Lecture 15: Hypothesis Testing Part II

Chapter 4.3

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There are two potential outcomes of a hypothesis test. Either we

- reject H_0
- fail to reject H_0

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Two kinds of errors:

- Type I Error: a false positive (test result)
- Type II Error: a false negative (test result)

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In this case:

- Type I error is putting an innocent person in jail (considered worse)
- Type II error is letting a guilty person go free.

Type II Errors: Airport Screening

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Failing to reject H_0 when H_A is true is not "patting down" passenger X when they have a weapon.

Hence the long lines at airport security.

Goals for Today

- Define significance level
- Tie-in p-Values with sampling distributions
- Example

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i.e. $\alpha = 0.05 = 5\%$ is the significance level.

With 95% confidence intervals from earlier, we expect it to miss the true population parameter 5% of the time. This corresponds to $\alpha = 0.05$.

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- 525 heads? Do you think the coin is biased?
- 900 heads? Do you think the coin is biased?

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Note the p-value is different than the population proportion p (bad historical choice).

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It is NOT the probability of H_0 being true. This is the most common misinterpretation of the p-value.

Thought experiment: Coin Flips

You have a coin that test for fairness with $n = 1000$ flips. Set $p_0 = 0.5$ (coin is fair) and define a "success" as getting heads.

$$H_0 : p = p_0$$
$$vs \qquad H_A : p \neq p_0$$

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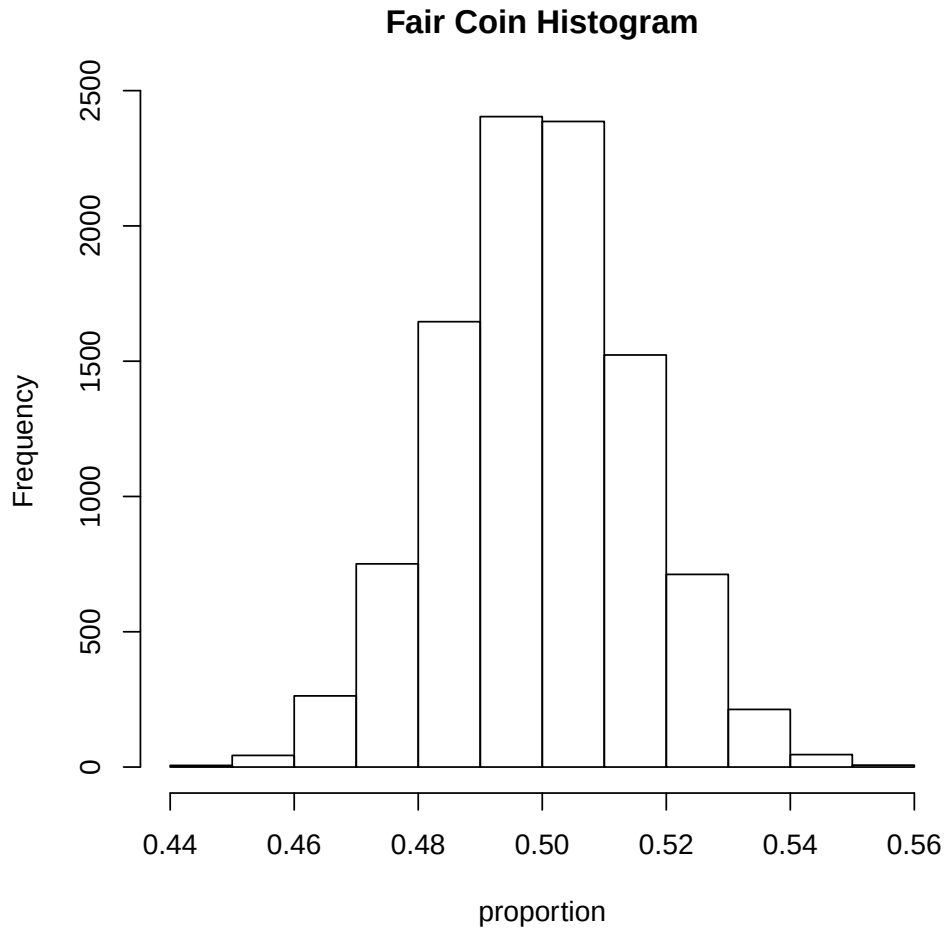
- Since it is based on a sample, $\widehat{p}$ has a sampling distribution
- The standard error is $\sqrt{\frac{p(1-p)}{n}}$ (Chapter 6).
- Furthermore, since conditions hold, the sampling distribution is Normal (CLT)

