Lecture 19: ANOVA Part I

Chapter 5.5

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- n is small.
- Independence: $n \leq 10\%$ rule
- Observations come from a nearly normal distribution:
  - Look at a histogram of the data (difficult when n is small)
  - Consider whether any previous experiences alert us that the data may be normal

Analysis of Variance (ANOVA)

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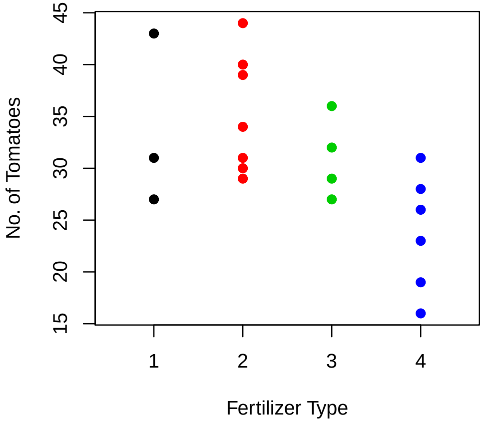

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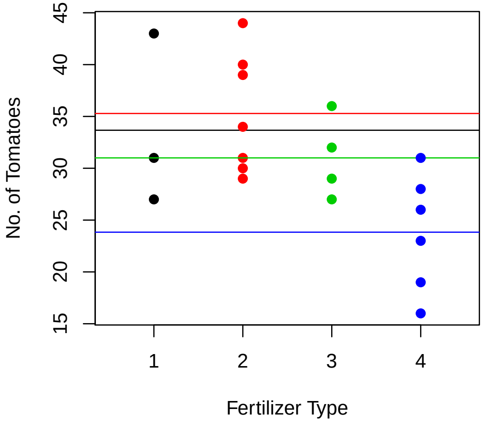

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- n_i plants assigned to each of the $k = 4$ fertilizers:

n_1	n_2	n_3	n_4	total n
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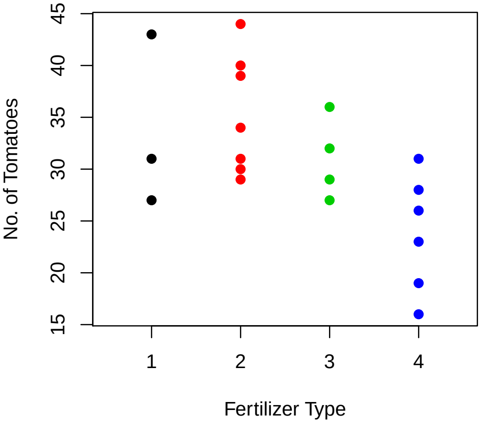
- Count the number of tomatoes on each plant

Tomato Fertilizer

We observe the following, where each point is one tomato plant.



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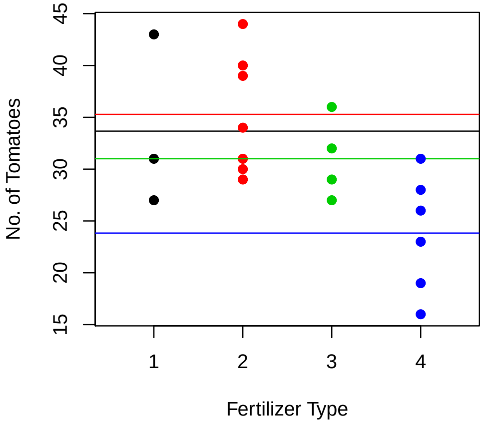
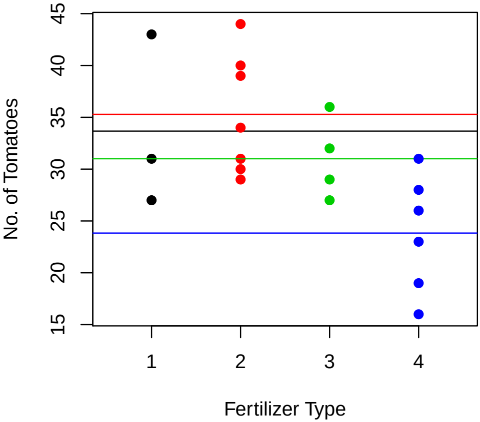
Plot the sample mean of each level.



Tomato Fertilizer

We observe the following, where each point is one tomato plant. Plot the sample mean of each level. Question: are the mean tomato yields different?



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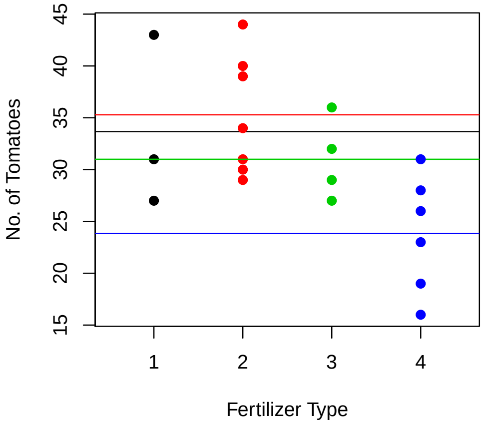
Ex. for groups 1 & 2:

$$\begin{aligned} H_0 : &\quad \mu_1 = \mu_2 \\ \text{vs. } H_a : &\quad \mu_1 \neq \mu_2 \end{aligned}$$

