# Lecture 8: Normal Distribution

Chapter 3.1

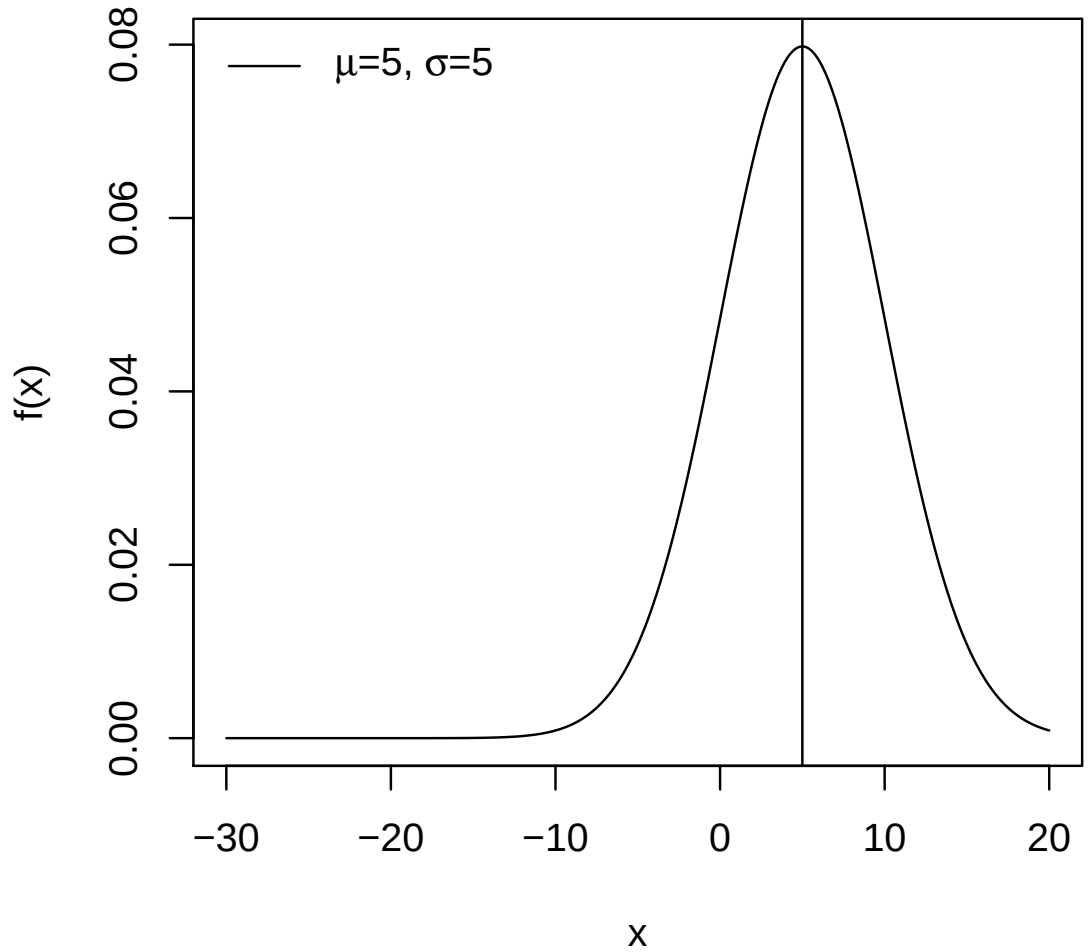
# Goals for Today

- Define the normal distribution in terms of its parameters
- Review: $\frac{2}{3}$ / 95% / 99.7% rule
- Standardizing normal observations to z-scores

# Normal Distribution

# Normal Distribution

$\mu$ (mean) specifies the center, $\sigma$ (standard deviation) the spread.



