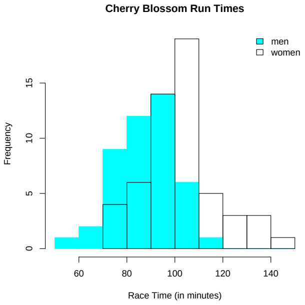
Lecture 31: t Distribution for Difference of Two Means

Chapter 5.4

Question for Today

In Chapter 5.2 we asked: Did men ($n_m = 45$) run faster than women ($n_w = 55$) in the Cherry Blossom Race?



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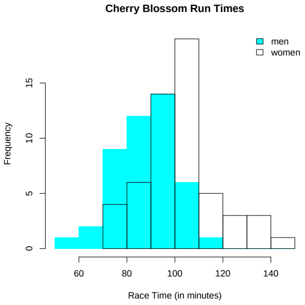

What can we say about $\mu_1 - \mu_2$ when n_1 and n_2 are both small?

Components

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4. Hypothesis tests using t-statistic

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The true formula for degrees of freedom is

$$df = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{(s_1^2/n_1)^2/(n_1 - 1) + (s_2^2/n_2)^2/(n_2 - 1)}$$

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Rather, for this class, use the smaller of n_1 and n_2 minus 1 i.e.

$$\min(n_1, n_2) - 1$$

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- The two samples are independent

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If so, we can leverage this fact to make the t distribution approach slightly more precise.

Pooled Standard Deviation Estimate

The pooled standard deviation estimate is

$$s^2_{pooled} = \frac{s_1^2 \times (n_1 - 1) + s_2^2 \times (n_2 - 1)}{n_1 + n_2 - 2}$$

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$$s_{pooled}^2 = \frac{s_1^2 \times (n_1 - 1) + s_2^2 \times (n_2 - 1)}{n_1 + n_2 - 2}$$

So use s_{pooled}^2 instead of s_1^2 and s_2^2 in SE:

$$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{s_{pooled}^2}{n_1} + \frac{s_{pooled}^2}{n_2}} = \sqrt{s_{pooled}^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

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The -1 and -2 are degrees of freedom corrections.

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Caveats: Only pool when background research/intuition indicates the population σ_1 and σ_2 of the two groups are nearly equal.