Lecture 17: Paired Data and Difference of Two Means

Chapter 5.1-5.2

Goals for Today

- Define statistical power
- Difference of Means
- Note on Practical vs Statistical Significance

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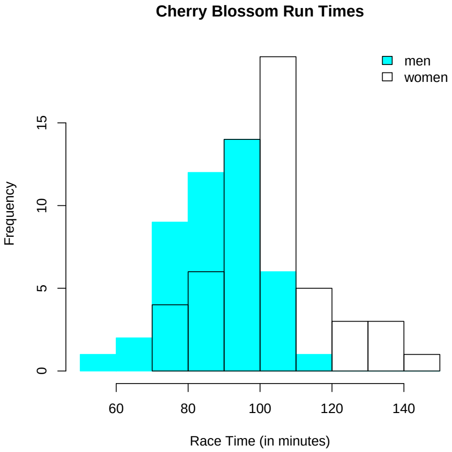
7. Are we observing counts that we were expected?
8. Are two categorical variables independent?

Are the means of two groups μ_1 and μ_2 equal or not?

Example from Chapter 5.2: Did men (n=45) run faster than women (n=55)?



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The data:

	men	women
$\overline{x}$	87.65	102.13
s	12.5	15.2
n	45	55

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First, the point estimate for $\mu_w - \mu_m$ is the sample difference of means

$$\bar{x}_w - \bar{x}_m = 102.13 - 87.65 = 14.48$$

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then the difference in sample means $\overline{x}_1 - \overline{x}_2$ will also have a nearly normal sampling distribution...

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- mean $\mu_1 - \mu_2$
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Note the different s^2's and sample sizes.

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the sampling distribution is Normal with mean=$\mu_w - \mu_m$ and

$$SE_{\bar{x}_w - \bar{x}_m} = \sqrt{\frac{15.2^2}{55} + \frac{12.5^2}{45}} = 2.77$$

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So for the Cherry Blossom Run data, a 95% CI for $\mu_w - \mu_m$ is:

$$14.48 \pm 1.96 \times 2.77 = [9.05, 19.91]$$

Next Time

- Hypothesis test for differences in means
- Paired differences
- One sample t-test