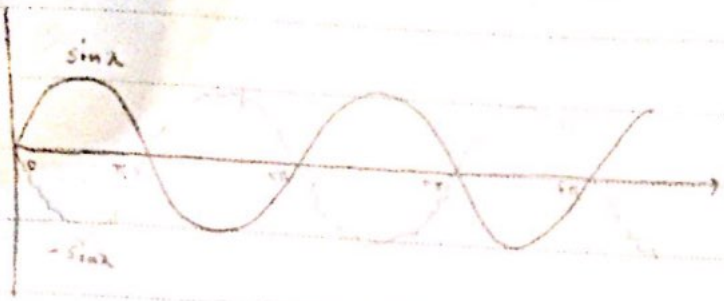
Subject:
Year. Month. Date.

۹۵۱۲۰۳۱ ریاضیات

بودن یا نبودن مسئله این است!



$g(x) = \sin x$ $\quad h(x) = -\sin x$

$g \in O(h) \Leftarrow \exists c \geq 0, n_0 \geq 1 \quad \forall n \geq n_0 \quad g(n) \leq c.h(n)$ فرض خلف

$n = 2n_0\pi + \frac{\pi}{2} \geq n_0 \Rightarrow g(n) = \sin(2n_0\pi + \frac{\pi}{2}) = \sin(\frac{\pi}{2}) = 1$

$c.h(n) = -c.\sin(2n_0\pi + \frac{\pi}{2}) = -c$

※ $1 > -c$

↳ فرض خلف باطل

$h \in O(g) \Leftarrow \exists c \geq 0, n_0 \geq 1 \quad \forall n \geq n_0 \quad h(n) \leq c.g(n)$ فرض خلف

$n = 2n_0\pi - \frac{\pi}{2} \geq n_0 \Rightarrow h(n) = -\sin(2n_0\pi - \frac{\pi}{2}) = -\sin(-\frac{\pi}{2}) = 1$

$c.g(n) = c.\sin(2n_0\pi - \frac{\pi}{2}) = -c$

※ $1 > -c$

↳ فرض خلف باطل

a. Case 1: $O(n^{\log_b a - \epsilon}) = O(n^{1-\epsilon})$ Master Theorem

$f(n) = \sqrt{n} = n^{\frac{1}{2}} \in O(n^{1-\epsilon}) = O(n^{\frac{1}{2}})$ $\Rightarrow T(n) = \Theta(n^{\log_2 2}) = \Theta(n)$

$\epsilon \le \frac{1}{2}$ $c \ge 2$ $n_0 = 2$

b. Case 3: $\Omega(n^{\log_4 3 + \epsilon})$ Master Theorem

$f(n) = n \log n \in \Omega(n^{\log_4 3 + \epsilon}) = \Omega(n)$ $n_0 = 100$ $c = 1$ $cn \log n \ge n$

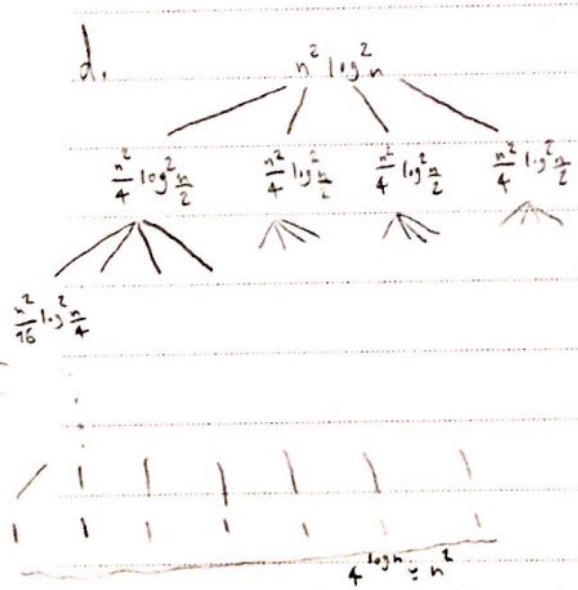
$\epsilon \le 1 - \log_4 3$

$\exists c < 1 \quad 3\left(\frac{n}{4}\log\frac{n}{4}\right) < cn \log n$ $\quad c = \frac{3}{4} \Rightarrow T(n) = \Theta(n \log n)$

c. Case 2: $\Theta(n^{\log_2 4}) = \Theta(n^2)$

$n^2 \in \Theta(n^2) \Rightarrow T(n) = \Theta(n^2 \lg n)$

d.



