# Lecture 28: Logistic Regression

Chapter 8.4

# Binary Outcome Variables

# Outcome Variable

# Outcome Variable

# Outcome Variable

# Outcome Variable

Figure 8.14 from page 369

# Simple Logistic Regression Example p.370

So say we fit a logistic regression with ($n = 3921$):

- $Y_i$ is `spam`: binary variable of whether message was classified as spam (1 if spam)
- $x_i$ is `to_multiple`: binary variable indicating if more than one recipient listed

# Simple Logistic Regression Example p.370

So say we fit a logistic regression with ($n = 3921$):

- $Y_i$ is `spam`: binary variable of whether message was classified as spam (1 if spam)
- $x_i$ is `to_multiple`: binary variable indicating if more than one recipient listed

| | Estimate | Std. Error | z value | Pr(>\|z\|) |
|---|---|---|---|---|
| (Intercept) | -2.1161 | 0.0562 | -37.67 | 0.0000 |
| `to_multiple` | -1.8092 | 0.2969 | -6.09 | 0.0000 |

# Inverse Logit Transformation

# Fitted Probabilities

# Fitted Model Using Backwards Regression

The following model was selected in the text using backwards selection using $\alpha = 0.05$.

| | Estimate | Std. Error | z value | Pr(>\|z\|) |
|---|---|---|---|---|
| (Intercept) | -0.8057 | 0.0880 | -9.15 | 0.0000 |
| to_multiple? | -2.7514 | 0.3074 | -8.95 | 0.0000 |
| word winner used? | 1.7251 | 0.3245 | 5.32 | 0.0000 |
| special formatting? | -1.5857 | 0.1201 | -13.20 | 0.0000 |
| 'RE:' in subject? | -3.0977 | 0.3651 | -8.48 | 0.0000 |
| attachment? | 0.2127 | 0.0572 | 3.72 | 0.0002 |
| word password used? | -0.7478 | 0.2956 | -2.53 | 0.0114 |

# Fitted Model Using Backwards Regression

The following variables increase the probability that the email is spam, since $b > 0$

| | Estimate | Std. Error | z value | Pr(>\|z\|) |
|---|---|---|---|---|
| (Intercept) | -0.8057 | 0.0880 | -9.15 | 0.0000 |
| word winner used? | 1.7251 | 0.3245 | 5.32 | 0.0000 |
| attachment? | 0.2127 | 0.0572 | 3.72 | 0.0002 |

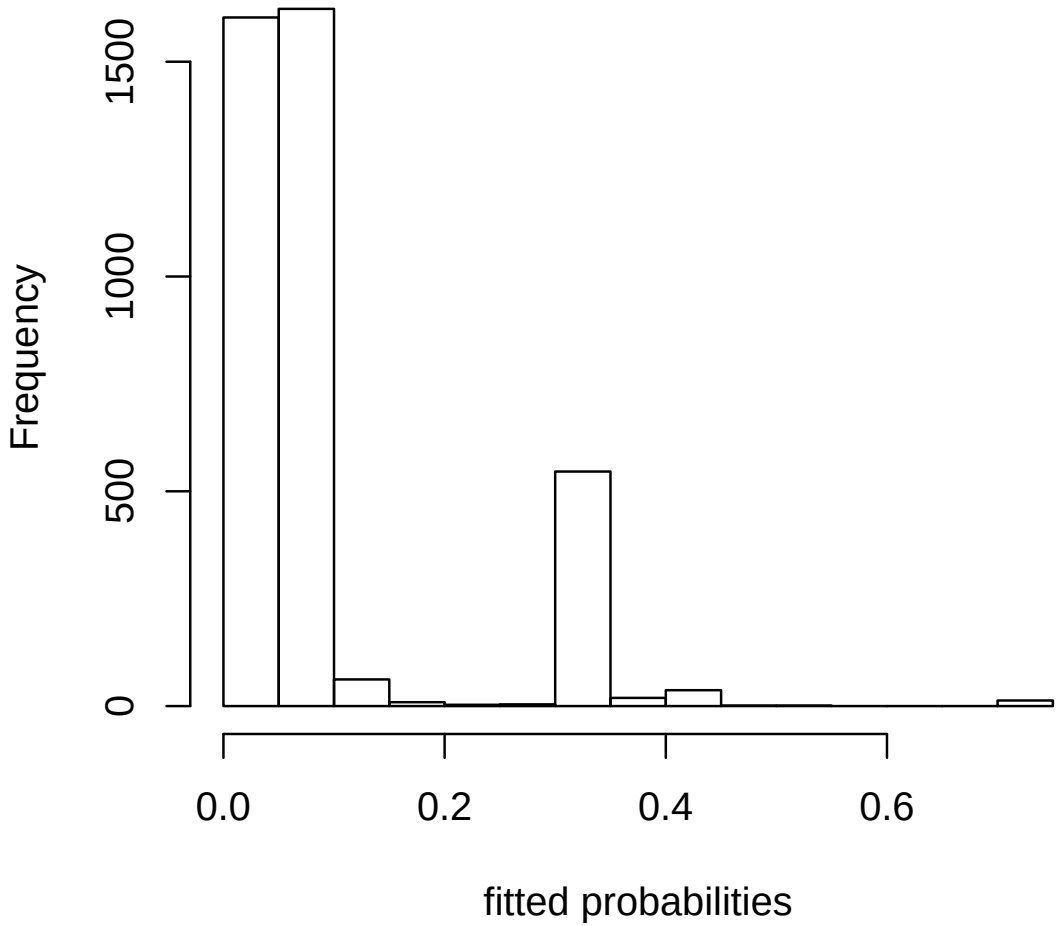
# Fitted Model Using Backwards Regression

