Determination, Separation, and Tracking of an Unknown Time Varying Number of Maneuvering Sources By Bayes Joint Decision-Estimation

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Abstract—**This paper proposes a solution for the problem of determination, separation, and tracking of an unknown time-varying number of maneuvering sources based on a mixture of signals received by some omnidirectional sensors located at different places. To our knowledge, this problem has not been addressed in its full range, except for some simplified versions of it (e.g., with a fixed known number of sources) or with sensor arrays, which is quite different from our case (with omnidirectional sensors). This problem has at least three main subproblems: determine the number of sources, estimate signals of different sources (separation), and estimate state vector of the sources (tracking), while the number of sources can change with time and their dynamic models are uncertain. In other words, this problem includes one decision (number of sources) and two estimation subproblems (signal estimation and state estimation). These three subproblems are highly interrelated that solving one requires solutions of the other two. Therefore, they have to be considered jointly. Optimal Bayes joint decision and estimation (JDE) based on a generalized Bayes risk can handle such problems. However, here we have several additional difficulties, including *two* interrelated estimation subproblems, dynamic model uncertainty, correlated states of different sources, dependent dynamic models of different sources, nonlinearity of observation model, and two involved Markov process types (one for the number of sources and the other for the dynamic model). An approximate linear minimum mean square error (LMMSE) estimator is derived to deal with the interrelated estimation subproblems. Having considered all the aforementioned difficulties, Bayes JDE required terms are derived based on a recursive calculation of some key terms. The proposed method is theoretically solid and simple for implementation. It is examined by simulations.**

Keywords: Joint decision and estimation, target tracking, LMMSE estimation, Markov process, source separation.[1]

I. Introduction

The problem of jointly deciding the number of concurrent sources, separating their signals, and tracking their state is a complicated task, which is of great interest in different applications including surveillance, teleconferencing, cocktail party, etc. One may categorize the existing literature on source separation and tracking roughly into three classes from the viewpoint of the assumptions on modeling of the problem. The first class assumes separation of signals of some static sources [1], [2], [3]. Generally, it is assumed that mixtures of signals from different sources are received by different sensors located at different places. Then they try to find an inverse of the mixing operator to apply to the received signals in order to recover the signals propagated by different sources. It is assumed that sources do not move. In the second class it is assumed that sources can move, but the number of sources is fixed and known. The third class addresses a more general problem in which the number of sources is unknown and may be time-varying [4], [5], [6], [7]; however, it is often assumed that there is no dynamic model uncertainty for the motion of the sources. Also, from the viewpoint of observation model and sensors, the existing publications are mostly based on sensor arrays located at different places, which can capture much more information about sources than omnidirectional sensors; however, they are much more complicated and expensive.

The problem in this paper belongs to the third class. Thus, the problem is simultaneously deciding on the number of concurrent sources while the number may change over time, estimating their signals (separation), and tracking their state in the presence of dynamic model uncertainty based on observations received by some omnidirectional sensors located at different places. We are not aware of any existing publication considering the whole problem, although there are papers on simplified versions of this problem (e.g., with a fixed known number of sources) or based on sensor arrays rather than omnidirectional sensors. Most publications on this topic are based on sensor arrays. The meaning of source separation and tracking in this problem is somewhat different from that with sensor arrays. For example, in the latter after estimating direction of arrivals, signals from different sources are already separated by their directions of arrival (DoAs). Then, signals can be associated to their corresponding sources and tracking can be done based on DoAs (similar to multitarget tracking). For omnidirectional sensors, however, an observation received by a sensor is a mixture of signals from all the sources plus noise. So, an observation corresponding to a specific source can not be distinguished from another. Therefore, one is supposed to estimate states and signals of sources. Thus, our problem has three major subproblems: decision about the number of sources, estimation of source signals, and estimation of the state vector of the sources. In other words, this problem includes one decision subproblem and two estimation subproblems. These three subproblems are highly interrelated — the result of one relies on those of the other two. Therefore, they have to be considered jointly. Furthermore, there are additional difficulties in this problem: There is uncertainty about the dynamic model of each source (unlike most publications in

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the third class) and given observations, dynamic models of different sources are not generally independent; given observations, states of different sources are not generally independent; observation models are nonlinear. According to the dimensions of the working space and the speed of signal propagation, it is assumed that propagation of signals is instantaneous (i.e. speed is infinity). Also, we assume that reverberations are negligible in working space. This problem is naturally complex and has multiple uncertainties. Thus, computational complexity is an issue in this problem. So, computationally demanding methods are not desirable for either the whole structure of the problem (e.g., joint density estimation for determining the number of sources and estimating their signals and states) or a part of the problem (e.g., particle filter for nonlinear filtering). Therefore, for different parts of the problem we need some approaches which are desirable for both performance and computation.

Bayes Joint decision and estimation based on a generalized Bayes risk [8] can optimally handle problems in which decision and estimation affect each other [9], [10], [11]. Unlike some other methods for solving such problems (e.g., density estimation based methods), Bayes JDE directly addresses the desired goal — joint decision and point estimation which is suitable for the current problem. However, Bayes JDE has not been considered for solving this problem, yet. So, it is a new framework for solving this complicated problem. Although Bayes JDE can conceptually handle our problem, there are several difficulties in the derivation of its required terms for this problem, two of which are interrelated estimation subproblems and measurement nonlinearity. In order to deal with these two issues a linear minimum mean square error (LMMSE) estimator is derived (using an approximation) for state estimation in the presence of unknown signals and with a nonlinear observation model. The derived LMMSE estimator is new for this problem. The LMMSE estimator, which is the best linear estimator and has a low computational complexity, is derived in two steps: first, we obtain a prior state estimate of the sources in the presence of unknown signals based on a linear approximation, and then the final states and signals are estimated based on a posterior joint density of the two. Another point in derivation of the LMMSE estimator is: given observations, states of different sources are generally correlated and can not be considered separately. This correlation makes the problem more difficult and has to be taken care of in the derivation.

