Lecture 19: ANOVA Part I

Chapter 5.5

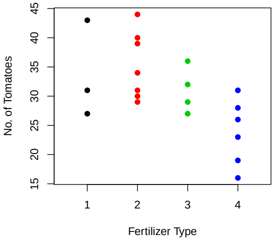
Previously: Conditions for Using t Distribution

We use the t distribution when you have

- n is small.
- Independence: $n \leq 10\%$ rule
- Observations come from a nearly normal distribution:
  - Look at a histogram of the data (difficult when n is small)
  - Consider whether any previous experiences alert us that the data may be normal

Analysis of Variance (ANOVA)

A farmer has the choice of four tomato fertilizers and wants to compare their performance in terms of crop yield.



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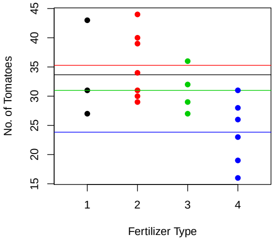


Analysis of Variance (ANOVA)

We have $k = 4$ groups AKA levels of a factor: the 4 types of fertilizer.

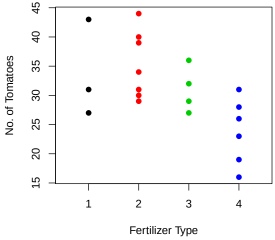
- n_i plants assigned to each of the $k = 4$ fertilizers:

n_1	n_2	n_3	n_4	total n
3	7	4	6	20

- Count the number of tomatoes on each plant

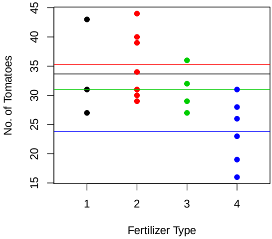
Tomato Fertilizer

We observe the following, where each point is one tomato plant.



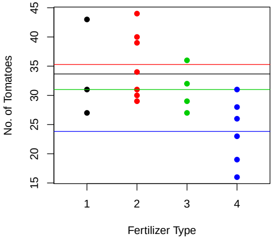
Tomato Fertilizer

We observe the following, where each point is one tomato plant. Plot the sample mean of each level.



Tomato Fertilizer

We observe the following, where each point is one tomato plant. Plot the sample mean of each level. Question: are the mean tomato yields different?



Analysis of Variance

Say we have k groups and want to compare the k means:

$$\mu_1, \mu_2, \ldots, \mu_k$$

We could do $\binom{k}{2}$ individual two-sample tests.

Ex. for groups 1 & 2:

$$\begin{aligned} H_0: &\quad \mu_1 = \mu_2 \\ \text{vs. } H_a: &\quad \mu_1 \neq \mu_2 \end{aligned}$$

Analysis of Variance

Or we do a single overall test via Analysis of Variance ANOVA:

The hypothesis test is:

$$\begin{aligned} H_0: &\quad \mu_1 = \mu_2 = \ldots = \mu_k \\ \text{vs. } H_a: &\quad \text{at least one of the } \mu_i\text{'s are different} \end{aligned}$$

Analysis of Variance

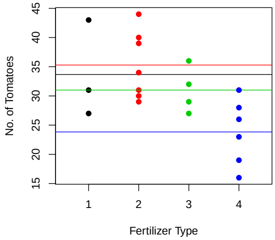
ANOVA asks: where is the overall variability of the observations originate from?

The test statistic used to compute a p-value is now the F-statistic:

$$F = \frac{\text{measure of between-group variability}}{\text{measure of within-group variability}}$$

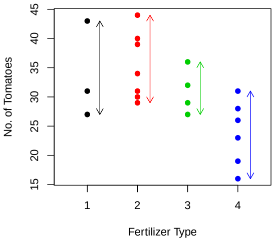
Tomato Fertilizer Example

Numerator: the between-group variation refers to the variability between the levels (the 4 horizontal lines):



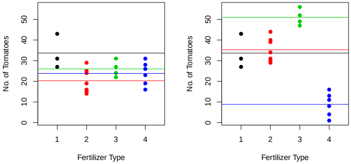
Tomato Fertilizer Example

Denominator: the within-group variation refers to the variability within each level (the 4 vertical arrows):



Tomato Fertilizer Example

Now compare the following two plots. Which has "more different" means?