# Normal Example

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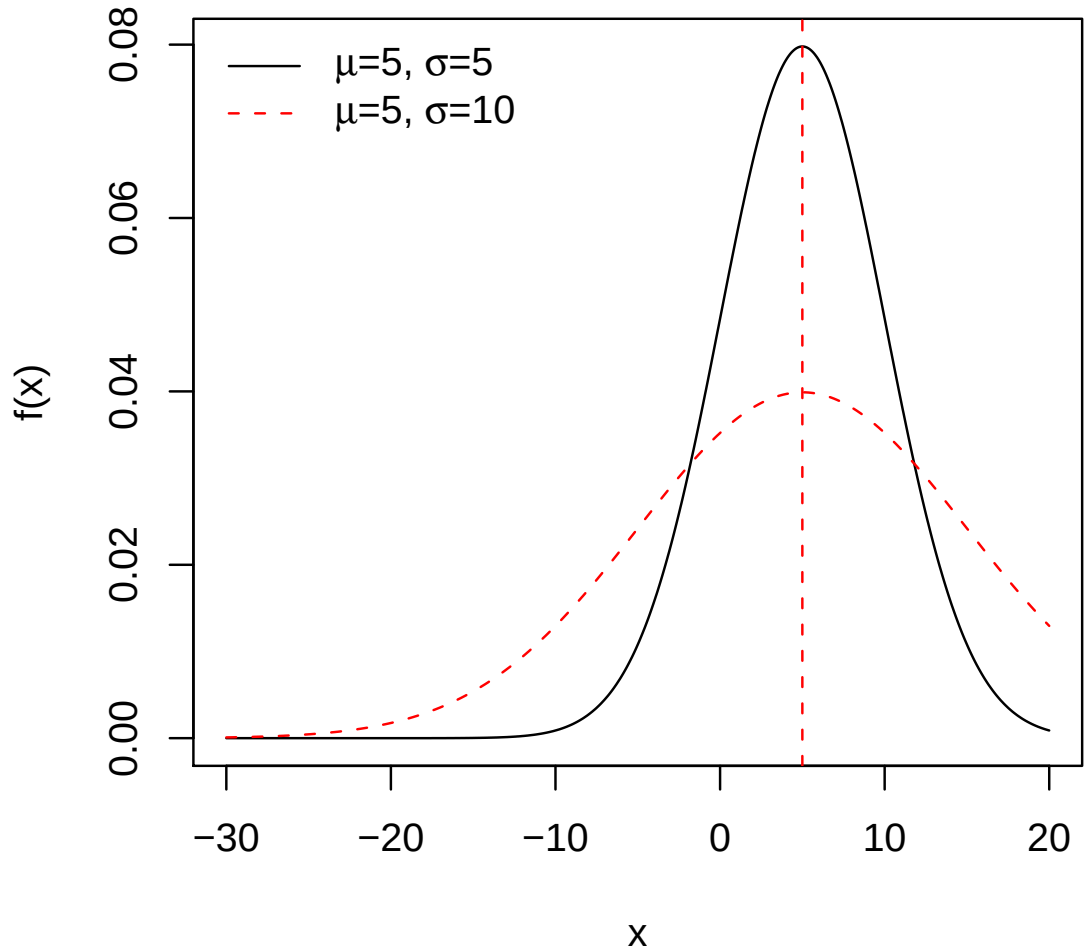