In order to deal with changes in the number of sources, we model the number of sources as a Markov process. Also, we model the evolution of the dynamic model of the sources as a group by Markov processes to cope with the dynamic model uncertainty. Thus, the multiple model approach can not be easily applied to this problem the way it is applied in a typical target tracking problem. One reason is that two types of Markov processes are involved in this problem, and another reason is that given observations, dynamic models of different sources are not generally independent. Thus, another difficulty in the derivation of the Bayes JDE required terms (including posterior state estimate given a hypothesis, and posterior hypothesis probability) is that two types of Markov processes are involved in this problem. Derivations of the essential terms based on these two types of Markov processes are new results for this problem. Finally, having considered all the issues, the required terms in the JDE framework are derived based on a recursive calculation of some key terms. The paper is organized as follows. Section II is for problem description and modeling. Section III considers a simple version of the whole problem in which the number of sources is fixed and known and the dynamic models are also known. An LMMSE estimator is derived for state estimation in the presence of unknown signals, based on an approximation. Section IV considers the whole problem. In this section a recursive JDE is presented for simultaneous decision about the number of concurrent sources and estimation of their states and signals. Due to space limitation, we omit details of the derivation of equations presented in section III and IV. Simulations are presented in section V.

II. Problem Description and Modeling

Consider a working space with N omnidirectional sensors located at different places and M_k sources at time k which move in the space. Each source may move following its own dynamic model and the number of sources is unknown and may change with time. A source may appear or disappear in time. Propagation of a signal from each source is omnidirectional and signals from different sources are independent. Also, observation noises of different sensors are Gaussian and uncorrelated. Each sensor receives a mixture (a linear combination) of signals from multiple sources. The coefficient corresponding to the signal from source $m \in \{1, ..., M_k\}$ received at sensor $n \in \{1, ..., N\}$ is a nonlinear function of the source location ($p^{so}_{m,k} : (x^{so}_{m,k}, y^{so}_{m,k}, z^{so}_{m,k})$) and sensor location ($p^{se}_n : (x^{se}_n, y^{se}_n, z^{se}_n)$). Note that we do not consider time index for the sensors. However, sensors can move as far as we know their locations. $s_{m,k}$ is the signal of source m at time k, and $v_{n,k}$ is the observation noise of sensor n at time k. The observation model is

$$o_{n,k} = \sum_{m=1}^{M_k} h(p^{se}_n, p^{so}_{m,k}) s_{m,k} + v_{n,k} \quad , \quad n = 1, ..., N \quad (1)$$

where $o_{n,k}$ is the observation received by sensor n at time k and $h(p^{se}_n, p^{so}_{m,k})$ is the mixing coefficient for the signal from source m at sensor n at time k. Equation (1) can be written in matrix form as

$$O_k = H_k S_k + V_k \quad (2)$$

where $O_k = (o_{1,k}, o_{2,k}, ..., o_{N,k})'$ is the observation vector, $H_k = (h_{1,k} \quad h_{2,k} \quad \dots \quad h_{M_k,k})$ is matrix of coefficients where $h_{m,k} = (h(p^{se}_1, p^{so}_{m,k}), h(p^{se}_2, p^{so}_{m,k}), \dots, h(p^{se}_N, p^{so}_{m,k}))'$, $S_k = (s_{1,k}, s_{2,k}, ..., s_{M_k,k})'$ is the vector of source signals, and $V_k = (v_{1,k}, v_{2,k}, ..., v_{N,k})'$ is the observation noise vector. In addition, we have

$$O_k = \sum_{m=1}^{M_k} h_{m,k} s_{m,k} + V_k \quad (3)$$

Let $X_{m,k}$ be the state vector of source m at time k. The linear dynamic equation for movement of source m in the Cartesian coordinates can be written as

$$X_{m,k} = F_{m,k} X_{m,k-1} + G_{m,k} w_{m,k-1} \quad (4)$$

where $w_{m,k-1}$ is zero mean Gaussian noise with variance $Q_{m,k-1}$. We assume the dynamic noises of all the sources have the same variance ($Q_{m,k} = Q_k$). In a 3D space we consider dynamic models along different axes being decoupled. So,

$$F_{m,k} = diag(F^x_{m,k}, F^y_{m,k}, F^z_{m,k})$$
$$G_{m,k} = ({G^x_{m,k}}', {G^y_{m,k}}', {G^z_{m,k}}')'$$

For example, for a nearly constant velocity model along x axis for source m at time k we have

$$F^x_{m,k} = \left[\begin{array}{cc} 1 & T \\ 0 & 1 \end{array} \right], \quad G^x_{m,k} = [\frac{T^2}{2}, T]' \tag{5}$$

We call all the sources together a *meta source* [2], and when we want to emphasize the number of sources in a meta source we denote it as MS_j which means a meta source consisting of j sources. So, the state vector of the meta source can be written as

$$X_k = F_k X_{k-1} + \Upsilon_k \tag{6}$$

where $F_k = diag(F_{1,k}, F_{2,k},..., F_{M_k,k})$, $\Upsilon_k = [\Upsilon'_{1,k}, \Upsilon'_{2,k},..., \Upsilon'_{M_k,k}]'$, and $\Upsilon_{m,k} = G_{m,k} w_{m,k-1}$. Also, $X_k = [X'_{1,k}, X'_{2,k}, ..., X'_{M_k,k}]'$ is the meta source state vector. A model of the signal from source m is assumed to be Gaussian with mean μ_m and variance σ^2_m: $s_{m,k} \sim N(\mu_m, \sigma^2_m)$. Since signals from different sources are independent, for the signal vector we have

$$S_k(\theta) \sim N(\mu, P_S) \tag{7}$$

where $\theta = (\mu, P_S)$, $P_S = diag(\sigma^2_1, \sigma^2_2, ..., \sigma^2_{M_k})$ and $\mu = (\mu_1, \mu_2, ..., \mu_{M_k})'$. We assume that the signal parameter θ is known. The obsrvation noise for all sensors is $V \sim N(0, R)$, where R is the covariance of observation noise. As mentioned, each mixing coefficient is usually assumed to be a function of the positions of the corresponding source and sensor. Here, we assume that the function is of the distance between them:

$$r_{m,n,k} = \left[(x^{se}_n - x^{so}_{m,k})^2 + (y^{se}_n - y^{so}_{m,k)})^2 + (z^{se}_n - z^{so}_{m,k})^2 \right]^{\frac{1}{2}}$$

