Lecture 8: Normal Distribution

Chapter 3.1

Goals for Today

- Define the normal distribution in terms of its parameters
- Review: $\frac{2}{3}$ / 95% / 99.7% rule
- Standardizing normal observations to z-scores

Normal Distribution

From text page 118:

Many variables are nearly normal, but none are exactly normal. Thus the normal distribution, while not perfect for any single problem, is very useful for a variety of problems.

We will use it in data exploration and to solve important problems in statistics.

Normal Distribution

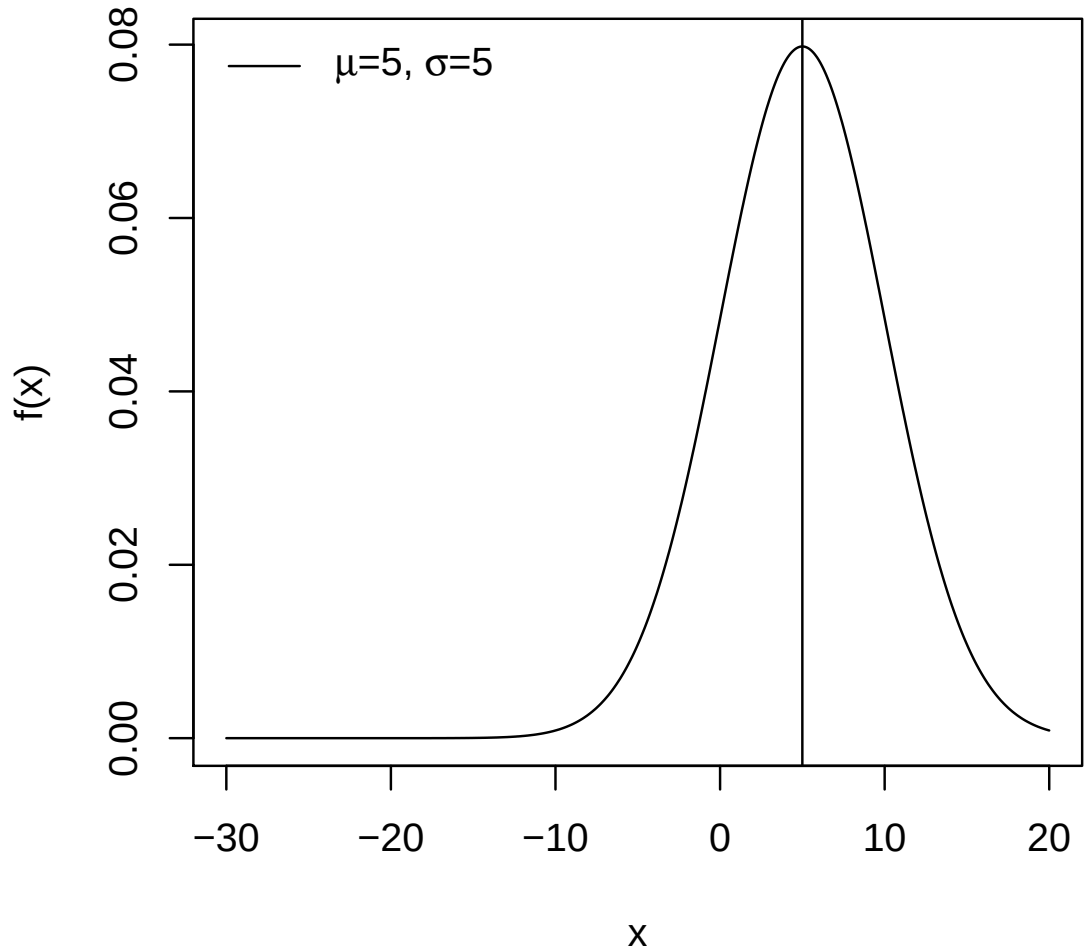
Normal distributions:

1. are symmetric
2. are unimodal and bell-shaped
3. have area under the curve 1

Normal Distribution

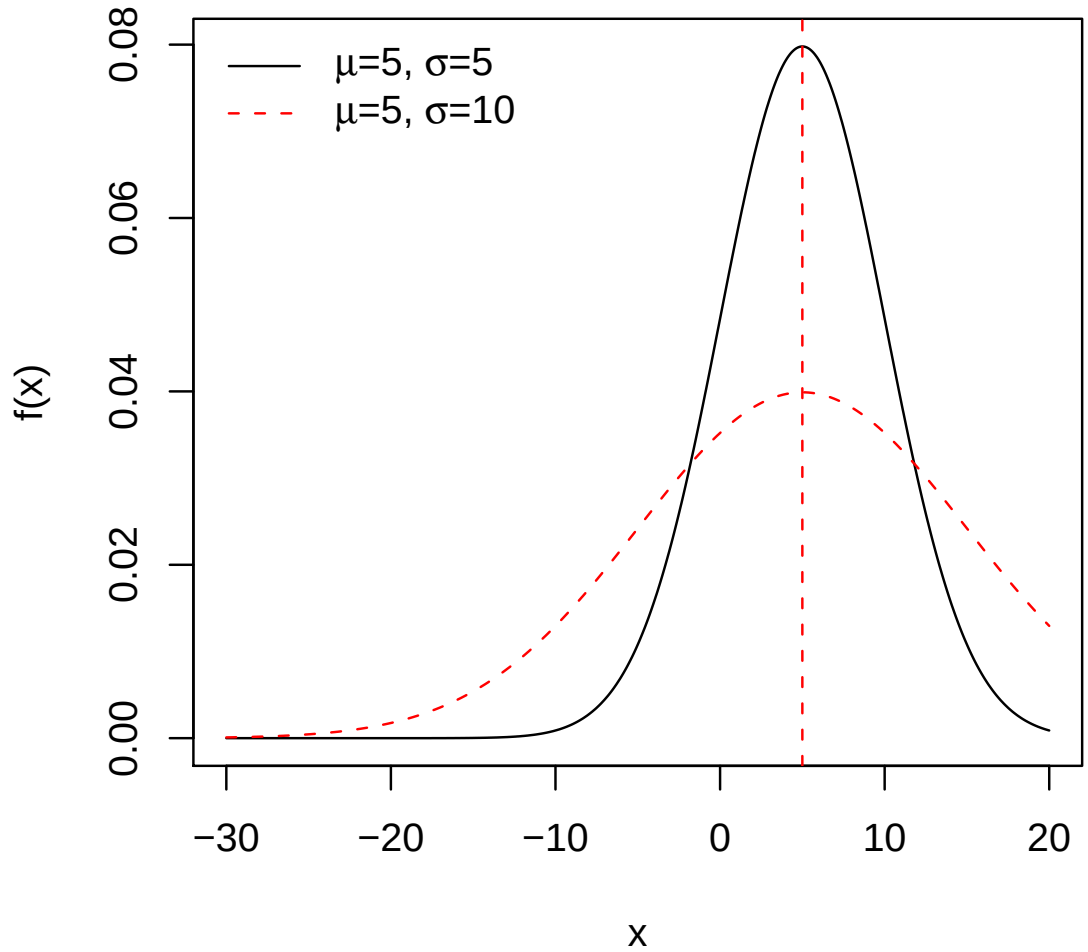
Normal Distribution

μ (mean) specifies the center, σ (standard deviation) the spread.



Normal Example

μ (mean) specifies the center, σ (standard deviation) the spread.



