Lecture 17: Paired Data and Difference of Two Means

Chapter 5.2, 5.1

Goals for Today

- Difference of means
- Note on Practical vs Statistical Significance
- Paired differences of means

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Today we look at 3 and 2.

General Outline

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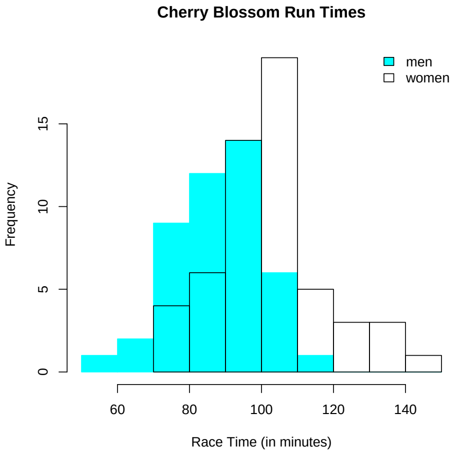

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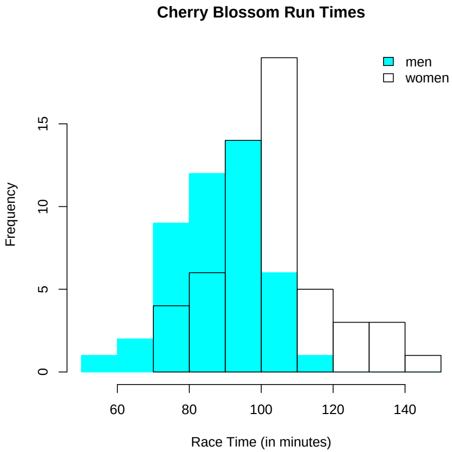

	men	women
$\overline{x}$	87.65	102.13
s	12.5	15.2
n	45	55

Difference in Means

Normality of Sampling Distribution

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Confidence Interval

Hypothesis Test

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However, the 95% CI might be:

$$[0.0005, 0.0015]$$

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- Hypothesis tests with "rejections of H_0" focus almost entirely on statistical significance.
- Confidence intervals allow you to also focus on practical significance.

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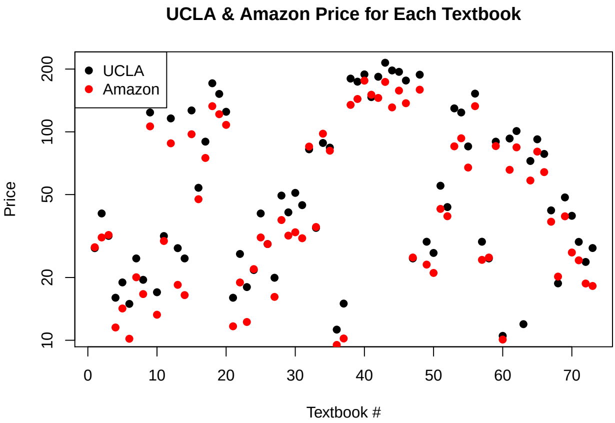
- Disease rates amongst pairs of twins
- In the text: price of the same textbook at the UCLA bookstore vs Amazon

Paired Differences

The methodology for paired data remains the same, except our observations are the difference in pairs. Example, for the UCLA Bookstore vs Amazon book price example in the text



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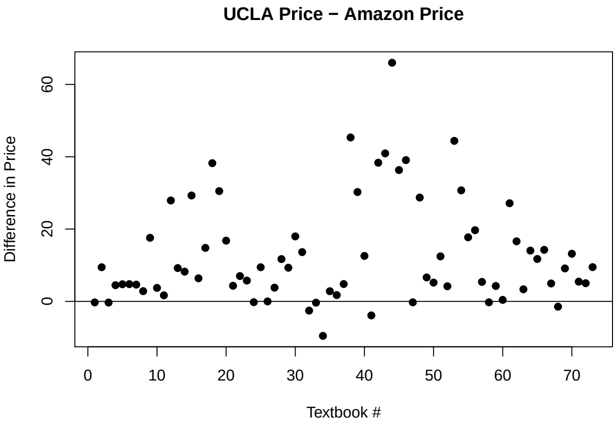

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- population parameter is μ_{diff} with point estimate $\overline{x}_{diff}$
- Check the conditions not on the original observations, but rather the differences.
- If met, $\overline{x}_{diff}$ has a normal sampling distribution
  - mean μ_{diff}
  - $SE_{diff} = \frac{\sigma_{diff}}{\sqrt{n_{diff}}} \approx \frac{s_{diff}}{\sqrt{n_{diff}}}$

Next Time

- t-test