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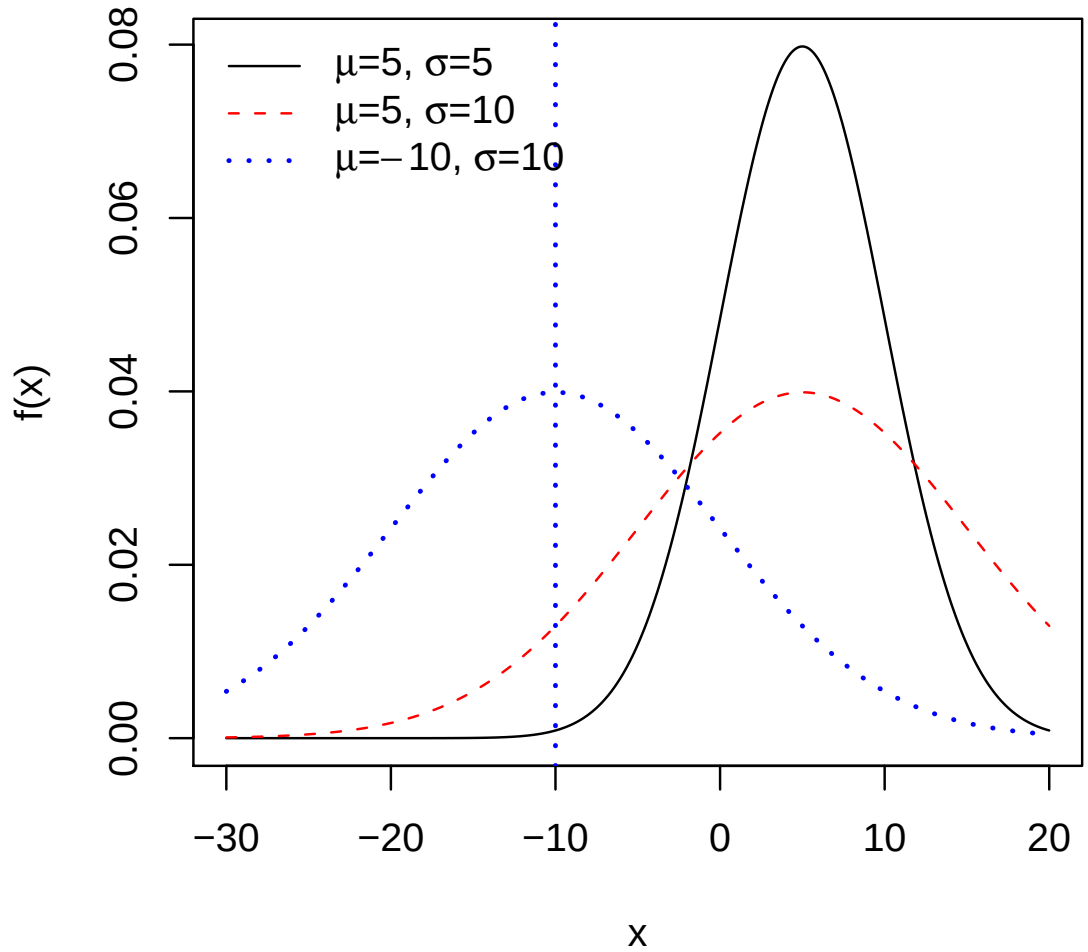
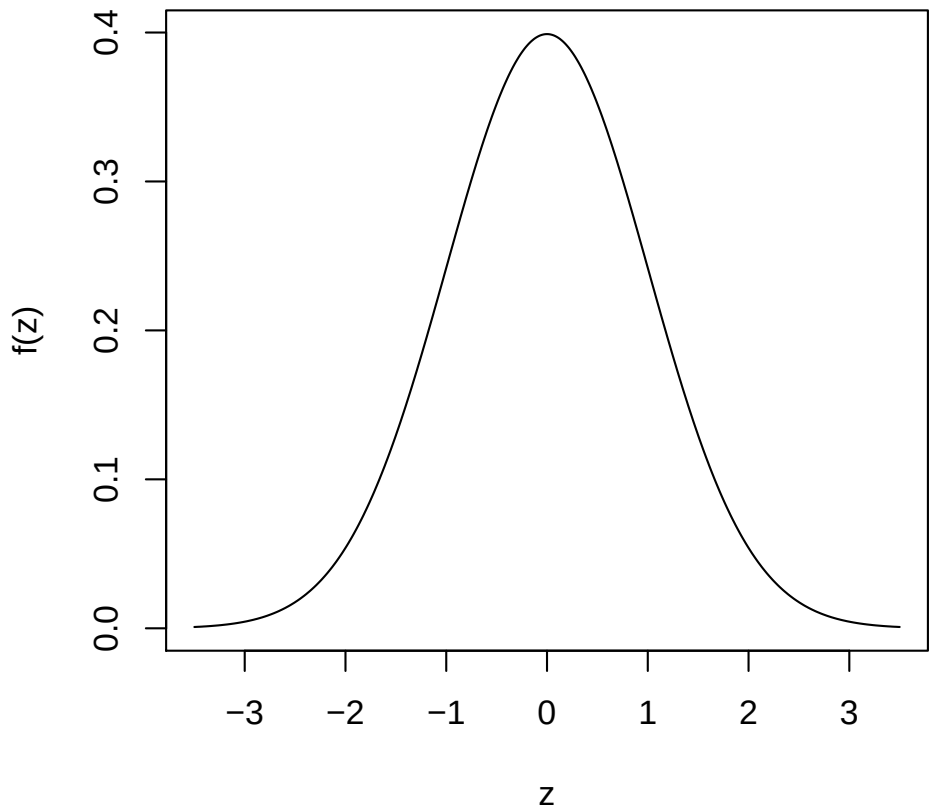