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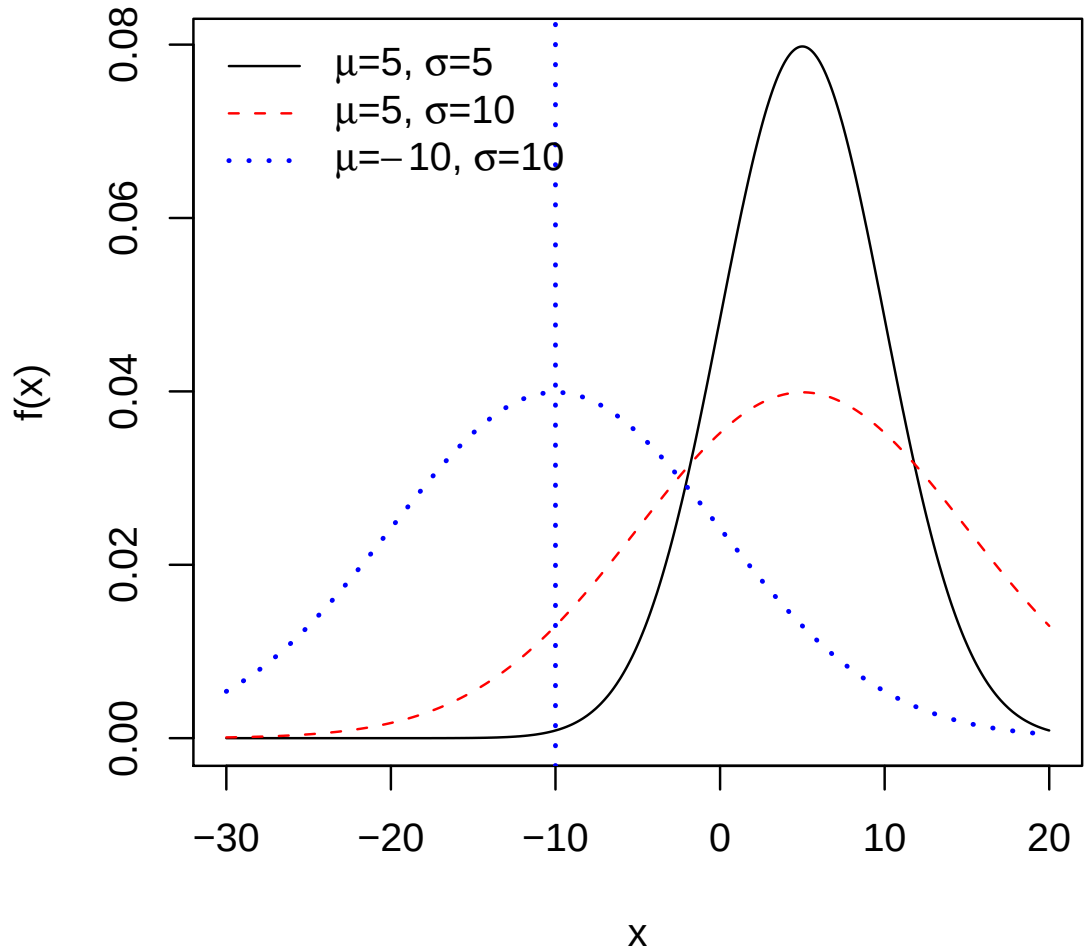
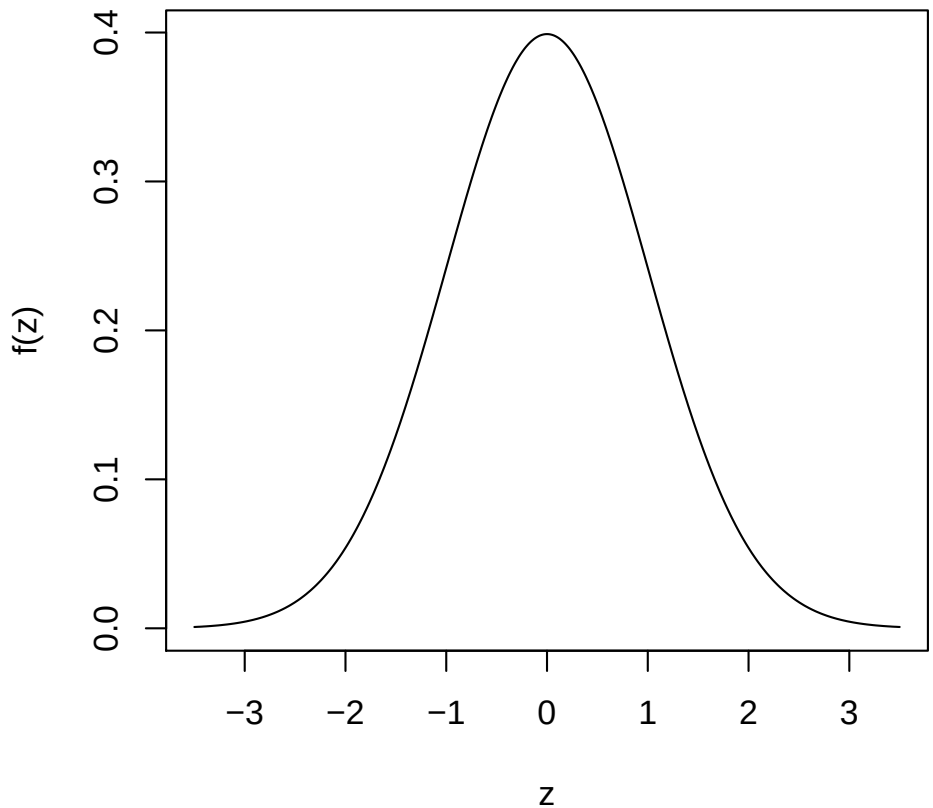


