# Lecture 29: Bayesian Statistics

Chapter 2.2.7



## Recall Conditional Probability

## Back to Midterm: New Notation



## Back to Midterm

## Back to Midterm

So recall from the midterm we have the following $2 \times 2$ table of possible outcomes:

| | | Test conclusion | |
|---|---|---|---|
| | | $\ominus$ | $\oplus$ |
| **Truth** | $H_0$ true | $(1 - 0.05) \times 900 = 855$ | $0.05 \times 900 = 45$ |
| | $H_A$ true | $(1 - 0.8) \times 100 = 20$ | $0.8 \times 100 = 80$ |



## Different Set-Up

Now say for the same machine $H_A$ is true 40% of the time. i.e. $P(H_A) = 0.4$

| | | Test conclusion | |
|---|---|---|---|
| | | $\ominus$ | $\oplus$ |
| **Truth** | $H_0$ true | $(1 - 0.05) \times 600 = 570$ | $0.05 \times 600 = 30$ |
| | $H_A$ true | $(1 - 0.8) \times 400 = 80$ | $0.8 \times 400 = 320$ |

## How Reliable Are Your Test Results?



## How Reliable Are Your Test Results?

The probability that a positive result is right depends on how likely $H_A$ is. Same goes for negative results.

Question 2 from Quiz 9: (Nature article) Say a scientist obtains a p-value of 0.01. An incorrect interpretation of this is that it is the probability of a "false alarm" (type I error)... If one wants to make a statement about this being a false alarm, what additional piece of information is required?

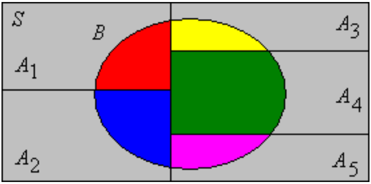
Answer 2: The plausibility of the hypothesis being tested for.

## Bayes Theorem



## Illustration

- The sample sample $S$ is the overall grey box
- $A_1, \ldots, A_5$ are the five blocks that partition $S$.
- $B$ is the oval



## Tailored to our Situation



## Tailored to our Situation

## The Debate

In the midterm you applied Bayes Theorem from scratch/intuition.

Why isn't everybody taking into account $P(H_A)$ when testing $H_0$ vs $H_A$?

In this example, we assumed we knew the true $P(H_A)$. In real life however, we don't.



## Statistics In General

Statistics is inferring about some unknown parameter $\theta$.

- Frequentist Statistics: the true $\theta$ is a single value.
- Bayesian Statistics: the true $\theta$ is a distribution of values that reflects our belief in the plausibility of different values.

Ex: Coin Flips

## The Bayesian Procedure

To express our belief about $\theta$ from as a Bayesian, we have:

1. A prior distribution $\Pr(\theta)$. It reflects our prior belief about $\theta$.
2. The likelihood function $\Pr(X|\theta)$. This is the mechanism that generates the data.
3. A posterior distribution $\Pr(\theta|X)$. We update our belief about $\theta$ after observing data $X$.

$$\Pr(\theta|X) = \frac{\Pr(X|\theta) \cdot \Pr(\theta)}{\Pr(X)}$$

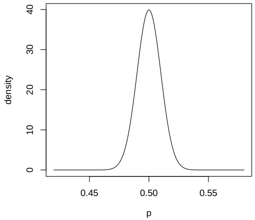


## The Issue: The Bayesian Procedure

Where do you come up with $\Pr(\theta)$? It's completely subjective! You decide!

## Prior Distribution

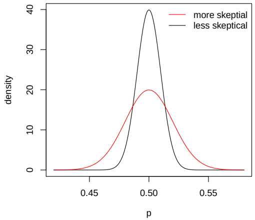
This distribution can reflect someone's prior belief of $p$.





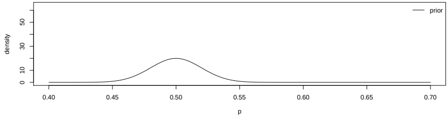

## Prior Distribution

Say someone is more skeptical that $p = 0.5$, we can lower it.



## The Bayesian Procedure

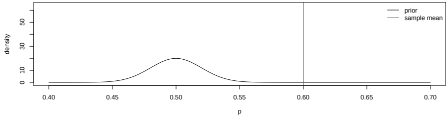
Say we have the following prior belief centered at $p = 0.5$





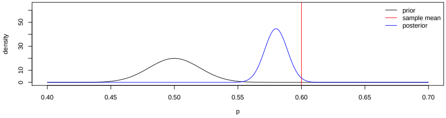

## The Bayesian Procedure

Say we collect data, represented by the red line, suggesting $p = 0.6$



## The Bayesian Procedure

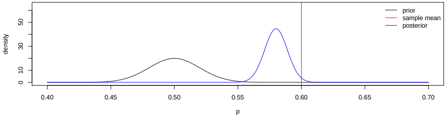
We then update our belief, as reflected in the posterior distribution!

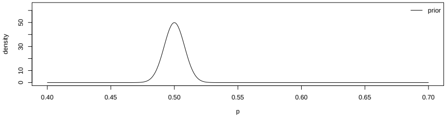





## The Bayesian Procedure

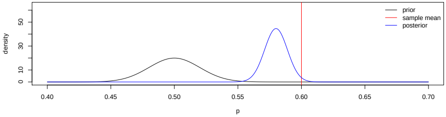
Now say we have a stronger prior belief that $p = 0.5$

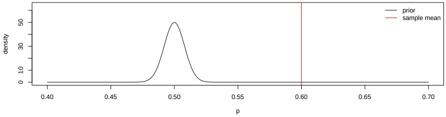


## The Bayesian Procedure

Say we observed the same data (as represented in red).

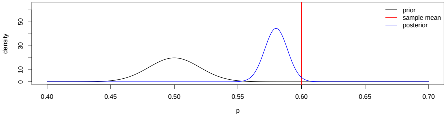


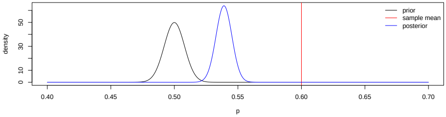






## The Bayesian Procedure

The posterior in this case is pulled left due to the sharper prior.



## Back to Debate

Frequentists believe statistics should be completely objective and therefore do not accept the premise of a subjective prior.

Back to Hypothesis Testing Machine example. In order to use such a procedure in real life, we would need to specify some prior belief $\Pr(H_A)$ that $H_A$ is true.