Lecture 15: Hypothesis Testing Part II

Chapter 4.3

Previously... Statistical Hypothesis Testing

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  i.e. the status quo that is initially assumed to be true, but will be tested.
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There are two potential outcomes of a hypothesis test. Either we

- reject H_0
- fail to reject H_0

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	H_A true	Type II Error	OK

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	H_A true	Type II Error	OK

Two kinds of errors:

- Type I Error: a false positive (test result)
- Type II Error: a false negative (test result)

Type I Errors: US Criminal Justice System

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In this case:

- Type I error is putting an innocent person in jail (considered worse)
- Type II error is letting a guilty person go free.

Type II Errors: Airport Screening

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Hence the long lines at airport security.

Goals for Today

- Define significance level
- Tie-in p-Values with sampling distributions
- Example

Significance Level

Thought experiment: p-Values

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- 525 heads? Do you think the coin is biased?

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Say you flip a coin you think is fair 1000 times. Say you observe

- 501 heads? Do you think the coin is biased?
- 525 heads? Do you think the coin is biased?
- 900 heads? Do you think the coin is biased?

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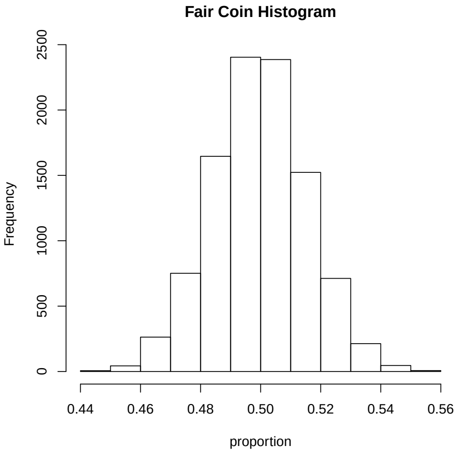
Note the p-value is different than the population proportion p (bad historical choice).

p-Values

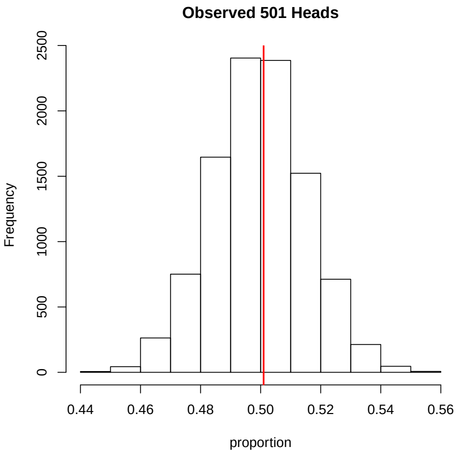
Recall our Coin Example

Sampling Distribution of $\widehat{p}$

Under H_0 that the coin is fair i.e. $p = p_0 = 0.5$, the sampling distribution of $\widehat{p}$ when $n = 1000$ is:



Say we observe...



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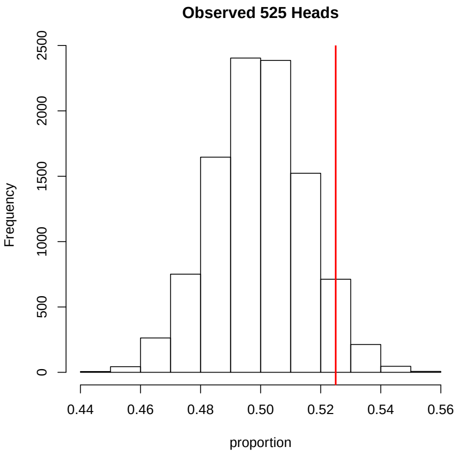

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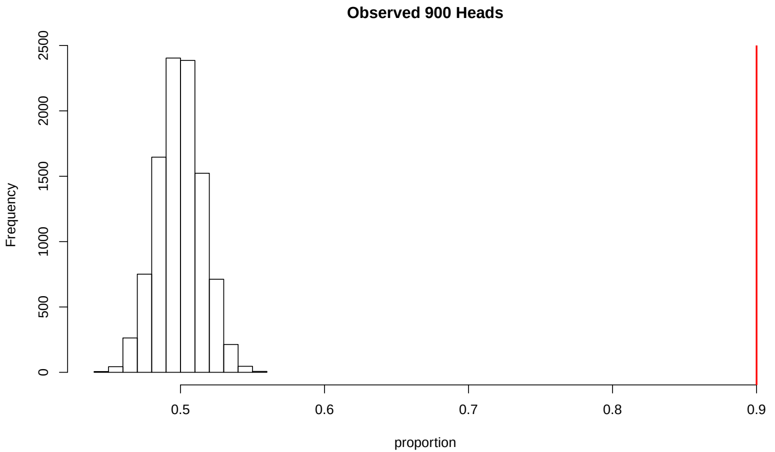

Example about Sleep Habits

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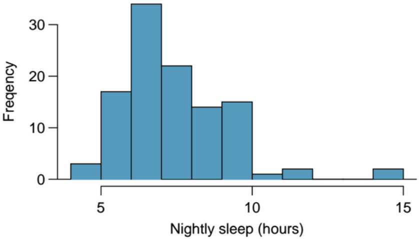

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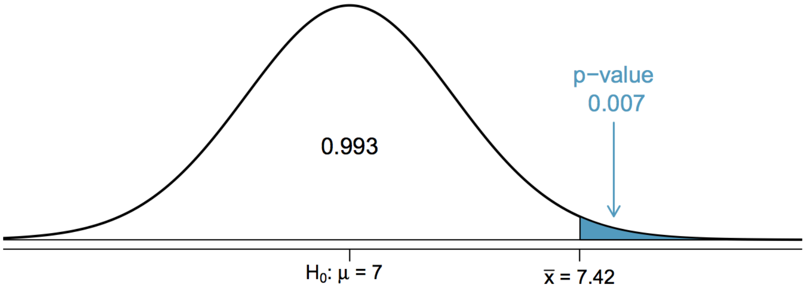
In our case, since $H_A : \mu > 7$, more extreme means to the right of $z = 2.47$.

Hence, the p-value is 0.007:



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Interpretation: we reject (at the $\alpha = 0.05$ significance level) the hypothesis that the average # of hours of Reedies sleep is 7, in favor of the hypothesis that sleep more.

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Correct interpretation of the p-value: If the null hypothesis is true ($\mu = 7$), the probability of observing a sample mean $\overline{x} = 7.42$ or greater is 0.007.

Incorrect interpretation of the p-value: The probability that the null hypothesis ($\mu = 7$) is true is 0.007.

Two-Sided Alternative Hypothesis

Say instead we had a two-sided alternative hypothesis:

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The the p-value would be double: $2 \times 0.007 = 0.014$. Picture:

Next Time

- How big a sample size to I need? i.e. power calculations
- Statistical vs practical significance