Standardized Normal Distribution

If $\mu = 0$ and $\sigma = 1$, this is the standard normal distribution:



Rules of Thumb

Recall if a distribution is normal, then:

1. Approx. $\frac{2}{3}$'s of the data are within ± 1 SD of the mean
2. Approx. 95% of the data are within ± 2 SD of the mean

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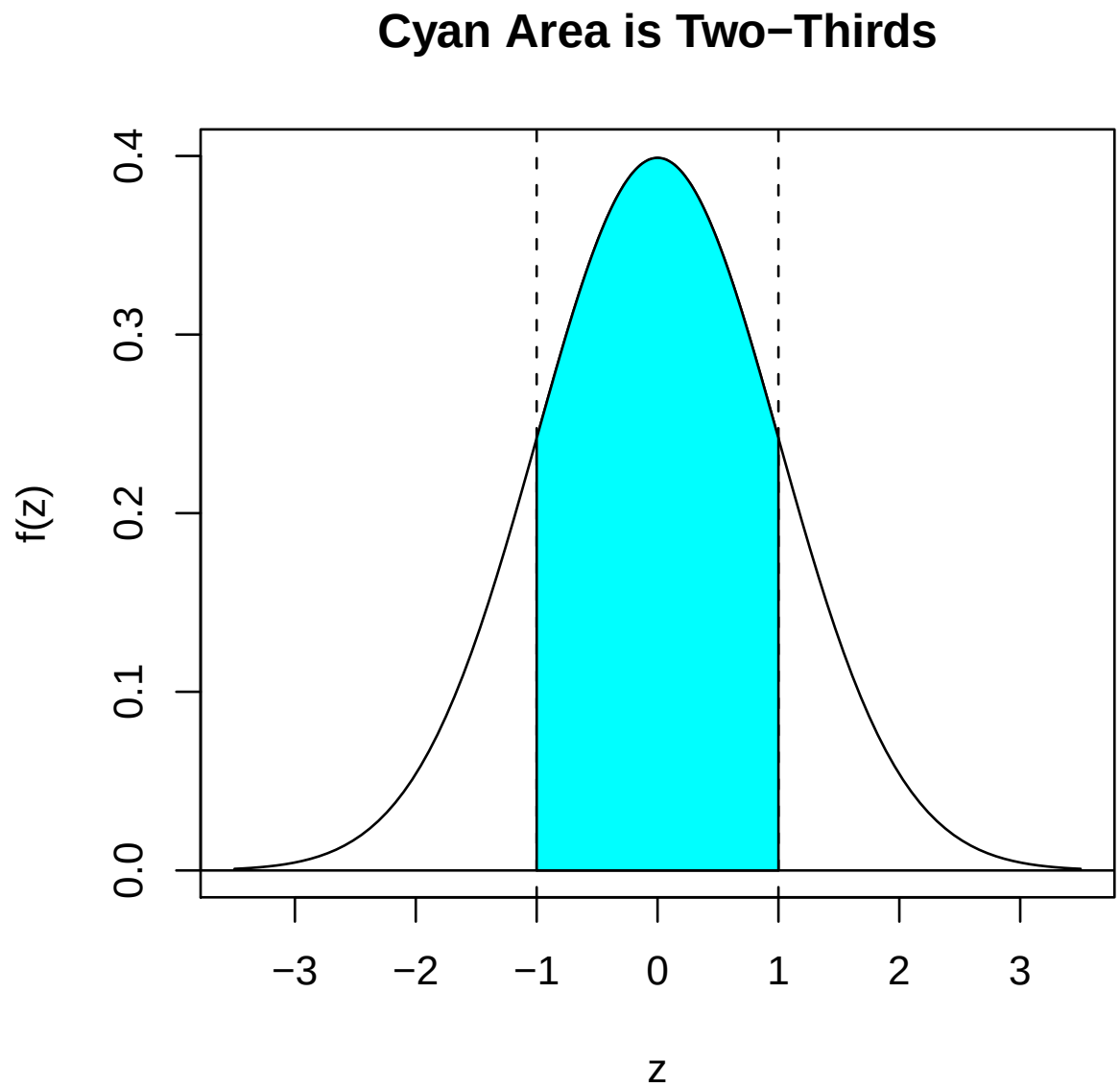
1. Approx. $\frac{2}{3}$'s of the data are within ± 1 SD of the mean
2. Approx. 95% of the data are within ± 2 SD of the mean
3. Also approx. 99.7% of the data are within ± 3 SD of the mean