# Standardized Normal Distribution

If $\mu = 0$ and $\sigma = 1$, this is the standard normal distribution:



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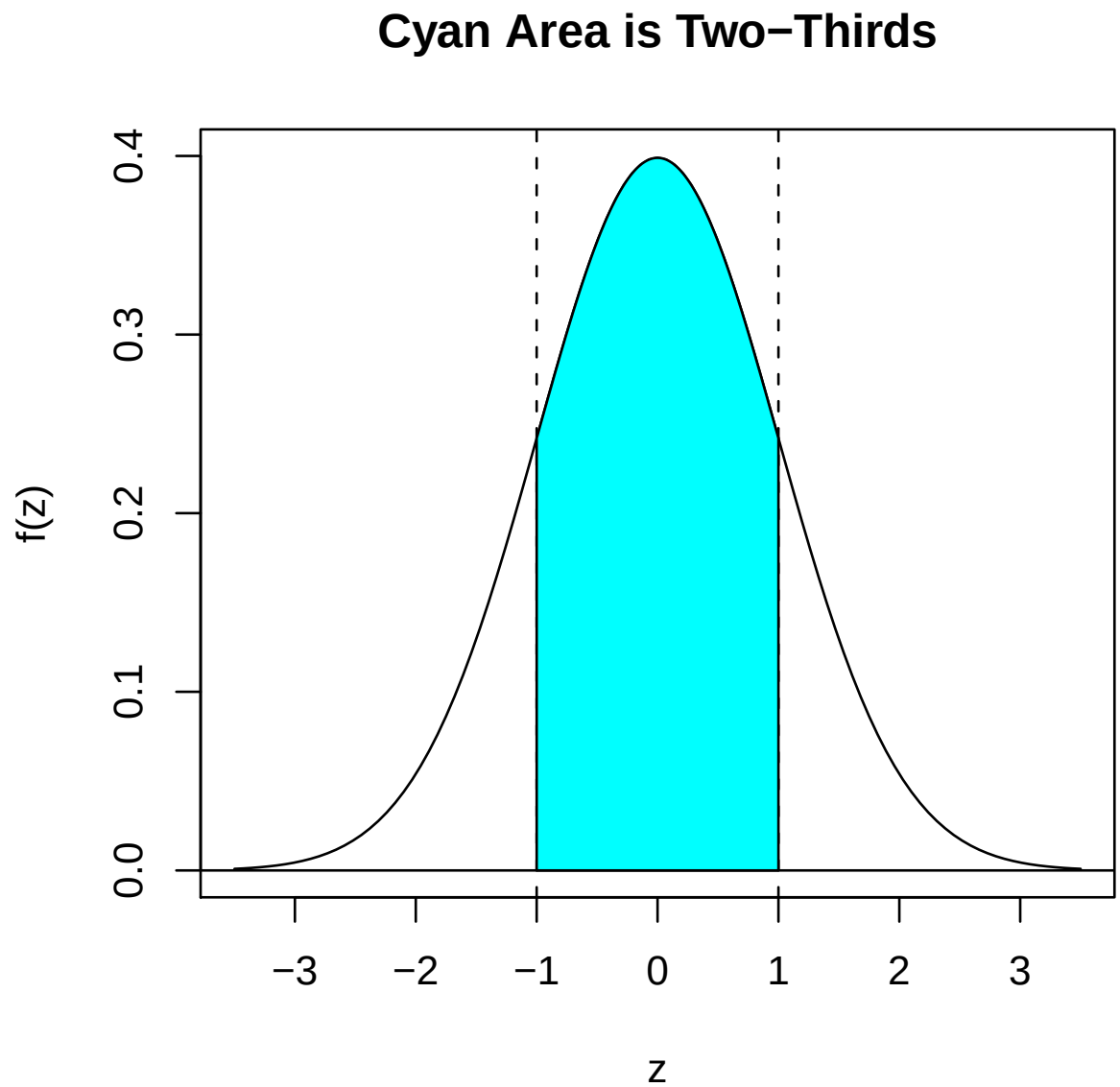
3. Approx. 99.7% of the data are within $\pm 3$ SD of the mean

