# Lecture 15: Hypothesis Testing Part II

Chapter 4.3



## Previously... Statistical Hypothesis Testing

A hypothesis test is a method for using sample data to decide between two competing hypotheses about the population parameter:

- A null hypothesis $H_0$.
  i.e. the status quo that is initially assumed to be true, but will be tested.
- An alternative hypothesis $H_A$. i.e. the challenger.

There are two potential outcomes of a hypothesis test. Either we

- reject $H_0$
- fail to reject $H_0$

## Previously... Decision Errors

Hypothesis tests will get things right sometimes and wrong sometimes:

| | | **Test conclusion** | |
|---|---|---|---|
| | | do not reject $H_0$ | reject $H_0$ in favor of $H_A$ |
| **Truth** | $H_0$ true | OK | Type I Error |
| | $H_A$ true | Type II Error | OK |

Two kinds of errors:

- Type I Error: a false positive (test result)
- Type II Error: a false negative (test result)



## Type I Errors: US Criminal Justice System

Defendants must be "guilty beyond a reasonable doubt": better to let a guilty person go free, than put an innocent person in jail.

- $H_0$: the defendant is innocent
- $H_A$: the defendant is guilty

thus "rejecting $H_0$" is a guilty verdict $\Rightarrow$ putting them in jail

In this case:

- Type I error is putting an innocent person in jail (considered worse)
- Type II error is letting a guilty person go free.

## Type II Errors: Airport Screening

An example of where Type II errors are more serious: airport screening.

$$
\begin{aligned}
H_0: &\quad \text{passenger X does not have a weapon} \\
H_A: &\quad \text{passenger X has a weapon}
\end{aligned}
$$

Failing to reject $H_0$ when $H_A$ is true is not "patting down" passenger X when they have a weapon.

Hence the long lines at airport security.



## Goals for Today

- Define significance level
- Tie-in p-Values with sampling distributions
- Example

## Significance Level

Hypothesis testing is built around rejecting or failing to reject the null hypothesis.
i.e. we do not reject $H_0$ unless we have strong evidence.

As a rule of thumb, when $H_0$ is true, we do not want to incorrectly reject $H_0$ more than 5% of the time.
i.e. $\alpha = 0.05 = 5\%$ is the significance level.

With 95% confidence intervals from earlier, we expect it to miss the true population parameter 5% of the time. This corresponds to $\alpha = 0.05$.



## Thought experiment: Coin Flips

Say you flip a coin you think is fair 1000 times. Say you observe

- 501 heads? Do you think the coin is biased?
- 525 heads? Do you think the coin is biased?
- 900 heads? Do you think the coin is biased?

## Thought experiment: p-Values

Intuitively, a p-value quantifies how extreme an observation is given the null hypothesis.

The smaller the p-value, the more extreme the observation, where the meaning of extreme depends on the context.

Note the p-value is different than the population proportion $p$ (bad historical choice).



## p-Values

Definition: The p-value or observed significance level is the probability of observing a test statistic as extreme or more extreme (in favor of the alternative) as the one observed, assuming $H_0$ is true.

It is NOT the probability of $H_0$ being true. This is the most common misinterpretation of the $p$-value.

## Thought experiment: Coin Flips

You have a coin that test for fairness with $n = 1000$ flips. Set $p_0 = 0.5$ (coin is fair) and define a "success" as getting heads.

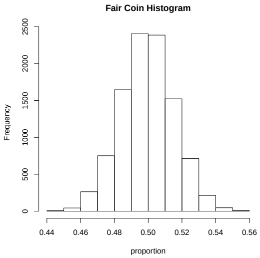
$$\begin{aligned} & H_0 : p = p_0 \\ vs \quad & H_A : p \neq p_0 \end{aligned}$$

- The point estimate $\widehat{p}$ of $p$ is $\frac{\#\text{ of successes}}{\#\text{ of trials}}$.
- Since it is based on a sample, $\widehat{p}$ has a sampling distribution
- The standard error is $\sqrt{\frac{p(1-p)}{n}}$ (Chapter 6).
- Furthermore, since conditions hold, the sampling distribution is Normal (CLT)