Ex: Standard Normal $\mu = 0$, $\sigma = 1$



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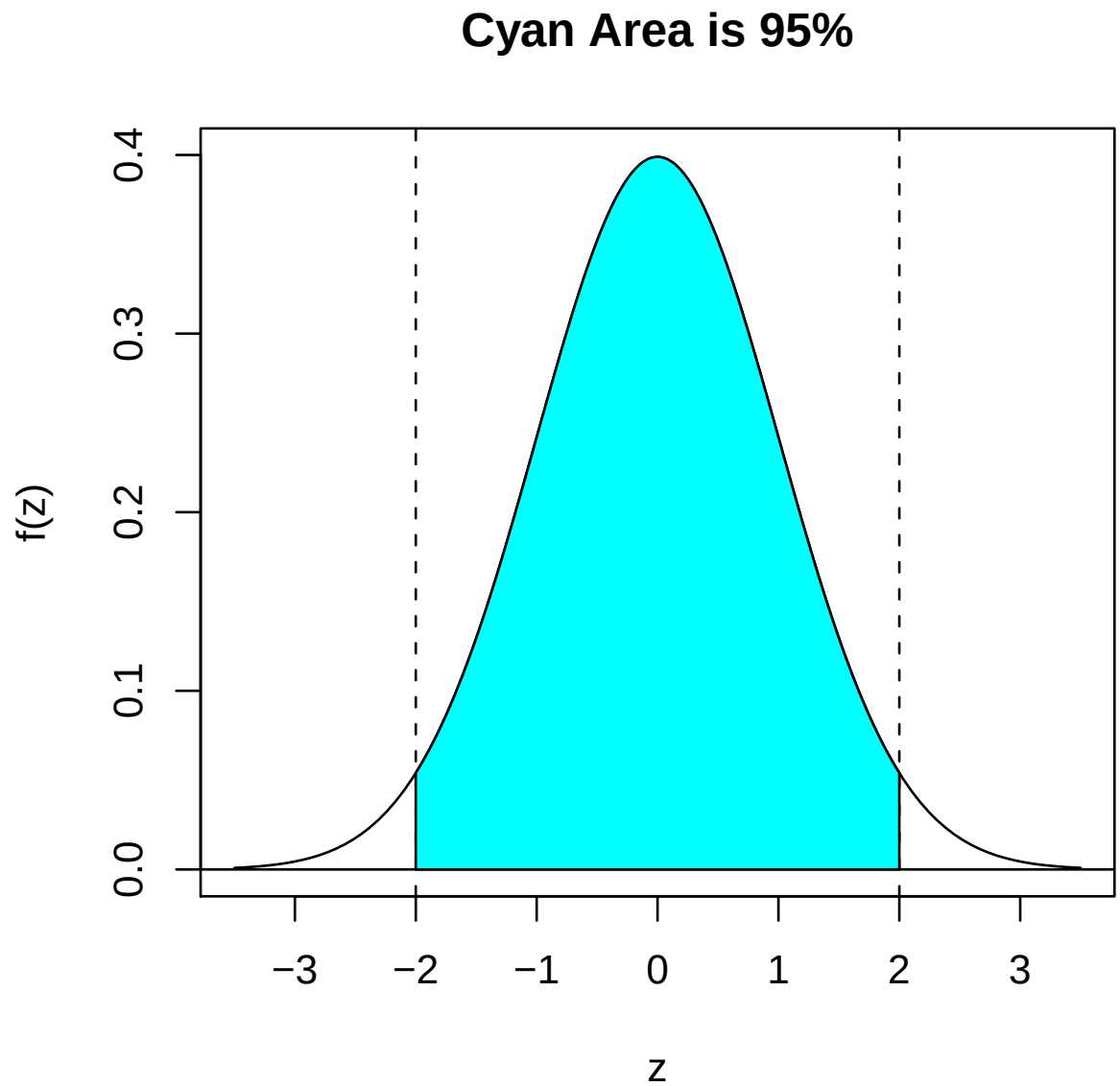

