Lecture 17: Paired Data and Difference of Two Means

Chapter 5.2, 5.1

Goals for Today

- Difference of means
- Paired differences of means
- Note on Practical vs Statistical Significance

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8. Are two categorical variables independent?

Chapter 5.2: Are the means of 2 groups μ_1 and μ_2 equal?

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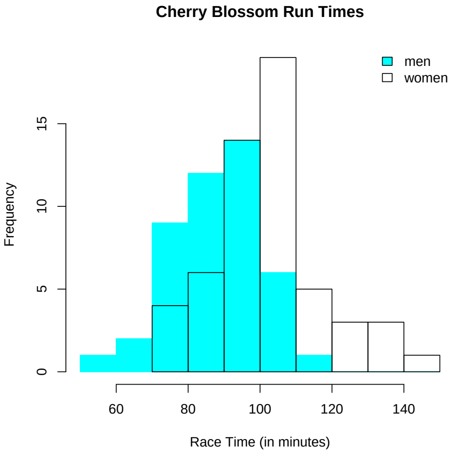

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The data:

	men	women
$\overline{x}$	87.65	102.13
s	12.5	15.2
n	45	55

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First, the point estimate for $\mu_w - \mu_m$ is the sample difference of means

$$\bar{x}_w - \bar{x}_m = 102.13 - 87.65 = 14.48$$

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then the difference in sample means $\overline{x}_1 - \overline{x}_2$ will also have a nearly normal sampling distribution...

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Note the different s^2's and sample sizes.

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the sampling distribution is Normal with mean=$\mu_w - \mu_m$ and

$$SE_{\bar{x}_w - \bar{x}_m} = \sqrt{\frac{15.2^2}{55} + \frac{12.5^2}{45}} = 2.77$$

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So for the Cherry Blossom Run data, a 95% CI for $\mu_w - \mu_m$ is:

$$14.48 \pm 1.96 \times 2.77 = [9.05, 19.91]$$

Hypothesis Test

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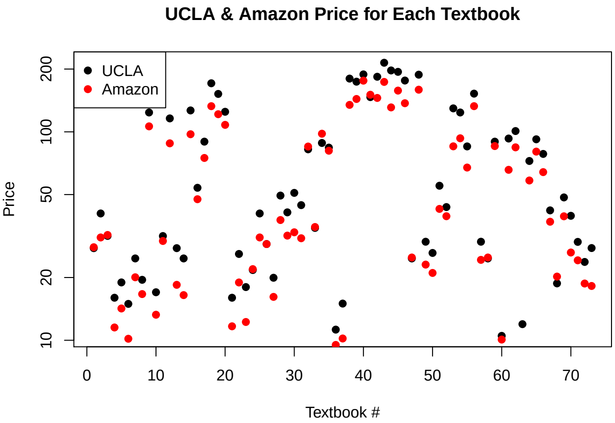
- In the text: price of the same textbook at the UCLA bookstore vs Amazon

Paired Differences

The methodology for paired data remains the same, except our observations are the difference in pairs. Example, for the UCLA Bookstore vs Amazon book price example in the text



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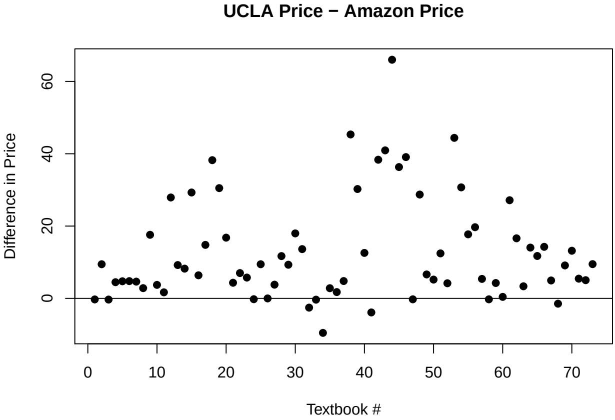

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- population parameter is μ_{diff}
- point estimate $\overline{x}_{diff}$ of μ_{diff}
- Conditions: not on the original observations, but rather the differences: 10% rule, sample size n, and not too skewed *differences*.
- If met, $\overline{x}_{diff}$ has a normal sampling distribution with mean μ_{diff} and $SE_{diff} = \frac{s_{diff}}{\sqrt{n_{diff}}}$.

Next Time

- One sample t-test