Lecture 12: Sampling Distributions & Standard Errors

Chapter 4.1

Goals for Today

Chapter 4: Arguably the most important chapter as it goes to the heart of statistical inference. Three important definitions:

1. point estimate
2. sampling distribution
3. standard error

Point Estimates

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We need to characterize the random error.

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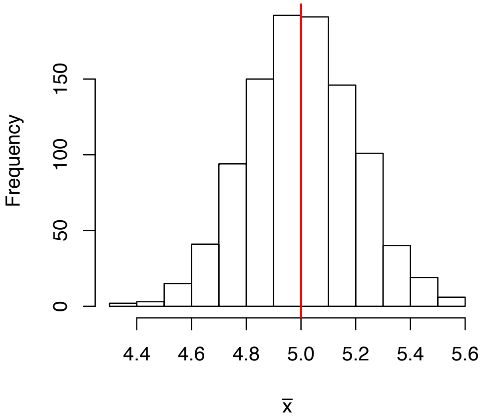
1st time	We get $\overline{x} = 4.831$
2nd time	We get $\overline{x} = 5.104$
3rd time	We get $\overline{x} = 4.965$
...	
1000th time	We get $\overline{x} = 4.957$

Sampling Distribution

This histogram is the 1000 instances of $\overline{x}$ i.e. the sampling distribution of $\overline{x}$:



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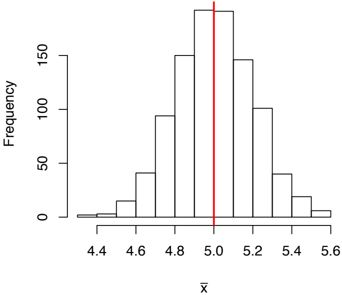
We will only focus on sample means, including the sample proportion $\widehat{p}$.

Measure of Spread

What about spread? $[4.6, 5.4]$ contains roughly 95% of the data.

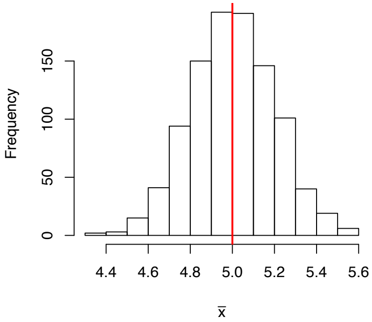


Standard Errors

Standard Error of $\overline{x}$

Back to Histogram

Samples were of size $n = 100$ with $\sigma = 2$. We estimated that the SD was 0.2.



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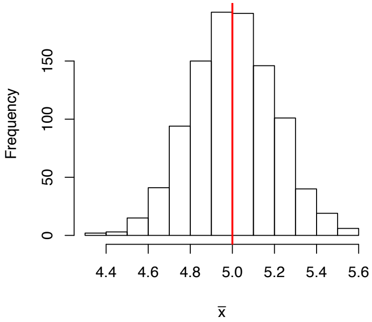

Using the formula:

$$SE = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = \frac{2}{10} = 0.2$$

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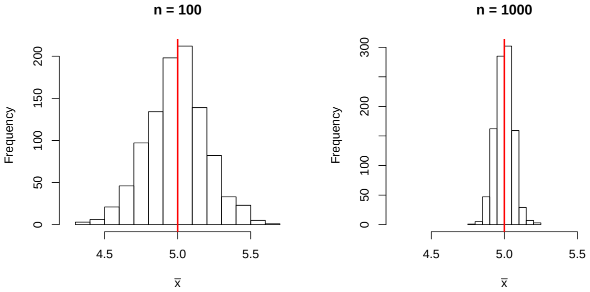

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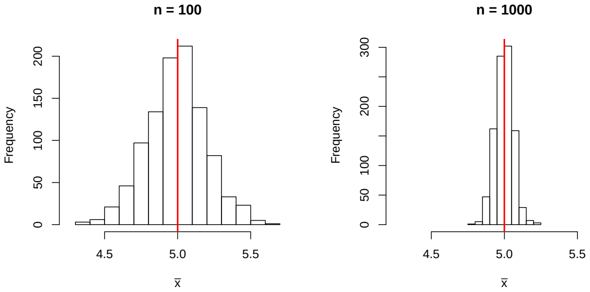

Both are "accurate", but estimates on the right are "more precise."

Repeated Sampling

Standard Error of the Sample Mean

Population Distribution vs Sampling Distribution

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- The sampling distribution characterizes the (random) behavior of point estimates.
- The standard deviation of a sampling distribution is the standard error: it quantifies the uncertainty/variability of point estimates.

Next Time

- Confidence Intervals
- When quoting survey results, what does: "the results of this survey are estimated to be accurate within 3.1 percentage points, 19 times out of 20" mean?
- Big One: Central Limit Theorem