Engineering Mathematics Project

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Magnetic Resonance Imaging (MRI) Reconstruction from K-Space

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Preface

Notes on the project:

- Due date: 1402/11/09

- The project must be done individually. Each individual will present his results in an online session on 1402/11/10.

- Please submit your project report as a `.pdf` file. Include all outputs and final results in the report. Make sure to list the practice text questions and provide a concise explanation of your problem-solving approach in each section.

- Ensure that all codes are provided in a separate `.m/.py/.ipynb` file. If a code cannot be tested accurately upon submission, the reported results will be considered invalid, and no points will be awarded in such cases.

- You have the flexibility to utilize either `MATLAB` or `Python` for your project.

- Ensure that you save all files, including your report, codes, helper functions, and any additional outputs, if required, in a compressed file format such as `.zip` or `.rar`. This compressed file should then be uploaded to the Coursework CW submission platform.

- Your file names must be in the following format:

  `Project_#StudentID.zip/.rar/.pdf/.m/.py/.ipynb`

- The details of the grading system of this project will be provided in the coming days. Generally, the project is worth a total of 1 point, which could account for the grades lost in the final exam.

- In this project, it is essential to uphold the principles of academic integrity and refrain from any form of cheating or copying. Cheating undermines the learning process, diminishes personal growth, and compromises the trust placed in us as students/researchers/professionals. It is crucial to recognize that engaging in dishonest practices not only tarnishes our own reputation but also has serious consequences, both ethically and academically. We want to emphasize that if anyone is found to have cheated, their results will not be accepted in this project, and they will receive a zero mark.

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Some Text [4]

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2.1 MRI

MRI (Magnetic Resonance Imaging) is a non-invasive medical imaging technique that uses a combination of magnetic fields and radio waves to create detailed images of the body's internal structures, such as organs, tissues, and the skeletal system. It does not involve exposure to ionizing radiation, making it a safe and valuable diagnostic tool. MRI machines have two main components: a magnet producing a magnetic field of 1.5 to 3.0 Tesla and a computer manipulating the signals emitted by hydrogen atoms in the body. The procedure is painless and can produce high-resolution images of soft tissues, making it particularly useful for examining the brain, spinal cord, nerves, muscles, ligaments, and tendons. MRI is widely used for diagnosis, staging, and follow-up of various medical conditions, including tumors, anomalies, and other irregularities in different parts of the body.

2.2 K-space and MRI

In the context of magnetic resonance imaging (MRI), k-space refers to an array of numbers representing spatial frequencies in the MR image. Each point in k-space contains spatial frequency and phase information about every pixel in the final image. The concept of k-space is closely related to the Fourier transform, where the MR signal is mathematically transformed into k-space. The individual points in k-space do not correspond one-to-one with individual pixels in the image, and the brightness of each point in k-space represents the relative contribution of that point's unique spatial frequency to the final image. Therefore, k-space is a fundamental concept in MRI image acquisition and reconstruction, playing a crucial role in forming MR images.

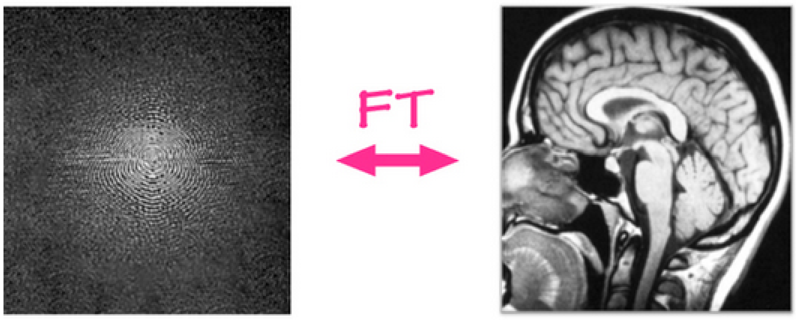


Figure 1: MRI image and its k-space representation
[3]

2.3 Questions

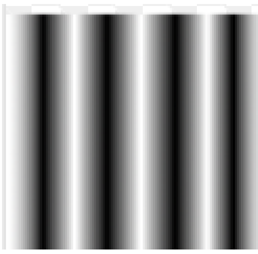
Answer the following questions.

1. Explain the concept of 2D Fourier transform. What are the bases of the 2D Fourier transform?

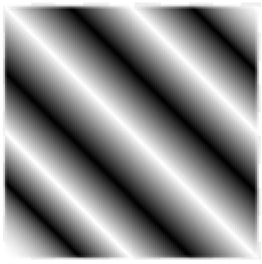
2. What is the connection between each point in k-space and raw image? What does each point in k-space represent?