Tomato Fertilizer Example

- They have the same within-group variability. Call this value W
- The right plot has higher between group variability b/c the 4 means are more different. Call these values B_{left} and B_{right} with $B_{left} < B_{right}$
- Recall $F = \frac{\text{measure of between-group variability}}{\text{measure of within-group variability}}$
- Since $\frac{B_{left}}{W} < \frac{B_{right}}{W}$, thus $F_{left} < F_{right}$ The right plot as a larger F-statistic

F Distributions

Assuming H_0 is true (that $\mu_1 = \mu_2 = \ldots = \mu_k$), the F-statistic

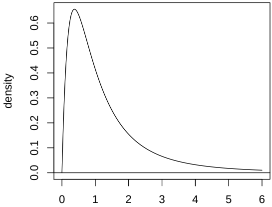
$$F = \frac{\text{measure of between-group variability}}{\text{measure of within-group variability}}$$

follows the F distribution with $df_1 = k - 1$ and $df_2 = n - k$ degrees of freedom where

- n = total number of observations
- k = number of groups

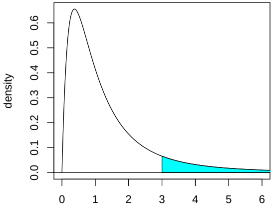
F Distributions

For $df_1 = 4$ and $df_2 = 6$, the F distribution looks like:



F Distributions

p-values are computed where "more extreme" means larger. Say the $F = 3$, the p-value is the area to the right of 3 and is computed in R: `pf(3,df1=4,df2=6,lower.tail=FALSE)`



Conducting An F-Test

The results are typically summarized in an ANOVA table:

Source of Variation	df	SS	MS	F	p-value
Between groups	$k-1$	$SSTr$	$MSTr = \frac{SSTr}{k-1}$	$\frac{MSTr}{MSE}$	p
Within groups	$n-k$	SSE	$MSE = \frac{SSE}{n-k}$		
Total	$n-1$	SST			

Conditions

1. The observations have to be independent. 10% rule.
2. Trade off of n and normality of observations within each group.
3. Each of the groups has constant variance $\sigma_1^2 = \ldots = \sigma_k^2 = \sigma^2$. Check via:
   - boxplots
   - comparing the sample standard deviations $s_1, \ldots, s_k$

Discussion of Quiz

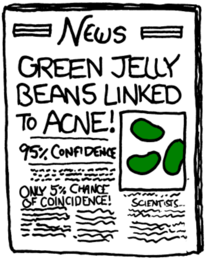
Question 1: Why did $\frac{1}{20}$ studies yield a positive/significant result i.e. that there is a link between jelly beans and acne?

Not that the p-value is 0.05, rather that $\alpha = 0.05$:

- significance level AKA
- type I error rate AKA
- false positive rate

i.e. we expect 1 out of 20 results to be significant even if there is no effect.

Publication Bias



Publication Bias

Publication bias: people only highlight significant/positive results. From Wikipedia: "Publication bias occurs when the publication of research results depends on their nature and direction."

To counter this, some prominent medical journals including

- New England Journal of Medicine
- The Lancet
- Journal of the American Medical Association

require registration of a trial before it starts so that unfavorable results are not withheld from publication.

Journal of Negative Results: `http://www.jnrbm.com/`

Publication Bias

From: Sterne JA, Davey Smith G (2001) Sifting the evidence - What's wrong with significance tests. BMJ 322: 226231.

What α to Use?

Should I use $\alpha = 0.05$ as my significance level? Before using it, put some thought into the balance between:

- Type I errors. Setting a smaller α yields a more conservative procedure: all things being equal, you will reject H_0 less often.
- Type II errors. Setting a bigger α yields a more liberal procedure: all things being equal, you will reject H_0 more often.

Multiple Testing

A related issue is the statistical concept of multiple testing.

Say we are conducting many experiments, and H_0 is true for all of them.

If you repeat experiments many times, you're bound to get a significant result eventually just by chance alone.

Multiple Testing

What do people do? Make the α stricter! i.e.

- make the α smaller
- i.e. less chance the p-value is smaller than α
- i.e. less chance of incorrectly rejecting H_0 when it is true

Use the Bonferroni correction to α: If you are conducting n tests, use $\alpha^* = \frac{\alpha}{n}$. You'll study its properties in HW8.