Lecture 20: Single Proportion Test

Chapter 6.1

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What are some next things to ask?

- What was n?
- What is the SE of $\widehat{p} = 44\% = 0.44$?
- What is the sampling distribution of $\widehat{p}$?

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Note:

$$\widehat{p} = \frac{x_1 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

where each of the x_i's are 0/1 success/failure Bernoulli random variables.

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- The observations are independent: the 10% rule
- We expect to see at least 10 successes and 10 failures in our sample. This is called the success-failure condition:
  - $np \geq 10$
  - $n(1-p) \geq 10$

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Note the similarity of the previous formula for the sample mean $\overline{x}$:

$$SE_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sigma^2}{n}}$$

What p to use?

But we don't know what p is. So what p do we use

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- Confidence intervals: plug in the point estimate $\widehat{p}$ of p
- Hypothesis tests: plug in the null value p_0 from $H_0 : p = p_0$

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- Defining a success as a person approving of the job done by the Supreme Court:
  - $976 \times \widehat{p} = 976 \times .44 = 429$ successes ≥ 10
  - $976 \times (1 - \widehat{p}) = 976 \times .56 = 547$ failures ≥ 10

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$$SE_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} = \sqrt{\frac{0.44(1-0.44)}{976}} = 0.016$$

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In our case

$$\widehat{p} \pm 1.96 \times SE_{\widehat{p}} = 0.44 \pm 1.96 \times 0.016 = (0.409, 0.471)$$

Hypothesis Tests

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Thomas Carcetti is running for mayor of Baltimore. His campaign manager claims he has more than 50% support of the electorate.

The Baltimore Sun collects a random sample of $n = 500$ likely voters and finds that 52% support him. Does this provide convincing evidence for the claim of Carcetti's manager at the 5% significance level?

Hypothesis Tests

The hypothesis test is, with the null value $p_0 = 0.5$

$$\begin{aligned} & H_0 : p = p_0 \\ \text{vs} \qquad & H_A : p > p_0 \end{aligned}$$

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  - $np_0 = 500 \times 0.5 = 250 \geq 10$
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$$SE_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(1-0.5)}{500}} = 0.022$$

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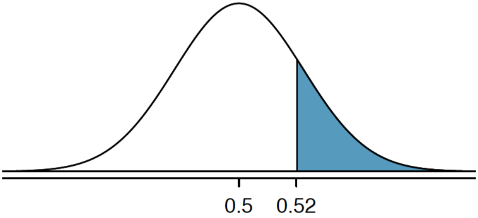
p-value is 0.1867. In the original %'age scale:



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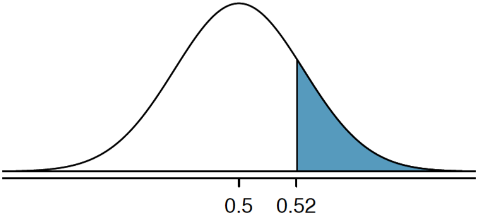

Hence we do not reject the null hypothesis, and we do not find convincing evidence to support the campaign manager's claim.

Next Time

Same as with the jump from

$$\mu \text{ to } \mu_1 - \mu_2$$

i.e. from one to two-sample tests for means, we make the jump from

$$p \text{ to } p_1 - p_2$$

i.e. from one to two-sample tests for proportions.