Lecture 11.3: Model Selection + Multiple Regression Assumption Verification

Chapter 8.2-8.3

Question for Today

Recall the Mario Kart analysis

```
Coefficients:
               Estimate Std. Error t value Pr(>|t|)
(Intercept)    41.34153    1.71167  24.153  < 2e-16 ***
condused       -5.13056    1.05112  -4.881 2.91e-06 ***
stockPhotoyes   1.08031    1.05682   1.022    0.308
duration       -0.02681    0.19041  -0.141    0.888
wheels          7.28518    0.55469  13.134  < 2e-16 ***
---

Residual standard error: 4.901 on 136 degrees of freedom
Multiple R-squared:  0.719, Adjusted R-squared:  0.7108
```

Question for Today

This was the full model: we included every explanatory variable provided.

Question for Today

This was the full model: we included every explanatory variable provided.

Recall the principle inspired by Occam's Razor: all other things being equal, simpler is better. In our case: less predictor variables included in the model!

Question for Today

This was the full model: we included every explanatory variable provided.

Recall the principle inspired by Occam's Razor: all other things being equal, simpler is better. In our case: less predictor variables included in the model!

Is there a systematic (or should I say, less unsystematic) way to pick which predictor variables to include?

Question for Today

This was the full model: we included every explanatory variable provided.

Recall the principle inspired by Occam's Razor: all other things being equal, simpler is better. In our case: less predictor variables included in the model!

Is there a systematic (or should I say, less unsystematic) way to pick which predictor variables to include?

Via model selection techniques.

Two Common Strategies

The following are two common stepwise regression methods because they add/subtract one variable at a time:

- Backward Elimination
- Forward Selection

Two Common Strategies

The following are two common stepwise regression methods because they add/subtract one variable at a time:

- Backward Elimination
- Forward Selection

We will discuss this in terms of a p-value approach. We can also use R^2_{adj} as a criterion.

Backward Elimination

1. Start with the full model

Backward Elimination

1. Start with the full model
2. While there still exists statistically non-significant variables
   2.1 Identify the variable with the largest p-value and drop it
   2.2 Refit the model

Backward Elimination

1. Start with the full model
2. While there still exists statistically non-significant variables
   2.1 Identify the variable with the largest p-value and drop it
   2.2 Refit the model
3. Report model once there are no more non-significant variables

Backward Elimination

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	41.3415	1.7117	24.15	0.0000
cond_used	-5.1306	1.0511	-4.88	0.0000
stockPhotoyes	1.0803	1.0568	1.02	0.3085
duration	-0.0268	0.1904	-0.14	0.8882
wheels	7.2852	0.5547	13.13	0.0000

Drop duration.

Backward Elimination

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	41.2245	1.4911	27.65	0.0000
`cond_used`	-5.1763	0.9961	-5.20	0.0000
`stockPhotoyes`	1.1177	1.0192	1.10	0.2747
`wheels`	7.2984	0.5448	13.40	0.0000

Drop `stockPhotoyes`.

Backward Elimination

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	42.3698	1.0651	39.78	0.0000
`cond_used`	-5.5848	0.9245	-6.04	0.0000
`wheels`	7.2328	0.5419	13.35	0.0000

Done.

Forward Selection

1. Start with the model with no variables

Forward Selection

1. Start with the model with no variables
2. Fit all models with one possible additional variable

Forward Selection

1. Start with the model with no variables
2. Fit all models with one possible additional variable
3. Add the additional variable with the smallest p-value if its significant

Forward Selection

1. Start with the model with no variables
2. Fit all models with one possible additional variable
3. Add the additional variable with the smallest p-value if its significant
4. Repeat steps 2 and 3 until there are no significant additional variables.

Criticisms of the Techniques

Data dredging is the use of data mining to uncover relationships in data.

Criticisms of the Techniques

Data dredging is the use of data mining to uncover relationships in data.

Critics regard stepwise regression as a paradigmatic example of data dredging, intense computation often being an inadequate substitute for subject area expertise.

Criticisms of the Techniques

Data dredging is the use of data mining to uncover relationships in data.

Critics regard stepwise regression as a paradigmatic example of data dredging, intense computation often being an inadequate substitute for subject area expertise.

The process of data mining involves automatically testing huge numbers of hypotheses about a single data set by exhaustively searching for combinations of variables that might show a correlation. Think of multiple testing issues!

