Lecture 21: Difference of two proportions

Chapter 6.2

Question for today

How do we infer about a difference in proportions $p_1 - p_2$?

Surveys

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Let's infer about the difference in proportion of people who approve. Any guesses which is higher?

Example from Text

	Sample size n_i	Approve (%)	Disapprove (%)	Other (%)
people who do not buy it will pay a penalty given first	771	47	49	3
people who cannot afford it will receive financial help from the gov't given first	732	34	63	3

Example from Text

	Sample size n_i	Approve (%)	Don't Approve (%)
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So $\widehat{p}_1 - \widehat{p}_2 = 0.47 - 0.34 > 0$: people are more likely to support Obamacare in the first scenario.

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- Both sample proportions $\widehat{p}_1$ and $\widehat{p}_2$ are approximately normal:

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  - success/failure condition: at least 10 successes and failures
- the two samples are independent from each other

... for Sampling Dist'n of $\widehat{p}_1 - \widehat{p}_2$ Being Normal

The sampling distribution of $\widehat{p}_1 - \widehat{p}_2$ is approximately Normal with

- mean $p_1 - p_2$
- standard error

$$SE_{\widehat{p}_1 - \widehat{p}_2} = \sqrt{SE^2_{\widehat{p}_1} + SE^2_{\widehat{p}_2}} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

Standard Error

Recall we showed that the SE for $\overline{x}_1 - \overline{x}_2$ was

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Compare this to

$$SE_{\widehat{p}_1 - \widehat{p}_2} = \sqrt{SE^2_{\widehat{p}_1} + SE^2_{\widehat{p}_2}} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

What p_1 & p_2?

What p_1 & p_2 do we

- Use to check success/failure condition?
- Use in $SE_{\widehat{p}_1 - \widehat{p}_2}$?

For

- Confidence intervals: plug in $\widehat{p}_1$ and $\widehat{p}_2$
- Hypothesis tests: plug in pooled estimate $\widehat{p}$

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    - Group 1: 362 successes and $771 - 362 = 409$ failures
    - Group 2: 249 successes and 483 failures

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$$\text{point estimate} \pm z^* \times SE = 0.13 \pm 1.65 \times 0.025 = (0.09, 0.17)$$

Interpretation

Two key observations:

- $(9\%, 17\%)$ does not contain 0, suggestive of a true difference.
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More support Obamacare if stated as follows:

People who do not buy it will pay a penalty while people who cannot afford it will receive financial help from the government.

Hypothesis Tests

Now we are interested in testing the difference of two proportions:

$$\begin{aligned} & H_0 : p_1 - p_2 = 0 \\ \text{vs} \qquad & H_1 : p_1 - p_2 \neq 0 \end{aligned}$$

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i.e. under H_0 the two proportions are both equal to some value p:

$$p_1 = p_2 = p$$

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we use a pooled estimate $\widehat{p}$ of the proportion p. i.e. as if there were no difference between them, so we can combine them:

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The SE to use is:

$$SE_{\widehat{p}_1 - \widehat{p}_2} = \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n_1} + \frac{\widehat{p}(1-\widehat{p})}{n_2}} = \sqrt{\widehat{p}(1-\widehat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Exercise 6.31 on Page 305

A 2010 survey asked 827 randomly sample voters in California "How do you feel about drilling for oil and natural gas off the coast of California?"

	College Grad	
	Yes	No
Support	154	132
Oppose	180	126
Don't Know	104	131
Total	438	389

Exercise 6.31 on Page 305

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Total	438	389

Test at the $\alpha = 0.10$ significance level if the proportion of college graduates who support off-shore drilling is different than that of non-college graduates.

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2. We assume that both groups are sampled independently.

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- Test statistic: z-score of $\widehat{p}_1 - \widehat{p}_2$ under $H_0 : p_1 - p_2 = 0$

$$z = \frac{\text{point estimate} - \text{null value}}{SE} = \frac{0.013 - 0}{0.033} = 0.392$$

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- p-value: 0.6922. i.e. we fail to reject H_0. We don't have strong evidence of a difference in support.

Jury Selection

Preview of next lecture: In many trials a big issue is the racial makeup of the jury.

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Preview of next lecture: In many trials a big issue is the racial makeup of the jury.

Question: is there a way to figure out if there is a racial bias in jury selection?

Jury Selection

Say we have a juror pool (registered voters) where the racial breakdown is:

Race	White	Black	Hispanic	Other	Total
Registered Voters	72%	7%	12%	9%	100%

Jury Selection

If we pick $n = 100$ jurors at random (i.e. unbiasedly), we expect the breakdown of counts to be:

Race	White	Black	Hispanic	Other	Total
Registered Voters	72%	7%	12%	9%	100%
Representation	72	7	12	9	$n = 100$

Jury Selection

Say we observe the following counts:

Race	White	Black	Hispanic	Other	Total
Registered Voters	72%	7%	12%	9%	100%
Representation	0	0	100	0	$n = 100$

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Representation	0	0	100	0	$n = 100$

Fairly obvious bias in juror selection!

Jury Selection

But what about the following? Is there a bias? i.e. a non-random mechanism at play?

Race	White	Black	Hispanic	Other	Total
Registered Voters	72%	7%	12%	9%	100%
Representation	75	6	11	8	$n = 100$

Next Two Lectures

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Two tests we'll see:

- Goodness-of-fit tests: for frequency tables
- Tests for independence: for contingency/two-way tables