Therefore, according to the propagation model of the signals, the following model is considered for the mixing coefficients:

$$h(p^{se}_n, p^{so}_{m,k}) = g(r_{m,n,k}) = \frac{1}{\sqrt{r^2_{m,n,k} + d^2}} \tag{8}$$

where d is a constant in order to prevent the denumerator from going to zero when a source is very close to a sensor. When a source is very close to a sensor the corresponding coefficient between the source and sensor is almost 1. Thus, we simply assume $d = 1$.

We clarify the proposed method for the whole problem in two steps in the following section. In the first step (subsection A), solution for the problem of separation and tracking of a fixed known number of non-maneuvering sources is presented. Then in subsection B, the whole problem is addressed.

[2]As we will see, since the observation vector is a mixture of data from multiple sources, tracking of the sources (specially the update step) should be done considering all of them together. So, this name is useful.

III. Determination, Separation, and Tracking of unknown Time-Varying Number of Maneuvering Sources

A. Separation and Tracking of Known Fixed Number of Non-maneuvering Sources

In this subsection, it is assumed that the number of sources is known and fixed, and dynamic models of sources are known. In other words, in equation (4) source signals and their mixing coefficients are unknown. $h_{m,k}$ is a vector-valued nonlinear function of the position of source m at time k. Thus, equation (4) is nonlinear in the state vector X_k of the meta source, and the signals ($s_{m,k}$) are also unknown. The goal is to estimate the state and its signal vector S_k. To do so, an idea is: consider the signals as nuisance parameters to be removed, estimate X_k, and then consider estimating S_k based on the estimate of X_k. However, it is not easy to handle the integrals involved in this approach. In addition, the integrals do not have a closed form and an approximation is needed. Most integral approximation methods are computationally demanding for such a problem. On the other hand, there are different kinds of uncertainties involved in our problem (regarding the source states, source signals, number of sources, and dynamic model for each source) which make it complicated already. Therefore, a less computationally complex approach is desired. We derive the LMMSE estimator [12], [13], [14] for estimating X_k in the presence of unknown signals, based on an approximation. Then, based on the estimate of X_k as prior information, we estimate X_k and S_k based on their joint posterior density. The LMMSE meta source state estimator is based on the following equations

$$\begin{aligned} \hat{X}^p_k &= E^*(X_k|O^k) \\ &= \hat{X}_{k|k-1} + C_{[X,O]_{k|k-1}} C^{-1}_{O_{k|k-1}} (O_k - \hat{O}_{k|k-1}) \end{aligned} \tag{9}$$

$$\begin{aligned} P^p_k &= E((X_k - \hat{X}^p_k)(X_k - \hat{X}^p_k)'|O^{k-1}) \\ &= P_{k|k-1} - C_{[X,O]_{k|k-1}} C^{-1}_{O_{k|k-1}} C'_{[X,O]_{k|k-1}} \end{aligned} \tag{10}$$

where "E" denotes expectation operator while the LMMSE estimator of X_k given observations O_k is denoted by $E^*(X_k|O^k)$ [12]. Also, the superscipt "p" indicates that this is a preliminary step for the final estimation of the state. Here,

$$\begin{aligned} \tilde{X}_k &= X_k - \hat{X}_{k|k-1} \\ \tilde{O}_k &= O_k - \hat{O}_{k|k-1} \\ \hat{X}_{k|k-1} &= E^*(X_k|O^{k-1}) \\ \hat{O}_{k|k-1} &= E^*(O_k|O^{k-1}) \\ P_{k|k-1} &= cov(\tilde{X}_k|O^{k-1}) \\ C_{O_{k|k-1}} &= cov(\tilde{O}_k|O^{k-1}) \\ C_{[X,O]_{k|k-1}} &= cov(\tilde{X}_k, \tilde{O}_k|O^{k-1}) \end{aligned}$$

In the following, we calculate the required terms in the LMMSE estimator. For prediction of the first two moments of the state vector we have

$$\begin{aligned} \hat{X}_{k|k-1} &= F_k \hat{X}_{k-1} \\ P_{k|k-1} &= F_k P_{k-1} F'_k + G_k Q_{k-1} G'_k \end{aligned}$$

and for observation prediction

$$\hat{O}_{k|k-1} \approx \sum_{m=1}^{M} (\hat{h}_{m,k|k-1})\mu_i \tag{11}$$

where a Taylor series expansion has been used to calculate the mean of $h_{m,k}$ and $\hat{h}_{m,k|k-1} = h_{m,k}|_{\hat{X}_{k|k-1}}$. Also, it should be noticed that since the number of sources is assumed known and fixed in this subsection, we drop time index k for M_k. It can be shown that

$$C_{[X,O]_{k|k-1}} \approx \sum_{m=1}^{M} P_{k|k-1}\breve{H}'_{m,k|k-1}\mu_m \tag{12}$$

$$\begin{aligned} C_{O_{k|k-1}} \approx & \sum_{m=1}^{M} [\sigma_m^2 \hat{h}_{m,k|k-1}\hat{h}'_{m,k|k-1} \\ & + \sigma_m^2 \breve{H}_{m,k|k-1}P_{k|k-1}\breve{H}'_{m,k|k-1}] \\ & + \sum_{m=1}^{M}\sum_{l=1}^{M} [\breve{H}_{m,k|k-1}P_{k|k-1}\breve{H}'_{l,k|k-1}\mu_m\mu_l] + R \end{aligned} \tag{13}$$

where $\breve{H}_{m,k|k-1} = \frac{\partial h_{m,k}}{\partial X'_k}|_{\hat{X}_{k|k-1}}$. Thus, the LMMSE meta source state estimator, based on the approximations above, is obtained by substituting the terms into (9) and (10).