Level جمع

$n^2 \log^2 n$

$4 \times \frac{n^2}{4}\log^2\frac{n}{2} = n^2 \log^2\frac{n}{2}$

$16 \times \frac{n^2}{16}\log^2\frac{n}{4} = n^2 \log^2\frac{n}{4}$

n^2

$\log n$ Level

$T(n) = n^2 \log^2 n + n^2 \log^2\frac{n}{2} + n^2 \log^2\frac{n}{4} + \dots + n^2 \cdot 9 + n^2 \cdot 4 + 1$

$= n^2(\lg^2 n + \lg^2\frac{n}{2} + \lg^2\frac{n}{4} + \dots + 9 + 4 + 1)$ تعریف $\log n = x$

$= n^2(x^2 + (x-1)^2 + (x-2)^2 + \dots + 9 + 4 + 1)$ جمع توان های ۲

$= n^2\left(\frac{x^3}{3} + \frac{x^2}{2} + \frac{x}{6}\right) = n^2\left(\frac{\lg^3 n}{3} + \frac{\lg^2 n}{2} + \frac{\lg n}{6}\right)$

$\frac{1}{3} n^2 \lg^3 n \le n^2\left(\frac{\lg^3 n}{3} + \frac{\lg^2 n}{2} + \frac{\lg n}{6}\right) \le n^2 \lg^3 n$

$c = \frac{1}{3}$ $n_0 = 5$ $\quad T(n) = \Theta(n^2 \log^3 n)$ $\quad c' = 3$ $n_0' = 5$

ه. $T(n) = 2T(\frac{n}{2}) + \frac{n}{\lg n} = 2\left(2T(\frac{n}{4}) + \frac{\frac{n}{2}}{\lg \frac{n}{2}}\right) + \frac{n}{\lg n}$

$= 4T(\frac{n}{4}) + \frac{n}{\lg n} + \frac{\frac{n}{2}}{\lg \frac{n}{2}} = 4\left(2T(\frac{n}{8}) + \frac{\frac{n}{4}}{\lg \frac{n}{4}}\right) = 8T(\frac{n}{8}) + \frac{n}{\lg n} + \frac{\frac{n}{2}}{\lg \frac{n}{2}} + \frac{\frac{n}{4}}{\lg \frac{n}{4}}$

$= 2^i T(\frac{n}{2^i}) + \frac{n}{\lg n} + \frac{\frac{n}{2}}{\lg \frac{n}{2}} + \cdots + \frac{\frac{n}{2^{i-1}}}{\lg \frac{n}{2^{i-1}}}$

$\frac{n}{2^i} = 1 \Rightarrow i = \lg n \quad \Rightarrow T(n) = 2^{\lg n} T(1) + n\left(\frac{1}{\lg n} + \frac{1}{\lg \frac{n}{2}} + \cdots + \frac{1}{\lg \frac{n}{2^{\lg n - 1}}}\right)$

$= n + n \sum_{i=0}^{\lg n - 1} \frac{1}{\lg \frac{n}{2^i}} = n + n \sum_{i=0}^{\lg n - 1} \frac{1}{\lg n - i} = n + n\left(\frac{1}{\lg n} + \frac{1}{\lg n - 1} + \frac{1}{\lg n - 2} + \cdots + \frac{1}{1}\right)$

سری هارمونیک

$= n + n \lg(\lg(n)) = \Theta(n \lg(\lg(n)))$

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۴. $T(n) = T(n^{\frac{1}{۲}}) + ۱ = (T(n^{\frac{1}{۴}}) + ۱) + ۱ = T(n^{\frac{1}{۴}}) + ۲$

$= T(n^{\frac{1}{۸}}) + ۳ = T(n^{\frac{1}{۲^i}}) + i$

$n^{\frac{1}{۲^i}} = ۲ \Rightarrow \frac{1}{۲^i} = \log_n^۲ \Rightarrow ۲^i = \frac{1}{\log_n^۲} \Rightarrow ۲^i = \lg n \Rightarrow i = \lg(\lg n)$

$T(n) = T(۲) + \lg(\lg(n)) = \Theta(\lg(\lg(n)))$