Lecture 27: Model Selection + Multiple Regression Conditions

Chapter 8.2-8.3

Question for Today

Recall the Mario Kart analysis

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)     41.34       1.71   24.15  < 2e-16 ***
condused        -5.13       1.05   -4.88 2.91e-06 ***
stockPhotoyes    1.08       1.06    1.02    0.308
duration        -0.03       0.19   -0.14    0.888
wheels           7.29       0.55   13.13  < 2e-16 ***
---

Residual standard error: 4.901 on 136 degrees of freedom
Multiple R-squared:  0.719, Adjusted R-squared:  0.7108
```

Question for Today

This is the full model: every explanatory variable provided is included.

Recall Occam's Razor: all other things being equal, simpler is better.

In our case: simpler = less predictor variables included in the model.

The act of choosing which predictor variables to include in your model is model selection.

Two Common Strategies

There are two stepwise regression methods that add/subtract one variable at a time:

- Backward Elimination
- Forward Selection

The criteria used will be p-values.

Backward Elimination

1. Start with the full model
2. While there still exists statistically non-significant variables
   - 2.1 Identify the variable with the largest p-value and drop it
   - 2.2 Refit the model
3. Report model once there are no more non-significant variables

Backward Elimination

Starting here:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	41.3415	1.7117	24.15	0.0000
`cond_used`	-5.1306	1.0511	-4.88	0.0000
`stockPhotoyes`	1.0803	1.0568	1.02	0.3085
`duration`	-0.0268	0.1904	-0.14	0.8882
`wheels`	7.2852	0.5547	13.13	0.0000

Backward Elimination

Drop `duration`.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	41.3415	1.7117	24.15	0.0000
`cond_used`	-5.1306	1.0511	-4.88	0.0000
`stockPhotoyes`	1.0803	1.0568	1.02	0.3085
`duration`	-0.0268	0.1904	-0.14	0.8882
`wheels`	7.2852	0.5547	13.13	0.0000

Backward Elimination

Drop `stockPhotoyes`.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	41.2245	1.4911	27.65	0.0000
`cond_used`	-5.1763	0.9961	-5.20	0.0000
`stockPhotoyes`	1.1177	1.0192	1.10	0.2747
`wheels`	7.2984	0.5448	13.40	0.0000

Backward Elimination

Done.

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	42.3698	1.0651	39.78	0.0000
cond_used	-5.5848	0.9245	-6.04	0.0000
wheels	7.2328	0.5419	13.35	0.0000

Forward Selection

