Lecture 20: Single Proportion Test

Chapter 6.1

Quiz 8

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Answer: One reason is that correlational studies are excellent starting points for deciding which hypotheses to evaluate with the more rigorous randomized controlled experiment.

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Answer:

3. preliminary result
1. large-scale observational study
4. large-sample randomized controlled test
2. well-established scientific law that we know how to apply in a wide range of conditions

Causality

What is causality? How do we establish it?

- http://nfs.unipv.it/nfs/minf/dispense/patgen/lectures/files/disease_causality.html
- http://bayes.cs.ucla.edu/BOOK-2K/

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Let's assume the sampling procedure was representative. What's the first thing to ask?

- What was n?
- In other words, what is the SE of $\widehat{p} = 44\%$?

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- conduct hypothesis tests via the normal tables

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This happens when the population distribution of 0's and 1's is not too strongly skewed. But as before, as the sample size $n \longrightarrow \infty$, this is less of an issue by the CLT.

Conditions for Sampling Dist'n of $\widehat{p}$ Being Nearly Normal

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- The observations are independent: the 10% rule
- We expect to see at least 10 successes and 10 failures in our sample. This is called the success-failure condition: that both
  - $np \geq 10$
  - $n(1-p) \geq 10$

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If conditions are met, then the sampling distribution of $\widehat{p}$ is nearly normal with

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Note the similarity of the previous formula for the sample mean $\overline{x}$:

$$SE_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \sqrt{\frac{\sigma^2}{n}}$$

Standard Error of $\widehat{p}$

But we don't know what p is. So what p do we use

- to check the conditions?
- for the $SE_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}}$?

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- Hypothesis tests: plug in the null value p_0 from $H_0 : p = p_0$

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  - $976 \times \widehat{p} = 976 \times .44 = 429$ successes ≥ 10
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$$SE_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} = \sqrt{\frac{0.44(1-0.44)}{976}} = 0.016$$

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In our case

$$\widehat{p} \pm 1.96 \times SE_{\widehat{p}} = 0.44 \pm 1.96 \times 0.016 = (0.409, 0.471)$$

Hypothesis Tests

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Thomas Carcetti is running for mayor of Baltimore. His campaign manager claims he has more than 50% support of the electorate.

The Baltimore Sun collects a random sample of $n = 500$ likely voters and finds that 52% support him. Does this provide convincing evidence for the claim of Carcetti's manager at the 5% significance level?

Hypothesis Tests

The hypothesis test is, with the null value $p_0 = 0.5$

$$\begin{aligned} & H_0 : p = p_0 \\ \text{vs} \qquad & H_A : p > p_0 \end{aligned}$$

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- Success-failure condition
  - $np_0 = 500 \times 0.5 = 250 \geq 10$
  - $n(1 - p_0) = 500 \times (1 - 0.5) = 250 \geq 10$

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$$SE_{\widehat{p}} = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.5(1-0.5)}{500}} = 0.022$$

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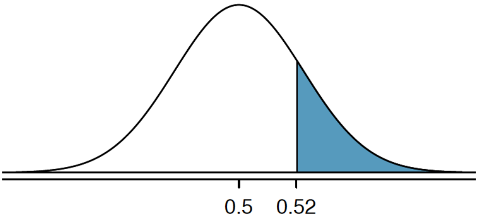
p-value is 0.1867.



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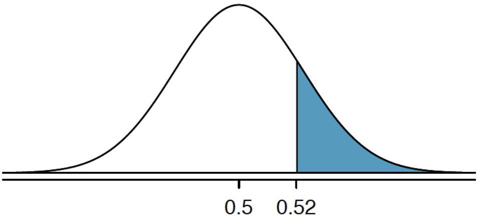

Hence we do not reject the null hypothesis, and we do not find convincing evidence to support the campaign manager's claim.

Next Time

Same as with the jump from

$$\mu \text{ to } \mu_1 - \mu_2$$

i.e. from one to two-sample tests for means, we make the jump from

$$p \text{ to } p_1 - p_2$$

i.e. from one to two-sample tests for proportions.