Lecture 25: Linear Regression Part II

Chapter 7.2-7.4

Questions for Today: Example From Text

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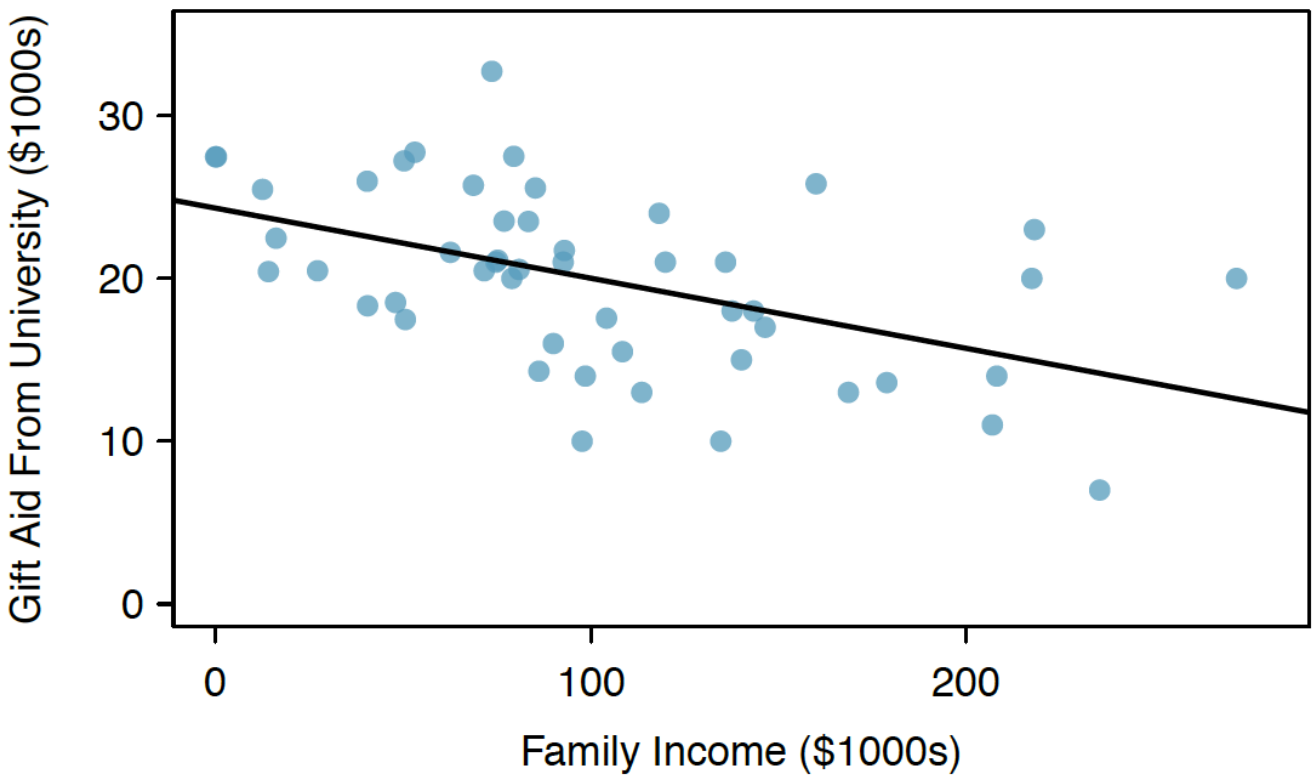
- Explanatory variable: family income
- Outcome variable: gift aid



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Using these values,

	family income in \$1000's (x)	gift aid in \$1000's (y)
mean	$\overline{x} = 101.8$	$\overline{y} = 19.94$
sd	$s_x = 63.2$	$s_y = 5.46$
		$R = -0.499$

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they fit the least-squares line:

$$\begin{aligned} \widehat{y} &= b_0 + b_1 x \\ \widehat{\text{aid}} &= 24.3 - 0.0431 \times \text{family\_income} \end{aligned}$$

What do 24.3 and -0.0431 mean?

Point Estimates of Intercept

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In this case it is relevant since some families make no income, but the intercept may have little or no practical value if there are no observations near $x = 0$.

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Even though we've labeled aid as the outcome variable, we are not positing a causal relationship; just an association.

