Lecture 29: Bayesian Statistics

Chapter 2.2.7

Recall Conditional Probability

New Notation

Previously

Previously

So recall from previously we have the following 2×2 table of possible outcomes:

		Test conclusion	
		$\ominus$	$\oplus$
Truth	H_0 true	$(1-0.05) \times 900 = 855$	$0.05 \times 900 = 45$
	H_A true	$(1-0.8) \times 100 = 20$	$0.8 \times 100 = 80$

Different Set-Up

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		Test conclusion	
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Truth	H_0 true	$(1 - 0.05) \times 600 = 570$	$0.05 \times 600 = 30$
	H_A true	$(1 - 0.8) \times 400 = 80$	$0.8 \times 400 = 320$

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Question 2 from Quiz 9: (Nature article) Say a scientist obtains a p-value of 0.01. An incorrect interpretation of this is that it is the probability of a “false alarm” (type I error)... If one wants to make a statement about this being a false alarm, what additional piece of information is required?

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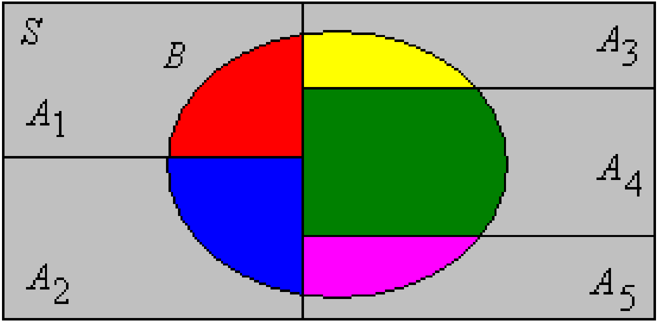
Question 2 from Quiz 9: (Nature article) Say a scientist obtains a p-value of 0.01. An incorrect interpretation of this is that it is the probability of a "false alarm" (type I error)... If one wants to make a statement about this being a false alarm, what additional piece of information is required?

Answer 2: The plausibility of the hypothesis being tested for.

Bayes Theorem

Illustration

- The sample sample S is the overall grey box
- $A_1, \ldots, A_5$ are the five blocks that partition S.
- B is the oval



Tailored to our Situation

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The Debate

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In this example, we assumed we knew the true $P(H_A)$. In real life however, we don't.

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Ex: Coin Flips

The Bayesian Procedure

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1. A prior distribution $\Pr(\theta)$. It reflects our prior belief about θ.
2. The likelihood function $\Pr(X|\theta)$. This is the mechanism that generates the data.
3. A posterior distribution $\Pr(\theta|X)$. We update our belief about θ after observing data X.

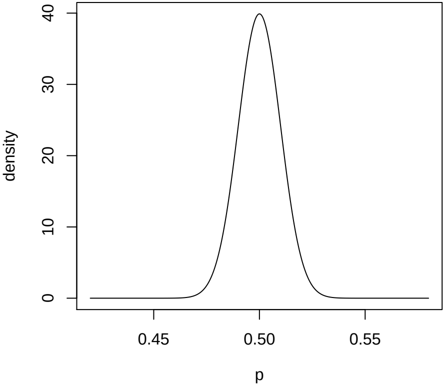
$$\Pr(\theta|X) = \frac{\Pr(X|\theta) \cdot \Pr(\theta)}{\Pr(X)}$$

The Issue: The Bayesian Procedure

Where do you come up with $\Pr(\theta)$? It's completely subjective! You decide!

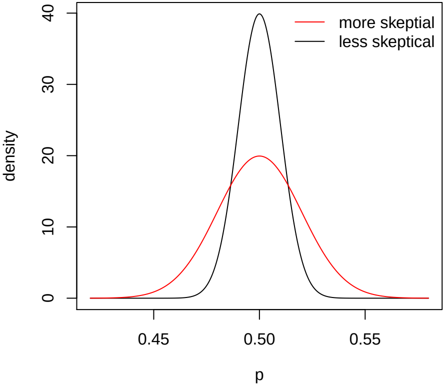
Prior Distribution

This distribution can reflect someone's prior belief of p.



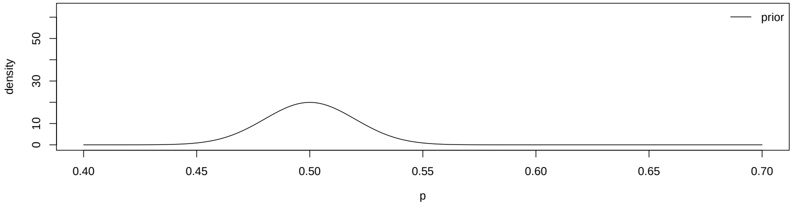
Prior Distribution

Say someone is more skeptical that $p = 0.5$, we can lower it.



