

Lecture 15: Hypothesis Testing Part II

Chapter 4.3

Previously... Statistical Hypothesis Testing (in this class)

A **hypothesis** is a claim about a population parameter.

A **hypothesis test** is a method for using sample data to decide between two competing hypotheses about the population parameter:

- ▶ A **null hypothesis** H_0 .
i.e. the **status quo** that is initially assumed to be true, but will be tested.
- ▶ An **alternative hypothesis** H_A . i.e. the **challenger**.

There are two potential outcomes of a hypothesis test. Either we

- ▶ reject H_0
- ▶ fail to reject H_0

Goals for Today

- ▶ Tie-in p-Values with sampling distributions
- ▶ Revisit sleep example and consider two-sided hypotheses
- ▶ Go over the General Hypothesis Testing Procedure

Recall our Coin Example

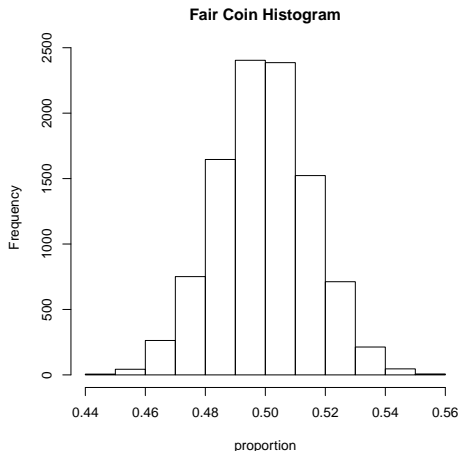
You have a coin that you want to test if it's fair via $n = 1000$ flips. Set $p_0 = 0.5$ and define a “success” as getting heads. i.e.

$$\begin{array}{ll} H_0 : p = p_0 \text{ i.e. coin is fair} \\ \text{vs} & H_A : p \neq p_0 \end{array}$$

- ▶ The point estimate \hat{p} of p is $\frac{\# \text{ of successes}}{\# \text{ of trials}}$.
- ▶ Since it is based on a sample, \hat{p} has a sampling distribution
- ▶ The standard error is $\sqrt{\frac{p(1-p)}{n}}$ (Chapter 6).
- ▶ Furthermore, the sampling distribution is Normal in this case (Central Limit Theorem)

Sampling Distribution of \hat{p}

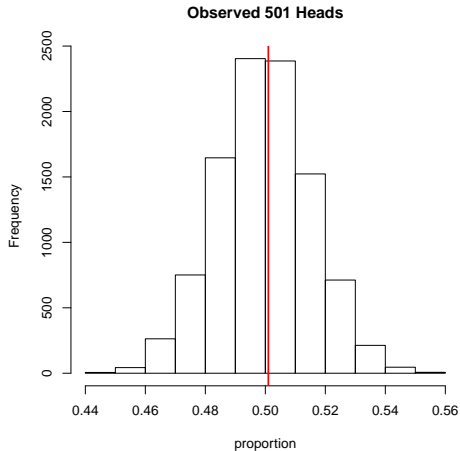
Under the null hypothesis that the coin is fair i.e. $p = p_0 = 0.5$, the sampling distribution of \hat{p} when $n = 1000$ is:



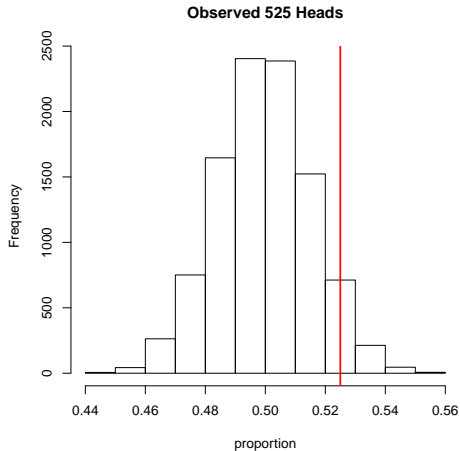
Say we observe...