The following variables decrease the probability that the email is spam, since $b < 0$

| | Estimate | Std. Error | z value | Pr(>\|z\|) |
|---|---|---|---|---|
| (Intercept) | -0.8057 | 0.0880 | -9.15 | 0.0000 |
| to_multiple? | -2.7514 | 0.3074 | -8.95 | 0.0000 |
| special formatting? | -1.5857 | 0.1201 | -13.20 | 0.0000 |
| 'RE:' in subject? | -3.0977 | 0.3651 | -8.48 | 0.0000 |
| word password used? | -0.7478 | 0.2956 | -2.53 | 0.0114 |

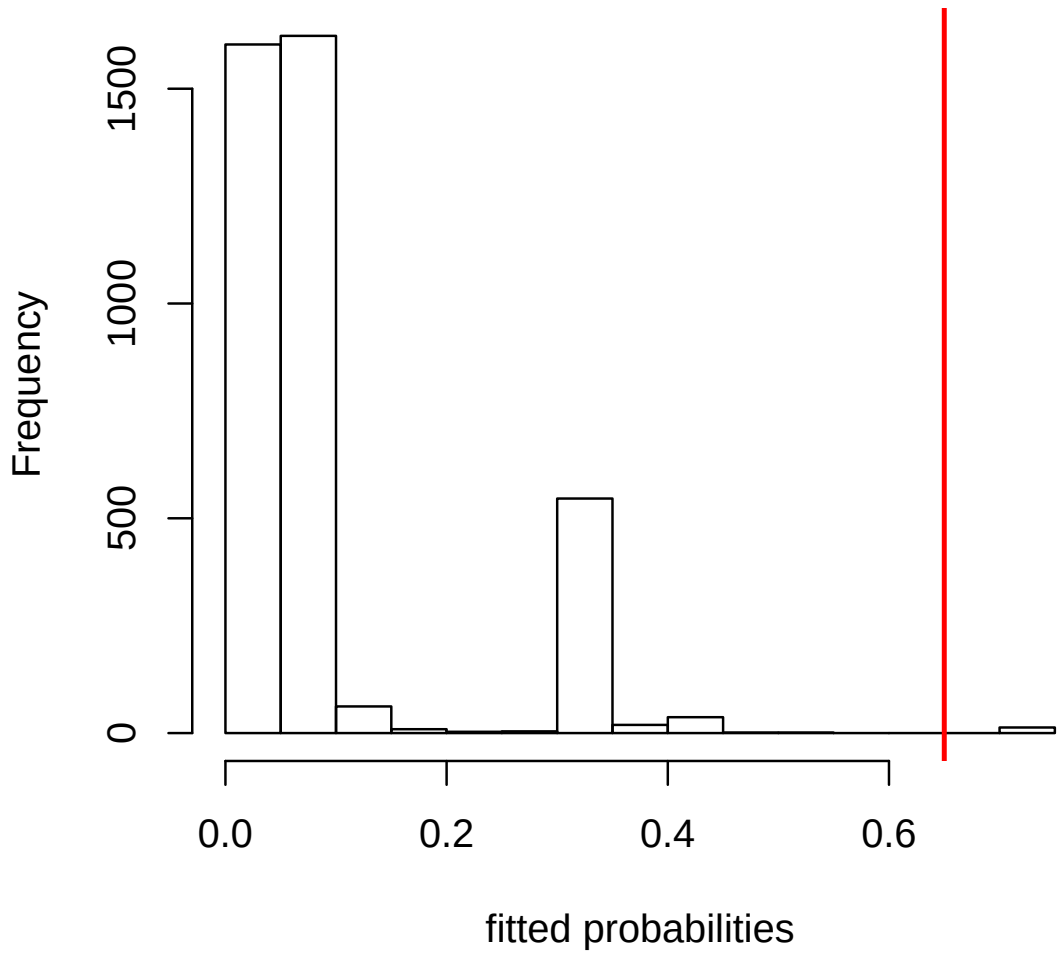
# Fitted Probabilities

These are all 3921 fitted probabilities:



# Using Cutoffs to Classify Emails as Spam

Say we use a cutoff of 65% to classify an email spam or not:



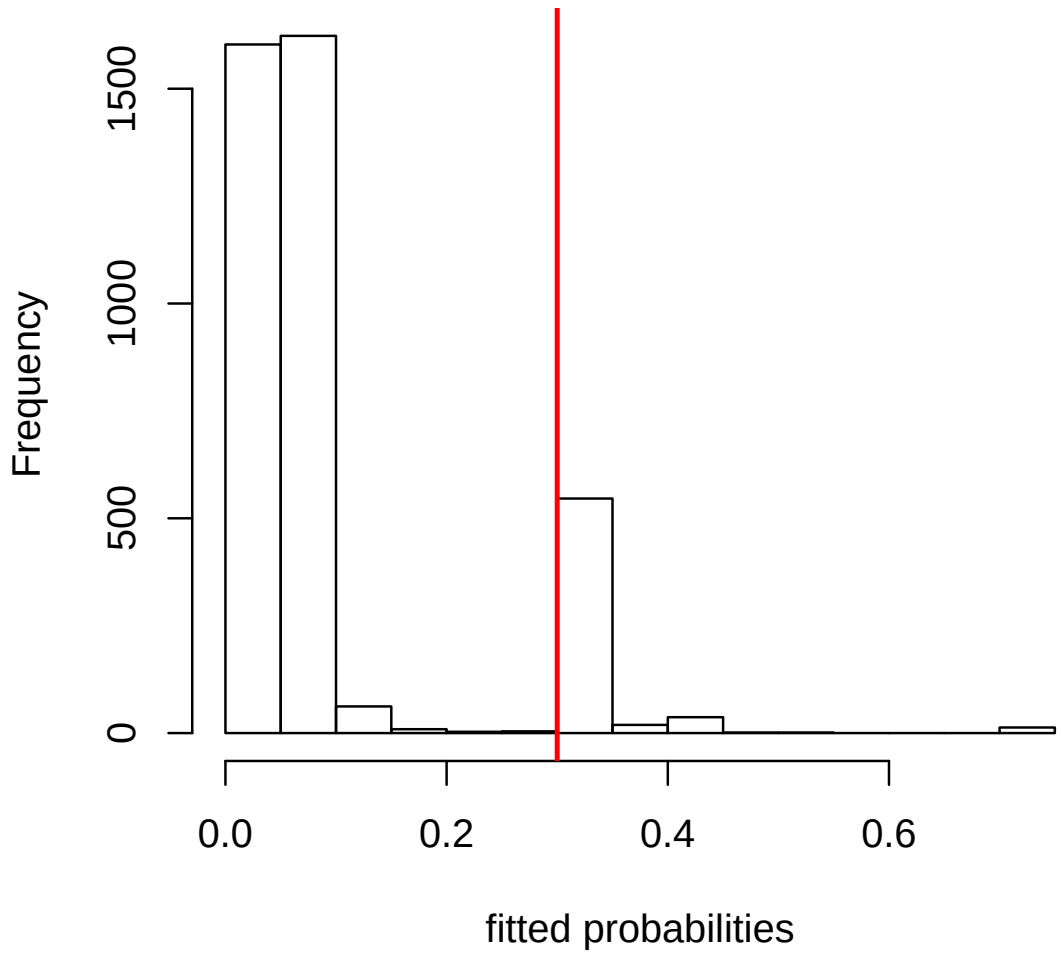
# Using Cutoffs to Classify Emails as Spam

Using a cutoff of 65%:

| | | **Classification** | |
|---|---|---|---|
| | | Not Spam | Spam |
| **Truth** | Not Spam | 3351 | 3 |
| | Spam | 357 | 10 |

# Using Cutoffs to Classify Emails as Spam

Now say we use a cutoff of 30% to classify an email spam or not:



# Using Cutoffs to Classify Emails as Spam

Using a cutoff of 30%:

| | | **Classification** | |
|---|---|---|---|
| | | Not Spam | Spam |
| **Truth** | Not Spam | 3138 | 416 |
| | Spam | 166 | 201 |

# Using Cutoffs to Classify Emails as Spam

# Assumptions for Logistic Regression