$$f(x,y) = x^3 + (x-4k)y^2 = 0$$

Region 1 :

$$F(P_1) = f(x_i+1, y_i + \frac{1}{2} + e_1) = 0$$

$$\Rightarrow (x_i+1)^3 + (x_i+1-4k)(y_i+\frac{1}{2}+e_1)^2 = 0$$

$$\Rightarrow (x_i+1)^3 + (x_i+1-4k)\left[(y_i+\frac{1}{2})^2 + e_1^2 + 2e_1(y_i+\frac{1}{2})\right] = 0$$

$$\Rightarrow f(M_1) = -(x_i+1-4k)\left[e^2 + 2e_1(y_i+\frac{1}{2})\right]$$

$$\Rightarrow d_{1_i} = -(x_i+1-4k)\left[e_1^2 + 2e_1(y_i+\frac{1}{2})\right]$$

$e_1 \geq 0 \Rightarrow e_1^2 + 2e_1(y_i+\frac{1}{2}) > 0$

Also we know $x < 4k \Rightarrow -(x_i+1-4k) > 0$ $\Bigg\} \Rightarrow d_{1_i} \geq 0$

$e_1 < 0 \Rightarrow e_1^2 + 2e_1(y_i+\frac{1}{2}) < 0 \Rightarrow d_{1_i} < 0$

Therefore.

$$d1_i \begin{cases} <0 & \text{select } B : x_{i+1} = x_i + 1 \quad y_{i+1} = y_i \\ \geq 0 & \text{select } C : x_{i+1} = x_i + 1 \quad y_{i+1} = y_i + 1 \end{cases}$$

$$d1_i = (x_i+1)^3 + (x_i+1-4k)(y_i+\frac{1}{2})^2$$

initial :

$$(x_0, y_0) = (0,0)$$

$$d1_0 = 1 + (1-4k) \times \frac{1}{4} \xRightarrow{\times 4} d1_0 = 5 - 4k$$

$$d1_i = 4 \times f(M1_{i+1})$$

ie B is selected :

$$d1_{i+1} = 4 \times \left[(x_{i+1}+1)^3 + (x_{i+1}+1-4k)(y_{i+1}+\frac{1}{2})^2 \right]$$

$$= 4 \times \left[(x_i + 1 + 1)^3 + (x_i + 1 - 4k + 1)(y_i + \tfrac{1}{2})^2 \right]$$

$$= 4 \times \left[(x_i+1)^3 + 1 + 3(x_i+1)^2 + 3(x_i+1) + (x_i+1-4k)(y_i+\tfrac{1}{2})^2 + (y_i+\tfrac{1}{2})^2 \right]$$

$$= d1_i + 4 \times \left(1 + 3(x_i^2 + 1 + 2x_i) + 3x_i + 3 + y_i^2 + \tfrac{1}{4} + y_i \right)$$

$$= d1_i + 4\left(3x_i^2 + 9x_i + y_i^2 + y_i + \tfrac{29}{4} \right)$$

$$= d1_i + 12x_i^2 + 36x_i + 4y_i^2 + 4y_i + 29$$

if c is selected:

$$d1_{i+1} = 4 \times \left[(x_i + 1 + 1)^3 + (x_i + 1 - 4k + 1)(y_i + \tfrac{1}{2} + 1)^2 \right]$$

$$= 4 \times \left[(x_i+1)^3 + 1 + 3(x_i+1)^2 + 3(x_i+1) + (x_i+1-4k+1) \left((y_i+\tfrac{1}{2})^2 + 1 + 2(y_i+\tfrac{1}{2}) \right) \right]$$

$$= d1_i + 4 \times \left[1 + 3(x_i^2 + 2x_i + 1) + 3(x_i+1) \right.$$

$$+ (x_i + 1 - 4k)(2 + 2y_i)$$

$$+ (y_i + \tfrac{1}{2})^2 + 2y_i + 2 \Big]$$

$$= d_{1_i} + 4 \times \Big[3x_i^2 + 11x_i + \frac{45}{4} - 8k + 2x_i y_i$$

$$y_i^2 + (-8k + 5) y_i \Big]$$

$$= d_{1_i} + 12x_i^2 + 44x_i + 45 - 32k + 8x_i y_i$$

$$+ 4y_i^2 + (20 - 32k) y_i$$

Region 2:

$$f(x, y) = x^3 + (x - 4k) y^2 = 0$$

$$F(P_2) = (x_i + \tfrac{1}{2} + e_2)^3 + (x_i + \tfrac{1}{2} + e_2 - 4k)(y_i + 1)^2 = 0$$

$$\Rightarrow (x_i + \tfrac{1}{2})^3 + e_2^3 + 3(x_i + \tfrac{1}{2})^2 e_2 + 3(x_i + \tfrac{1}{2}) e_2^2$$

$$+ (x_i + \tfrac{1}{2} - 4k)(y_i + 1)^2 + e_2 (y_i + 1)^2 = 0$$

$$\Rightarrow d2_i = F(M_2) = -e_2^3 - 3(x_i+\frac{1}{2})^2 e_2 - 3(x_i+\frac{1}{2})e_2^2$$
$$- e_2(y_i+1)^2$$

$\Rightarrow$ $e_2 > 0 \longrightarrow d2_i < 0$ select C

$e_2 \leq 0 \longrightarrow d2_i \geq 0$ select D

$3(x_i+\frac{1}{2})^2 > 3e(x_i+\frac{1}{2})$

Initial:

$(x_0, y_0) = (0,0)$

$$d2_0 = (x_0+\frac{1}{2})^3 + (x_0+\frac{1}{2} - 4K)(y_0+1)^2$$

$$= \frac{1}{8} + (\frac{1}{2} - 4K)$$

$$d_{2_0} = 8 \times F(M_{2_1}) = 5 - 32K$$

if C is selected $x_{i+1} = x_i + 1$ $y_{i+1} = y_i + 1$

$$d_{2_{i+1}} = 8\times\left[\left(x_{i+1}+\frac{1}{2}\right)^3 + \left(x_{i+1}+\frac{1}{2}-4k\right)\left(y_{i+1}+1\right)^2\right]$$

$$= 8\times\left[\left(x_i+\frac{1}{2}+1\right)^3 + \left(\left(x_i+\frac{1}{2}-4k\right)+1\right)\left(\left(y_i+1\right)+1\right)^2\right]$$

$$= 8\times\left[\left(x_i+\frac{1}{2}\right)^3 + 1 + 3\left(x_i+\frac{1}{2}\right)^2 + 3\left(x_i+\frac{1}{2}\right) + \right.$$

$$\left(x_i+\frac{1}{2}-4k\right)\left(y_i+1\right)^2 + \left(x_i+\frac{1}{2}-4k\right)\left(2y_i+3\right)$$

$$\left. + \left(y_i^2+4+4y_i\right)\right]$$

$$= d_{2_i} + 8\times\left[1 + 3\left(x_i^2+\frac{1}{4}+x_i\right) + 3x_i + \frac{3}{2}\right.$$

$$+ 2x_iy_i + 3x_i + y_i + \frac{3}{2} - 8ky_i - 12k$$

$$\left. + y_i^2 + 4 + 4y_i\right]$$

$$= d_{2_i} + 8\times\left[3x_i^2 + 9x_i + y_i^2 + (5-8k)y_i + 2x_iy_i\right.$$

$$\left. 1+\frac{3}{4}+\frac{3}{2}+\frac{3}{2}-12k+4\right]$$

$$= d_{2_i} + 24x_i^2 + 72x_i + 8y_i^2 + (40-64k)y_i + 16x_iy_i$$

$+ 70 - 96k$

if D is selected ; $x_{i+1} = x_i \quad y_{i+1} = y_i + 1$

$$d2_{i+1} = 8 \times \left[\left(x_{i+1} + \frac{1}{2}\right)^3 + \left(x_{i+1} + \frac{1}{2} - 4k\right)\left(y_{i+1} + 1\right)^2 \right]$$

$$= 8 \times \left[\left(x_i + \frac{1}{2}\right)^3 + \left(x_i + \frac{1}{2} - 4k\right)\left(y_i + 1 + 1\right)^2 \right]$$