3. Consider the images below:

(a) Image 1



(b) Image 2



(c) Image 3

(d) Image 4

Figure 2: sample images

We transform each image into k-space, each point in the plot below represents one of the images in the k-space. For each picture, select the correct point in the k-space.

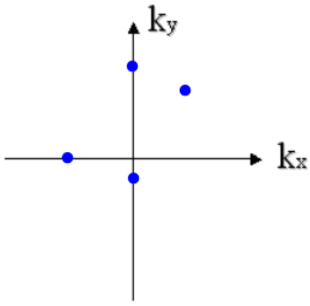
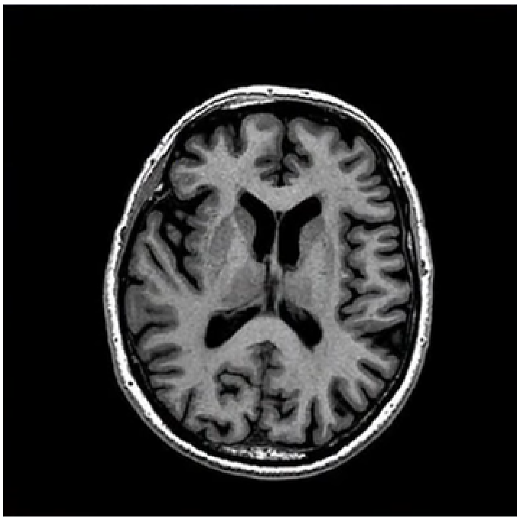
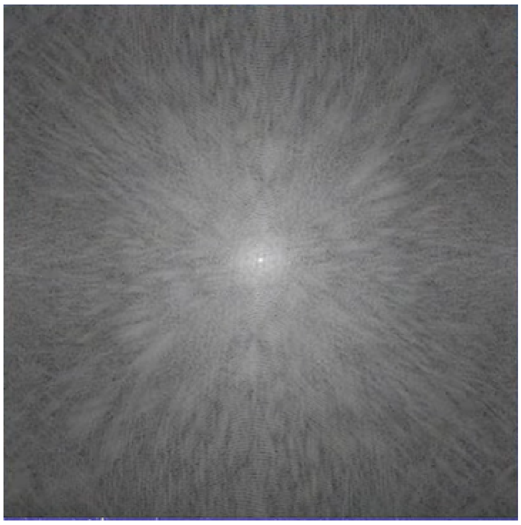


Figure 3: k-space with the transformed images

4. Consider the MRI image and its k-space transformed image below:



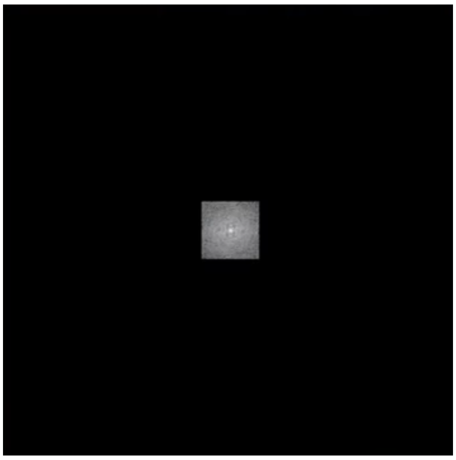
(a) MRI image



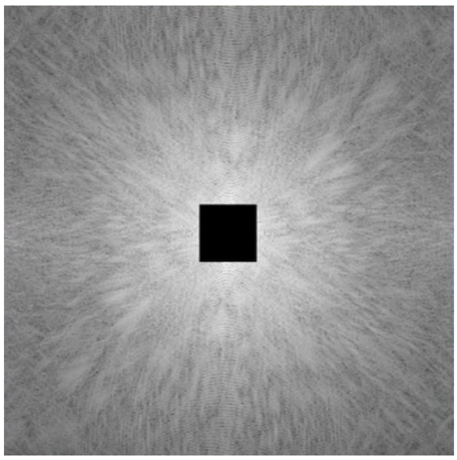
(b) k-space representation

Figure 4: raw MRI image and its corresponding k-space image [2]

What happens if we delete high-frequency bases in k-space? what about low-frequency bases? which one would preserve the original image more?



(a) k-space representation when we delete high-frequency elements



(b) k-space representation when we delete low-frequency elements

Figure 5: k-space images with changes

3 MRI Reconstruction

3.1 K-space and its properties

Magnetic Resonance Imaging (MRI) is a powerful medical imaging modality that provides detailed anatomical and functional information non-invasively. At the heart of the MRI imaging process lies a mathematical space known as k-space, where the raw data from an MRI scan is initially acquired. The transformation from k-space to the final clinical image involves a complex and crucial step known as image reconstruction.
Reconstructing MRI data from k-space involves transforming the raw k-space data, which is acquired during the MRI scan, into a meaningful image. Several techniques and algorithms are employed for this purpose.Like: Filtered Back Projection(FBP),Inverse Fourier Transform,Parallel Imaging and Compressed Sensing (CS),Iterative Reconstruction. In this project we are going to work on the second method (Inverse Fourier Transform) and learn how the parameters effect the final image.
For the purpose of this project,search and explain two of the above methods that has been mentioned.