Now, meta source signal and state vectors can be estimated based on their joint posterior density given observations and the available prior information about the state as follows:

$$p(X_k, S_k|O^k, \mathcal{P}_k) \propto p(X_k|S_k, O^k, \mathcal{P}_k)p(S_k|O^k, \mathcal{P}_k) \tag{14}$$

where $\mathcal{P}_k = \{\hat{X}_k^p, P_k^p\}$ denotes the prior information available based on the output of the LMMSE estimator (9) and (10). Then, we have

$$\begin{aligned} p(S_k|O^k, \mathcal{P}_k) &= \frac{p(O_k|S_k, O^{k-1}, \mathcal{P}_k)}{p(O_k|O^{k-1}, \mathcal{P}_k)} p(S_k|O^{k-1}, \mathcal{P}_k) \\ &\propto N(O_k; \hat{H}_k^p S_k, R)N(S_k; \mu, P_s) \propto N(S_k; \hat{S}_k, \hat{P}_{S,k}) \end{aligned} \tag{15}$$

where $\hat{H}_k^p = H_k|_{\hat{X}_k^p}$ is an estimate of the mixing coefficient matrix in (2), and

$$\hat{S}_k = \mu + \hat{P}_{S,k}\hat{H}_k^{p\prime}R^{-1}(O_k - \hat{H}_k^p\mu) \tag{16}$$

$$\hat{P}_{S,k} = P_S - P_S\hat{H}_k^{p\prime}(\hat{H}_k^p P_S \hat{H}_k^{p\prime} + R)^{-1}\hat{H}_k^p P_S \tag{17}$$

The final point estimate of the meta source state can be obtained based on the posterior density in (14) conditioned on the estimated signal vector $\hat{S}_k$ as

$$\begin{aligned} & p(X_k|\hat{S}_k, O^k, \mathcal{P}_k) \propto \\ & p(O_k|X_k, O^{k-1}, \hat{S}_k, \mathcal{P}_k)p(X_k|O^{k-1}, \hat{S}_k, \mathcal{P}_k) \end{aligned} \tag{18}$$

Therefore, similar to (9) and (10) we can calculate the LMMSE estimator of X_k, but this time based on the new prior which is the output of the previous LMMSE estimator $\{\hat{X}_k^p, P_k^p\}$:

$$\begin{aligned} \hat{X}_k &= E^*(X_k|O^k, \hat{S}_k, \mathcal{P}_k) \\ &= \hat{X}_k^p + \breve{C}_{[X,O]_{k|k-1}}\breve{C}^{-1}_{O_{k|k-1}}(O_k - \bar{O}_k) \end{aligned} \tag{19}$$

$$\begin{aligned} P_k &= E((X_k - \hat{X}_k)(X_k - \hat{X}_k)'|O^{k-1}, \hat{S}_k, \mathcal{P}_k) \\ &= P_k^p - \breve{C}_{[X,O]_{k|k-1}}\breve{C}^{-1}_{O_{k|k-1}}\breve{C}'_{[X,O]_{k|k-1}} \end{aligned} \tag{20}$$

where

$$\begin{aligned} \breve{X}_k &= X_k - \hat{X}_k^p \\ \breve{O}_k &= O_k - \bar{O}_k \\ \bar{O}_k &= E^*(O_k|O^{k-1}, \hat{S}_k, \mathcal{P}_k) \\ \breve{C}_{[X,O]_{k|k-1}} &= cov(\breve{X}_k, \breve{O}_k|O^{k-1}, \hat{S}_k, \mathcal{P}_k) \\ \breve{C}_{O_{k|k-1}} &= cov(\breve{O}_k|O^{k-1}, \hat{S}_k, \mathcal{P}_k) \end{aligned}$$

Then, the final estimate of X_k is

$$\hat{X}_k = \hat{X}_k^p + A_k B_k \Big(O_k - \sum_{m=1}^{M}[\hat{h}_{m,k}\hat{s}_{m,k}]\Big) \tag{21}$$

$$P_k = P_k^p - A_k B_k A'_k \tag{22}$$

where

$$\begin{aligned} A_k &= \sum_{m=1}^{M} P_k^p \breve{H}'_{m,k}\hat{s}_{m,k} \\ B_k &= \Big(\sum_{m=1}^{M}\sum_{l=1}^{M}[\breve{H}_{m,k}P_k^p\breve{H}'_{l,k}\hat{s}_{l,k}\hat{s}_{m,k}] + R\Big)^{-1} \end{aligned}$$

where $\hat{s}_{m,k}$ is the mth element of $\hat{S}_k$ in (16), $\breve{H}_{m,k} = \frac{\partial h_{m,k}}{\partial X'_k}|_{\hat{X}_k^p}$, and $\hat{h}_{m,k} = h_{m,k}|_{\hat{X}_k^p}$.

Therefore, with estimation of X_k, a solution for the problem of separation and tracking of a known fixed number of non-maneuvering sources is complete. In the next subsection, we consider the whole problem.

B. Determination, Separation, and Tracking of Unknow Time Varying Number of Maneuvering Sources

In this subsection, the problem presented in subsection A is extended to the case in which the number of sources is unknown and time-varying and the meta source dynamic model is uncertain. As explained, this is a joint decision and estimation (JDE) problem. We solve it based on optimal Bayes JDE method [8].

