Lecture 22: Chi-Square Tests for Goodness-of-Fit

Chapter 6.3

Question for Today

Say we had $n = 100$ people picked as jurors, we expect the breakdown to be:

Race	White	Black	Hispanic	Other	Total
Registered Voters	72%	7%	12%	9%	100%
Representation	72	7	12	9	$n = 100$

Question for Today

Say we observe the following. Is there a bias? i.e. a non-random mechanism?

Race	White	Black	Hispanic	Other	Total
Registered Voters	72%	7%	12%	9%	100%
Representation	75	6	11	8	$n = 100$

Chi-Square Tests

Chi-square χ^2 tests allow us to compare

- Observed counts
- Expected counts

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i.e. What is the "goodness" of the fit of the observed counts to the expected counts?

The Data

Let's use $n = 275$ people. Assuming the same proportions as above, we compute the expected counts. Ex: $198 = 275 \times 0.72$.

Race	White	Black	Hispanic	Other	Total
Expected Counts	198	19.25	33	24.75	275

The Data

Let's use $n = 275$ people. Assuming the same proportions as above, we compute the expected counts. Ex: $198 = 275 \times 0.72$. Now say we observe the following counts:

Race	White	Black	Hispanic	Other	Total
Expected Counts	198	19.25	33	24.75	275
Observed Counts	205	26	25	19	275

Hypothesis Test in General

Hypothesis Test in Our Case

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2. t-test:
   - test statistic: t-statistic
   - null distribution: t-distribution with $df = n - 1$

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3. AVOVA:
   - test statistic: F-statistic
   - null distribution: F-distribution with $df_1 = k - 1$ and $df_2 = n - k$

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3. AVOVA:
   - test statistic: F-statistic
   - null distribution: F-distribution with $df_1 = k - 1$ and $df_2 = n - k$
4. Goodness-of-fit:
   - test statistic: χ^2-statistic
   - null distribution: χ^2 distribution with $df = k - 1$

Deviations

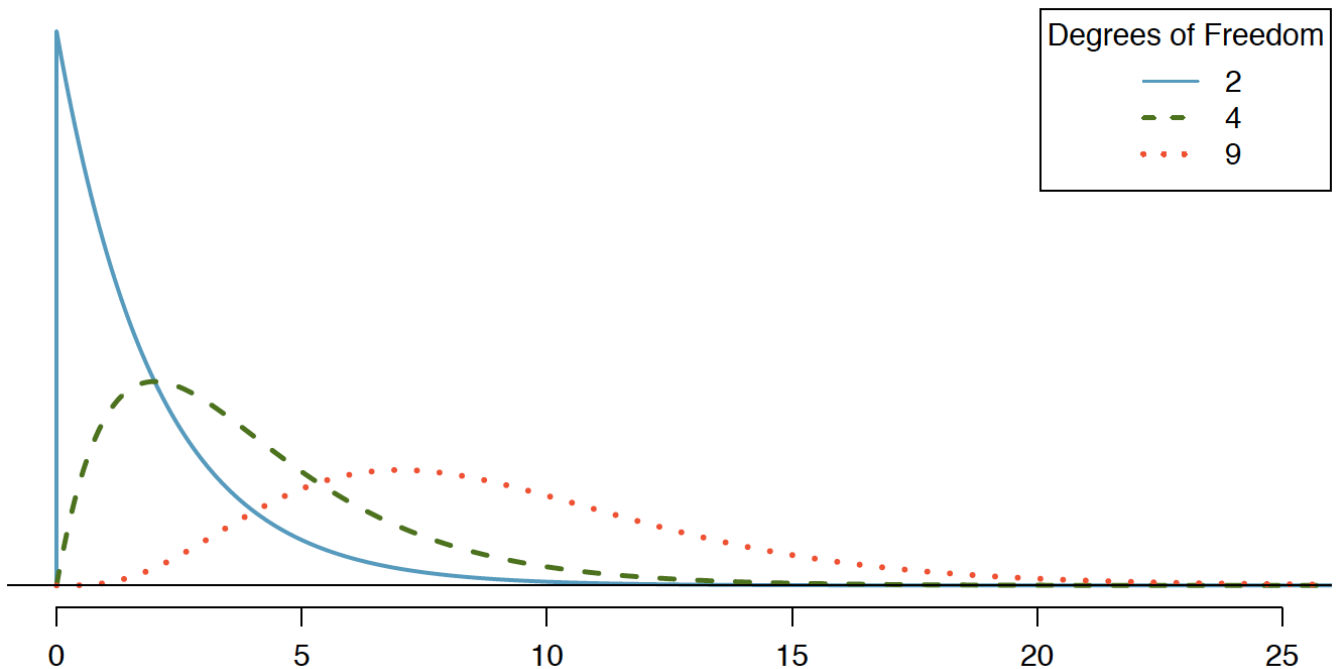
Deviations

Chi-Square Test Statistic

Chi-Square Test Statistic

p-values

We compare the test statistic to a χ^2 distribution with $df = k - 1$ degrees of freedom.
Note: not $df = n - 1$ like with t-test.



p-values

The *p*-value is the area to the right of the test statistic. Use p.412:

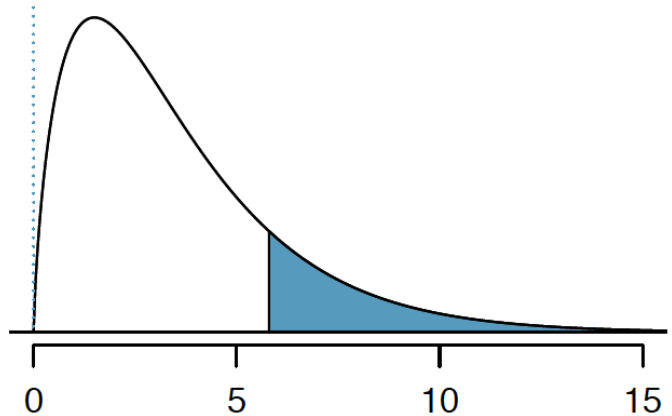


Figure B.2: Areas in the chi-square table always refer to the right tail.

Upper tail		0.3	0.2	0.1	0.05	0.02	0.01	0.005	0.001
df	2	2.41	3.22	4.61	5.99	7.82	9.21	10.60	13.82
	3	3.66	4.64	6.25	7.81	9.84	11.34	12.84	16.27
	4	4.88	5.99	7.78	9.49	11.67	13.28	14.86	18.47
	5	6.06	7.29	9.24	11.07	13.39	15.09	16.75	20.52

Hypothetical Scenarios

Say we have two hypothetical scenarios of observed counts:

Race	White	Black	Hispanic	Other	Total
Expected Counts	198	19.25	33	24.75	275
Observed Counts					275

Assumptions for Chi-Square Test

Next Time

We look at chi-square tests for two-way tables to test for independence. i.e. are two variables independent from each other?