# Lecture 17: Paired Data and Difference of Two Means

Chapter 5.2, 5.1

# Goals for Today

- Difference of means
- Note on Practical vs Statistical Significance
- Paired differences of means

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Today we look at 3 and 2.

# General Outline

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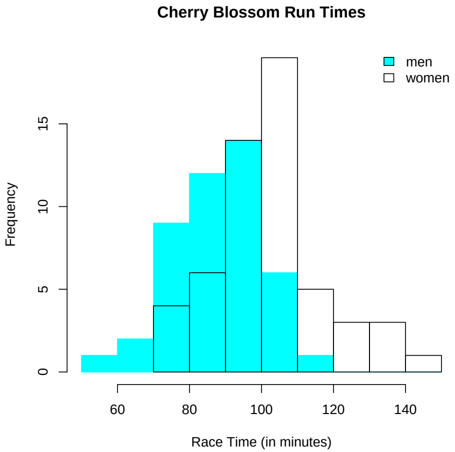

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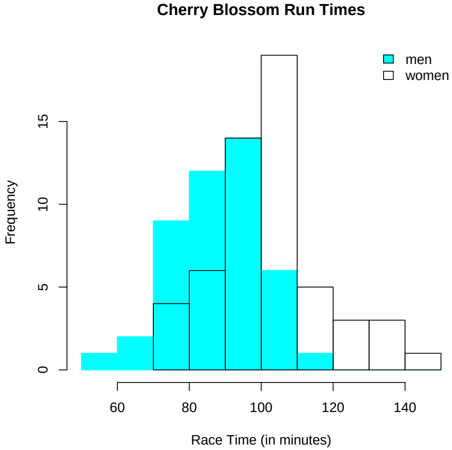


| | men | women |
|---|---|---|
| $\bar{x}$ | 87.65 | 102.13 |
| $s$ | 12.5 | 15.2 |
| $n$ | 45 | 55 |

# Difference in Means

# Normality of Sampling Distribution

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# Confidence Interval

# Hypothesis Test

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However, the 95% CI might be:

$$[0.0005, 0.0015]$$

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- Hypothesis tests with "rejections of $H_0$" focus almost entirely on statistical significance.
- Confidence intervals allow you to also focus on practical significance.

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- Disease rates amongst pairs of twins
- In the text: price of the same textbook at the UCLA bookstore vs Amazon

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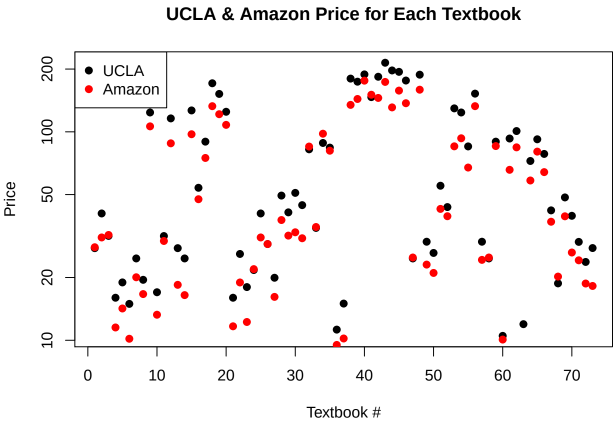
The methodology for paired data remains the same, except our observations are the difference in pairs. Example, for the UCLA Bookstore vs Amazon book price example in the text



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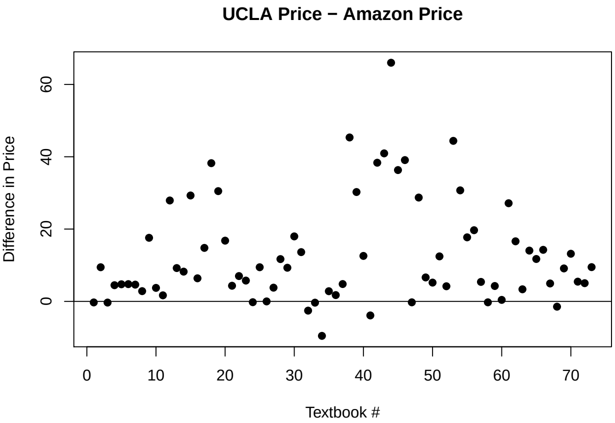

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- population parameter is $\mu_{diff}$ with point estimate $\bar{x}_{diff}$
- Check the conditions not on the original observations, but rather the differences.
- If met, $\bar{x}_{diff}$ has a normal sampling distribution
  - mean $\mu_{diff}$
  - $SE_{diff} = \frac{\sigma_{diff}}{\sqrt{n_{diff}}} \approx \frac{s_{diff}}{\sqrt{n_{diff}}}$

# Next Time

- t-test