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What would be the gift aid given to a family with \$1,000,000 (i.e. $x = 1000$) in family income?

$$24.3 - 0.0431 \times 1000 = -18.8$$

The school will take \$18,800 dollars away from you?

Categorical Predictor x With Two Levels

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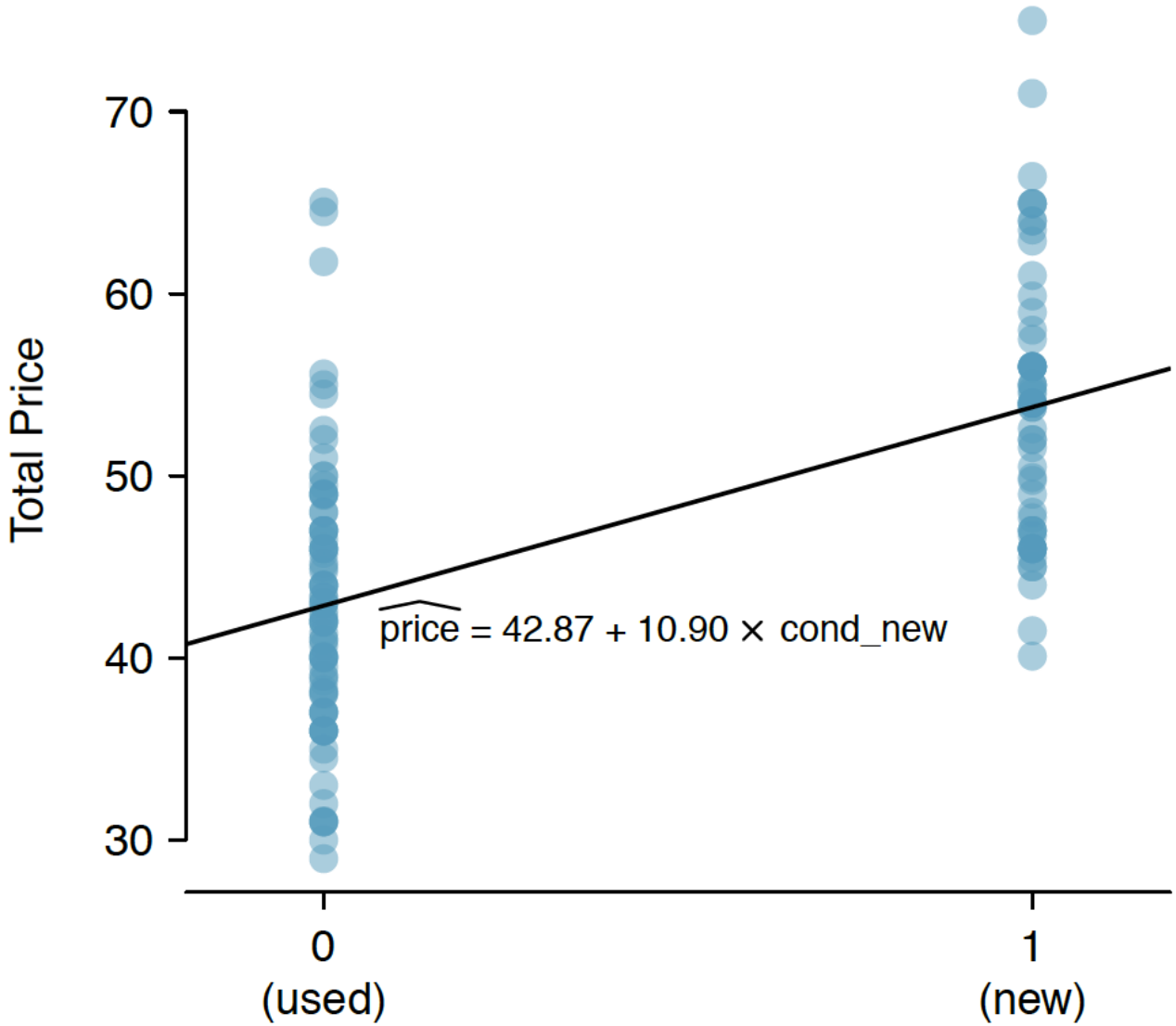
The linear model is thus

