Lecture 17: Paired Data and Difference of Two Means

Chapter 5.2, 5.1

Goals for Today

- Difference of means
- Note on Practical vs Statistical Significance
- Paired differences of means

6 Types of Questions

Here are the 6 broad types of questions about population parameters we'll be answering with statistical methods: confidence intervals and hypothesis tests

1. What is the mean value μ?
2. Are the means μ_1 and μ_2 of two groups different?
3. What is the mean paired difference μ_{diff}?
4. What is the proportion p of "successes"?
5. Are the proportions of "successes" p_1 and p_2 of two groups different?
6. Are the means $\mu_1, \ldots, \mu_k$ of k groups different?

Today we look at 3 and 2.

General Outline

We now generalize what we did in Chapter 4:

1. Define the population parameter and determine its point estimate
2. Show that the sampling distribution of the point estimate is Normal
   - Verify CLT & any additional conditions
   - Find the SE

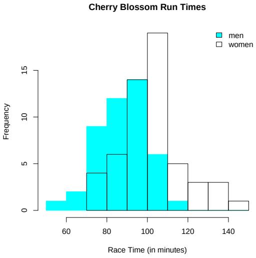
Then we either:

- Build a confidence interval: point estimate $\pm z^* SE$
- Conduct a hypothesis test with test statistic: z-score of the point estimate

$$z = \frac{\text{point estimate} - \text{null value}}{SE}$$

Chapter 5.2: Are Two Means μ_1 & μ_2 Different?

We randomly sample 45 men (of 7192) and 55 women (of 9732) runners in the 2012 Cherry Blossom Run. Did men run faster than women?



	men	women
$\bar{x}$	87.65	102.13
s	12.5	15.2
n	45	55

Difference in Means

We want the difference of two population means:

- μ_w: mean time for women
- μ_m: mean time for men

Thus:

- Population parameter: $\mu_w - \mu_m$.
  i.e. if men run faster, this is positive
- Point estimate: $\bar{x}_w - \bar{x}_m = 102.13 - 87.65 = 14.48$
  i.e. difference of sample means

Normality of Sampling Distribution

If two sample means $\overline{x}_1$ and $\overline{x}_2$

- each satisfy the 3 CLT conditions
- Additionally: the two samples are independent from each other

Then the sampling distribution of $\overline{x}_1 - \overline{x}_2$ will be approximately normal with

- mean $\mu_1 - \mu_2$
- estimated standard error

$$SE_{\overline{x}_1 - \overline{x}_2} = \sqrt{SE^2_{\overline{x}_1} + SE^2_{\overline{x}_2}} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \approx \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Normality of Sampling Distribution

We verify the conditions:

1. Each sample consists of $\leq$10% of their respective populations.
2. Both histograms don't look too skewed.
3. Each sample has at least 30 observations (rule of thumb).
4. Additionally: the samples are independent (not paired or linked in any way).

Thus the sampling distribution is Normal with mean=$\mu_w - \mu_m$ and

$$SE_{\overline{x}_w - \overline{x}_m} = \sqrt{\frac{15.2^2}{55} + \frac{12.5^2}{45}} = 2.77$$

Confidence Interval

A 95% confidence interval for $\mu_1 - \mu_2$ is

$$\begin{aligned}(\text{point estimate for } \mu_1 - \mu_2) &\pm z^* \times SE \\ (\overline{x}_1 - \overline{x}_2) &\pm 1.96 \times SE_{\overline{x}_1 - \overline{x}_2}\end{aligned}$$

For the Cherry Blossom Run data, a 95% CI for $\mu_w - \mu_m$ is:

$$14.48 \pm 1.96 \times 2.77 = [9.05, 19.91]$$

Hypothesis Test

For $\alpha = 0.001$ (i.e. we want reject with high confidence) we test

- $H_0 : \mu_w - \mu_m = 0$
- $H_A : \mu_w - \mu_m > 0$

Test statistic: z-score of $\overline{x}_w - \overline{x}_m$ under H_0:

$$\begin{aligned}\frac{\text{point estimate} - \text{null value}}{SE} &= \frac{(\overline{x}_w - \overline{x}_m) - \text{null value}}{SE_{\overline{x}_1 - \overline{x}_2}} \\ &= \frac{14.48 - 0}{2.77} = 5.23\end{aligned}$$

The p-value is 0, hence we reject H_0 and declare that men ran significantly faster than women.

Practical vs Statistical Significance

When rejecting H_0, we call this a statistically significant result. But statistically significant results aren't always practically significant.

Say for very large n_M & n_F we observe $\overline{x}_M = 87.65$ and $\overline{x}_F = 87.651$ and reject H_0.

The point estimate of the difference $\overline{x}_M - \overline{x}_F = 0.001$. Near negligible!

However, the 95% CI might be:

$$[0.0005, 0.0015]$$

Practical vs Statistical Significance

Moral of the story

- Hypothesis tests with "rejections of H_0" focus almost entirely on statistical significance.
- Confidence intervals allow you to also focus on practical significance.

Chapter 5.1: Paired Data

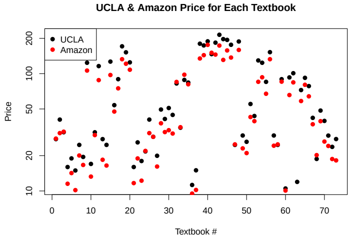
Two sets of observations are paired if each observation in one set has a special correspondence or connection with exactly one observation in the other data set.

Examples:

- Cholesterol levels before and after some intervention for the same person
- Disease rates amongst pairs of twins
- In the text: price of the same textbook at the UCLA bookstore vs Amazon

Paired Differences

The methodology for paired data remains the same, except our observations are the difference in pairs. Example, for the UCLA Bookstore vs Amazon book price example in the text



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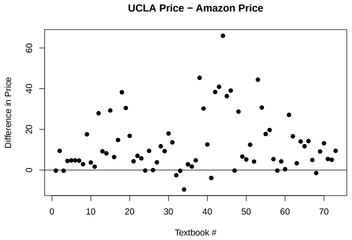

Paired Differences

We have

- population parameter is μ_{diff} with point estimate $\overline{x}_{diff}$
- Check the conditions not on the original observations, but rather the differences.
- If met, $\overline{x}_{diff}$ has a normal sampling distribution
  - mean μ_{diff}
  - $SE_{diff} = \frac{\sigma_{diff}}{\sqrt{n_{diff}}} \approx \frac{s_{diff}}{\sqrt{n_{diff}}}$

Next Time

- t-test