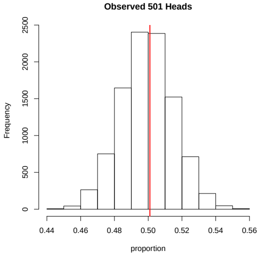
## Sampling Distribution of $\widehat{p}$

Under $H_0$ that the coin is fair, i.e. $p = p_0 = 0.5$, the sampling distribution of $\widehat{p}$ when $n = 1000$ is:



## Say we observe...

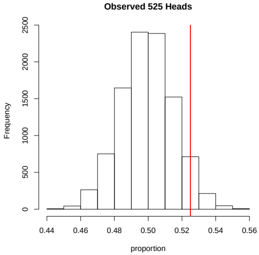
$$\widehat{p} = \frac{501}{1000}$$





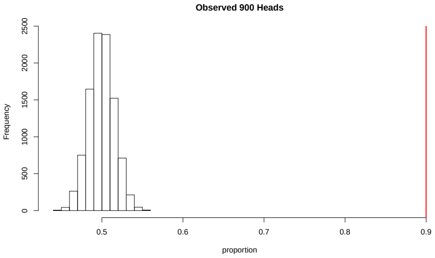

## Say we observe...

$$\widehat{p} = \frac{525}{1000}$$



## Say we observe...

$\widehat{p} = \frac{900}{1000}$

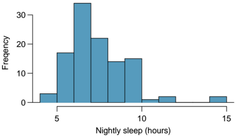





## Example about Sleep Habits

A poll found that college students sleep about 7 hours a night. Researchers suspect that Reedies sleep more. They want to investigate this claim at a pre-specified $\alpha = 0.05$ level.

They sample $n = 110$ Reedies and find that $\overline{x} = 7.42$ and $s = 1.75$ and the histogram looks like:



## Example about Sleep Habits

Let $\mu =$ true population mean # of hours Reedies sleep a night. Then $\mu_0 = 7$ and:

- $H_0 : \mu = \mu_0 = 7$
- $H_A : \mu > 7$



## Example about Sleep Habits

We check the 3 conditions to use the Normal model:

1. Independence: $n = 110 \leq 10\%$ of 1,453 (Reed enrollment)
2. $n \geq 30$
3. The distribution of the $n = 110$ observations (Figure 4.14) is not too skewed.

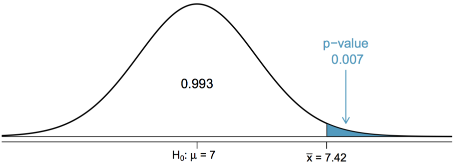
If $H_0$ is true, then $\overline{x}$ has mean $\mu_0$. So in our case for $\overline{x}$

$$z = \frac{\overline{x} - \text{null value}}{SE} = \frac{7.42 - 7}{\frac{1.75}{\sqrt{110}}} = 2.47$$

## Example about Sleep Habits

In our case, since $H_A : \mu > 7$, more extreme means to the right of $z = 2.47$.

If $H_0$ is true, then the null distribution looks like this:




Hence, the p-value is 0.007.



## Example about Sleep Habits

Since the p-value $0.007 < 0.05 = \alpha$, the pre-specified significance level, it has a high degree of extremeness, and thus we reject $H_0$.

Conclusion: we reject (at the $\alpha = 0.05$ significance level) the hypothesis that the average # of hours of Reedies sleep is 7, in favor of the hypothesis that sleep more.

## Example about Sleep Habits

Correct interpretation of the p-value: If the null hypothesis is true ($\mu = 7$), the probability of observing a sample mean $\overline{x} = 7.42$ or greater is 0.007.

Incorrect interpretation of the p-value: The probability that the null hypothesis ($\mu = 7$) is true is 0.007.



## Next Time

- How big a sample size to I need? i.e. power calculations
- Statistical vs practical significance