Lecture 15: Hypothesis Testing Part II

Chapter 4.3

Goals for Today

- Define significance level
- Tie-in p-Values with sampling distributions
- Example

Type I Errors: US Criminal Justice System

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In this case:

- Type I error $=$ jailing an innocent person (worse)
- Type II error $=$ letting a guilty person go free.

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Hence the long lines at airport security.

Significance Level

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- 525 heads? Do you think the coin is biased?

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Say you flip a coin you think is fair 1000 times. Say you observe

- 501 heads? Do you think the coin is biased?
- 525 heads? Do you think the coin is biased?
- 900 heads? Do you think the coin is biased?

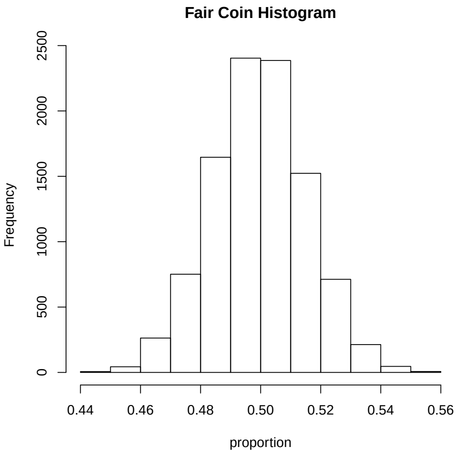
Thought experiment: p-Values

p-Values

Thought experiment: Coin Flips

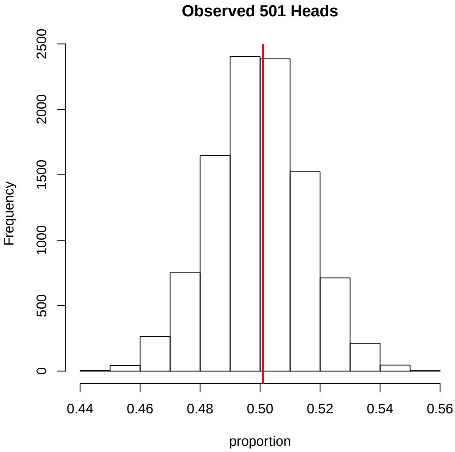
Sampling Distribution of $\widehat{p}$

Under H_0 the sampling distribution of $\widehat{p}$ when $n = 1000$ is:



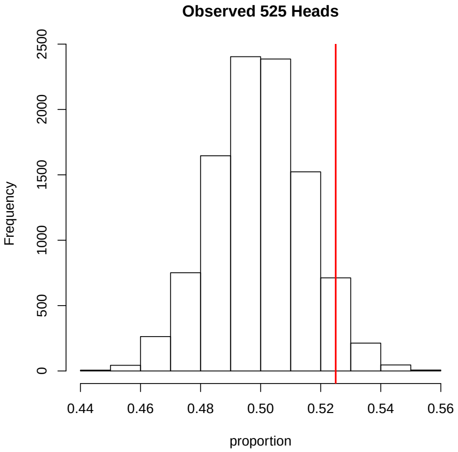
Say we observe...

$$\widehat{p} = \frac{501}{1000}$$



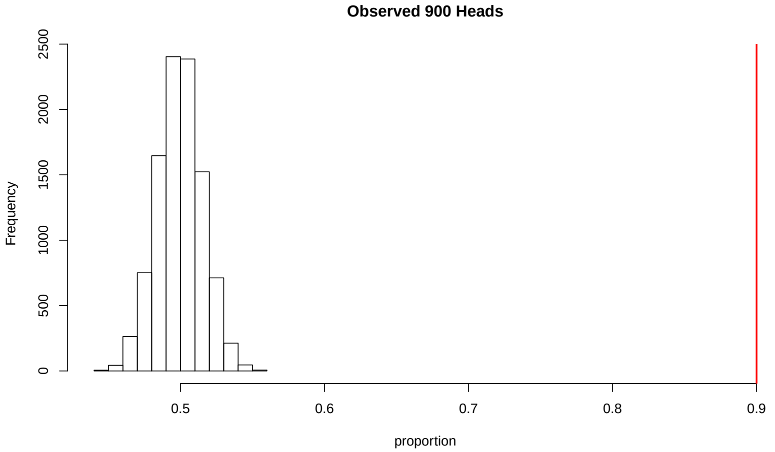
Say we observe...

$$\widehat{p} = \frac{525}{1000}$$



Say we observe...

$$\widehat{p} = \frac{900}{1000}$$



Example about Sleep Habits

A US-wide poll found that college students sleep about 7 hours a night. You suspect that Midd Kids sleep more and investigate this claim at a pre-specified $\alpha = 0.01$ level.

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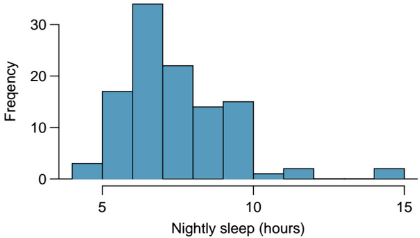
You sample $n = 110$ Midd Kids and find that $\overline{x} = 7.42$ and $s = 1.75$ with a histogram that looks like:



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Correct interpretation of the p-value: If the null hypothesis is true ($\mu = 7$), the probability of observing a sample mean $\overline{x} = 7.42$ or greater is 0.007 (small).

Incorrect interpretation of the p-value: The probability that the null hypothesis ($\mu = 7$) is true is 0.007.