Lecture 12: Sampling Distributions & Standard Errors

Chapter 4.1

Goals for Today

Start Chapter 4: Arguably the most important chapter as it goes to the heart of what statistical inference is. Three important definitions today:

1. point estimate
2. sampling distribution
3. standard error

Point Estimates

Behavior of Point Estimates

Ex: Say we draw a random sample of size $n = 100$ from a large population that is normally distributed with $\mu = 5$ and $\sigma = 2$.

Behavior of Point Estimates

Ex: Say we draw a random sample of size $n = 100$ from a large population that is normally distributed with $\mu = 5$ and $\sigma = 2$.

Two Important Questions:

Behavior of Point Estimates

Ex: Say we draw a random sample of size $n = 100$ from a large population that is normally distributed with $\mu = 5$ and $\sigma = 2$.

Two Important Questions:

1. Is $\overline{x}$ going to be exactly 5?

Behavior of Point Estimates

Ex: Say we draw a random sample of size $n = 100$ from a large population that is normally distributed with $\mu = 5$ and $\sigma = 2$.

Two Important Questions:

1. Is $\overline{x}$ going to be exactly 5?
2. Say we get $\overline{x} = 5.025$. If we repeat this procedure: i.e. generate a new sample of size $n = 100$ and compute $\overline{x}$), will we get $\overline{x} = 5.025$?

Behavior of Point Estimates

Ex: Say we draw a random sample of size $n = 100$ from a large population that is normally distributed with $\mu = 5$ and $\sigma = 2$.

Two Important Questions:

1. Is $\overline{x}$ going to be exactly 5?
2. Say we get $\overline{x} = 5.025$. If we repeat this procedure: i.e. generate a new sample of size $n = 100$ and compute $\overline{x}$), will we get $\overline{x} = 5.025$?

We need to characterize this random error.

Behavior of Point Estimates

Let's repeat this procedure, say, 1000 times:

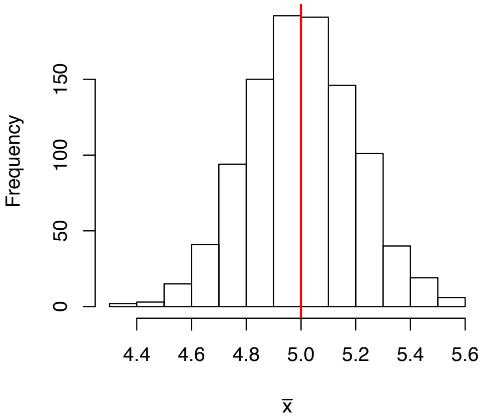
Behavior of Point Estimates

Let's repeat this procedure, say, 1000 times:

1st time	We get $\overline{x} = 4.831$
2nd time	We get $\overline{x} = 5.104$
3rd time	We get $\overline{x} = 4.965$
...	
1000th time	We get $\overline{x} = 4.957$

Sampling Distribution

This histogram is the 1000 instances of $\overline{x}$, where each $\overline{x}$ is based on a sample of $n = 100$. This is the sampling distribution of $\overline{x}$:



Sampling Distributions

Sampling Distributions

We can define the sampling distributions for any point estimate, not just $\overline{x}$:

Sampling Distributions

We can define the sampling distributions for any point estimate, not just $\overline{x}$:

- s
- the sample median
- etc.

Sampling Distributions

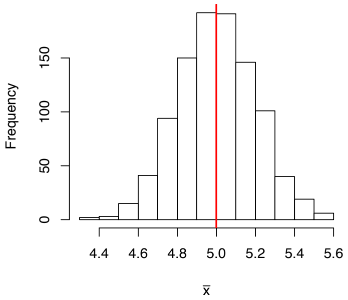
We can define the sampling distributions for any point estimate, not just $\overline{x}$:

- s
- the sample median
- etc.

We will only focus on sample means, including the sample proportion $\widehat{p}$.

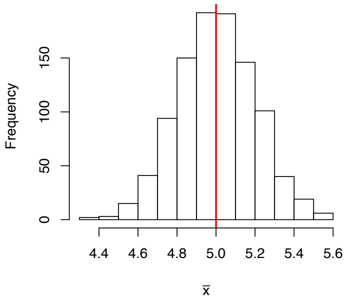
Measure of Spread

What about spread? $[4.6, 5.4]$ contains roughly 95% of the data.