Optimal Bayes JDE: We briefly explain the optimal Bayes JDE framework [8] for a generic problem. Consider $\mathcal{N}$ hypotheses and $\mathcal{M}$ decisions. $\mathcal{H}^j$ stands for the jth hypothesis and $\mathcal{D}^i$ stands for the ith decision. The Bayes JDE risk is

$$\bar{R} = \sum_{i}^{\mathcal{M}}\sum_{j}^{\mathcal{N}}(\alpha_{ij}c_{ij} + \beta_{ij}E(C^e_{ij}(u,\hat{u})|\mathcal{D}^i, \mathcal{H}^j)P(\mathcal{D}^i, \mathcal{H}^j) \tag{23}$$

where u is the estimand and $\hat{u}$ is the estimate. c_{ij} is the cost of the ith decision while the jth hypothesis is true. α_{ij} and β_{ij} are weights of decision and estimation costs, respectively. $C^e_{ij}(u,\hat{u})$ is estimation cost fucntion.

Optimal JDE Solution: To minimize $\bar{R}$ in (23), the optimal decision $\mathcal{D}$ is

$$\mathcal{D} = \mathcal{D}^i \quad if \quad C_i(o) \le C_l(o), \forall l \tag{24}$$

where o is the observation and the cost is given by

$$C_i(o) = \sum_{j=1}^{\mathcal{N}}(\alpha_{ij}c_{ij} + \beta_{ij}E(C^e_{ij}(u,\hat{u})|\mathcal{D}^i, \mathcal{H}^j)P(\mathcal{H}^j|o) \tag{25}$$

Given any set of regions $\{\Gamma_1, ..., \Gamma_M\}$ (not necessarily a partition) of the observation space, the optimal estimate is available. However, for simplicity, if we consider the set of decision regions being a partition, the optimal estimator based on (23) with $C^e_{i,j}(u,\hat{u}) = (u-\hat{u})'(u-\hat{u})$ is given by

$$\hat{u} = \sum_{i=1}^{\mathcal{M}} 1(o;\Gamma_i)\check{u}_i \tag{26}$$

where

$$\check{u}_i = \sum_{j=1}^{\mathcal{N}} E(X|o,\mathcal{H}_j)\frac{\beta_{ij}P(\mathcal{H}_j|o)}{\sum_{l=1}^{N}\beta_{il}P(\mathcal{H}_l|o)}, \quad o \in \Gamma_i$$

$$1(o;\Gamma_i) = \begin{cases} 1 & \text{o} \in \Gamma_i \\ 0 & \text{else} \end{cases}$$

and $\check{u}_i$ is undefined if $o \notin \Gamma_i$. The optimal Bayes joint decision-estimate $(\mathcal{D},\hat{u})$ is the joint of the above optimal decision and optimal estimate.

Determination, separation, and tracking of unknown time-varying number of maneuvering sources based on recursive Bayes JDE: Provided that the number of sources is known, the meta source state and signal vectors can be estimated based on the approach presented in subsection A ((21) and (22)). Since we have a dynamic system in this problem, it is better to use a recursive version of Bayes JDE [10] for implementation. We assume that at most $\mathcal{M}$ concurrent sources are possible in the working space and model the number of sources with a Markov process having known initial probabilities and known transition probabilities:

$$P(\mathcal{H}^i_k|\mathcal{H}^j_{k-1}) = [\Pi_{\mathcal{H}}]_{ij} \quad i,j \in \{1,...,\mathcal{M}\}$$

where $\mathcal{H}^i_k$ denotes the event that the number of concurrent sources at time k is i, and $\Pi_{\mathcal{H}}$ is the Markov transition probability matrix. For simplicity, we assume that at each time at most one change happens in the number of sources, although an extension of our formulation to any number of changes is straightforward. Also, in order to deal with the uncertainty in the meta source dynamic model we define a Markov process to model the evolution of the dynamic model [15], where the initial probabilities are assumed known. Also, its transition probabilities are known and for an MS_j are

$$P(m^u_k|m^v_{k-1}) = [\Pi^j_m]_{u,v} \quad u,v \in \{1,...,jN_d\}$$

where m^u_k denotes the event that the dynamic model of the MS_j at time k is the uth one. N_d is the number of possible dynamic models for each source. One can consider each possible dynamic model of an MS_j as a combination of dynamic models of j different sources. We assume that the set of possible decisions is the same as that of hypotheses. Based on (23), the estimation cost function should be determined for the cases in which decision and hypothesis are the same and also the cases in which they are different. We consider the estimation cost function as

$$C^e_{ij}(X_k,\hat{X}_k) = \frac{1}{j}(X_k-\hat{X}_k)'(X_k-\hat{X}_k), \quad i=j$$

and equal to γ for $i \neq j$, where γ is a design parameter. Also, normalization by the number of sources (j) is because we do not want to have a higher cost for tracking more sources [16].

It should be noticed that Bayes JDE equations should be appropriately derived considering two Markov processes (for the number of sources and evolution of dynamic models) so that they can be recursively calculated. Due to space limitation we skip the details of the derivations and just present the final results. Through the following steps we explain our recursive Bayes JDE for the problem of determination, separation, and tracking of an unknown time-varying number of maneuvering sources.

1) Initialization:
$\hat{X}_0^{\mathcal{H}^j,m^u} \equiv \hat{X}_0^{j,u}$, $\forall j$, $\forall u$: State estimate of MS_j with the uth dynamic model at time 0, where $j \in \{1,2,...,\mathcal{M}\}$ and $u \in \{1,2,...,jN_d\}$.
$P_0^{j,u}$, $\forall j$, $\forall u$: Corresponding error covariance.
$P(\mathcal{H}^j_0)$, $\forall j$: Probability that the number of concurrent sources is j at time zero.
$\xi^{ij}_0 = E(C^e_{ij}(X_0,\hat{X}_0)|\mathcal{D}^i_0,\mathcal{H}^j_0)$: Expected estimation cost if the hypothesis is j and decision is i about the number of concurrent sources, where $i \in \{1,2,...,\mathcal{M}\}$.
$P(m^u_0|\mathcal{H}^j_0)$: Probability of the dynamic model u being the true one for MS_j at time zero.