- ▶ 501 heads
- ▶ 525 heads
- ▶ 900 heads

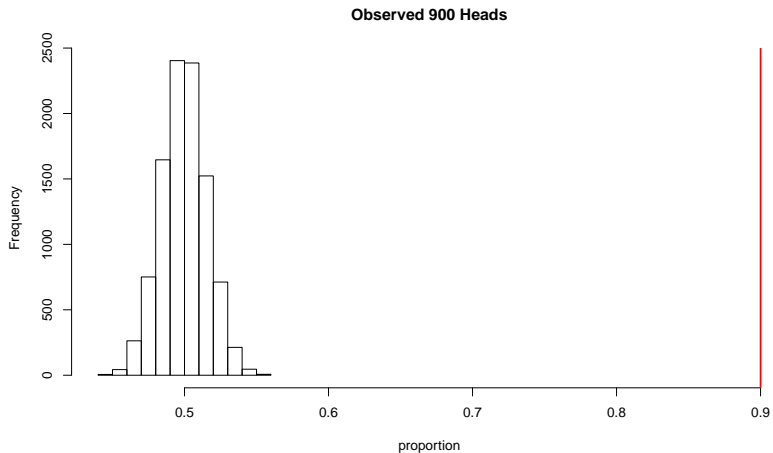
Say we observe...



Say we observe...



Say we observe...



Recall Example about Reenie Sleep Habits

A poll found that college students sleep about 7 hours a night. Researchers suspect that Reenies sleep more.

Let μ be the true **population mean** # of hours Reenies sleep a night. Then:

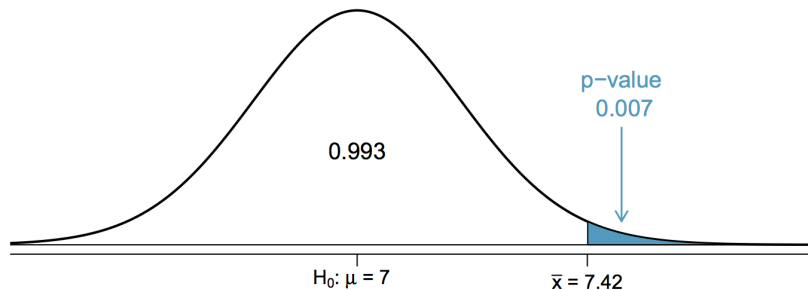
- ▶ $H_0 : \mu = \mu_0 = 7$
- ▶ $H_A : \mu > 7$

Researchers sample $n = 110$ Reenies and found that $\bar{x} = 7.42$ and $s = 1.75$, hence $z = \frac{7.42 - 7}{\frac{1.75}{\sqrt{110}}} = 2.47$.

Recall Example about Reddie Sleep Habits

In our case, since $H_A : \mu > 7$, more extreme means to the right of $z = 2.47$.

Hence, the p-value is 0.007:



Two-Sided Alternative Hypothesis

Say instead we had a two-sided alternative hypothesis:

- ▶ $H_0 : \mu = 7$

- ▶ $H_A : \mu \neq 7$

The the p-value would be double: $2 \times 0.007 = 0.014$

In General: Hypothesis Testing Procedure

1. Construct your hypothesis testing framework:
 - ▶ Define H_0 , H_A and if applicable a null value.
 - ▶ Set your significance level α
2. Verify that the conditions hold
3. Compute your test statistic
4. Compute the p-value
 - ▶ Identify the appropriate distribution to compare the test statistic to
 - ▶ Depending on H_A , determine what constitutes being more extreme and compute the p-value using the appropriate probability table.
5. If the p-value is $< \alpha$, reject H_0 . Otherwise do not.

In Reddie Sleep Example

1. Construct your hypothesis testing framework:
 - ▶ Define H_0 , H_A and if applicable a null value.
 - ▶ Set your significance level α
2. Verify that the conditions hold
3. Compute your test statistic: the z-score of $\bar{x} = 7.42$
4. Compute the p-value
 - ▶ Identify the appropriate distribution to compare the test statistic to: normal distribution
 - ▶ Depending on H_A , determine what constitutes being more extreme and compute the p-value using the appropriate probability table: z-table on page 409
5. If the p-value is $< \alpha$, reject H_0 . Otherwise do not.

Next Time

- ▶ How big a sample size do I need?
- ▶ Power Calculations