Measure of Spread

What about spread? $[4.6, 5.4]$ contains roughly 95% of the data.



$$
\begin{aligned}
& [\mu - 2SD, \mu + 2SD] = [4.6, 5.4] \\
\Rightarrow \quad & \text{length of interval is } 4SD = 5.4 - 4.6 \\
\Rightarrow \quad & SD = 0.2
\end{aligned}
$$

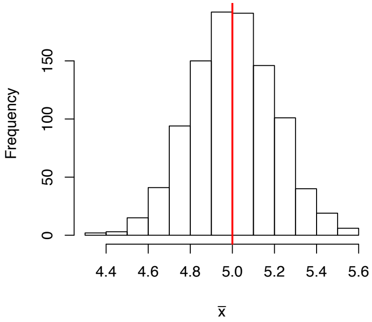
Standard Errors

Standard Error of $\overline{x}$

Back to Histogram

Samples were of size $n = 100$ with $\sigma = 2$. We estimated that the SD of the sampling distribution was 0.2. Using the formula:

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = \frac{2}{10} = 0.2$$



Standard Error of the Sample Mean $\overline{x}$

Compare 1000 instances of $\overline{x}$ when

Standard Error of the Sample Mean $\overline{x}$

Compare 1000 instances of $\overline{x}$ when

- $n = 100$. $SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = 0.2$

Standard Error of the Sample Mean $\overline{x}$

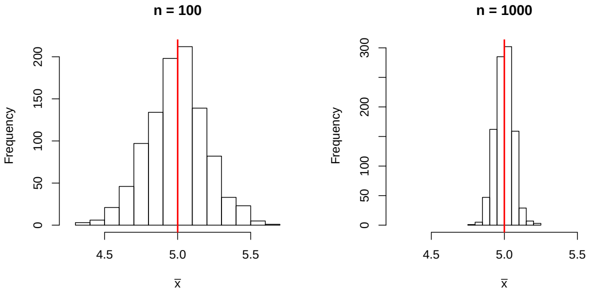
Compare 1000 instances of $\overline{x}$ when

- $n = 100$. $SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = 0.2$
- $n = 1000$. $SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{1000}} = 0.0632$. Smaller!

Standard Error of the Sample Mean $\overline{x}$

Compare 1000 instances of $\overline{x}$ when

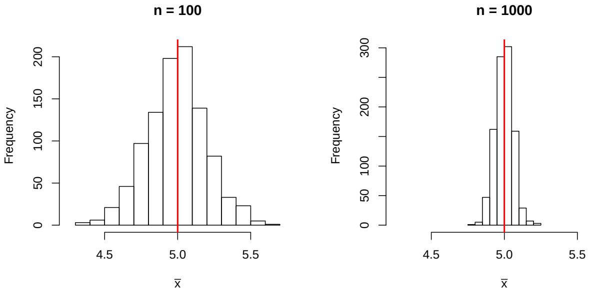
- $n = 100$. $SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = 0.2$
- $n = 1000$. $SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{1000}} = 0.0632$. Smaller!



Standard Error of the Sample Mean $\overline{x}$

Compare 1000 instances of $\overline{x}$ when

- $n = 100$. $SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = 0.2$
- $n = 1000$. $SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{1000}} = 0.0632$. Smaller!



Both are "accurate", but the estimates on the right are "more precise."

Repeated Sampling

Standard Error of the Sample Mean

Example

Say in you take a simple random sample of 100 runners in a race and you are interested in their ages:

- $\overline{x} = 35.05$
- $s = 8.97$

Example

Say in you take a simple random sample of 100 runners in a race and you are interested in their ages:

- $\overline{x} = 35.05$
- $s = 8.97$

Assuming that the 100 runners consist of less than 10% of the population, the standard error of $\overline{x}$ is

$$SE = \frac{s}{\sqrt{100}} = \frac{8.97}{10} = 0.897$$

Population Distribution vs Sampling Distribution

Recap

- Point estimates are based on a sample $x_1, \ldots, x_n$ and are used to estimate population parameters.

Recap

- Point estimates are based on a sample $x_1, \ldots, x_n$ and are used to estimate population parameters.
- The sampling distribution characterizes the (random) behavior of point estimates.

Recap

- Point estimates are based on a sample $x_1, \ldots, x_n$ and are used to estimate population parameters.
- The sampling distribution characterizes the (random) behavior of point estimates.
- The standard deviation of a sampling distribution is the standard error: it quantifies the uncertainty/variability of point estimates.

Next Time

- Confidence Intervals
- When quoting survey results, what does: "the results of this survey are estimated to be accurate within 3.1 percentage points, 19 times out of 20" mean?
- Big One: Central Limit Theorem