Lecture 12: Sampling Distributions & Standard Errors

Chapter 4.1

Goals for Today

Start Chapter 4: Arguably the most important chapter as it goes to the heart of what statistical inference is. Three important definitions today:

1. point estimate
2. sampling distribution
3. standard error

Point Estimates

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Ex: the sample mean

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i = \frac{x_1 + \ldots + x_n}{n}$$

is a point estimate of the true population mean μ

Behavior of Point Estimates

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We need to characterize this random error.

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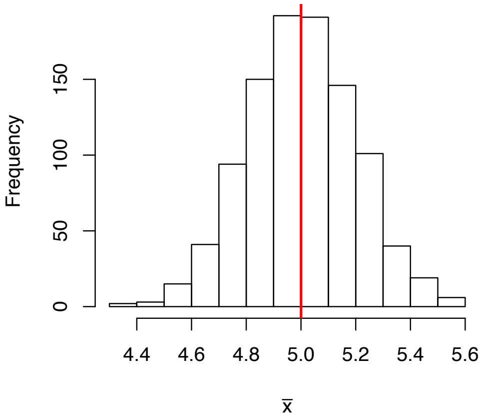
1st time	We get $\overline{x} = 4.831$
2nd time	We get $\overline{x} = 5.104$
3rd time	We get $\overline{x} = 4.965$
...	
1000th time	We get $\overline{x} = 4.957$

Sampling Distribution

This histogram is the 1000 instances of $\overline{x}$, where each $\overline{x}$ is based on a sample of $n = 100$. This is the sampling distribution of $\overline{x}$:



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If the sampling is representative (unbiased) then the sampling distribution will be centered around the true population parameter (in our case μ).

Sampling Distributions

Measure of Spread

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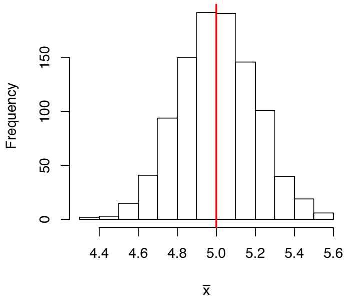

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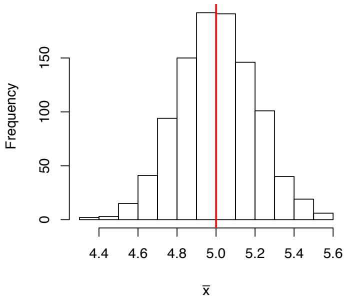

$$
\begin{aligned}
& [\mu - 2SD, \mu + 2SD] = [4.6, 5.4] \\
\Rightarrow \quad & \text{length of interval is } 4SD = 5.4 - 4.6 \\
\Rightarrow \quad & SD = 0.2
\end{aligned}
$$

Standard Errors

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Confusing: the standard error is a specific kind of standard deviation.

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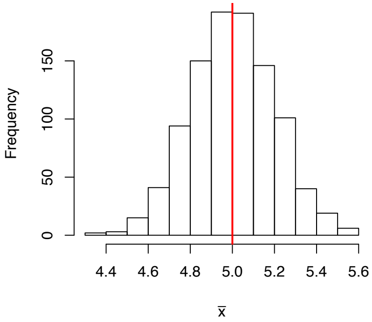
Rule of thumb for independence: You need a simple random sample consisting of less than 10% of the population.

Notice: $\sqrt{n}$ in the denominator: as n increases, SE decreases! This is why sample size matters.

Back to Histogram

Samples were of size $n = 100$ with $\sigma = 2$. We estimated that the SD of the sampling distribution was 0.2. Using the formula:

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = \frac{2}{10} = 0.2$$



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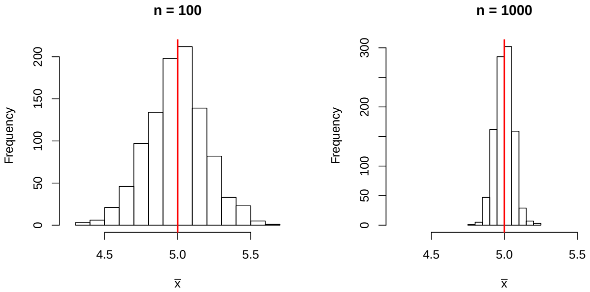

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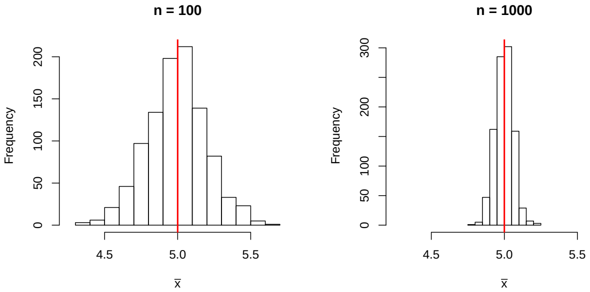

Both are "accurate", but the estimates on the right are "more precise."

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Answer: No, in practice you would not sample repeatedly: you do this only once for the largest n possible.

Rather the 1000 instances of $\overline{x}$ is a theoretical exercise to illustrate that $\overline{x}$'s are random and we characterize its randomness by its sampling distribution and its standard error.

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we can use the point estimate of σ. i.e. plug in s in place of σ:

$$SE = \frac{s}{\sqrt{n}}$$

Example

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Assuming that the 100 runners consist of less than 10% of the population, the standard error of $\overline{x}$ is

$$SE = \frac{s}{\sqrt{100}} = \frac{8.97}{10} = 0.897$$

Population Distribution vs Sampling Distribution

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- The sampling distribution characterizes the (random) behavior of point estimates.
- The standard deviation of a sampling distribution is the standard error: it quantifies the uncertainty/variability of point estimates.

Next Time

- Confidence Intervals
- When quoting survey results, what does: "the results of this survey are estimated to be accurate within 3.1 percentage points, 19 times out of 20" mean?
- Big One: Central Limit Theorem