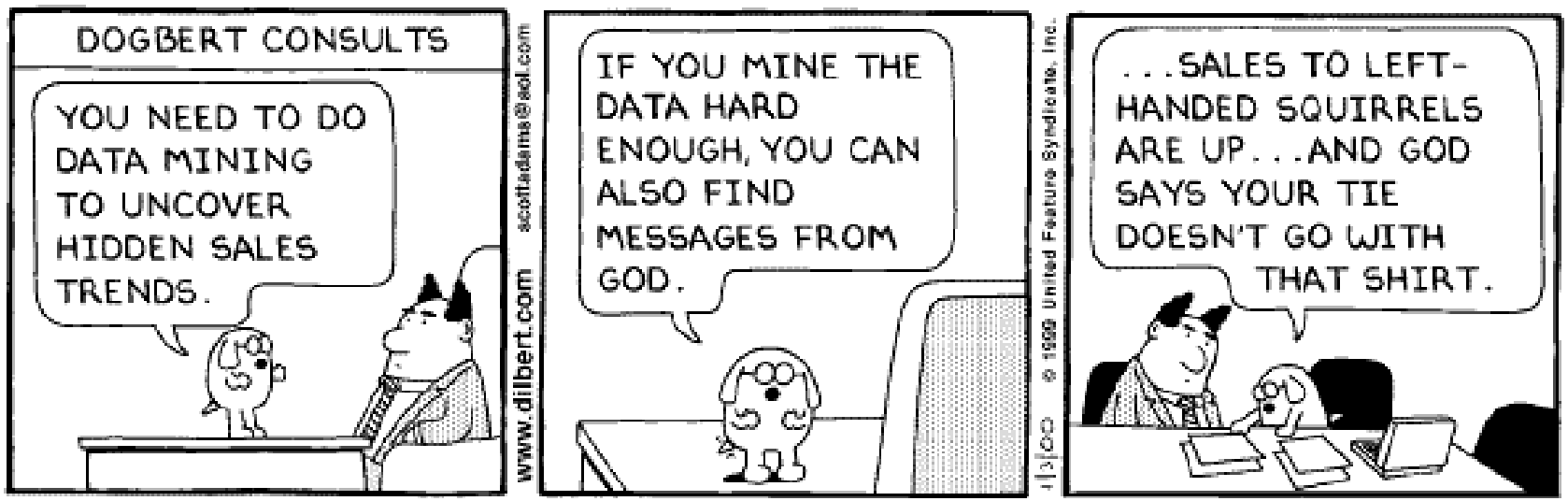
1. Start with the model with no variables
2. Fit all models with one possible additional variable
3. Add the additional variable with the smallest p-value if its significant
4. Repeat steps 2 and 3 until there are no significant additional variables.

Criticisms of the Techniques

Critics regard stepwise regression as data dredging, where intense computation is used as a substitute for subject area expertise when deciding on a model.

Data mining involves automatically testing huge numbers of hypotheses about a single data set by exhaustively searching for combinations of variables that might show a correlation.