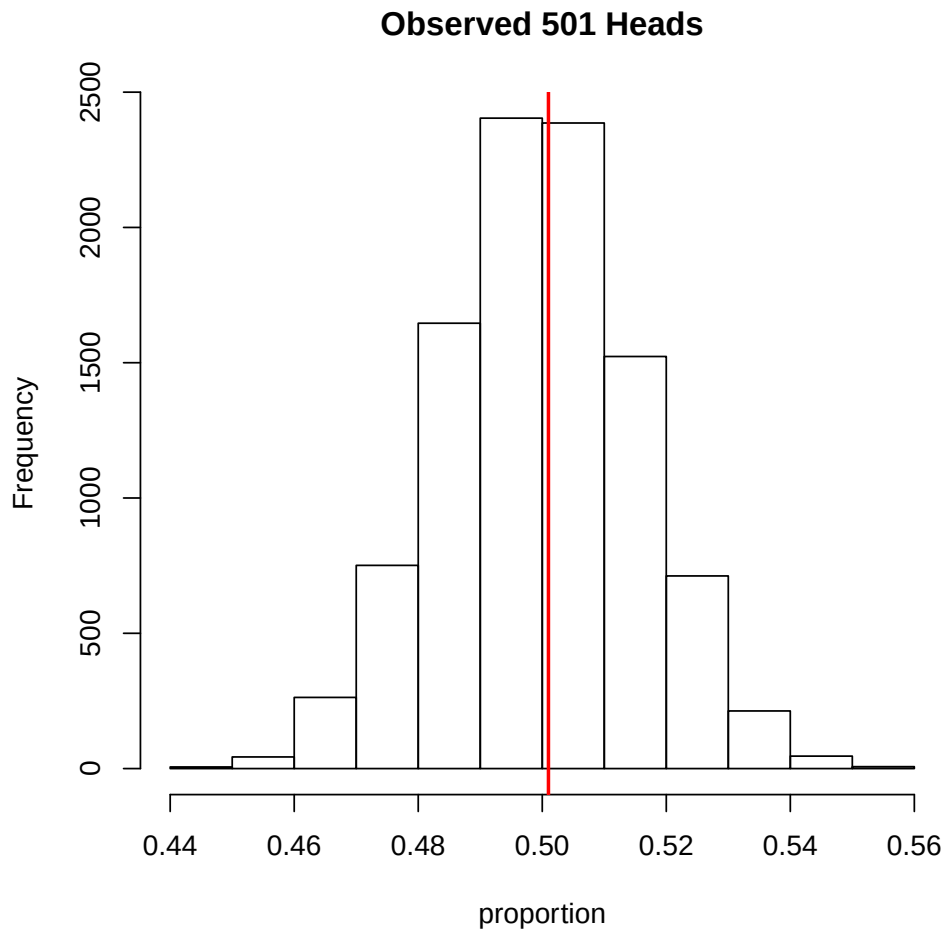
Sampling Distribution of $\widehat{p}$

Under H_0 that the coin is fair, i.e. $p = p_0 = 0.5$, the sampling distribution of $\widehat{p}$ when $n = 1000$ is:



