end{array}\right] E_{21(-2)} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & -1 & 1 \\ 3 & 7 & 6 & -4 \end{array}\right] E_{31(-3)} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & -3 & -13 \end{array}\right]$$

$$E_{32(1)} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & -4 & -12 \end{array}\right]$$

$\Rightarrow -4z = -12$
$\boxed{z = 3}$

$\Rightarrow -y - z = 1$
$-y - 3 = 1$
$-y = 4$
$\boxed{y = -4}$

$\Rightarrow u + 2y + 3z = 3$
$u + 2(-4) + 3(3) = 3$
$u - 8 + 9 = 3$
$\boxed{u = 2}$

$\therefore \vec{v} = 2\vec{a} + (-4)\vec{b} + 3\vec{c}$, Sehingga V kombinasi linier dari $\vec{a}, \vec{b}, \vec{c}$

7) Diketahui $\vec{w} = [0, 2, -2]$, $\vec{a} = [1, 2, -4]$, $\vec{b} = [-1, 1, 0]$, $\vec{c} = [1, 1, -3]$
Apakah $\vec{w}$ merupakan kombinasi linier dari $\vec{a}, \vec{b}, \vec{c}$?

$\vec{w} = u\vec{a} + v\vec{b} + z\vec{c}$

$$\begin{bmatrix} 0 \\ 2 \\ -2 \end{bmatrix} = u\begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix} + v\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}$$

$0 = u - v + z$
$2 = 2u + v + z$
$-2 = -4u - 3z$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 2 & 1 & 1 & 2 \\ -4 & 0 & -3 & -2 \end{array}\right] E_{21(-2)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -1 & 2 \\ -4 & 0 & -3 & -2 \end{array}\right]$$

$$E_{31(4)} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -1 & 2 \\ 0 & -4 & 1 & -2 \end{array}\right] E_{32(\frac{4}{3})} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & -1 & 2 \\ 0 & 0 & -\frac{1}{3} & \frac{2}{3} \end{array}\right]$$

$\Rightarrow -\frac{1}{3}z = \frac{2}{3}$
$\boxed{z = -2}$

$\Rightarrow 3y - z = 2$
$3y + 2 = 2$
$3y = 0$
$\boxed{y = 0}$

$\Rightarrow u - y + z = 0$
$u - 0 + (-2) = 0$
$\boxed{u = 2}$

$\therefore \vec{w} = 2\vec{a} + 0\vec{b} + (-2)\vec{c}$, Sehingga $\vec{w}$ kombinasi linier dengan $\vec{a}, \vec{b}, \vec{c}$

8) Diketahui $\vec{w} = [4, -2, -4]$, $\vec{a} = [1, 3, 6]$, $\vec{b} = [2, 1, 2]$, $\vec{c} = [3, 6, 12]$
Apakah $\vec{w}$ merupakan kombinasi linier dari $\vec{a}, \vec{b}, \vec{c}$?

$\vec{w} = u\vec{a} + v\vec{b} + z\vec{c}$

$$\begin{bmatrix} 4 \\ -2 \\ -4 \end{bmatrix} = u\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} + v\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + z\begin{bmatrix} 3 \\ 6 \\ 12 \end{bmatrix}$$

$4 = u + 2v + 3z$
$-2 = 3u + v + 6z$
$-4 = 6u + 2v + 12z$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 3 & 1 & 6 & -2 \\ 6 & 2 & 12 & -4 \end{array}\right] E_{21(-3)} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -5 & -3 & -14 \\ 6 & 2 & 12 & -4 \end{array}\right]$$

$$E_{31(-6)} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -5 & -3 & -14 \\ 0 & -10 & 6 & -28 \end{array}\right] E_{32(-2)} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -6 & -3 & -14 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$n = 3$
$P(A) = P(A|B) = 2$ } konsisten, banyak penyelesaian

$\therefore$ Karena hasilnya konsisten, maka $\vec{w}$ kombinasi linier dari $\vec{a}, \vec{b}, \vec{c}$

9) Diketahui $\vec{w} = [3,2,0]$, $\vec{a} = [2,1,1]$, $\vec{b} = [-4,0,-2]$, $\vec{c} = [6,5,3]$
Apakah $\vec{w}$ merupakan kombinasi linier dari $\vec{a}, \vec{b}, \vec{c}$?

$\vec{w} = x\vec{a} + y\vec{b} + z\vec{c}$

$$\begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = x\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} + y\begin{bmatrix} -4 \\ 0 \\ -2 \end{bmatrix} + z\begin{bmatrix} 6 \\ 5 \\ 3 \end{bmatrix}$$

$3 = 2x - 4y + 6z$

$2 = x + 5z$

$0 = x - 2y + 3z$

$$\left[\begin{array}{ccc|c} 2 & -4 & 6 & 3 \\ 1 & 0 & 5 & 2 \\ 1 & -2 & 3 & 0 \end{array}\right] E_{21}(-\tfrac{1}{2}) \left[\begin{array}{ccc|c} 2 & -4 & 6 & 3 \\ 0 & 2 & 2 & \frac{1}{2} \\ 1 & -2 & 3 & 0 \end{array}\right] E_{31}(-\tfrac{1}{2}) \left[\begin{array}{ccc|c} 2 & -4 & 6 & 3 \\ 0 & 2 & 2 & 1/2 \\ 0 & 0 & 0 & -\frac{3}{2} \end{array}\right]$$

$n = 3$
$P(A) = 2$
$P(A|B) = 3$
} Tidak konsisten

$\therefore \vec{w}$ bukan kombinasi linier $\vec{a}, \vec{b}, \vec{c}$ karena tidak konsisten