3.2 Questions

1. Load the rawkneedata.mat that is attached to this file.
2. Reconstruct the MRI image using inverse Fourier transform .
3.

4 Metrics

The noise in unfiltered magnetic resonance (MR) images is usually distributed, spatially invariant, and white. This noise, which can be Gaussian or salt and pepper, affects the quality of MR images and subsequently impacts image processing tasks such as feature extraction and classification. To address this, various denoising methods have been developed. [5]

4.1 Histogram

Use the *matplotlib.pyplot.hist* in Python on *imhist* in MATLAB commands to display the histograms of the noisy and noiseless images. Explain the characteristics of the filter used to reduce noise in the frequency domain based on the two histograms obtained. Is it a high-pass, middle-pass, or low-pass filter?

4.2 Low Pass Filters

We want to reduce image noise by applying spatial filters. Read carefully about the Mean, median, and Gaussian filters, and briefly explain each in your report.

4.3 Mean Filter

Write a function that applies a Mean filter to an input image and returns the filtered image. You are not allowed to use any pre-built Mean filter functions.7 (kernel size is up to you.)

4.4 Median Filter

Write a function that, upon receiving the kernel size, applies the median filter to the input image and returns the filtered image. (Hint: Use *scipy.signal.medfilt* in Python or *medfilt2* in MATLAB)

4.5 Gaussian Filter

Write a function that applies a Gaussian filter to an input image and returns the filtered image. You are not allowed to use any pre-built Mean filter functions.6 (kernel size is up to you.)

4.6 Non-Local Means

The non-local means (NLM) noise reduction algorithm is well known as an excellent technique for removing noise from a magnetic resonance (MR) image to improve diagnostic accuracy. [6] Research this algorithm and describe its method. Then reduce the noise of the image using the pre-built functions *imnlmfilt* in MATLAB or *skimage.restoration.denoise-nl-means* in Python.

$$\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

Figure 6: Mean filter 3x3

$$1/16\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}\quad 1/273\begin{bmatrix}1&4&7&4&1\\4&16&26&16&4\\7&26&41&26&7\\4&16&26&16&4\\1&4&7&4&1\end{bmatrix}\quad 1/1003\begin{bmatrix}0&0&1&2&1&0&0\\0&3&13&22&13&3&0\\1&13&59&97&59&13&1\\2&22&97&159&97&22&2\\1&13&59&97&59&13&1\\0&3&13&22&13&3&0\\0&0&1&2&1&0&0\end{bmatrix}$$

Figure 7: Discrete approximation of the Gaussian kernels 3x3, 5x5, 7x7 [1]

4.7 Evaluation

Evaluate the output of the four methods discussed in this section using SNR and PSNR criteria and present the results in a table. Also, calculate these two criteria for the initial noisy image and declare which method performed better in noise reduction.

The SNR equation is defined as:

$$SNR = 10.log_{10}\left(\frac{\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j)]^2}{\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j)-K(i,j)]^2}\right) \tag{1}$$

I is the noise-reduced image(or the noisy image), and K is the noiseless image.
PSNR is most simply defined via the mean squared error (MSE) which is defined as:

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I(i,j)-K(i,j)]^2 \tag{2}$$

The PSNR equation is defined as:

$$PSNR = 10.log_{10}\left(\frac{MAX^2}{MSE}\right) \tag{3}$$

Here, MAX is the maximum possible pixel value of the MR image. [5]

References

[1] Discrete approximation of the gaussian kernels. https://www.researchgate.net/publication/325768087_Gaussian_filtering_for_FPGA_based_image_processing_with_High-Level_Synthesis_tools/figures?lo=1.

[2] Mri and k-space. https://www.youtube.com/watch?v=GF7Z8Sd9qYE.

[3] Mri raw image and its corresponding k-space image. https://mriquestions.com/what-is-k-space.html.

[4] Neurodegenerative diseases. https://www.niehs.nih.gov/research/supported/health/neurodegenerative/index.cfm. Last Reviewed: June 09, 2022.

[5] Hanafy M. Ali. Mri medical image denoising by filters. *intechopen*, 2018.

[6] Kyuseok Kim Yeong-Cheol Heo and Youngjin Lee. Image denoising using non-local means (nlm) approach in magnetic resonance (mr) imaging: A systematic review. *MDPI*, 2020.