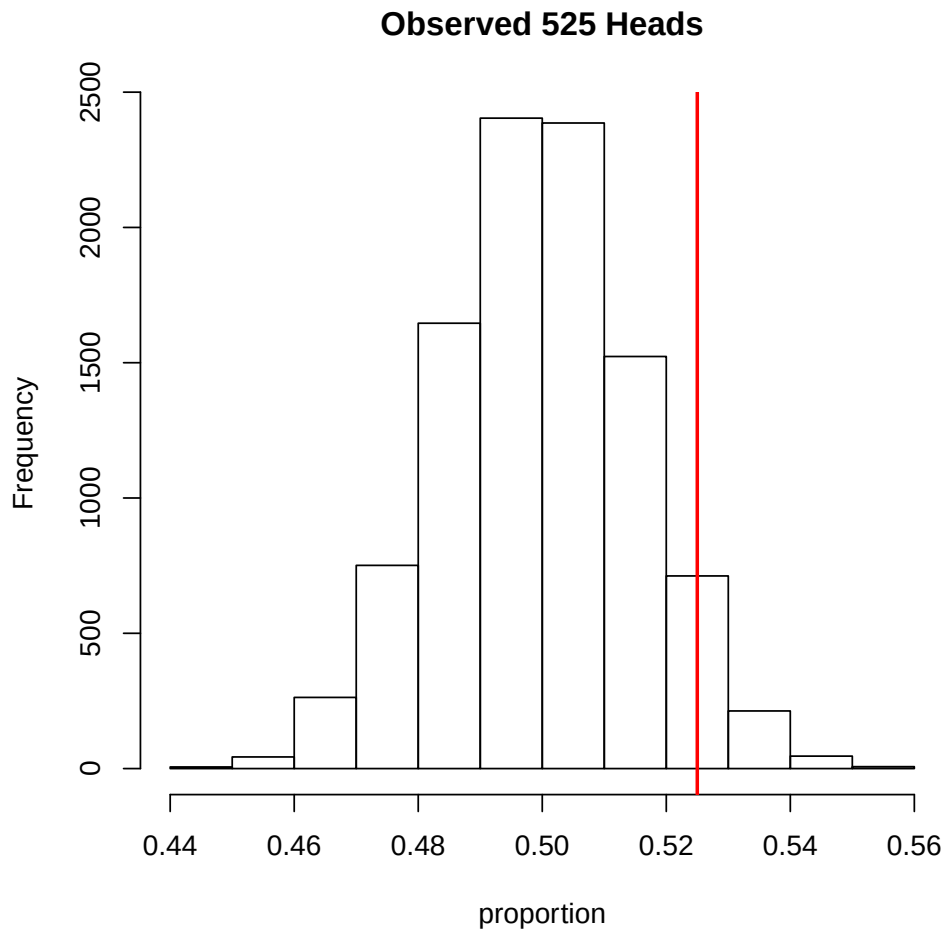
Say we observe...

