Lecture 7: Probability

Chapter 2.x

Outcomes

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Typical examples

- Die roll: 6 outcomes
- Coin Flip: 2 outcomes

Disjoint AKA Mutually Exclusive Outcomes

Addition Rule of Probability

General Addition Rule of Probability

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Sample Space and the Complement of Events

Independence

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2. You get a movie recommendation from your friend Robin, but then their significant other Sam also recommends it.

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Consider:

1. Die rolls
2. You get a movie recommendation from your friend Robin, but then their significant other Sam also recommends it.
3. You compare test scores from two Grade 9 students in the same class. Then same school. Then same school district. Then same city. Then same state.

Independence

Conditional Probability

Example

Let's suppose I take a random sample of 100 Reed students to study their smoking habits.

	Smoker	Not Smoker	Total
Male	19	41	60
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Total	31	69	100

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- What is the probability of a randomly selected male smoking?

$$P(S|M) = \frac{P(S \text{ and } M)}{P(M)} = \frac{19/100}{60/100} = \frac{19}{60}$$

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$$P(S|M) = \frac{P(S \text{ and } M)}{P(M)} = \frac{19/100}{60/100} = \frac{19}{60}$$

- What is the probability that a randomly selected smoker is female?

$$P(F|S) = \frac{P(F \text{ and } S)}{P(S)} = \frac{12/100}{31/100} = \frac{12}{31}$$

Put It Together! Independence and Conditional Prob.

Gambler's Fallacy: Roulette

You can bet on individual numbers, sets of numbers, or red vs black. Let's assume no 0 or 00, so that $P(R) = P(B) = \frac{1}{2}$.

Gambler's Fallacy: Roulette

One of the biggest cons in casinos: spin history boards.

Let's ignore the numbers and just focus on what color occurred.
Note: the white values on the left are black spins.

Gambler's Fallacy: Roulette

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Ex. on the 5th spin people think:

$$P(B_5 \mid R_1 \text{ and } R_2 \text{ and } R_3 \text{ and } R_4) >$$
$$P(R_5 \mid R_1 \text{ and } R_2 \text{ and } R_3 \text{ and } R_4)$$

Gambler's Fallacy: Roulette

But assuming the wheel is not rigged, spins are independent i.e. $P(A|B) = P(A)$. So:

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$$
\begin{aligned}
P(B_5|R_1 \text{ and } R_2 \text{ and } R_3 \text{ and } R_4) = P(B_5) &= \frac{1}{2} \\
P(R_5|R_1 \text{ and } R_2 \text{ and } R_3 \text{ and } R_4) = P(R_5) &= \frac{1}{2}
\end{aligned}
$$

Next Time

Discuss the Normal Distribution