

# Lecture 12: Sampling Distributions & Standard Errors

## Chapter 4.1

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## Goals for Today

Chapter 4: Arguably the most important chapter as it goes to the heart of statistical inference. Three important definitions:

1. point estimate
2. sampling distribution
3. standard error

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## Point Estimates

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## Behavior of Point Estimates

Ex: Say we draw samples of size  $n = 100$  from a large population with  $\mu = 5$  and  $\sigma = 2$ .

Two Important Questions:

1. Is  $\bar{x}$  going to be exactly 5?
2. Say we get  $\bar{x} = 5.025$ . If we repeat this procedure, will we get  $\bar{x} = 5.025$  again?

We need to characterize the random error.

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## Behavior of Point Estimates

Let's repeat this procedure, say, 1000 times:

1st time      We get  $\bar{x} = 4.831$

2nd time      We get  $\bar{x} = 5.104$

3rd time      We get  $\bar{x} = 4.965$

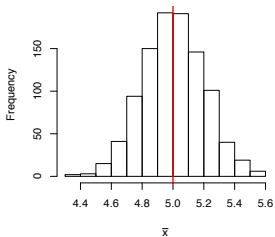
...

1000th time   We get  $\bar{x} = 4.957$

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## Sampling Distribution

This histogram is the 1000 instances of  $\bar{x}$  i.e. the **sampling distribution** of  $\bar{x}$ :



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## Sampling Distributions

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## Sampling Distributions

We can define the sampling distributions for **any** point estimate, not just  $\bar{x}$ :

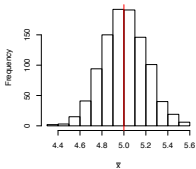
- ▶  $s$
- ▶ the sample median
- ▶ etc.

We will only focus on sample means, including the sample proportion  $\hat{p}$ .

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## Measure of Spread

What about spread?  $[4.6, 5.4]$  contains roughly 95% of the data.



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## Standard Errors

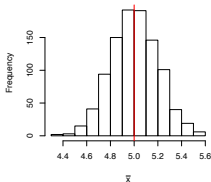
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## Standard Error of $\bar{x}$

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## Back to Histogram

Samples were of size  $n = 100$  with  $\sigma = 2$ . We estimated that the SD was 0.2.



Using the formula:

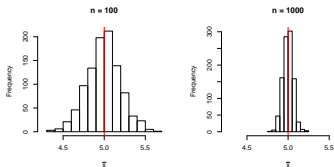
$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = \frac{2}{10} = 0.2$$

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## Standard Error of the Sample Mean $\bar{x}$

Compare 1000 instances of  $\bar{x}$  when

- ▶  $n = 100$ .  $SE = \frac{2}{\sqrt{100}} = 0.2$
- ▶  $n = 1000$ .  $SE = \frac{2}{\sqrt{1000}} = 0.0632$ . **Smaller!**



Both are “accurate”, but estimates on the right are “more precise.”

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## Repeated Sampling

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## Standard Error of the Sample Mean

## Population Distribution vs Sampling Distribution



## Recap

- ▶ **Point estimates** are based on a sample  $x_1, \dots, x_n$  and are used to estimate population parameters.
- ▶ The **sampling distribution** characterizes the (random) behavior of point estimates.
- ▶ The standard deviation of a sampling distribution is the **standard error**: it quantifies the uncertainty/variability of point estimates.

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## Next Time

- ▶ Confidence Intervals
- ▶ When quoting survey results, what does: “the results of this survey are estimated to be accurate within 3.1 percentage points, 19 times out of 20” mean?
- ▶ **Big One**: Central Limit Theorem

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