2) Assume the following terms are available from the previous time $k-1$:

$$\hat{X}^{j,u}_{k-1}, P^{j,u}_{k-1}, P(\mathcal{H}^j_{k-1}|O^{k-1}), P(m^u_{k-1}|O^{k-1},\mathcal{H}^j_{k-1})$$

along with the posterior cost for decision at time $k-1$:

$$C_i(O^{k-1}) = \sum_{j=1}^{\mathcal{M}} c^{ij}_{k-1}P(\mathcal{H}^j_{k-1}|O^{k-1}) \tag{27}$$

$$c^{ij}_{k-1} = \alpha_{ij}c_{ij} + \beta_{ij}\xi^{ij}_{k-1}$$

The decision regions based on decision costs (27) are denoted as $\{\Gamma^1_{k-1},...,\Gamma^{\mathcal{M}}_{k-1}\}$, which is a partition, where Γ^i_{k-1} denotes the region for decision i at time $k-1$.

3) After receiving O_k, the terms in step 2 should be updated to obtain

$$\hat{X}^{j,u}_k, P^{j,u}_k, P(\mathcal{H}^j_k|O^k), P(m^u_k|O^k,\mathcal{H}^j_k) \tag{28}$$

4) To modify the decision regions, hypothesis probabilities in (27) are updated in (28). So, we have

$$C^*_i(O^k) = \sum_{j=1}^{\mathcal{M}} c^{ij}_{k-1}P(\mathcal{H}^j_k|O^k) \tag{29}$$

where C^* denotes the intermediate cost in which the hypothesis probabilities have been updated to $P(\mathcal{H}^j_k|O^k))$, but the expected estimation costs c^{ij}_{k-1} have not been updated to c^{ij}_k, yet. Decision regions are also modified based on the intermediate cost as $\{\Gamma^{*1}_k,...,\Gamma^{*\mathcal{M}}_k\}$, where

$$\Gamma^{*i}_k = \{O_k : C^*_i(O^k) \leq C^*_l(O^k), \forall l\}$$

5) According to our estimation cost function, it can be shown that expected estimation cost given a hypothesis and decision is calculated as

$$\xi^{ij}_k = \frac{1}{j}E\Big[(X_k-\hat{X}_k)'(X_k-\hat{X}_k)\Big|\mathcal{D}^i_k,\mathcal{H}^j_k\Big], \quad i=j$$

and it is equal to γ for $i \neq j$, where $E\Big[(X_k - \hat{X}_k)'(X_k - \hat{X}_k)\Big|\mathcal{D}_k^i, \mathcal{H}_k^j\Big]$ for $i = j$ is available via the tracking filter.

6) Decision regions are updated using the updated decision costs

$$
\begin{aligned}
C_i(O^k) &= \sum_{j=1}^{\mathcal{M}} c_k^{ij} P(\mathcal{H}_k^j | O^k) \\
c_k^{ij} &= \alpha_{ij} c_{ij} + \beta_{ij} \xi_k^{ij} \\
\Gamma_k^i &= \{O_k : C_i(O^k) \leq C_l(O^k), \forall l\}
\end{aligned}
$$

7) Finally, the decision-estimate output is $(\mathcal{D}_k^d, (\hat{X}_k, P_k))$, where

$$C_d(O^k) \leq C_i(O^k) \quad \forall i \neq d$$

and

$$\hat{X}_k = \arg\min_X \sum_{j=1}^{\mathcal{M}} \Big[\beta_{dj} E(C_{dj}(X_k, X)|\mathcal{D}_k^d, \mathcal{H}_k^j) \\ .P(\mathcal{D}_k^d, \mathcal{H}_k^j | O^k)\Big]$$

based on our estimation cost function it can be shown that $\hat{X}_k = \hat{X}_k^{\mathcal{D}_k^d}$, and $P_k = P_k^{\mathcal{D}_k^d}$, which can be calculated based on the terms in (28). The superscript $\mathcal{D}_k^d$ means "decision d has been made about the number of concurrent sources at time k".

8) For the next time $k+1$, go to step 2, with $X_k^{j,u}$, $P_k^{j,u}$, $P(\mathcal{H}_k^j | O^k)$, and $P(m_k^u | O^k, \mathcal{H}_k^j)$.

IV. Performance Evaluation Measures

Some measures are considered for performance evaluation of the proposed method. The first one is joint performance measure (JPM) [10] for simultaneous evaluation of decision and estimation. We also consider the average over the outputs of the decision part over different Monte Carlo runs to evaluate decision performance. Furthermore, we use average Euclidean-error (AEE) measure [17] for evaluating the estimation part (source tracking and signal estimation). However, it should be noticed that JPM is the best one because in this problem joint performance is the goal. The JPM is the expectation of a distance between observation vector and the predicted observation vector by the JDE algorithm as follows:

$$\rho(O_k, \hat{O}_{k|k-1}) = E[d(O_k, \hat{O}_{k|k-1})] \tag{30}$$

where d is the Euclidean difference between two vectors. Joint performance measure (30) can be computed using sample average over N_{JDE} number of predicted observations generated by JDE and N_{MC} Monte Carlo runs of the algorithm as

$$\rho(O_k, \hat{O}_{k|k-1}) \approx \frac{1}{N_{MC}} \frac{1}{N_{JDE}} \sum_{j=1}^{N_{MC}} \sum_{i=1}^{N_{JDE}} d(O_k^j, \hat{O}_{k|k-1}^{i,j}) \tag{31}$$

Also, AEE measure for the position, velocity, or signal estimate evaluation is

$$d_{AEE}(Y_k, \hat{Y}_k) = \frac{1}{N_{MC}} \sum_{j=1}^{N_{MC}} d(Y_k^j, \hat{Y}_k^j) \tag{32}$$

where Y can be the position, velocity, or signal vector of the meta source and the superscript j denotes the jth Monte Carlo run.