$$= 8 \times \left[\left(x_i + \frac{1}{2}\right)^3 + \left(x_i + \frac{1}{2} - 4k\right)\left((y_i+1)^2 + (2y_i + 3)\right) \right]$$

$$= d2_i + 8\left[2x_iy_i + 3x_i + y_i + \frac{3}{2} - 8ky_i - 12k \right]$$

$$= d_{2_i} + 24x_i + (8 - 64k)y_i + 16x_iy_i + 12 - 96k$$

if B is selected ; $x_{i+1} = x_i + 1 \quad y_{i+1} = y_i$

$$d2_{i+1} = 8 \times \left[\left(x_{i+1} + \frac{1}{2}\right)^3 + \left(x_{i+1} + \frac{1}{2} - 4k\right)\left(y_{i+1} + 1\right)^2 \right]$$

$$= 8 \times \left[\left(x_i + \frac{1}{2} + 1\right)^3 + \left(x_i + \frac{1}{2} - 4k + 1\right)(y_i + 1)^2 \right]$$

$$= 8 \times \left[\left(x_i + \frac{1}{2}\right)^3 + 1 + 3\left(x_i + \frac{1}{2}\right)^2 + 3\left(x_i + \frac{1}{2}\right) + \left(x_i + \frac{1}{2} - 4k\right)(y_i + 1)^2 + (y_i + 1)^2 \right]$$

$$= d2_i + 8 \times \left[1 + 3\left(x_i^2 + \frac{1}{4} + x_i\right) + 3x_i + \frac{3}{2} + y_i^2 + 2y_i + 1 \right]$$

$$= d2_i + 8 \times \left(3x_i^2 + 6x_i + y_i^2 + 2y_i + 2 + \frac{3}{4} + \frac{3}{2} \right]$$

$$= d2_i + 24x_i^2 + 48x_i + 8y_i^2 + 16y_i + 34$$

$$d1_i \begin{cases} < 0 & B \quad e_1 < 0 \\ \geq 0 & C \quad e_1 \geq 0 \end{cases} \qquad d2_i \begin{cases} < 0 & C \quad e_2 > 0 \\ \geq & D \quad e_2 \leq 0 \end{cases}$$

while curve is being generated in region 1 using $d1_i$, $e_2 > 0 \Rightarrow d2_i < 0$

when $d2_i$ changes sign we know that curve has come to region 2

Algorithm:

$x_0 = 0 \quad y_0 = 0 \quad d1_0 = 5 - 4k \quad d2_0 = 5 - 32k$

$i = 0$

while $d2_i < 0$ {

display (x_i, y_i)

$x_{i+1} = x_i + 1$

if $d1_i < 0$ {

$y_{i+1} = y_i$

$d1_{i+1} =$

$d1_i + 12x_i^2 + 36x_i + 4y_i^2 + 4y_i + 29$

$d2_{i+1} =$

$d2_i + 24x_i^2 + 48x_i + 8y_i^2 + 16y_i + 34$

}

else{

$y_{i+1} = y_i + 1$

$d1_{i+1} =$

$d1_i + 12x_i^2 + 44x_i + 45 - 32k + 8x_iy_i$

$+ 4y_i^2 + (20 - 32k)y_i$

$d2_{i+1} =$

$d2_i + 24x_i^2 + 72x_i + 8y_i^2 + (40 - 64k)y_i + 16x_iy_i$

$+ 70 - 96k$

}

$i = i + 1$

}

while $x_i < 4k$ {

display(x_i, y_i)

$y_{i+1} = y_i + 1$

if $d_{2_i} < 0$ {

$x_{i+1} = x_i + 1$

$d_{2_{i+1}} =$

$d_{2_i} + 24x_i^2 + 72x_i + 8y_i^2 + (40 - 64k)y_i + 16x_iy_i$

$+ 70 - 96k$

}

else{

$x_{i+1} = x_i$

$d_{2_{i+1}} =$

$= d_{2_i} + 24x_i + (8 - 64k) y_i + 16 x_i y_i + 12 - 96k$

}

i++

}