Lecture 23: Tests for Independence in Two-Way Tables

Chapter 6.4

Today's Example

Example from Chapter 6.4 page 297: Google is always tinkering with its search ranking algorithm. Say we want to compare the following 3 algorithms:

1. the current version
2. new test algorithm 1
3. new test algorithm 2

Today's Example

User satisfaction is measured with the `new search` variable:

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User satisfaction is measured with the `new search` variable:

- no new search: User clicked on a result. Suggests user is satisfied with result.
- new search: User did not click on a result and tried a new related search. Suggests user is dissatisfied with result.

Today's Example

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Say we observe the following contingency table:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	4000	2000	2000	8000
New search	1000	500	500	2000
Total	5000	2500	2500	10000

Today's Example

Say we observe the following contingency table:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	4000	2000	2000	8000
New search	1000	500	500	2000
Total	5000	2500	2500	10000

For all 3 algorithms, there is a new search $\frac{1}{5}$ of the time.

Today's Example

Say we observe the following contingency table:

`new search`	`algorithm` Current	Test 1	Test 2	Total
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For all 3 algorithms, there is a new search $\frac{1}{5}$ of the time.

The two levels of `new search` are independent of `algorithm`: regardless of which algorithm used, the proportion of new searches stays the same.

Today's Example

Now say instead we observed the following results:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	4000	2500	1500	8000
New search	1000	0	1000	2000
Total	5000	2500	2500	10000

Today's Example

Now say instead we observed the following results:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	4000	2500	1500	8000
New search	1000	0	1000	2000
Total	5000	2500	2500	10000

In this case, they are dependent: depending on which algorithm used, the proportion of new search is different.

Hypothesis Test

Different Names

Example from Textbook

Let's make the values match the example from the textbook on page 299:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	3511	1749	1818	7078
New search	1489	751	682	2922
Total	5000	2500	2500	10000

Example from Textbook

Before we start, let's make each column reflect a proportion and not a count.

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	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	0.7022	0.6996	0.7272	0.7078
New search	0.2978	0.3004	0.2728	0.2922
Total	1	1	1	1

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If all algorithms performed the same, we'd expect

- 0.7078 for all 3 values in the top row
- 0.2922 for all 3 values in the bottom row

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- 0.7078 for all 3 values in the top row
- 0.2922 for all 3 values in the bottom row

Are we observing what we expect? i.e. What is the degree of this deviation?

What's Expected

We expect:

`new search`	`algorithm` Current	Test 1	Test 2	Total
No new search				$7078 = 0.7078 \times 10000$
New search				$2922 = 0.2922 \times 10000$
Total	5000	2500	2500	10000

What's Expected

We expect:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search			$1769.5 = 0.7078 \times 2500$	7078
New search			$730.5 = 0.2922 \times 2500$	2922
Total	5000	2500	2500	10000

What's Expected

We expect:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search		$1769.5 = 0.7078 \times 2500$	1769.5	7078
New search		$730.5 = 0.2922 \times 2500$	730.5	2922
Total	5000	2500	2500	10000

What's Expected

We expect:

new search	algorithm Current	Test 1	Test 2	Total
No new search	$3539 = 0.7078 \times 5000$	1769.5	1769.5	7078
New search	$1461 = 0.2922 \times 5000$	730.5	730.5	2922
Total	5000	2500	2500	10000

Observed vs. Expected

Expected Counts:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	3539	1769.5	1769.5	7078
New search	1461	730.5	730.5	2922
Total	5000	2500	2500	10000

Observed vs. Expected

Expected Counts:

new search	algorithm			Total
	Current	Test 1	Test 2	
No new search	3539	1769.5	1769.5	7078
New search	1461	730.5	730.5	2922
Total	5000	2500	2500	10000

Observed Counts:

new search	algorithm			Total
	Current	Test 1	Test 2	
No new search	3511	1749	1818	7078
New search	1489	751	682	2922
Total	5000	2500	2500	10000

Chi-Square Statistic

Chi-Square Distribution

Chi-Square Distribution

Conditions/Assumptions

Why Are They Called Degrees of Freedom?

In the case of χ^2 tests, the degrees of freedom is the number of values needed before you specify all values in the cells of the table.

Why Are They Called Degrees of Freedom? Rows

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Each row has $df = 2$ because if we specify 2 values, all values in the row are specified.

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Example:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	X	Y		7078
New search				2922
Total	5000	2500	2500	10000

Why Are They Called Degrees of Freedom? Rows

Each row has $df = 2$ because if we specify 2 values, all values in the row are specified.

Example:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	X	Y		7078
New search				2922
Total	5000	2500	2500	10000

then the missing value is $7078 - X - Y$.
i.e. the wiggle room we have is $C - 1$ two cells

Why Are They Called Degrees of Freedom? Columns

Each column has $df = 1$ because if we specify 1 value, all values in the column are specified.

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Example:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	X			7078
New search				2922
Total	5000	2500	2500	10000

Why Are They Called Degrees of Freedom? Columns

Each column has $df = 1$ because if we specify 1 value, all values in the column are specified.

Example:

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	X			7078
New search				2922
Total	5000	2500	2500	10000

then the missing value is $5000 - X$.
i.e. the wiggle room we have is $R - 1$ one cell

Why Are They Called Degrees of Freedom? Columns

So the overall df is $(C-1) \times (R-1)$, in our case $df = 2$.

	algorithm			
new search	Current	Test 1	Test 2	Total
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Total	5000	2500	2500	10000

Why Are They Called Degrees of Freedom? Columns

So the overall df is $(C-1) \times (R-1)$, in our case $df = 2$.

	algorithm			
new search	Current	Test 1	Test 2	Total
No new search	X	Y		7078
New search				2922
Total	5000	2500	2500	10000

i.e. if we know these two values, we can fill the rest of the table.