V. Simulations

In order to demonstrate performance of the proposed method we consider the following scenario. In this scenario there are 8 omnidirectional sensors located at $(0, 0)$, $(0, 50)$, $(0, 100)$, $(50, 0)$, $(50, 100)$, $(100, 0)$, $(100, 50)$, and $(100, 100)$. For simplicity, we assume that the maximum number of concurrent sources is 2, and the diagonal elements of the transition probability matrix of the Markov process for the number of concurrent sources are 0.99 and off-diagonals 0.01. Also, to handle the uncertainty in the dynamic model, for each source we consider three models along each axis: one nearly constant velocity (M_1) and two nearly constant acceleration models with specified accelerations -1 (M_2) and 1 (m/s^2) (M_3) [18], [19]. The transition probability matrix of a Markov process corresponding to the dynamic model of a meta source is determined considering the transition probability matrix for the dynamic model of each source along each axis as

$$\begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.05 & 0.95 & 0 \\ 0.05 & 0 & 0.95 \end{bmatrix}$$

where the probability of direct transition from M_2 to M_3 (and vice versa) is zero. In the simulated scenario, source 1 exists in the space at the begining (Fig. 1). Then, at time 10 source 2 appears and exists until time 20 when it disappears at time 21. From time 1 to 10 source 1 moves at a nearly constant velocity, at time 11 it changes to have a nearly constant acceleration until time 25 when it switches back to a nearly constant velocity. Source 2 follows a model with a constant acceleration all the time during its presence. The parameter values in this scenario are as follows. Sampling interval is 0.5 (second), the dynamic noise standard deviation is 0.003, standard deviation of the observation noise of each sensor is 0.2. We consider signals of different sources being independent with the same Gaussian distribution with parameters $\theta = (220, 3500)$. Note that these are parameters of signals at the sources, not at the sensors, because a sensor receives signal from a source multiplied by a corresponding coefficient (attenuation), plus some observation noise. Decision cost coefficients in JDE are $c_{ij} = 1$ for $i \neq j$ and 0 otherwise. For relative weights of decision and estimation we consider $\alpha_{ij} = 1$ for every i and j and $\beta_{ij} = 0.8$ for $i = j$ and 0.7 otherwise. The number of predicted observations generated by JDE at each time (for JPM computation) is $N_{JDE} = 10$, the number of Monte Carlo runs is $N_{MC} = 100$. The design parameter γ in estimation cost function is set to 8. In the derivation of the JDE equations, we assume that at each time at most one change happens in the number of concurrent sources. Then, a prior disappearance probability of each source is required. These probabilities can be automatically calculated according to the distances of sources to the exit area of the working space (one can also incorporate other aspects into these probabilities). In other words, the farther from the exit area, the less probability of disappearance. In the simulation, we consider disappearance probability being inversely proportional to the distance to the exit area. Also, the exit area is around the corner $(100, 0)$. Moreover, in the derivation of the JDE equations a prior density for the initial state $(x_0, v_{x0}, y_0, v_{y0})$ of a newborn source is required. In the simulation, we assume that this prior density is a multivariate Gaussian with mean equal to the true value and covariance matrix $diag(10, 1, 10, 1)$ (the entrance area is around the origin corner). Also, prior probabilities of the number of sources (at the beginning) are set to be equal for both hypotheses. We are not aware of any other existing method proposed for solving this problem of determination, separation, and tracking of an unknown time varying number of maneuvering sources using omnidirectional sensors. In order to compare performance of our method with a benchmark, we run an algorithm in which the number of sources is known at each time.

Fig. 2 shows the JPM result for our Bayes JDE based method in comparison with the algorithm that knows the number of sources (ideal). As it can be seen, at the beginning the error of JDE is larger than that of the ideal one, because JDE has no prior information about the number of sources. The difference between the two algorithms is negligible until time 10, at which the number of sources changes. Due to this change, JDE error increases since its prediction about the number of sources is not good at time 10. Since the ideal algorithm knows the number of sources all the time, its error does not change much. The reason for its relative increase in the error at time 10 is that estimating signals and states of two concurrent sources is more difficult than that of one source. Then, at time 21 source 2 disappears and the performance of JDE degrades. However, JDE can recognize the change in the number of sources within a couple of time steps and again its performance gets better. Except for time steps at which the number of sources changes and transient time steps after a change, the difference between JDE and the ideal algorithm is not much and JDE performed very well. Fig. 3 shows the average decision output of JDE (over Monte Carlo runs) about the number of concurrent sources at each time. Also, Fig. 4 shows AEE of position estimates and Fig. 5 shows AEE of signal estimates for the proposed method in comparison with the ideal one. When there exists one source in the space and JDE decides correctly, AEE computation for position or signal is straightforward. Also, when there exist two sources and JDE decides correctly, in AEE computation we consider the true and estimate state pairs, which lead to less position estimate error, and then compute their corresponding signal AEEs. However, when there exists one source, and JDE does not decide correctly, the pair with a smaller distance (based on position) is considered for AEE. In addition, when there exist two sources and JDE does not decide correctly, the pair with less distance is considerd for AEE computation while for the other source no AEE is computed. Therefore, it is clear that AEE of the state and signal estimates, or decision output, can not illustrate the whole performance of the method well, and JPM is the best measure for this problem.