Criticisms of the Techniques

Assumptions of Multiple Regression

Assumptions of Multiple Regression

- The residuals e_i of the model

Assumptions of Multiple Regression

- The residuals e_i of the model
  - are nearly normal

Assumptions of Multiple Regression

- The residuals e_i of the model
  - are nearly normal
  - have nearly constant variance

Assumptions of Multiple Regression

- The residuals e_i of the model
  - are nearly normal
  - have nearly constant variance
  - are independent

Assumptions of Multiple Regression

- The residuals e_i of the model
  - are nearly normal
  - have nearly constant variance
  - are independent
- Each variable is linearly related to the outcome

Example Model

We investigate plots for the following model:

$$\widehat{\texttt{price}} = b_0 + b_1 \times \texttt{cond\_new} + b_2 \times \texttt{wheels}$$

Example Model

We investigate plots for the following model:

$$\widehat{\texttt{price}} = b_0 + b_1 \times \texttt{cond\_new} + b_2 \times \texttt{wheels}$$

Example Model

We investigate plots for the following model:

$$\widehat{\texttt{price}} = b_0 + b_1 \times \texttt{cond\_new} + b_2 \times \texttt{wheels}$$

- Normal probability plot of residuals
- Absolute values of residuals against fitted values: look for non-constant variance

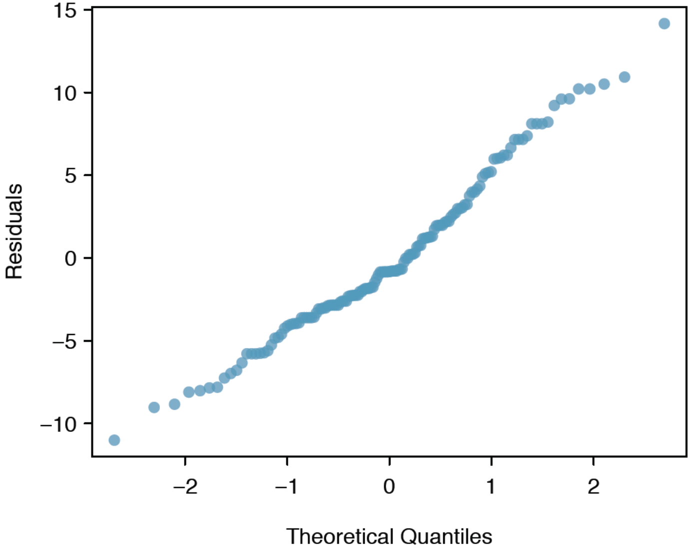
Example Model

We investigate plots for the following model:

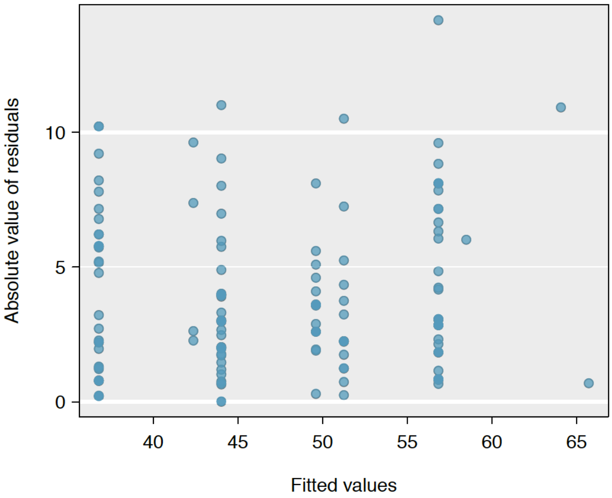
$$\widehat{\texttt{price}} = b_0 + b_1 \times \texttt{cond\_new} + b_2 \times \texttt{wheels}$$

- Normal probability plot of residuals
- Absolute values of residuals against fitted values: look for non-constant variance
- Residuals against each predictor variable

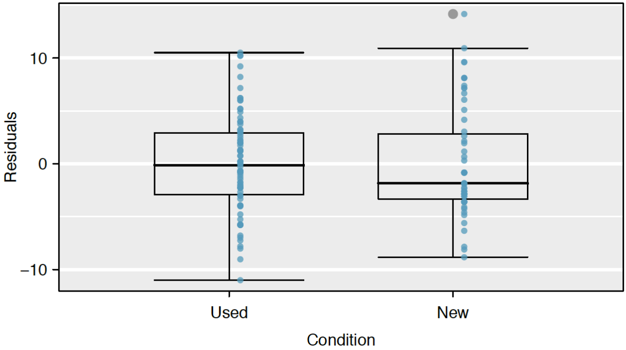
Normal Probability Plot of Residuals



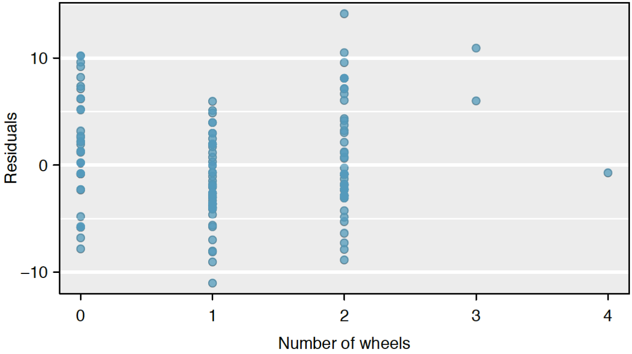
Absolute Values of Residuals Against Fitted Values



Residuals Against Each Predictor Variable: Condition



Residuals Against Each Predictor Variable: Wheels



George E.P. Box

There was a famous statistician named Box

famous for the Box/Cox Transformation.