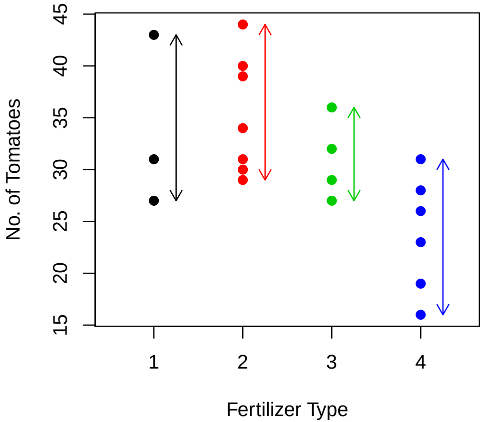
Tomato Fertilizer Example

Numerator: the between-group variation refers to the variability between the levels (the 4 horizontal lines):



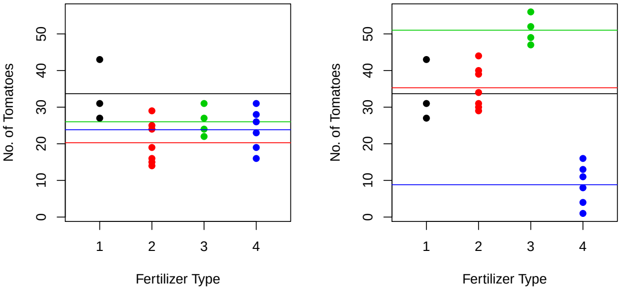
Tomato Fertilizer Example

Denominator: the within-group variation refers to the variability within each level (the 4 vertical arrows):



Tomato Fertilizer Example

Now compare the following two plots. Which has "more different" means?



Tomato Fertilizer Example

F Distributions

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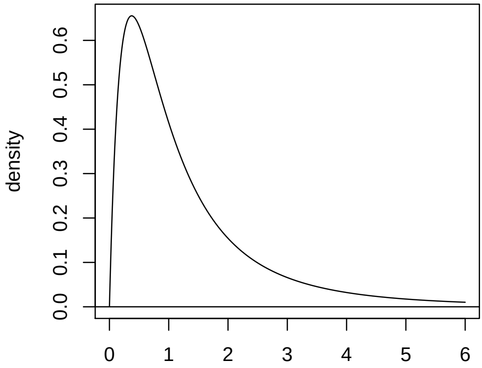
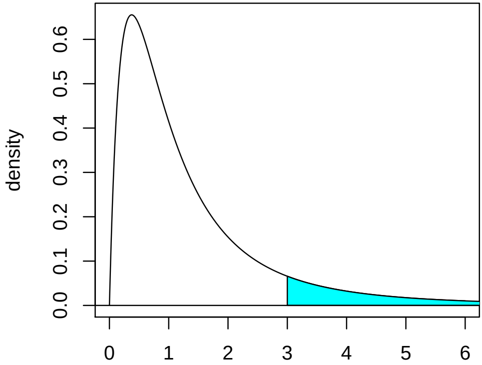
For $df_1 = 4$ and $df_2 = 6$, the F distribution looks like:



F Distributions

p-values are computed where "more extreme" means larger. Say the $F = 3$, the p-value is the area to the right of 3 and is computed in R: `pf(3,df1=4,df2=6,lower.tail=FALSE)`



Conducting An F-Test

The results are typically summarized in an ANOVA table:

Source of Variation	df	SS	MS	F	p-value
Between groups	$k-1$	$SSTr$	$MSTr = \frac{SSTr}{k-1}$	$\frac{MSTr}{MSE}$	p
Within groups	$n-k$	SSE	$MSE = \frac{SSE}{n-k}$		
Total	$n-1$	SST			

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2. Trade off of n and normality of observations within each group.
3. Each of the groups has constant variance $\sigma_1^2 = \ldots = \sigma_k^2 = \sigma^2$. Check via:
   - boxplots
   - comparing the sample standard deviations $s_1, \ldots, s_k$

Discussion of Quiz

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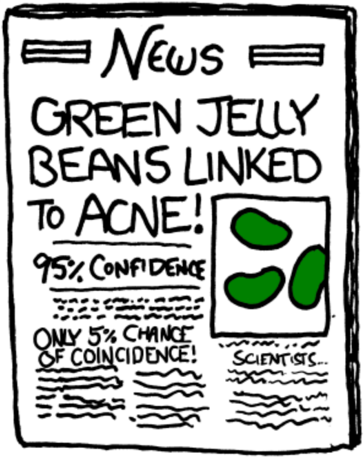
Not that the p-value is 0.05, rather that $\alpha = 0.05$:

- significance level AKA
- type I error rate AKA
- false positive rate

i.e. we expect 1 out of 20 results to be significant even if there is no effect.

Publication Bias



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Journal of Negative Results: `http://www.jnrbm.com/`

Publication Bias

From: Sterne JA, Davey Smith G (2001) Sifting the evidence - What's wrong with significance tests. BMJ 322: 226231.

What α to Use?

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Should I use $\alpha = 0.05$ as my significance level? Before using it, put some thought into the balance between:

- Type I errors. Setting a smaller α yields a more conservative procedure: all things being equal, you will reject H_0 less often.
- Type II errors. Setting a bigger α yields a more liberal procedure: all things being equal, you will reject H_0 more often.

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If you repeat experiments many times, you're bound to get a significant result eventually just by chance alone.

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Use the Bonferroni correction to α: If you are conducting n tests, use $\alpha^* = \frac{\alpha}{n}$. You'll study its properties in HW8.