VI. Sammary

A solution has been proposed for the problem of determination, separation, and tracking of an unknown time-varying number of maneuvering sources using observations received by omnidirectional sensors. To our knowledge, this is the first such successful attempt. This problem is complex and thus computationally demanding approaches are not desirable here. In other words, a theoretically solid and simply implementable

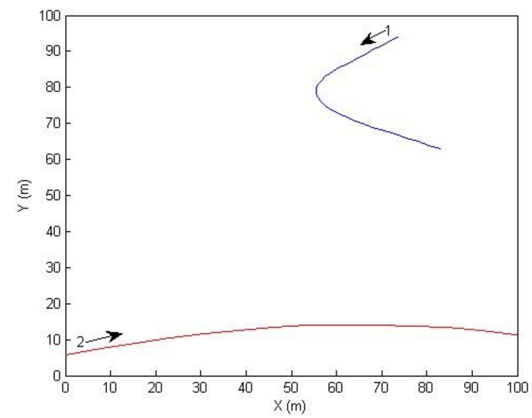

Fig. 1. Working space with two sources

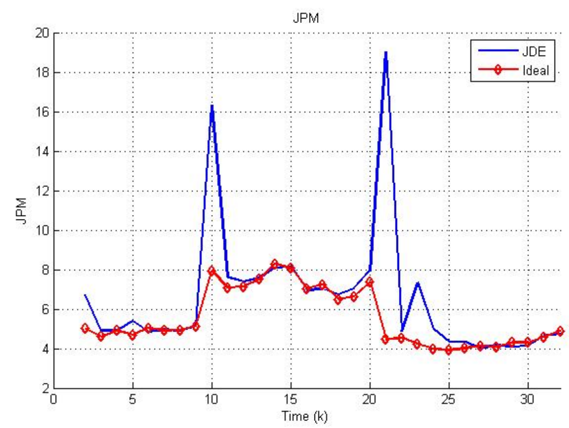

Fig. 2. Joint performance measure, JDE in comparison with ideal algorithm that knows the number of sources over time. Changes in the number of sources happen at time 10 and 21.

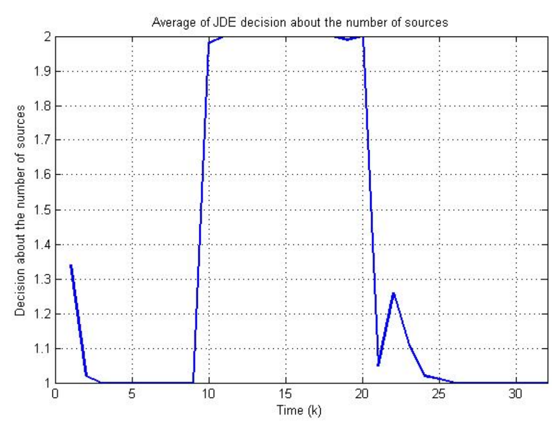

Fig. 3. Average over JDE decision about number of sources over time. The number of sources: one (time 1-9), two (time 10-20), one (time 21 on)

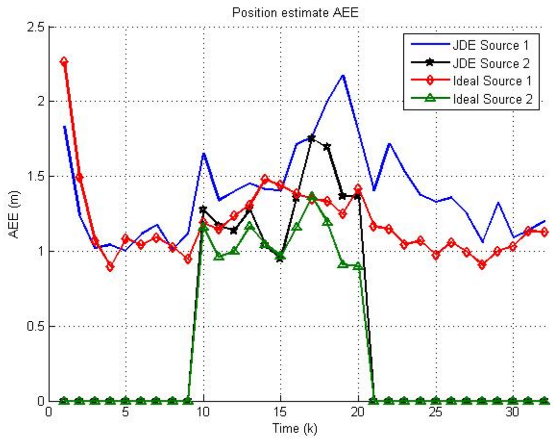

Fig. 4. AEE of position estimates of sources. Source 2 exists only over time 10 to 20.

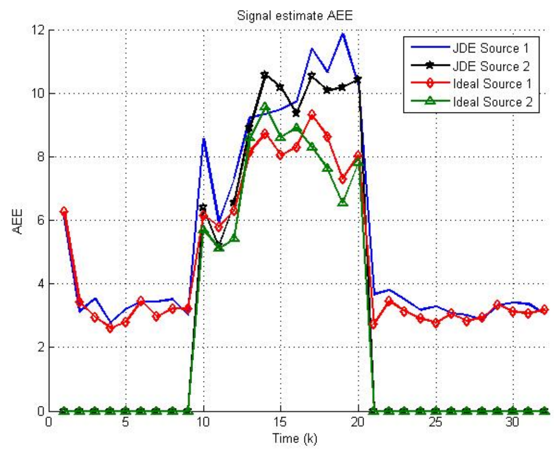

Fig. 5. AEE of signal estimates of sources. Source 2 exists only over time 10 to 20.

approach is desirable. Determination of the number of sources and estimation of their state and signal vectors by minimizing Bayes joint decision and estimation risk is theoretically solid and simple. So, Bayes JDE is quite desirable for this complicated problem. Also, the Bayes JDE method is flexible enough to address the problem. Moreover, Bayes JDE is a new approach in the area of source separation and tracking.

An LMMSE estimator has been derived for the estimation part of the problem, which is a new result for this problem. This estimator is theoretically solid and the best among linear estimators. According to the results, the approximation is adequate for this case, although a higher order approximation is possible without much difficulty. The derived estimators for the state and signal vectors are not complex at all, which is desirable for this complicated problem. The multiple model approach is a powerful method for handling dynamic model uncertainty of a maneuvering target. However, it is not easily applicable in this problem due to uncertainty in the number of sources. Therefore, the corresponding equations for handling changes in the number of sources and dealing with maneuvers of the sources have been jointly derived in a recursive form which is also a new result.

In order to evaluate performance of the proposed method, a comprehensive joint performance measure (JPM) has been used, since other measures can not illustrate the whole performance and they just consider some aspects separately [17]. The results show that the proposed method is effective and performs well in comparison with the ideal one. Therefore, the proposed method has different desirable properties we have been looking for: theoretically solid and simple for implementation.

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