

Lecture 8: Normal Distribution

Chapter 3.1

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Goals for Today

- ▶ Define the normal distribution in terms of its **parameters**
- ▶ Review: $\frac{2}{3}$ / 95% / 99.7% rule
- ▶ Standardizing normal observations to **z-scores**

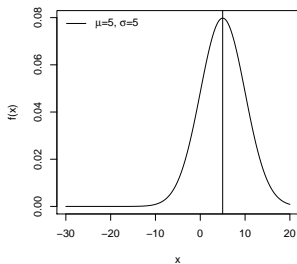
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Normal Distribution

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Normal Distribution

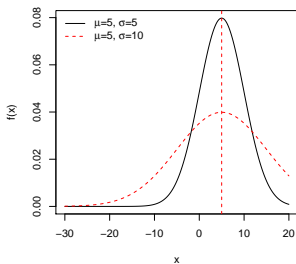
μ (mean) specifies the center, σ (standard deviation) the spread.



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Normal Example

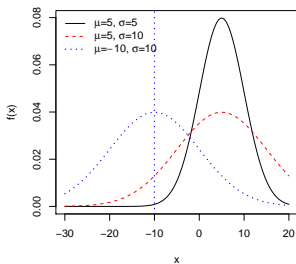
μ (mean) specifies the center, σ (standard deviation) the spread.



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Normal Example

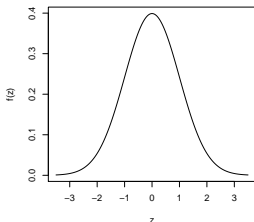
μ (mean) specifies the center, σ (standard deviation) the spread.



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Standardized Normal Distribution

If $\mu = 0$ and $\sigma = 1$, this is the **standard normal distribution**:



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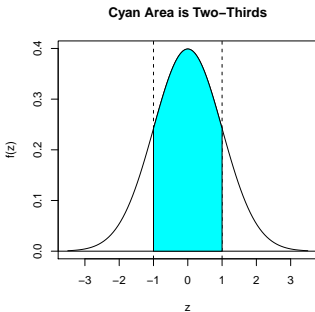
Rules of Thumb

Recall if a distribution is bell-shaped i.e. normal, then:

1. Approx. $\frac{2}{3}$'s of the data are within ± 1 SD of the mean
2. Approx. 95% of the data are within ± 2 SD of the mean
3. Approx. 99.7% of the data are within ± 3 SD of the mean

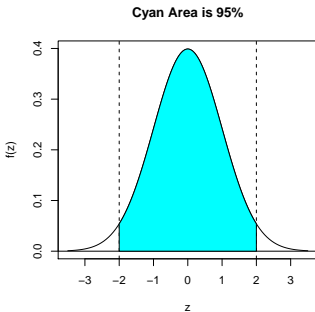
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Ex: Standard Normal $\mu = 0, \sigma = 1$



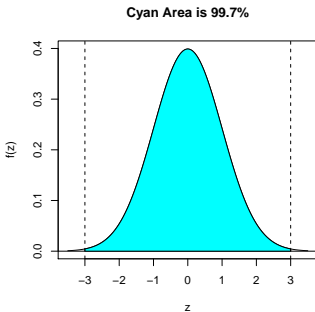
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Ex: Standard Normal $\mu = 0, \sigma = 1$



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Ex: Standard Normal $\mu = 0, \sigma = 1$



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Motivating Example

From text: Say

- ▶ Ann scores 1800 on the SAT
- ▶ Tom scores 24 on the ACT

Say both tests scores were normally distributed with:

	SAT	ACT
Mean μ	1500	21
SD σ	300	5

Question: Who did relatively better?

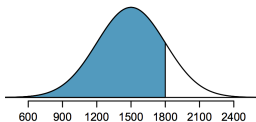
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z-scores

Back to Example

Percentiles

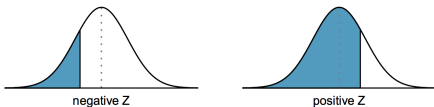
Question: What %'ile is Ann's SAT score of 1800?
i.e. what is the blue shaded area?



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Percentiles

Because the area under the curve is 1, the area to the left of z is the %'ile.



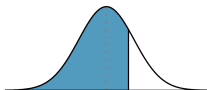
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Normal Probability Table

A normal probability table allows you to:

- ▶ identify the %'ile corresponding to a z-score
- ▶ or vice versa: the z-score corresponding to a %'ile

The normal probability tables on page 429 represent z-scores and %'iles corresponding to area to the left:



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Normal Probability Table

Z	Second decimal place of Z									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830

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Back to Ann and Tom

Next Time

Next time we will:

- ▶ Re-iterate the motivation for the normal curve.
- ▶ Go over examples using z-scores.
- ▶ Evaluating the normal approximation.