Lecture 11.1: Linear Regression Part II

Chapter 7.2-7.4

2014/04/14

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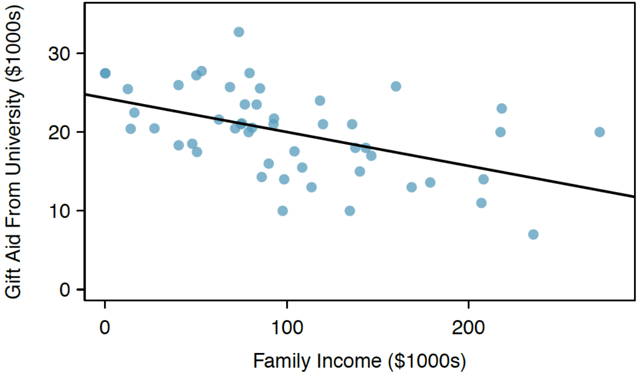
- Explanatory variable: Family income
- Outcome variable: Gift Aid (financial aid that is a gift, not a loan)



Questions for Today: Example From Text

Using these values,

	family income in \$1000's (x)	gift aid in \$1000's (y)
mean	$\overline{x} = 101.8$	$\overline{y} = 19.94$
sd	$s_x = 63.2$	$s_y = 5.46$
		$R = -0.499$

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they fit the least-squares line:

$$\begin{aligned} \widehat{y} &= b_0 + b_1 x \\ \text{i.e. } \widehat{\text{aid}} &= 24.3 - 0.0431 \times \text{family\_income} \end{aligned}$$

What do 24.3 and -0.0431 mean?

Point Estimates of Intercept

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In this case it is relevant since some families make no income, but the intercept may have little or no practical value if there are no observations near $x = 0$.

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Again, even though we've labeled aid as the outcome variable, we are not positing a causal relationship, just an association.

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What would be the gift aid given to a family with one million dollars ($x = 1000$) in family income?

$$24.3 - 0.0431 \times 1000 = -18.8$$

The school will take $18,800 dollars away from you?

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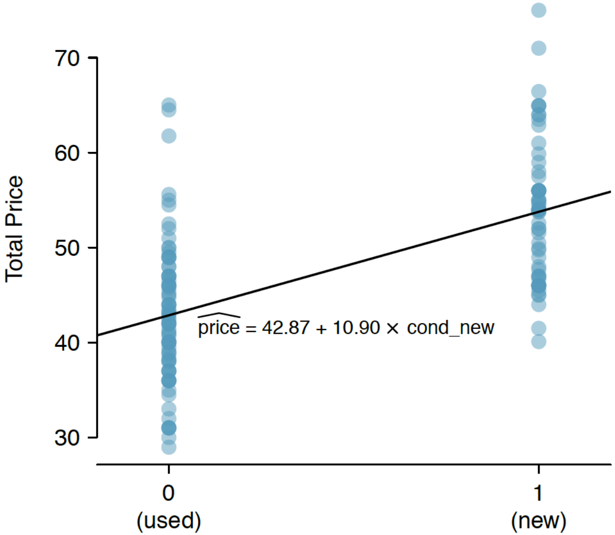
The linear model is thus

$$\widehat{\text{price}} = b_0 + b_1 \times \text{cond\_new}$$

Categorical Predictor x With Two Levels



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This can be generalized for predictor variables x with more than two levels, but this requires a different encoding of x.

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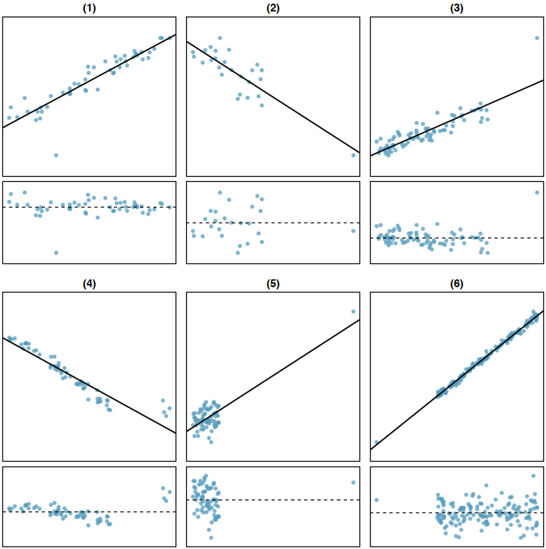

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Points that fall horizontally away from the center of the cloud tend to pull harder on the line, so we call them points with high leverage, i.e. large influence.

Next Example

Are Higher Movie Budgets Associated with Higher IMDB Ratings for Movies Made from 1980-2005? Guesses?

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But first, are the changes from

- 100 to 200
- 100,100 to 100,200

the same?

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We consider a $\log_{10}$ transformation:

x	y	$y-x$	$\frac{y}{x}$	$\log_{10}\left(\frac{y}{x}\right)$
100	200	100	2	0.301
100100	100200	100	1.000999	0.00004342

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So we are considering multiplicative changes, and not additive changes.

Next Time

Multiple Regression: As opposed to simple linear regression where there is only one predictor/explanatory variable x, we now consider many variables $x_1, x_2, \ldots$