Lecture 14: Hypothesis Testing Part I

Chapter 4.3

Goals for Today

- Introduce Hypothesis Testing Framework
- Testing Hypotheses Using Confidence Intervals
- Types of Errors
- Testing Hypotheses Using p-Values

Statistical Hypothesis Testing

Example

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- H_0: the coin is fair. i.e. the probability of heads is $p = 0.5$
- H_A: the coin is not fair. i.e. $p \neq 0.5$

Crucial Concept: Conclusions of Hypothesis Tests

Analogy: US Criminal Justice System

In the criminal justice system, the jury's verdict does NOT make any statement about the defendant being innocent, rather that there was not enough evidence to prove beyond a reasonable doubt that they were guilty.

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- Truth about the defendant: innocent vs guilty
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Decision:

- Verdict: not guilty vs guilty
- Test outcome: "Do not reject H_0" vs "Reject H_0"

Testing Hypotheses Using Confidence Intervals

Example on page 173: The average 10 mile run time for the Cherry Blossom Run in 2006 μ_{2006} was 93.29 min. Researchers suspect μ_{2012} was different:

Testing Hypotheses Using Confidence Intervals

Example on page 173: The average 10 mile run time for the Cherry Blossom Run in 2006 μ_{2006} was 93.29 min. Researchers suspect μ_{2012} was different:

- H_0: average time was the same. i.e. $\mu_{2012} = 93.29$
- H_A: average time was different. i.e. $\mu_{2012} \neq 93.29$

Testing Hypotheses Using Confidence Intervals

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    - procedures with lower type I error rates typically have higher type II error rates
    - vice-versa
- In other words, there is almost never a procedure that makes no type I errors and no type II errors. Some sort of balance between the two is required

Next Time

- More Hypothesis Testing