Lecture 20: Single Proportion Test

Chapter 6.1

Quiz 8

Question 1: According to the article, why do scientists even bother with correlational/observational studies, when no notions of causality can be established?

Answer: One reason is that correlational studies are excellent starting points for deciding which hypotheses to evaluate with the more rigorous randomized controlled experiment.

Quiz 8

Question 2: The article argues that various scientific disciplines should set professional labeling standards for material discussed in the media... Rank the four possible labels in order of how much creedence the public should give them, from lowest to highest.

Answer:

3. preliminary result
1. large-scale observational study
4. large-sample randomized controlled test
2. well-established scientific law that we know how to apply in a wide range of conditions

Causality

What is causality? How do we establish it?

- http://nfs.unipv.it/nfs/minf/dispense/patgen/lectures/files/disease_causality.html
- http://bayes.cs.ucla.edu/BOOK-2K/

Question for Today

According to a (representatively sampled) poll done by the New York Times/CBS News in June 2012, only about 44% of the American public approved of the Supreme Court's performance.

The sample proportion $\widehat{p} = 0.44$ is point estimate of p: the true (population) proportion of the American public who approves.

What are some next things to ask?

- What was n?
- What is the SE of $\widehat{p} = 44\% = 0.44$?
- What is the sampling distribution of $\widehat{p}$?

Question for Today

Just like with $\overline{x}$, if we want to use the normal model to

- build confidence intervals via z^*
- conduct hypothesis tests via the normal tables

we need the sampling distribution of $\widehat{p}$ to be nearly normal.

This happens when the population distribution of 0's and 1's is not too strongly skewed. As the sample size $n \longrightarrow \infty$, this is less of an issue by the CLT.

Note:

$$\widehat{p} = \frac{x_1 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

where each of the x_i's are 0/1 success/failure Bernoulli random variables.

Conditions for Sampling Dist'n of $\widehat{p}$ Being Nearly Normal

The sampling distribution of the sample proportion $\widehat{p}$ based on sample size n is nearly normal when

- The observations are independent: the 10% rule
- We expect to see at least 10 successes and 10 failures in our sample. This is called the success-failure condition:
  - $np \geq 10$
  - $n(1-p) \geq 10$

Conditions for Sampling Dist'n of $\widehat{p}$ Being Nearly Normal

If conditions are met, then the sampling distribution of $\widehat{p}$ is nearly normal with

- mean p (the true population proportion)
- standard error

$$SE_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

Note the similarity of the previous formula for the sample mean $\overline{x}$:

$$SE_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sigma^2}{n}}$$

What p to use?

But we don't know what p is. So what p do we use

- to check the success/failure condition?
- for the $SE_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}}$?

For

- Confidence intervals: plug in the point estimate $\widehat{p}$ of p
- Hypothesis tests: plug in the null value p_0 from $H_0 : p = p_0$

Confidence Intervals

Going back to the poll: $\widehat{p} = 0.44$ based on $n = 976$. What is a 95% confidence interval?

Check the conditions and find SE using $p = \widehat{p}$

- $976 < 10\%$ of 313 million $\Rightarrow$ independence
- Defining a success as a person approving of the job done by the Supreme Court:
  - $976 \times \widehat{p} = 976 \times .44 = 429$ successes ≥ 10
  - $976 \times (1 - \widehat{p}) = 976 \times .56 = 547$ failures ≥ 10

$$SE_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} = \sqrt{\frac{0.44(1-0.44)}{976}} = 0.016$$

Confidence Intervals

A 95% confidence interval using the normal model has $z^* = 1.96$, thus:

$$\text{point estimate} \pm 1.96 \times SE$$

In our case

$$\widehat{p} \pm 1.96 \times SE_{\widehat{p}} = 0.44 \pm 1.96 \times 0.016 = (0.409, 0.471)$$

Hypothesis Tests

Thomas Carcetti is running for mayor of Baltimore. His campaign manager claims he has more than 50% support of the electorate.

The Baltimore Sun collects a random sample of $n = 500$ likely voters and finds that 52% support him. Does this provide convincing evidence for the claim of Carcetti's manager at the 5% significance level?

Hypothesis Tests

The hypothesis test is, with the null value $p_0 = 0.5$

$$
\begin{aligned}
& H_0 : p = p_0 \\
\text{vs} \quad & H_A : p > p_0
\end{aligned}
$$

Check the conditions and find SE using $p = p_0$

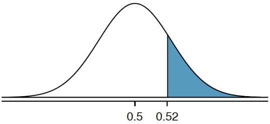
- $500 < 10\%$ of the population of Baltimore $\Rightarrow$ independence
- Success-failure condition
  - $np_0 = 500 \times 0.5 = 250 \geq 10$
  - $n(1 - p_0) = 500 \times (1 - 0.5) = 250 \geq 10$

$$SE_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(1-0.5)}{500}} = 0.022$$

Hypothesis Tests

$$z = \frac{\text{point estimate } \widehat{p} - \text{ null value } p_0}{SE_{\widehat{p}}} = \frac{0.52 - 0.50}{0.022} = 0.89$$

p-value is 0.1867. In the original %'age scale:



Hence we do not reject the null hypothesis, and we do not find convincing evidence to support the campaign manager's claim.

Next Time

Same as with the jump from

$$\mu \text{ to } \mu_1 - \mu_2$$

i.e. from one to two-sample tests for means, we make the jump from

$$p \text{ to } p_1 - p_2$$

i.e. from one to two-sample tests for proportions.