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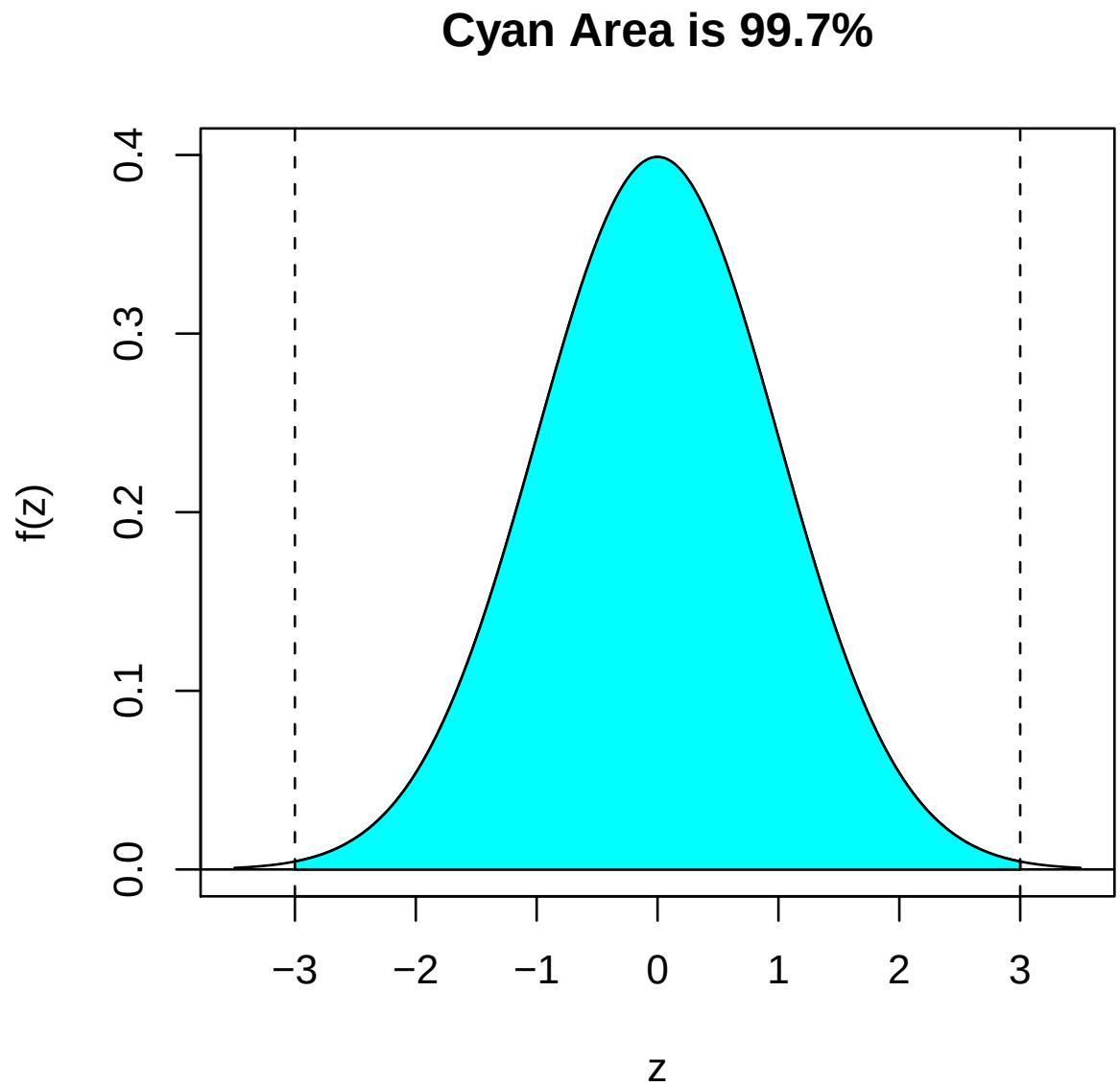