$$\widehat{p} = \frac{501}{1000}$$



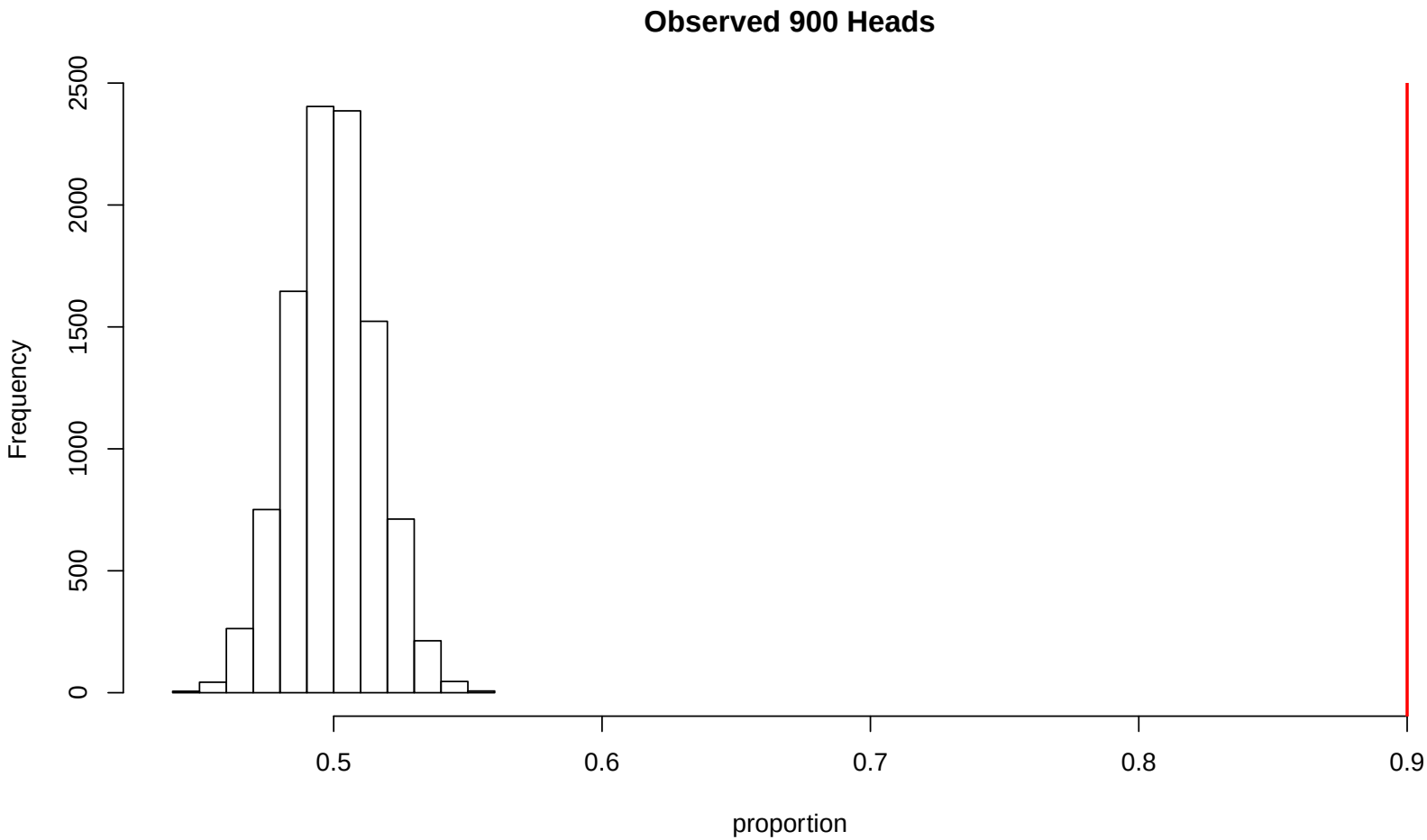
Say we observe...

$$\widehat{p} = \frac{525}{1000}$$



Say we observe...

$$\widehat{p} = \frac{900}{1000}$$



Example about Sleep Habits

A poll found that college students sleep about 7 hours a night. Researchers suspect that Reedies sleep more. They want to investigate this claim at a pre-specified $\alpha = 0.05$ level.

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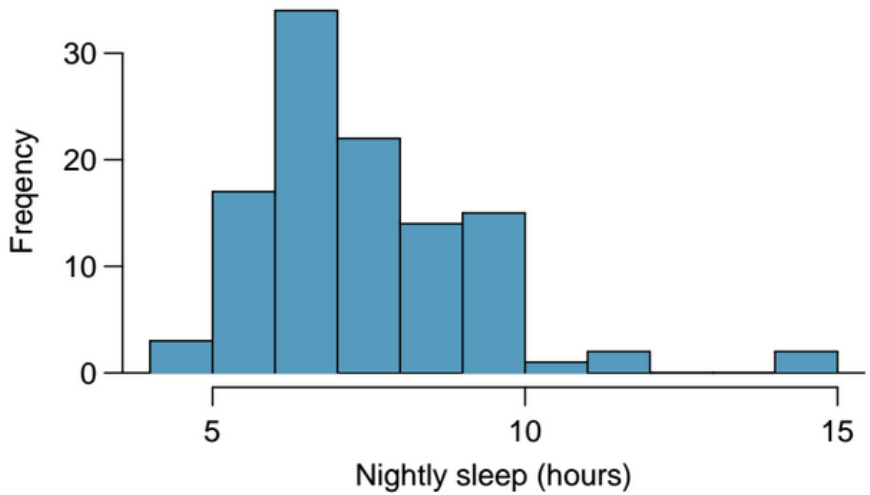
A poll found that college students sleep about 7 hours a night. Researchers suspect that Reedies sleep more. They want to investigate this claim at a pre-specified $\alpha = 0.05$ level.

