Lecture 29: Expected Value and Variance

Chapter 2.4

Random Variable

Intuitively Thinking: Expected Value

Intuitively Thinking: Expected Value

Say you have a random variable X:

x	2	3	4	10	11
$\Pr(X = x)$	$\frac{15}{100}$	$\frac{25}{100}$	$\frac{10}{100}$	$\frac{30}{100}$	$\frac{20}{100}$

E.g. We observe $X = 3$ with prob .25

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Is the value we expect to observe:

$$\frac{2+3+4+10+11}{5} \quad = \quad 6\ ?$$

Intuitively Thinking: Expected Value

No, each of the x's have different probability of occurring.

Intuitively Thinking: Expected Value

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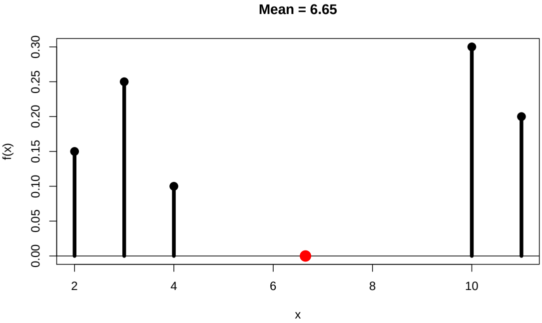
For each x, we assign different weights $\Pr(X = x)$ and not $\frac{1}{5}$:

$$2 \times \frac{15}{100} + 3 \times \frac{25}{100} + 4 \times \frac{10}{100} + 10 \times \frac{30}{100} + 11 \times \frac{20}{100} = 6.65$$

Expected Value

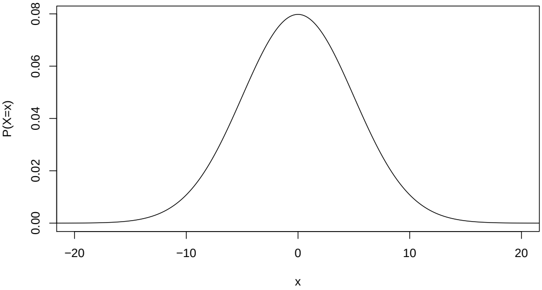
Expected Value

You can also think of the mean as the center of mass or balance point. It is 6.65 (marked with red point):



Intuitively Thinking: Measures of Spread

Consider the following distribution with $\mu = 0$. Let's build a measure of expected "spread".



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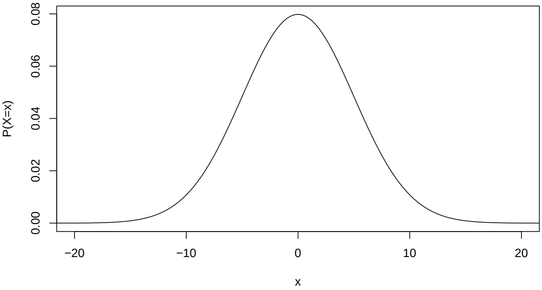
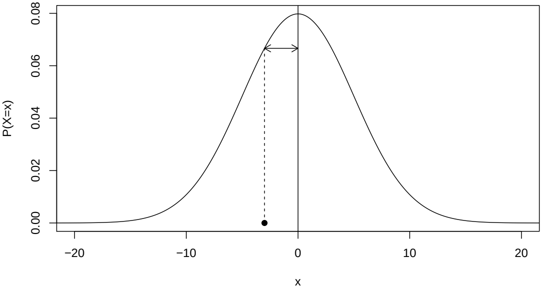


Let's define "spread" as the absolute deviation from μ: $|x - \mu|$.
i.e. $+$'ve & $-$'ve deviations are treated the same.

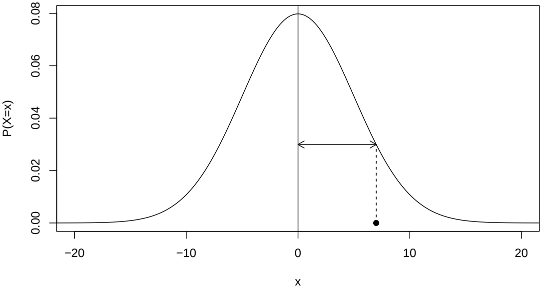
Intuitively Thinking: Measures of Spread

When $x = -3.0$, the abs. deviation from μ is $|-3.0 - \mu| = 3.0$.
Note $P(X = x) = 0.066$.



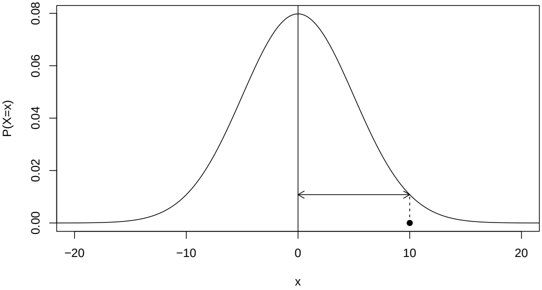
Intuitively Thinking: Measures of Spread

When $x = 7.0$, the abs. deviation from μ is $|7.0 - \mu| = 7.0$.
Note $P(X = x) = 0.030$.



Intuitively Thinking: Measures of Spread

When $x = 10.0$, the abs. deviation from μ is $|10.0 - \mu| = 10.0$.
Note $P(X = x) = 0.011$.



Intuitively Thinking: Measures of Spread

x	Abs Deviation $\lvert x - \mu \rvert$	Weight $P(X = x)$
-3.0	$\lvert -3.0 - 0 \rvert = 3.0$	0.066
7.0	$\lvert 7.0 - 0 \rvert = 7.0$	0.030
10.0	$\lvert 10.0 - 0 \rvert = 10.0$	0.011

So say we do this for all x and take a weighted average of the $|x - \mu|$ where the weights are $P(X = x)$.

Voilà: Our notion of expected spread.

Variance

Estimators

Sample Mean as an Estimator