Motivating Example

From text: Say Ann scores 1800 on the SAT and Tom scores 24 on the ACT.

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From text: Say Ann scores 1800 on the SAT and Tom scores 24 on the ACT. Say both tests scores were normally distributed with:

	SAT	ACT
Mean μ	1500	21
SD σ	300	5

Question: Who did relatively better?

z-scores

z-scores

Back to Example

So Ann did relatively better.

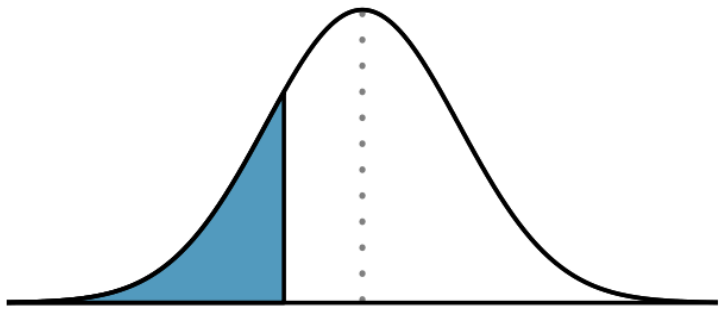
Percentiles

Recall a percentile (%'ile) indicates the value below which a given %'age of observations fall below.

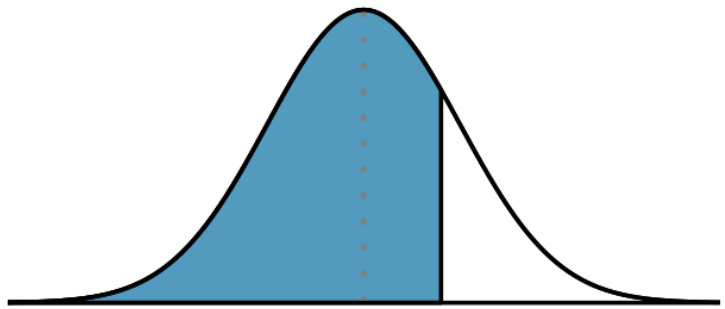
Question: What %'ile is Ann's SAT score of 1800?

Percentiles

Because the total area under the curve is 1, the area to the left of z represents the %'ile of the observation:



negative Z



positive Z

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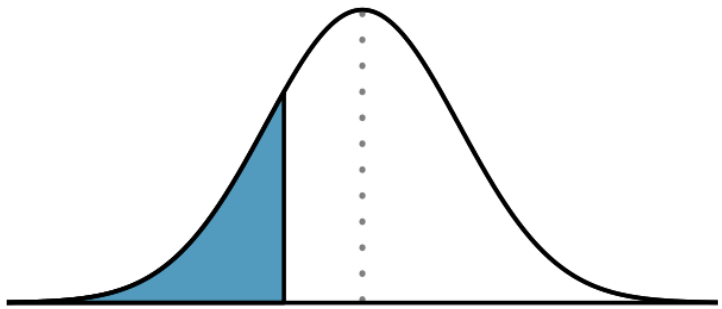