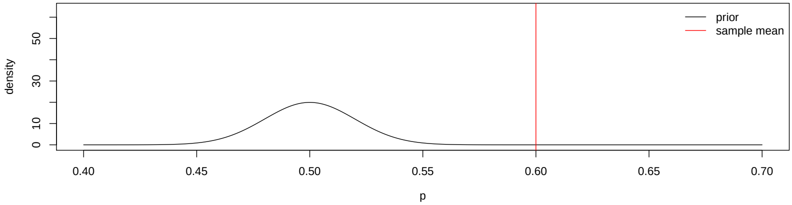
The Bayesian Procedure

Say we have the following prior belief centered at $p = 0.5$



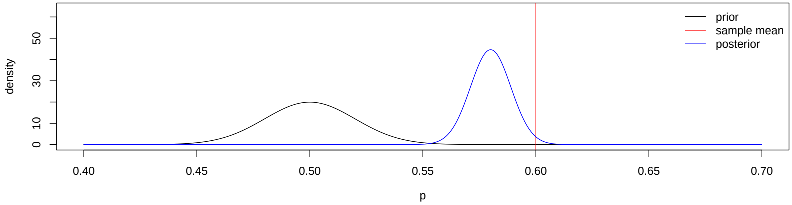
The Bayesian Procedure

Say we collect data, represented by the red line, suggesting $p = 0.6$



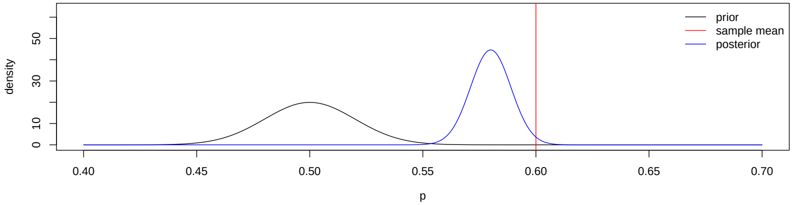
The Bayesian Procedure

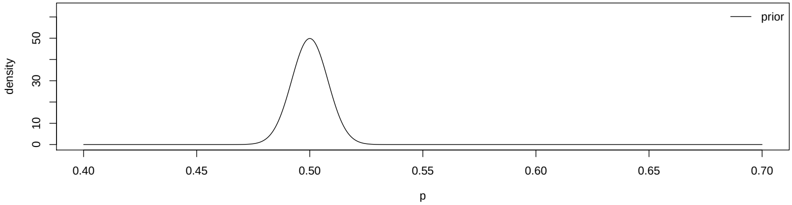
We then update our belief, as reflected in the posterior distribution!



The Bayesian Procedure

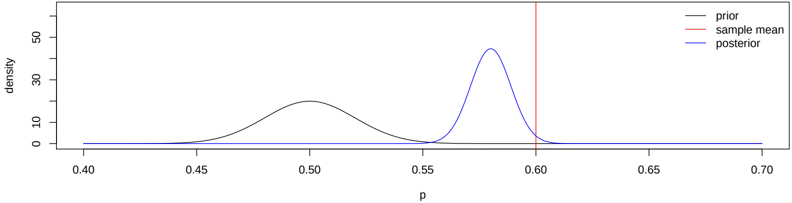
Now say we have a stronger prior belief that $p = 0.5$

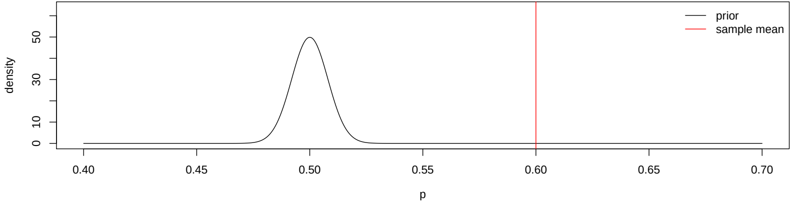




The Bayesian Procedure

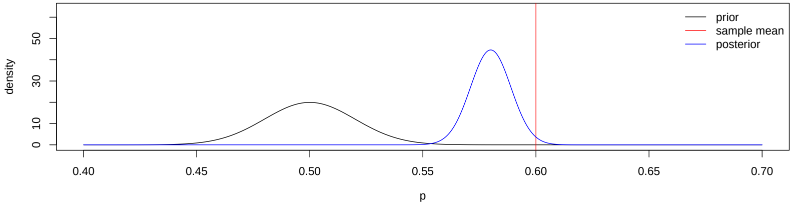
Say we observed the same data (as represented in red).

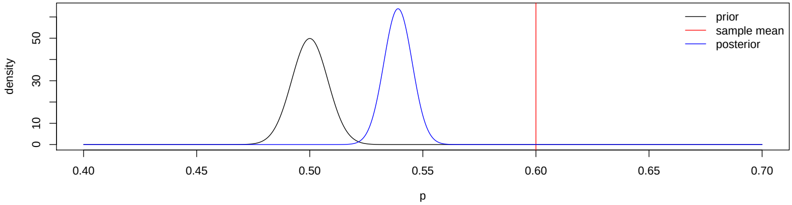




The Bayesian Procedure

The posterior in this case is pulled left due to the sharper prior.





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Back to Hypothesis Testing Machine example. In order to use such a procedure in real life, we would need to specify some prior belief $\Pr(H_A)$ that H_A is true.