They sample $n = 110$ Reedies and find that $\overline{x} = 7.42$ and $s = 1.75$ and the histogram looks like:



Example about Sleep Habits

Let $\mu =$ true population mean # of hours Reedies sleep a night.
Then $\mu_0 = 7$ and:

- $H_0 : \mu = \mu_0 = 7$
- $H_A : \mu > 7$

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1. Independence: $n = 110 \leq 10\%$ of 1,453 (Reed enrollment)
2. $n \geq 30$
3. The distribution of the $n = 110$ observations (Figure 4.14) is not too skewed.

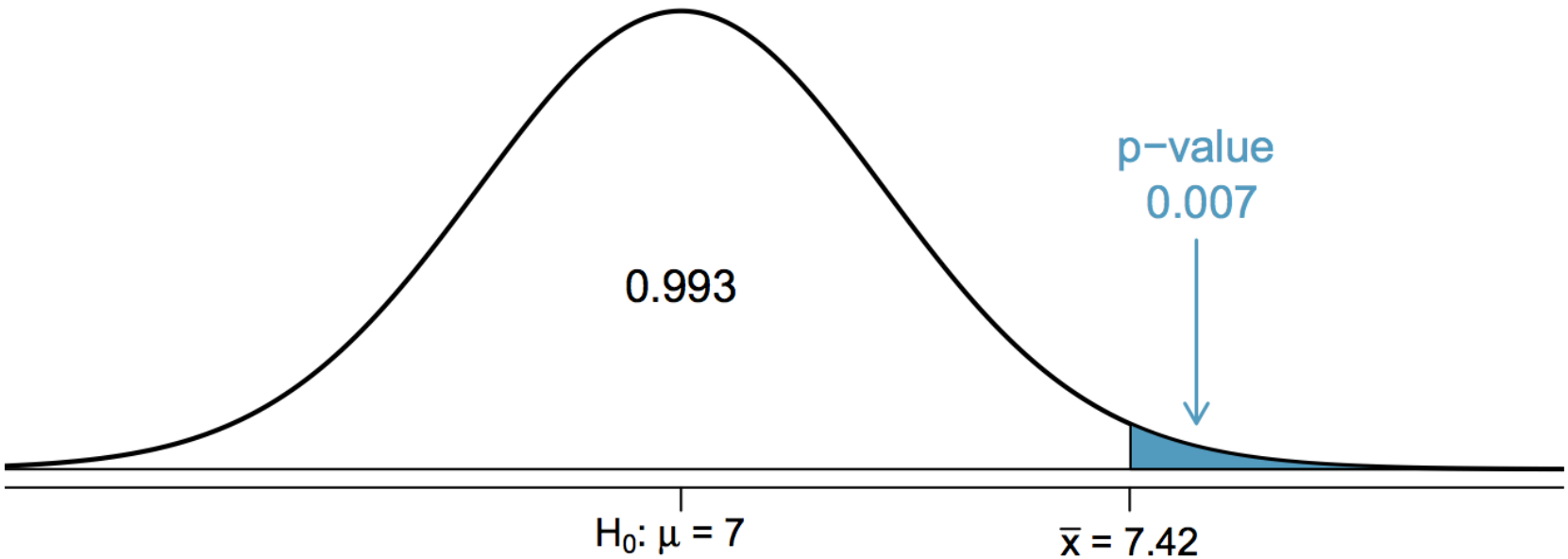
If H_0 is true, then $\overline{x}$ has mean μ_0. So in our case for $\overline{x}$

$$z = \frac{\overline{x} - \text{null value}}{SE} = \frac{7.42 - 7}{\frac{1.75}{\sqrt{110}}} = 2.47$$

Example about Sleep Habits

In our case, since $H_A : \mu > 7$, more extreme means to the right of $z = 2.47$.

If H_0 is true, then the null distribution looks like this:



Hence, the p-value is 0.007.

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Conclusion: we reject (at the $\alpha = 0.05$ significance level) the hypothesis that the average # of hours of Reedies sleep is 7, in favor of the hypothesis that sleep more.

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Correct interpretation of the p-value: If the null hypothesis is true ($\mu = 7$), the probability of observing a sample mean $\overline{x} = 7.42$ or greater is 0.007.

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Incorrect interpretation of the p-value: The probability that the null hypothesis ($\mu = 7$) is true is 0.007.

Next Time

- How big a sample size to I need? i.e. power calculations
- Statistical vs practical significance