# Ex: Standard Normal $\mu = 0$, $\sigma = 1$



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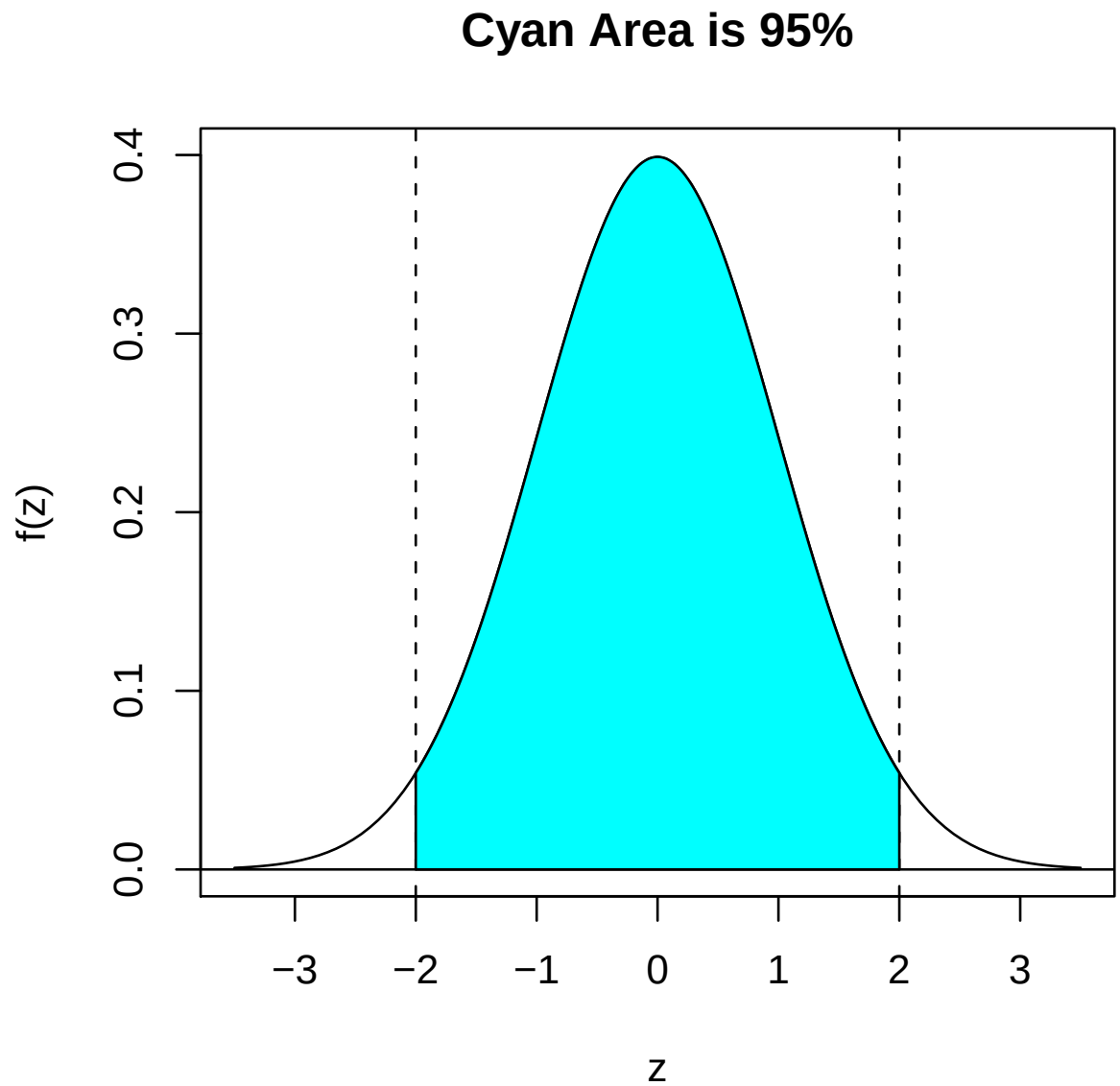