Criticisms of the Techniques



Assumptions of Multiple Regression

- The residuals e_i of the model
  - are nearly normal
  - have nearly constant variance
  - are independent
- Each variable is linearly related to the outcome
- No pattern in residuals relative to dependent variables.

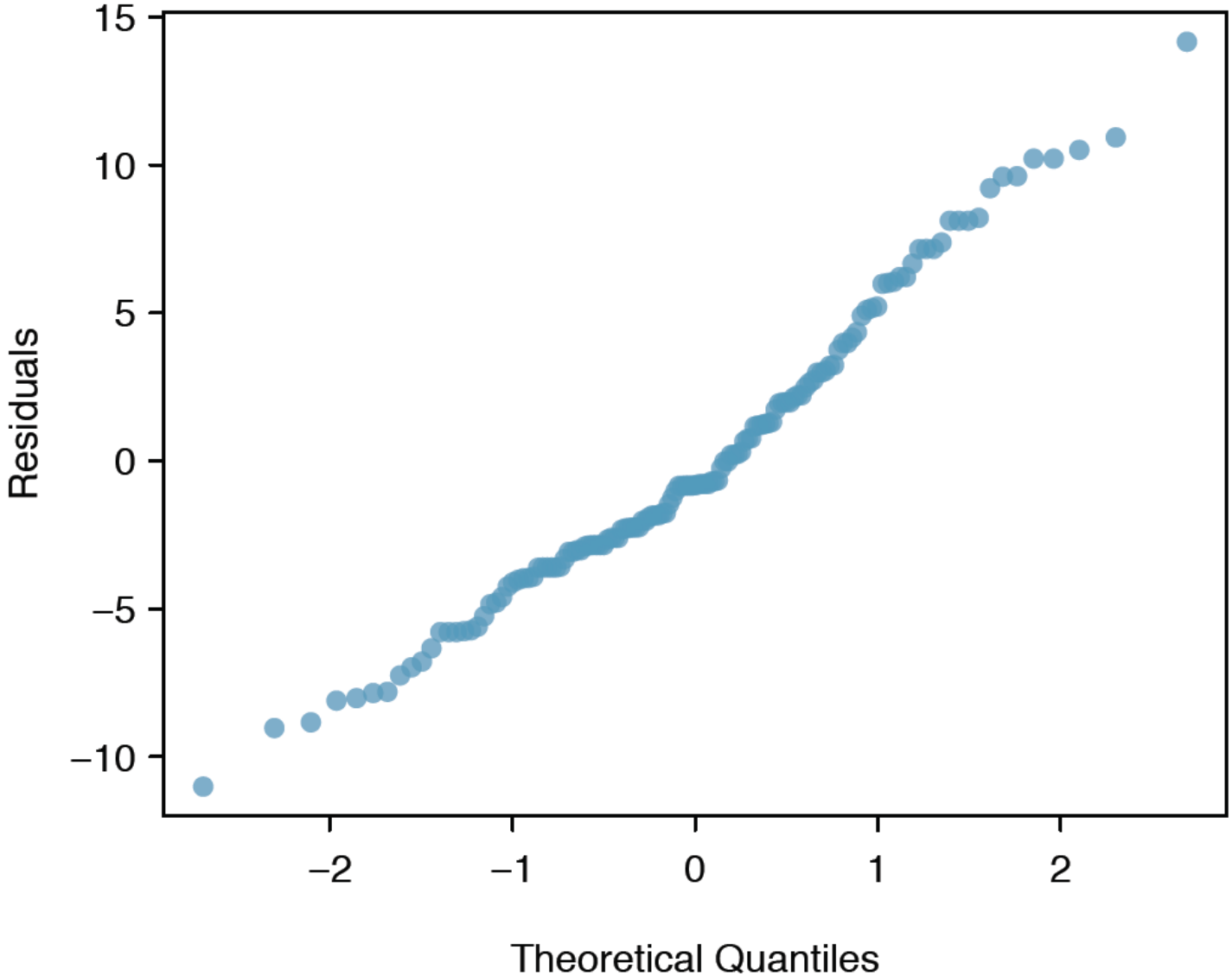
Example Model

We investigate plots for the following model:

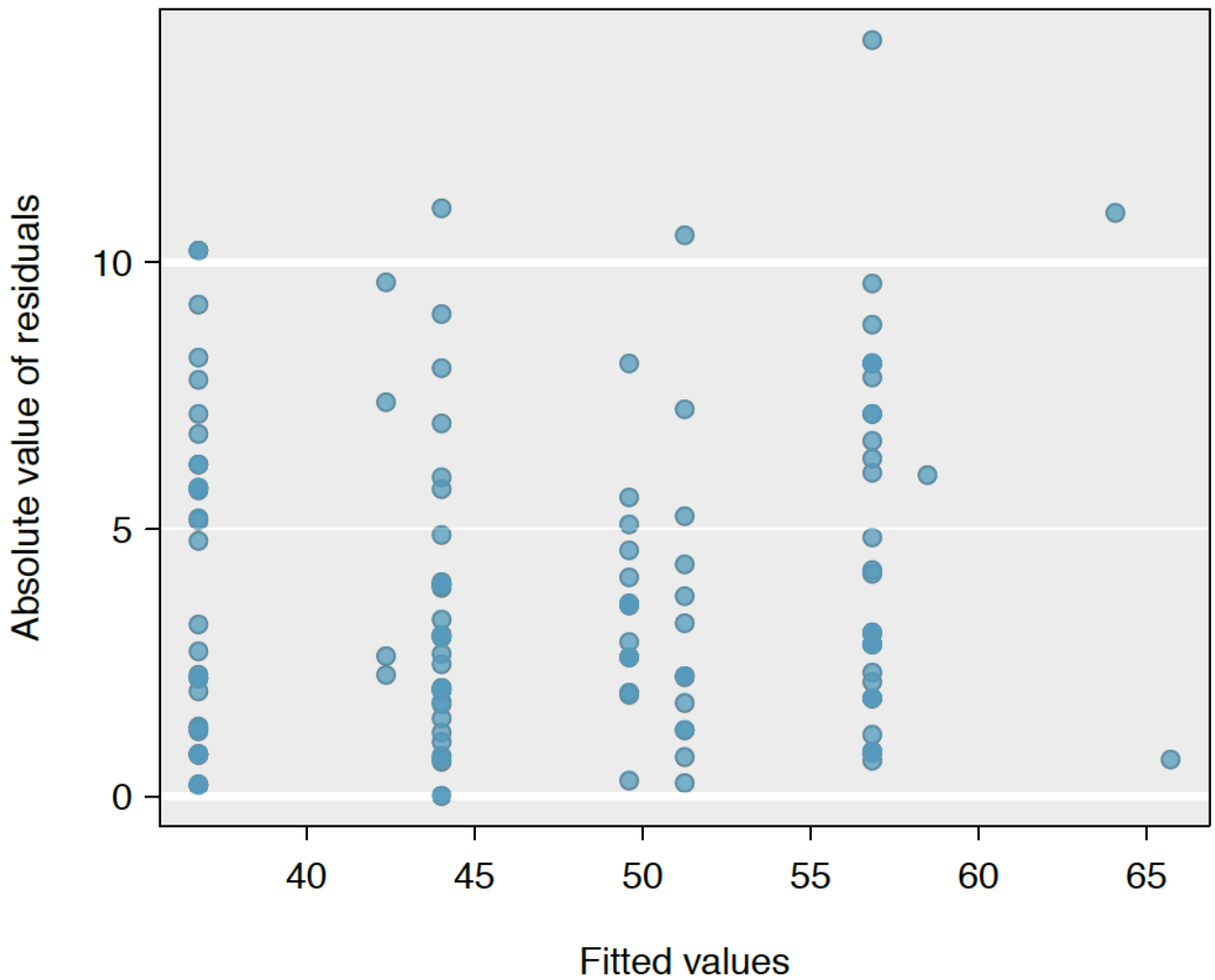
$$\widehat{\texttt{price}} = b_0 + b_1 \times \texttt{cond\_new} + b_2 \times \texttt{wheels}$$

