Lecture 7: Probability

Chapter 2.x

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Outcomes

Probability forms the theoretical backbone of statistics. We use probability to characterize randomness.

We often frame probability in terms of a random process giving rise to an outcome.

Typical examples

- Die roll: 6 outcomes
- ► Coin Flip: 2 outcomes

Disjoint AKA Mutually Exclusive Outcomes

Two outcomes are disjoint (AKA mutually exclusive) if they cannot both occur at the same time.

Die example:

- ▶ Rolling a 1 and a 2 are disjoint.
- ▶ Rolling a 1 and rolling "an odd number" are not disjoint.

Addition Rule of Probability

If A_1 and A_2 are disjoint outcomes, then

$$P(A_1 \text{ or } A_2) = P(A_1) + P(A_2)$$

Ex: Rolling 1 and 2 are disjoint, so:

$$P(\text{rolling 1 or 2}) = P(\text{rolling 1}) + P(\text{rolling 2}) = \frac{1}{6} + \frac{1}{6}$$

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General Addition Rule of Probability

If A_1 and A_2 are two outcomes (not necessarily disjoint), then

$$P(A_1 \text{ or } A_2) = P(A_1) + P(A_2) - P(A_1 \text{ and } A_2)$$

Venn diagram:

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General Addition Rule of Probability

Events are just combinations of outcomes. Ex: Deck of cards

- ▶ A₁ = event we draw a diamond
- $ightharpoonup A_2 = {\sf event}$ we draw a face card

These two events are not disjoint, as there are 3 diamond face cards. Venn diagram:

General Addition Rule of Probability

$$\begin{array}{ll} P(A_1 \text{ or } A_2) &=& P(\text{diamond or a face card}) \\ &=& P(\text{diamond}) + P(\text{face card}) - \\ && P(\text{diamond AND face card}) \\ &=& \frac{13}{52} + \frac{3 \times 4}{52} - \frac{3}{52} = \frac{22}{52} = 42.3\% \end{array}$$

Sample Space and the Complement of Events

A die has 6 possible outcomes. The sample space is the set of all possible outcomes $S = \{1, 2, \dots, 6\}$.

Say event A is the event of rolling an even number i.e $A=\{2,4,6\}$. The complement of event A is $A^c=\{1,3,5\}$ i.e. getting an odd number.

Thm

$$P(A) + P(A^c) = 1$$

Independence

Two processes are independent if knowing the outcome of one provides no useful information about the outcome of the other. Otherwise they are dependent.

Consider:

- 1 Die rolls
- You get a movie recommendation from your friend Robin, but then their significant other Sam also recommends it.
- You compare test scores from two Grade 9 students in the same class. Then same school. Then same school district. Then same city. Then same state.

Independence

We say that events A and B are independent if

$$P(A \text{ and } B) = P(A) \times P(B)$$

Ex: Dice rolls are independent:

$$\begin{array}{ll} \textit{P}(\textit{rolling 1} \; \textit{and then 6}) & = & \textit{P}(\textit{rolling 1}) \times \textit{P}(\textit{rolling 6}) \\ & = & \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \end{array}$$

Conditional Probability

The conditional probability of an event A given the event B, is defined by

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Example

Let's suppose I take a random sample of 100 Reed students to study their smoking habits.

		Smoker	Not Smoker	Total
	Male	19	41	60
	Female	12	28	40
	Total	31	69	100

What is the probability of a randomly selected male smoking?

$$P(S|M) = \frac{P(S \text{ and } M)}{P(M)} = \frac{19/100}{60/100} = \frac{19}{60}$$

▶ What is the probability that a randomly selected smoker is female?

$$P(F|S) = \frac{P(F \text{ and } S)}{P(S)} = \frac{12/100}{31/100} = \frac{12}{31}$$

Put It Together! Independence and Conditional Prob.

If A and B are independent events, then

$$P(A \text{ and } B) = P(A) \times P(B)$$

then

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$$

i.e. P(A|B) = P(A): the event B occurring has no bearing on the probability of A

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Gambler's Fallacy: Roulette



You can bet on individual numbers, sets of numbers, or red vs black. Let's assume no 0 or 00, so that $P(\text{red}) = P(\text{black}) = \frac{1}{2}$.

Gambler's Fallacy: Roulette

One of the biggest cons in casinos: spin history boards.



Let's ignore the numbers and just focus on what color occurred. Note: the white values on the left are black spins.

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Gambler's Fallacy: Roulette

Let's say you look at the board and see that the last 4 spins were red.

You will always hear people say "Black is due!"

Ex. on the 5th spin people think:

 $P(\text{black}_5 \mid \text{red}_1 \text{ and } \text{red}_2 \text{ and } \text{red}_3 \text{ and } \text{red}_4) > P(\text{red}_5 \mid \text{red}_1 \text{ and } \text{red}_2 \text{ and } \text{red}_3 \text{ and } \text{red}_4)$

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Gambler's Fallacy: Roulette

But assuming the wheel is not rigged, spins are independent i.e. P(A|B) = P(A). So:

$$P(\text{black}_5|\text{red}_1 \text{ and red}_2 \text{ and red}_3 \text{ and red}_4) = P(\text{black}_5) = \frac{1}{2}$$

 $P(\text{red}_5|\text{red}_1 \text{ and red}_2 \text{ and red}_3 \text{ and red}_4) = P(\text{red}_5) = \frac{1}{2}$

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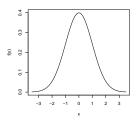
Next Week's Lab

Basketball players who make several baskets in succession are described as having a "hot hand." This refutes the assumption that each shot is independent of the next.

We are going to investigate this claim with data from a particular basketball player: Kobe Bryant of the Los Angeles Lakers in the 2009 NBA finals.



Discuss the Normal Distribution



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