Lecture 11: Binomial and Poisson Random Variables

Chapter 3.3-3.5

Goals for Today

Define

- Binomial random variables
- Poisson random variables

Binomial Distribution

Say instead of P(1st W in 5th game), we want the probability that they win exactly one out of the five games. Five ways:

Binomial Distribution

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Pattern	Probability	Equals
WLLLL	$p \times (1-p)^4$	$= p \times (1-p)^4$
LWLLL	$(1-p) \times p \times (1-p)^3$	$= p \times (1-p)^4$
LLWLL	$(1-p)^2 \times p \times (1-p)^2$	$= p \times (1-p)^4$
LLLWL	$(1-p)^3 \times p \times (1-p)$	$= p \times (1-p)^4$
LLLLW	$(1-p)^4 \times p$	$= p \times (1-p)^4$

Binomial Distribution

Step Back... Example of n choose x

Say I give you $n = 3$ balls labeled 1 thru 3. How many different ways can you choose $x = 2$ of them? 3 ways:

$$(1, 2), \ (1, 3), \ \text{and} \ \ (2, 3)$$

Step Back... n choose x in General

Binomial Distribution

Conditions for Binomial Distribution

Back to Soccer Example

Probability of exactly one win?

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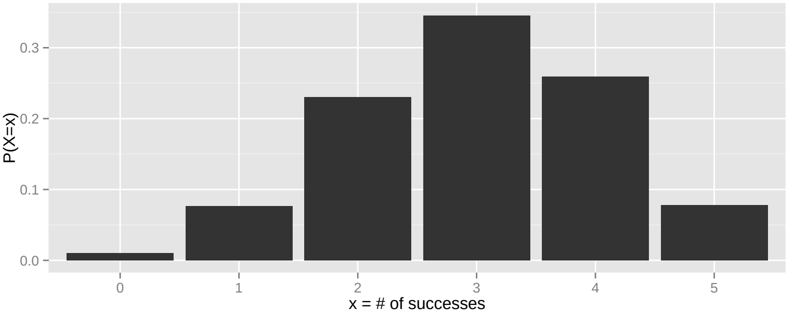
Back to Soccer Example

Back to Soccer Example



Poisson Distribution

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Poisson Distribution

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The Poisson distribution helps us model such counts.

Poisson Distribution

Conditions for Poisson Distribution

Exercise 3.47 on Page 158

Exercise 3.47 on Page 158

Next Time

Chapter 4: Foundations for Inference

- Variability in estimates $\overline{x}$, $\widehat{p}$, etc.
- In fact, we can associate a distribution to these estimates