- Normal probability plot of residuals
- Absolute values of residuals against fitted values: look for non-constant variance
- Residuals against each predictor variable

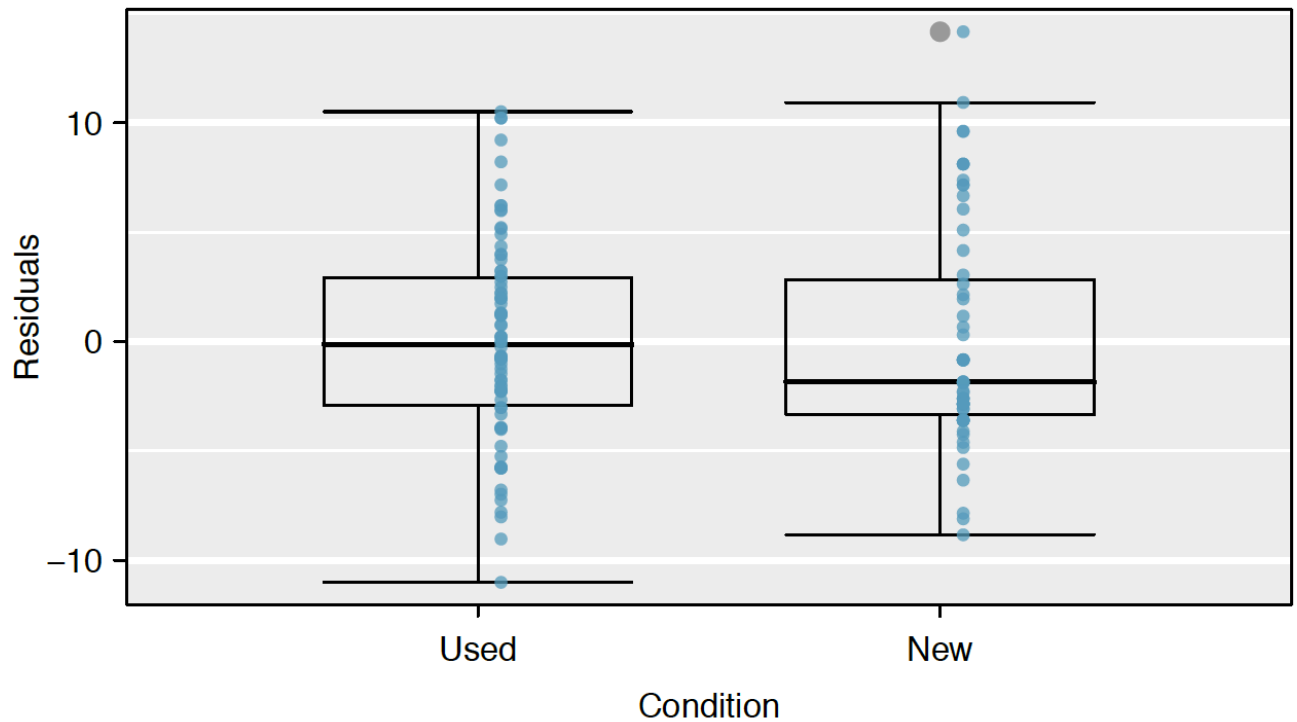
Normal Probability Plot of Residuals



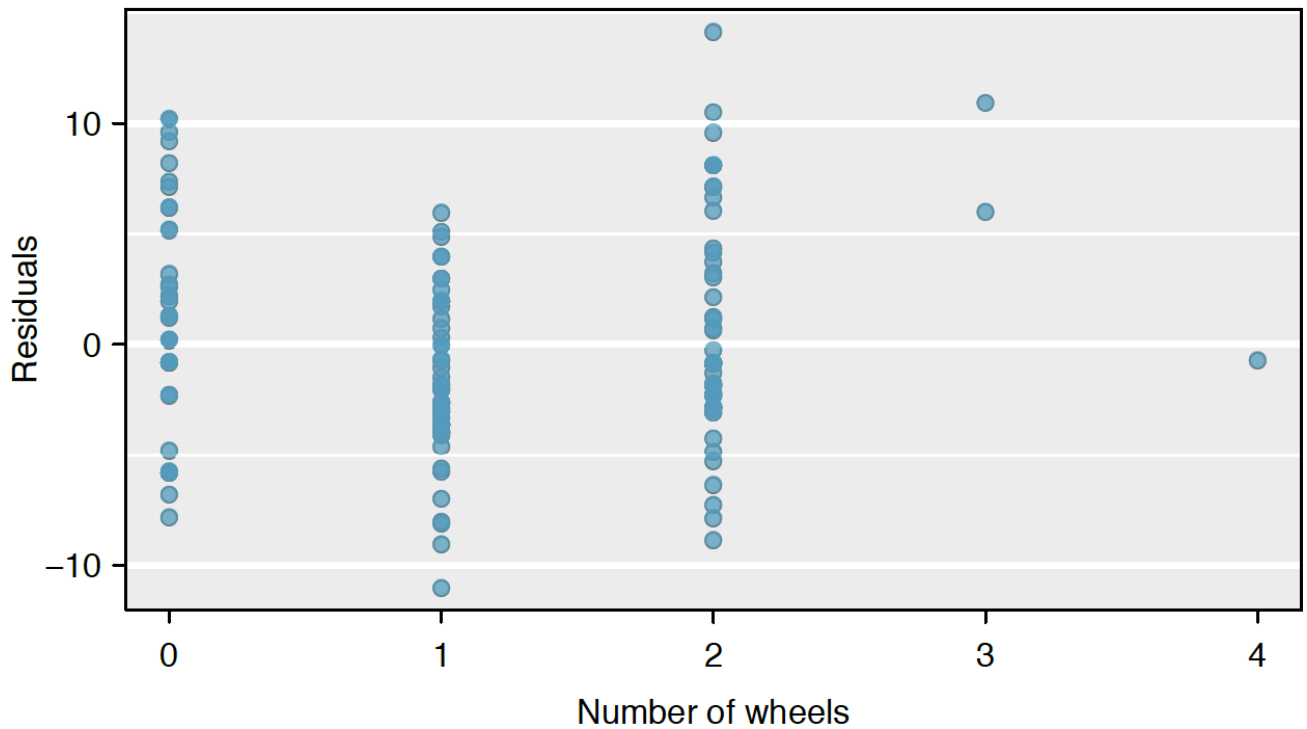
Absolute Values of Residuals Against Fitted Values



Residuals Against Each Predictor Variable: Condition

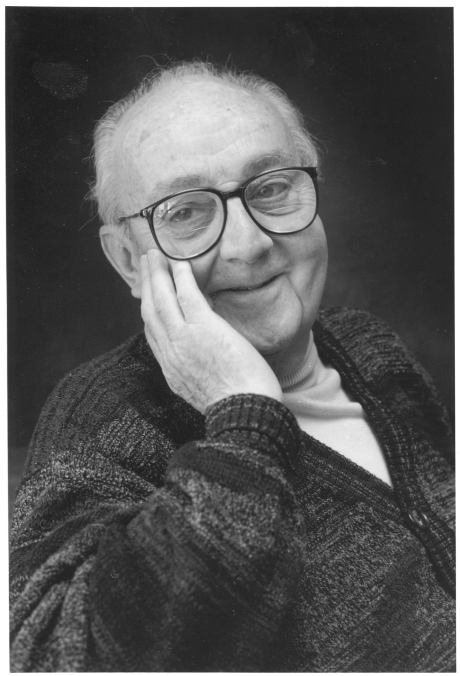


Residuals Against Each Predictor Variable: Wheels



George E.P. Box

There was a famous statistician named Box



famous for the Box/Cox Transformation.

George E.P. Box's Famous Quote

"All models are wrong, but some are useful."

Caution

We can tolerate a little leeway with model assumptions, but when they are grossly violated we have to be skeptical of any confidence intervals/p-values. If model assumptions are clearly violated

- consider a new model
- get the assistance of someone who can help

Next Time

What if the outcome variable is not numerical, but rather a yes/no response variable?

- Was an email spam or not?
- Will someone develop cancer or not?
- Is a person female?

We use logistic regression.