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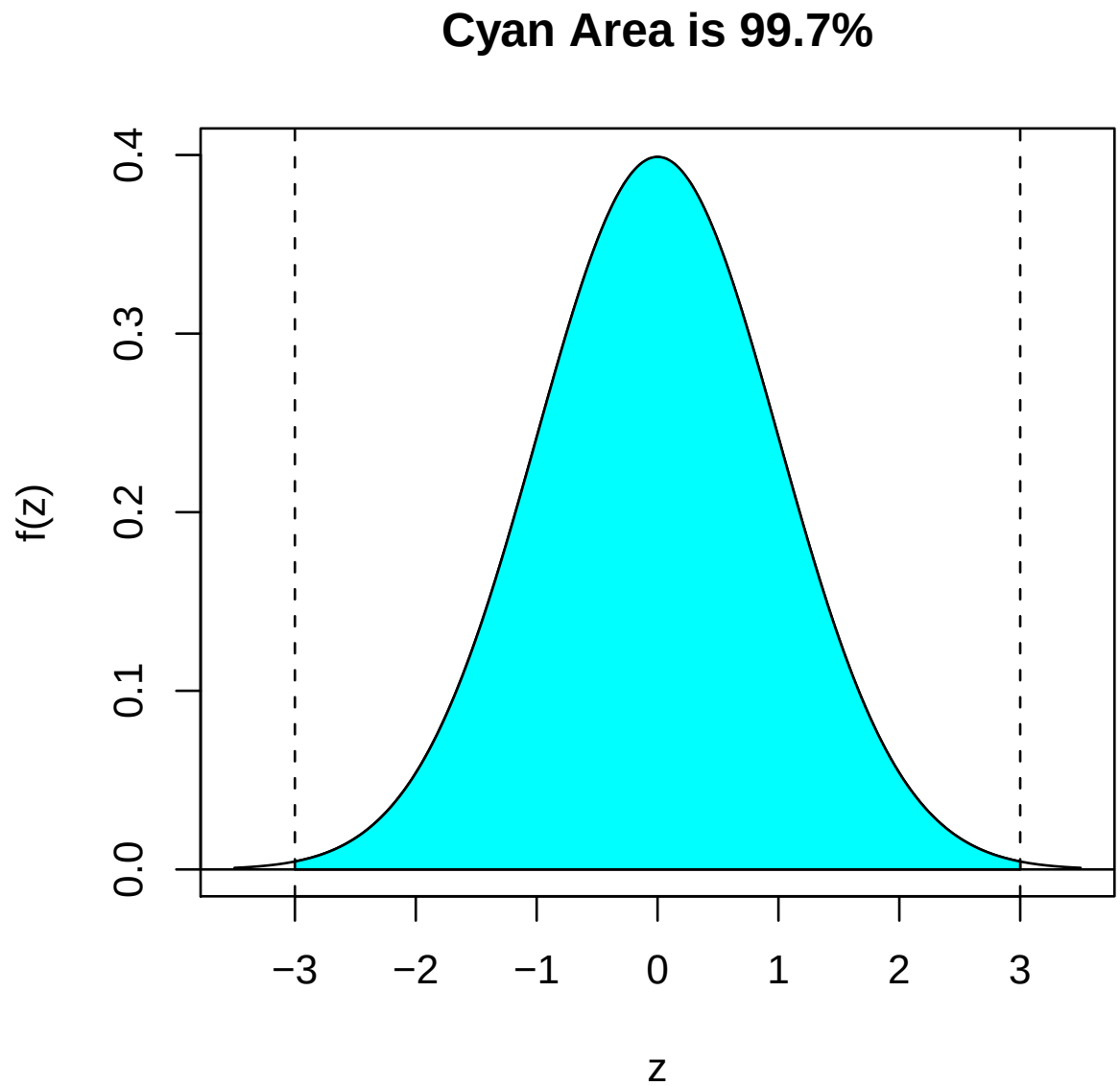

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From text: Say

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Say both tests scores were normally distributed with:

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Question: Who did relatively better?

# z-scores

# Back to Example

# Percentiles

Question: What %'ile is Ann's SAT score of 1800?