George E.P. Box's Famous Quote

"All models are wrong, but some are useful."

Caution

That being said, while we can tolerate a little leeway with model assumptions, don't report results when the assumptions are grossly violated. If model assumptions are clearly violated

Caution

That being said, while we can tolerate a little leeway with model assumptions, don't report results when the assumptions are grossly violated. If model assumptions are clearly violated

- consider a new model
- get the assistance of someone who can help

Collinearity Example

Analyzing SAT scores... (See `collinearity.Rmd`)

Analysis in Real Life

My friend JoAnne is a PhD student in Sociology studying the National Parks system. She fits a multiple regression model where:

Analysis in Real Life

My friend JoAnne is a PhD student in Sociology studying the National Parks system. She fits a multiple regression model where:

- data: results of a government funded survey

Analysis in Real Life

My friend JoAnne is a PhD student in Sociology studying the National Parks system. She fits a multiple regression model where:

- data: results of a government funded survey
- outcome variable: level of support increasing funding for the National parks

Analysis in Real Life

My friend JoAnne is a PhD student in Sociology studying the National Parks system. She fits a multiple regression model where:

- data: results of a government funded survey
- outcome variable: level of support increasing funding for the National parks
- predictor variables include
  - typical demographic info
  - other info like education, income, etc.

Missing Data

One problem she was having was a lot of the income data were missing data.

Missing Data

One problem she was having was a lot of the income data were missing data.

Her advisor suggested using the multiple imputation method, which attempts to fill in these missing values. She doesn't understand this method, nor do I very well.

Missing Data

One problem she was having was a lot of the income data were missing data.

Her advisor suggested using the multiple imputation method, which attempts to fill in these missing values. She doesn't understand this method, nor do I very well.

The income variable was 3-level categorical, so a summary of her data was:

- n in low-income
- n in medium-income
- n in high-income
- n of missing data

Missing Data

But then I asked her were a lot of the other variables missing as well? She said no.

Missing Data

But then I asked her were a lot of the other variables missing as well? She said no.

So I surmised that the data wasn't "missing" because of clerical errors, people forgetting to enter the values, etc.

Missing Data

But then I asked her were a lot of the other variables missing as well? She said no.

So I surmised that the data wasn't "missing" because of clerical errors, people forgetting to enter the values, etc.

Rather, these people were deliberately not volunteering this information.

Missing Data

Maybe they are a class of people who have privacy concerns? Maybe this has an impact on their ultimate support of the National Parks? So instead of looking at income as follows:

Missing Data

Maybe they are a class of people who have privacy concerns? Maybe this has an impact on their ultimate support of the National Parks? So instead of looking at income as follows:

- *n* in low-income
- *n* in medium-income
- *n* in high-income
- *n* of missing data

Missing Data

Maybe they are a class of people who have privacy concerns? Maybe this has an impact on their ultimate support of the National Parks? So instead of looking at income as follows:

- n in low-income
- n in medium-income
- n in high-income
- n of missing data

look at it as

- n in low-income
- n in medium-income
- n in high-income
- n who are sensitive about reporting income

Lesson

The moral of the story: statistics is just a tool that always comes second to the scientific question of interest and the data at hand.

Lesson

The moral of the story: statistics is just a tool that always comes second to the scientific question of interest and the data at hand.

Rather than using fancy statistical machinery that we didn't understand, we thought about the question/data and found a work around.

Lesson

The moral of the story: statistics is just a tool that always comes second to the scientific question of interest and the data at hand.

Rather than using fancy statistical machinery that we didn't understand, we thought about the question/data and found a work around.

In this case, the missing data was a blessing in disguise: we found a work around that lead to a new model that might potentially yield better inference than the original model.

Next Time

What if the outcome variable is not numerical, but rather a binary yes vs no response variable?

Next Time

What if the outcome variable is not numerical, but rather a binary yes vs no response variable?

- Was an email spam or not?
- Will someone develop cancer or not?
- Will a car pass an emission test?

Next Time

What if the outcome variable is not numerical, but rather a binary yes vs no response variable?

- Was an email spam or not?
- Will someone develop cancer or not?
- Will a car pass an emission test?

We use logistic regression.