Lecture 13: Central Limit Theorem + Confidence Intervals

Chapter 4.4 + 4.2

Goals for Today

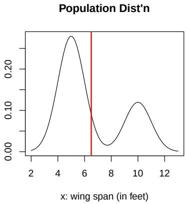
- Discuss the Central Limit Theorem
- Introduce confidence intervals
- Interpretation

Central Limit Theorem

Question 1: Why do we care about the CLT?

Answer: We want the sampling distribution of $\overline{x}$ to be Normal regardless of the shape of population distribution.

Example: The bimodal (population) distribution of dragon wing spans has a mean of 6.5:



Central Limit Theorem

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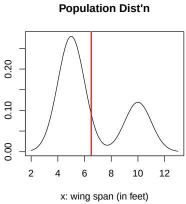
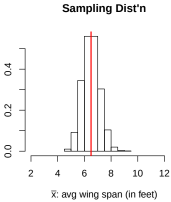




Central Limit Theorem

Question 2: Why do we care that the sampling distribution of $\overline{x}$ is Normal?

Answer: So we can use the Normal model. In other words, use the Normal table on p.429 of the book to calculate areas/percentiles!

	Second decimal place of Z									
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Central Limit Theorem

Question 3: Why do we care that we can use the Normal table?

So we can

- Build confidence intervals
- Conduct hypothesis tests

Central Limit Theorem

Recap: By the CLT

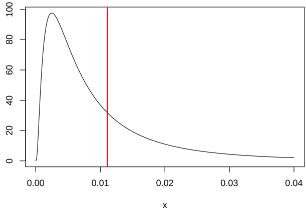
1. The sampling distribution of $\overline{x}$ is Normal regardless of the population distribution $\Longrightarrow$
2. We can use the Normal table on p.429 of the book to calculate areas/percentiles $\Longrightarrow$
3. We can build confidence intervals and conduct hypothesis tests

Definition

Conditions for the Normal Model

Example of Skew vs n

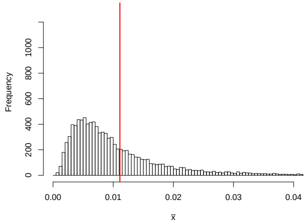
Let's say your observations come from the following very skewed population distribution with mean $\mu = 0.011109$.



This is where your individual observations x_i come from. Now compare 10000 values of $\overline{x}$'s based on different n: 2, 10, 30, 75.

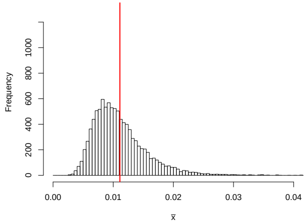
Example of Skew vs n

For 10000 values of $\overline{x}$ based on samples of size $n = 2$, the sampling distribution is:



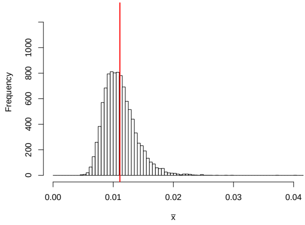
Example of Skew vs n

For 10000 values of $\overline{x}$ based on samples of size $n = 10$, the sampling distribution is:



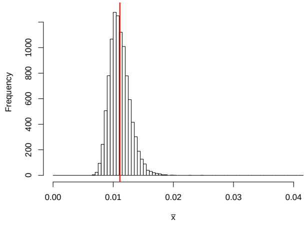
Example of Skew vs n

For 10000 values of $\overline{x}$ based on samples of size $n = 30$, the sampling distribution is:



Example of Skew vs n

For 10000 values of $\overline{x}$ based on samples of size $n = 75$, the sampling distribution is:



i.e. more normal and more narrow

Intuition of a Confidence Interval

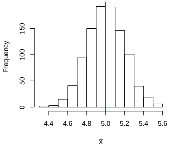
Our Goal: we want estimate a population parameter (e.g. μ).
Analogy: imagine μ is a fish in a murky river that we want to capture:

Using just the point estimate:

Using a confidence interval:

Intuition of a Confidence Interval

Recall example of 1000 instances of $\overline{x}$ based on $n = 100$. Each observation is from a population distribution that was Normal with $\mu = 5$ & $\sigma = 2$.



We observed the sampling distribution

- is centered at μ
- has spread $SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = 0.2$

Intuition of a Confidence Interval

A plausible range of values for the population parameter is called a confidence interval (CI). Since

- the SE is the standard deviation of the sampling distribution
- roughly 95% of the time $\overline{x}$ will be within 2 SE of μ if the sampling distribution is normal

If the interval spreads out 2 SE from $\overline{x}$, we can be roughly "95% confident" that we have captured the true parameter μ.

Intuition of a Confidence Interval

Confidence Intervals

Crucial: How to Interpret a Confidence Interval

The confidence interval has nothing to say about any particular calculated interval; it only pertains to the method used to construct the interval:

- Wrong, yet common, interpretation: There is a 95% chance that the C.I. captures the true population mean μ. The probability is 0 or 1: either it does or it doesn't.
- Correct, interpretation: If we were to repeat this sampling procedure 100 times, we expect 95 of calculated C.I.'s to capture the true μ

Illustration: How to Interpret a Confidence Interval

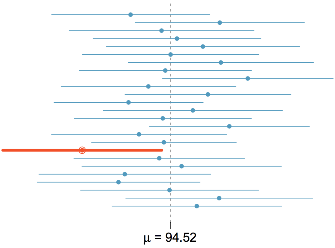
Ch 4 Ex: Times from 2012 Cherry Blossom 10 mile run with $n = 16,924$. We know the true population mean $\mu = 94.52$.

Say we take 25 (random) samples of size $n = 100$ and for each sample we compute:

- $\overline{x}$
- s
- and hence the 95% CI: $\left[\overline{x} - 1.96 \times \frac{s}{\sqrt{n}},\ \overline{x} + 1.96 \times \frac{s}{\sqrt{n}}\right]$

How to Interpret a Confidence Interval

Of the 25 CI's based on 25 different samples of size $n = 100$, one of them (in red) did not capture the true population mean μ:



Political Polls

We polled the electorate and found that 45% of voters plan to vote for candidate X. The margin of error for this poll is ± 3.4 percentage points 19 times out of 20.

Interpretation: the interpretation is not that there is a 95% chance that $[41.6, 48.4]$ captures the true %'age. Rather, that if we were to take 20 such polls, 19 of them would capture the true %'age.

Next Time

Hypothesis Testing: we can perform statistical tests on population parameters such as μ:

Define:

- Null and alternative hypotheses.
- Testing hypotheses using confidence intervals.
- Types of errors