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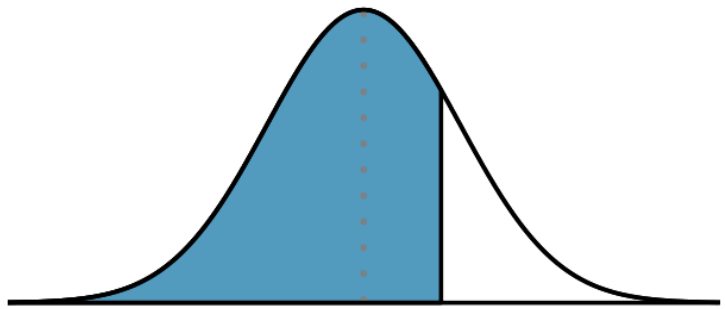

positive Z

- The blue shaded area on the left plot will be less than 0.5. We have %'iles less than the 50th %'ile.

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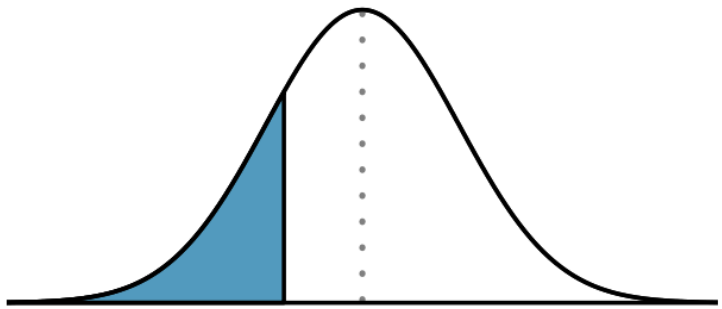
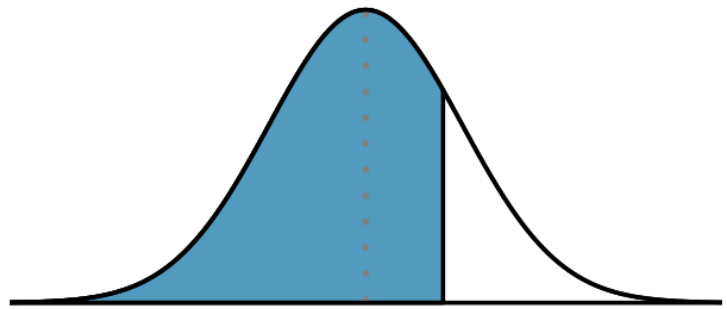


negative Z



positive Z

- The blue shaded area on the left plot will be less than 0.5. We have %'iles less than the 50th %'ile.
- The blue shaded area on the right plot will be greater than 0.5. We have %'iles greater than the 50th %'ile.

Normal Probability Table

A normal probability table allows you to:

- identify the %'ile corresponding to a z-score
- or vice versa: the z-score corresponding to a %'ile

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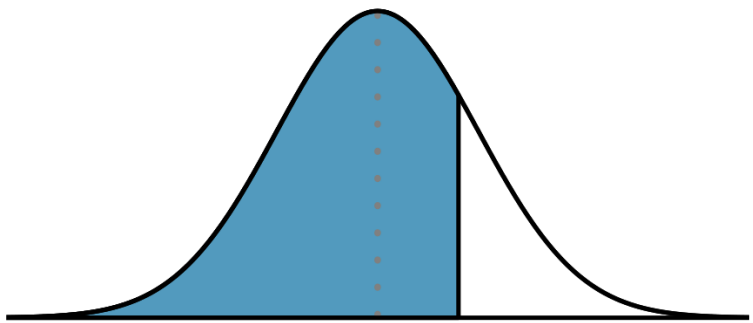
The normal probability tables on page 409 represent z-scores and %'iles corresponding to area to the left:



Normal Probability Table

	Second decimal place of Z									
Z	0.00	0.01	0.02	*0.03*	**0.04**	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	*0.6664*	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	**0.7995**	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Back to Ann and Tom

Next Time

Next time we will:

- Re-iterate the motivation for the normal curve.
- Go over examples using z-scores.
- Evaluating the normal approximation.