$$\widehat{\text{price}} = b_0 + b_1 \times \text{cond\_new}$$

Categorical Predictor x With Two Levels



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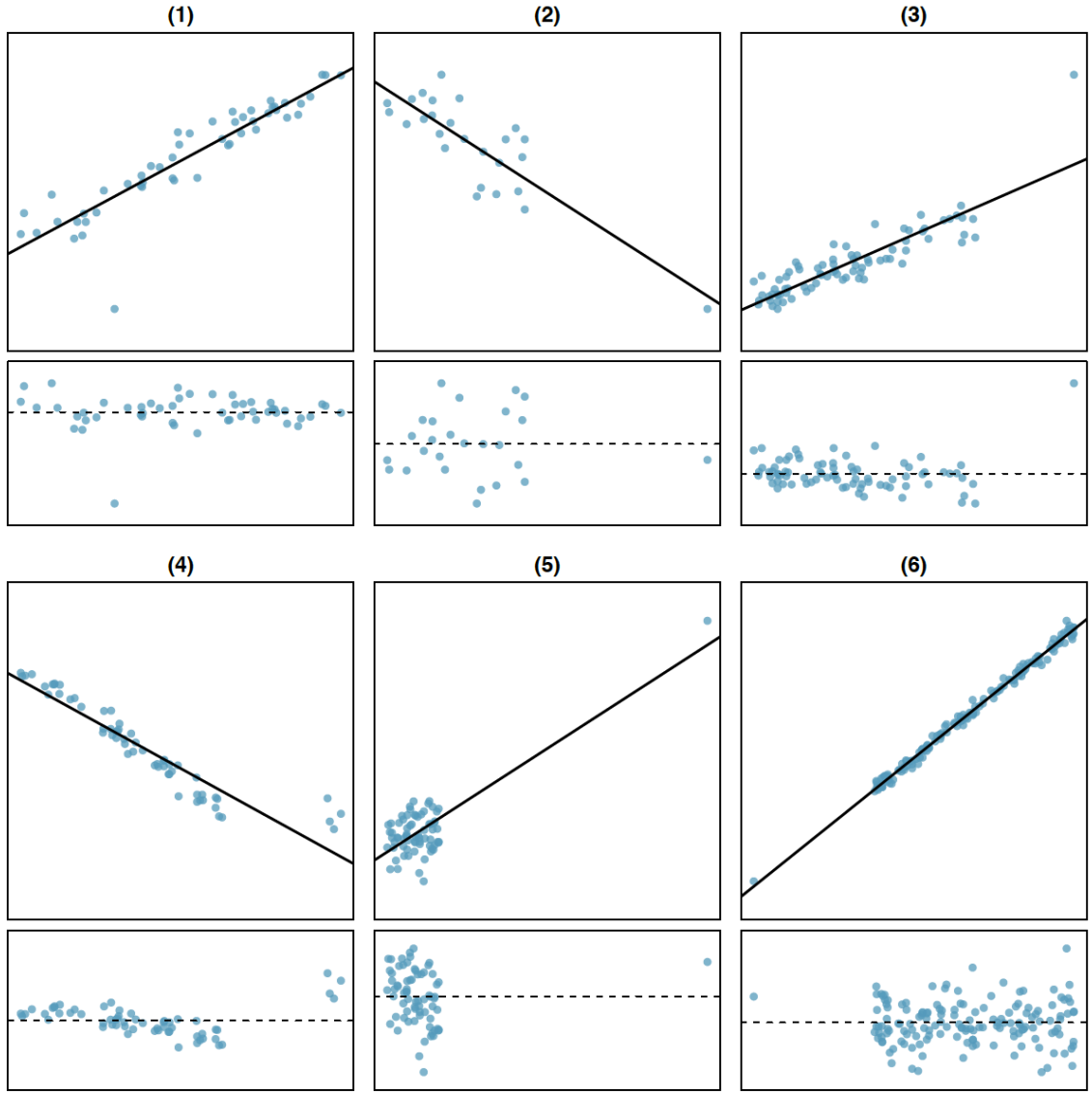
This can be generalized for predictor variables x with more than two levels.

Types of Outliers in Linear Regression



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Points that fall horizontally away from the center of the cloud tend to pull harder on the line, so we call them points with high leverage, i.e. large influence.

Next Example

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But first, are the changes from

- 100 to 200
- 100,100 to 100,200

the same?

Next Time

Multiple Regression: As opposed to simple linear regression where there is only one predictor/explanatory variable x, we now consider many variables $x_1, x_2, \ldots$