Lecture 16: Sample Size and Power

Chapter 4.6

Last Time: Reedie Sleep Example

Tested number of hours of sleep:

- $H_0 : \mu = 7$
- $H_A : \mu > 7$

Two-Sided Alternative Hypothesis

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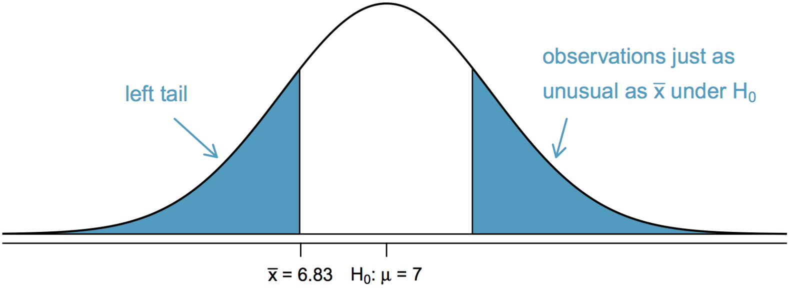
The the p-value would be double: $2 \times 0.007 = 0.014$. Picture:



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Ronald Fisher, the creator of p-values, never intended for them to be used this way: http://en.wikipedia.org/wiki/P-value#Criticisms

Goals for Today

- More in depth discussion of
  - 10% sampling rule
  - Skew condition to check to use the normal model
- How big a sample size do I need?
- Statistical power
- Statistical vs practical significance

10% Sampling Rule

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Issue: Sampling without vs with replacement.

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i.e.

- the sampling distribution is just one point: the true μ.
- if we repeat this procedure many times, we get the same value each time: 0 variability.

Sampling and the SE

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Answer: If not

- the *SE* in confidence intervals is off
- the z-scores of $\overline{x}$ have the wrong denominator

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- On page 164: the population distribution is not strongly skewed
- On page 167: the data are not strongly skewed

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- On page 167: the data are not strongly skewed
- On page 168: the distribution of sample observations is not strongly skewed

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- On page 167: the data are not strongly skewed
- On page 168: the distribution of sample observations is not strongly skewed
- On page 185: the population data are not strongly skewed

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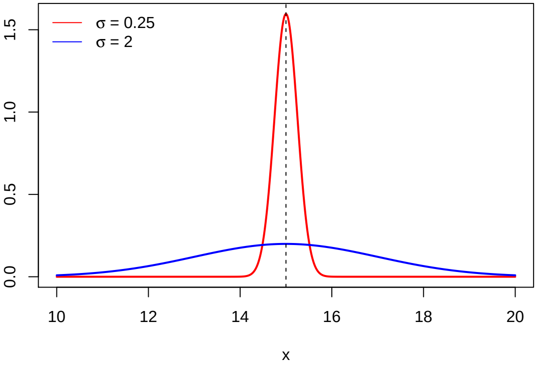
1. The true population distribution from which you are drawing your sample observations/data $x_1, \ldots, x_n$ is not too skewed.
2. The histogram (visual estimate) of the sample observations/data $x_1, \ldots, x_n$ is not too skewed.

Sample Size: Thought Experiment

Say you have two distributions with $\mu = 15$ but different σ.



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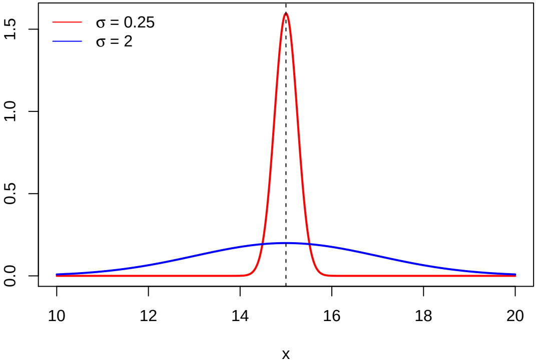

Which of the two distributions do you think will require a bigger *n* to estimate μ "well"?

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The margin of error is half the width of the CI.

Say we knew the true standard deviation σ, then

$$\text{Margin of Error} = 1.96\frac{\sigma}{\sqrt{n}}$$

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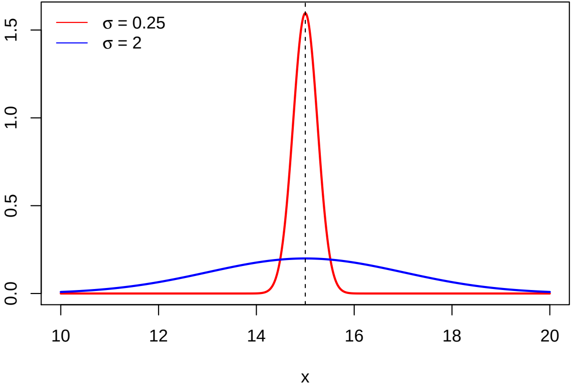
So

- As σ goes up, you need more n
- As z^* goes up, i.e. higher confidence level, you need more n
- As the desired margin of error goes down, you need more n

Back to Thought Experiment

For the same desired maximal margin of error m and same confidence level, we need a larger n to estimate the mean of the blue curve:



Type II Error Rate and Power

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