

# Lecture 21: Difference of two proportions

Chapter 6.2

## Question for today

How do we infer about a difference in proportions  $p_1 - p_2$ ?

# Surveys

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Let's infer about the difference in proportion of people who approve. Any guesses which is higher?

## Example from Text

	Sample size $n_i$	Approve (%)	Disapprove (%)	Other (%)
people who do not buy it will pay a penalty given first	771	47	49	3
people who cannot afford it will receive financial help from the gov't given first	732	34	63	3

## Example from Text

	Sample size $n_i$	Approve (%)	Don't Approve (%)
people who do not buy it will pay a penalty given first	771	47	53
people who cannot afford it will receive financial help from the gov't given first	732	34	66



## Example from Text

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So  $\hat{p}_1 - \hat{p}_2 = 0.47 - 0.34 > 0$ : people are more likely to support Obamacare in the first scenario.

Conditions...

... for Sampling Dist'n of  $\hat{p}_1 - \hat{p}_2$  Being Normal

# Standard Error

What  $p_1$  &  $p_2$ ?

# Confidence Intervals

What is a 90% confidence interval for the difference in proportions?

# Confidence Intervals

# Interpretation

Two key observations:

- ▶ (9%, 17%) does not contain 0, suggestive of a true difference.
- ▶ The sign of the difference:  $\hat{p}_1 - \hat{p}_2 = 0.13 > 0$



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- ▶ The sign of the difference:  $\hat{p}_1 - \hat{p}_2 = 0.13 > 0$

More support Obamacare if stated as follows:

*People who do not buy it will pay a penalty while people who cannot afford it will receive financial help from the government.*

# Hypothesis Tests

Now we are interested in testing the difference of two proportions:

$$\begin{array}{l} H_0 : p_1 - p_2 = 0 \\ \text{vs} \quad H_1 : p_1 - p_2 \neq 0 \end{array}$$

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i.e. under  $H_0$  the two proportions are both equal to some value  $p$ :

$$p_1 = p_2 = p$$

# Hypothesis Tests

## Exercise 6.31 on Page 305

A 2010 survey asked 827 randomly sample voters in California  
“How do you feel about drilling for oil and natural gas off the  
coast of California?”

	College Grad	
	Yes	No
Support	154	132
Oppose	180	126
Don't Know	104	131
Total	438	389

## Exercise 6.31 on Page 305

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	College Grad	
	Yes	No
Support	154	132
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Test at the  $\alpha = 0.10$  significance level if the proportion of college graduates who support off-shore drilling is different than that of non-college graduates.

## Exercise 6.31 on Page 305



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# Jury Selection

Preview of next lecture: In many trials a big issue is the racial makeup of the jury.

# Jury Selection

Preview of next lecture: In many trials a big issue is the **racial makeup** of the jury.

Question: is there a way to figure out if there is a **racial bias** in jury selection?

# Jury Selection

Say we have a juror pool (registered voters) where the racial breakdown is:

Race	White	Black	Hispanic	Other	Total
Registered Voters	72%	7%	12%	9%	100%

# Jury Selection

If we pick  $n = 100$  jurors **at random** (i.e. unbiasedly), we **expect** the breakdown of counts to be:

Race	White	Black	Hispanic	Other	Total
Registered Voters	72%	7%	12%	9%	100%
Representation	72	7	12	9	$n = 100$

# Jury Selection

Say we **observe** the following counts:

Race	White	Black	Hispanic	Other	Total
Registered Voters	72%	7%	12%	9%	100%
Representation	0	0	100	0	$n = 100$

# Jury Selection

Say we **observe** the following counts:

Race	White	Black	Hispanic	Other	Total
Registered Voters	72%	7%	12%	9%	100%
Representation	0	0	100	0	$n = 100$

Fairly obvious bias in juror selection!

# Jury Selection

But what about the following? Is there a bias? i.e. a non-random mechanism at play?

Race	White	Black	Hispanic	Other	Total
Registered Voters	72%	7%	12%	9%	100%
Representation	75	6	11	8	$n = 100$



## Next Two Lectures

Chi-square tests are used to compare **expected** counts with **observed** counts.

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Two tests we'll see:

- ▶ Goodness-of-fit tests: for frequency tables
- ▶ Tests for independence: for contingency/two-way tables