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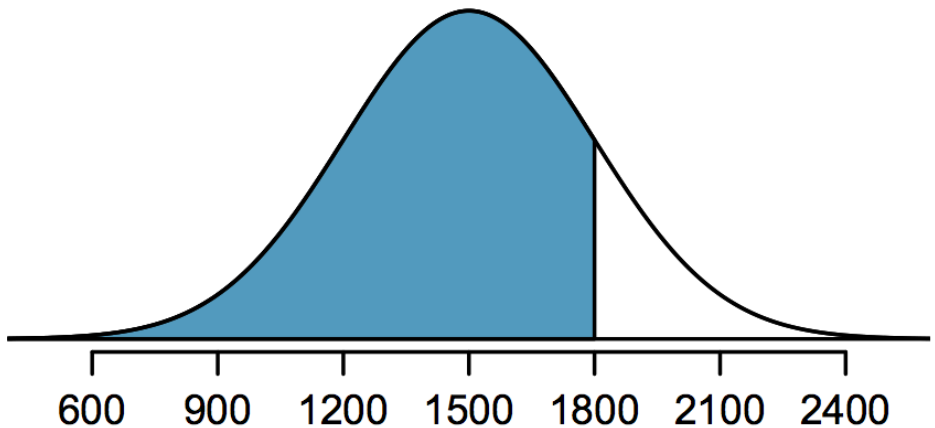
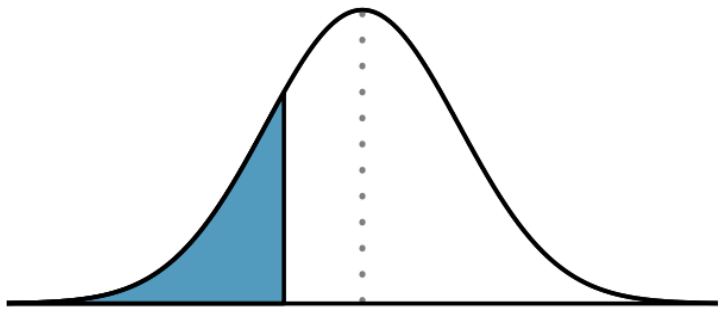
i.e. what is the blue shaded area?

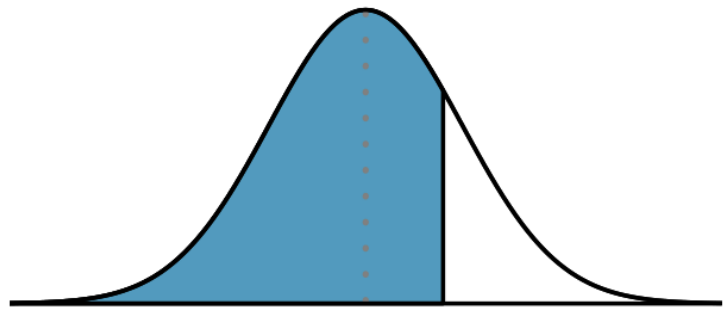


# Percentiles

Because the area under the curve is 1, the area to the left of z is the %'ile.



negative Z



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# Normal Probability Table

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- identify the %'ile corresponding to a z-score
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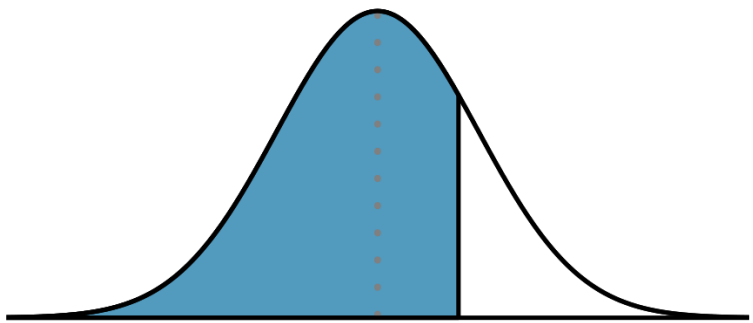
The normal probability tables on page 429 represent z-scores and %'iles corresponding to area to the left:



# Normal Probability Table

| | Second decimal place of $Z$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $Z$ | 0.00 | 0.01 | 0.02 | *0.03* | **0.04** | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| *0.4* | 0.6554 | 0.6591 | 0.6628 | *0.6664* | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | **0.7995** | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

# Back to Ann and Tom

# Next Time

Next time we will:

- Re-iterate the motivation for the normal curve.
- Go over examples using z-scores.
- Evaluating the normal approximation.