HW4 Solutions

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Disclaimer:
This is the solution manual to the homework assigned to students of Digital Control - Dr.Talebi.
We do not guarantee that this solution is precise and thorough so please contact your TA to propose your innovative solutions and/or any probable mistakes.

سوال اول

We have the following:
We have used a better approach in analyzing the stability of the equation in this question.
$a_0 = 0.3, a_1 = -0.1, a_2 = -1.1, a_3 = 1$
First check the stability criterion below:

$$1.D(1) > 0$$
$$2.(-1)^n D(-1) > 0$$
$$3.|a_3| > |a_0|$$
$$4.|b_0| > |b_2|$$

$1.D(1) = 1 - 1.1 - 0.1 + 0.3 = 0.1 > 0$
$2.(-1)^3 D(-1) = (-1)(-1 - 1.1 + 0.1 + 0.3) > 0$
3. $|a_3| > |a_0| \to 1 > 0.3$
Construct the Jury's array:

$$\begin{array}{cccc} z^0 & z^1 & z^2 & z^3 \\ 0.3 & -0.1 & -1.1 & 1 \\ 1 & -1.1 & -0.1 & 0.3 \\ b_0 & b_1 & b_2 & \end{array}$$

$b_0 = \begin{vmatrix} 0.3 & 1 \\ 1 & 0.3 \end{vmatrix} = 0.91$
$b_2 = \begin{vmatrix} 0.3 & -0.1 \\ 1 & -1.1 \end{vmatrix} = -0.23$
$|b_0| > |b_2|$
Thus the system is stable.

سوال دوم

We have the following:
$a_0 = -0.35, a_1 = 1.55, a_2 = -2.2, a_3 = 1$
First check the stability criterion below:

$$1.D(1) > 0$$
$$2.(-1)^n D(-1) > 0$$
$$3.|a_3| > |a_0|$$
$$4.|b_0| > |b_2|$$

$1.D(1) = 1 - 2.2 + 1.55 - 0.35 = 0$
The condition is not strictly satisfied, and for a definitive answer it must be checked.
$2.(-1)^n D(-1) = (-1)^3(-1 - 2.2 - 1.55 - 0.35) > 0$
$3.|a_3| > |a_0|, 1 > 0.35$
$4.|b_0| > |b_2|$
Construct the Jury array:

$$\begin{array}{cccc} z^0 & z^1 & z^2 & z^3 \\ -0.35 & 1.55 & -2.2 & 1 \\ 1 & -2.2 & 1.55 & -0.35 \\ b_0 & b_1 & b_2 & \end{array}$$

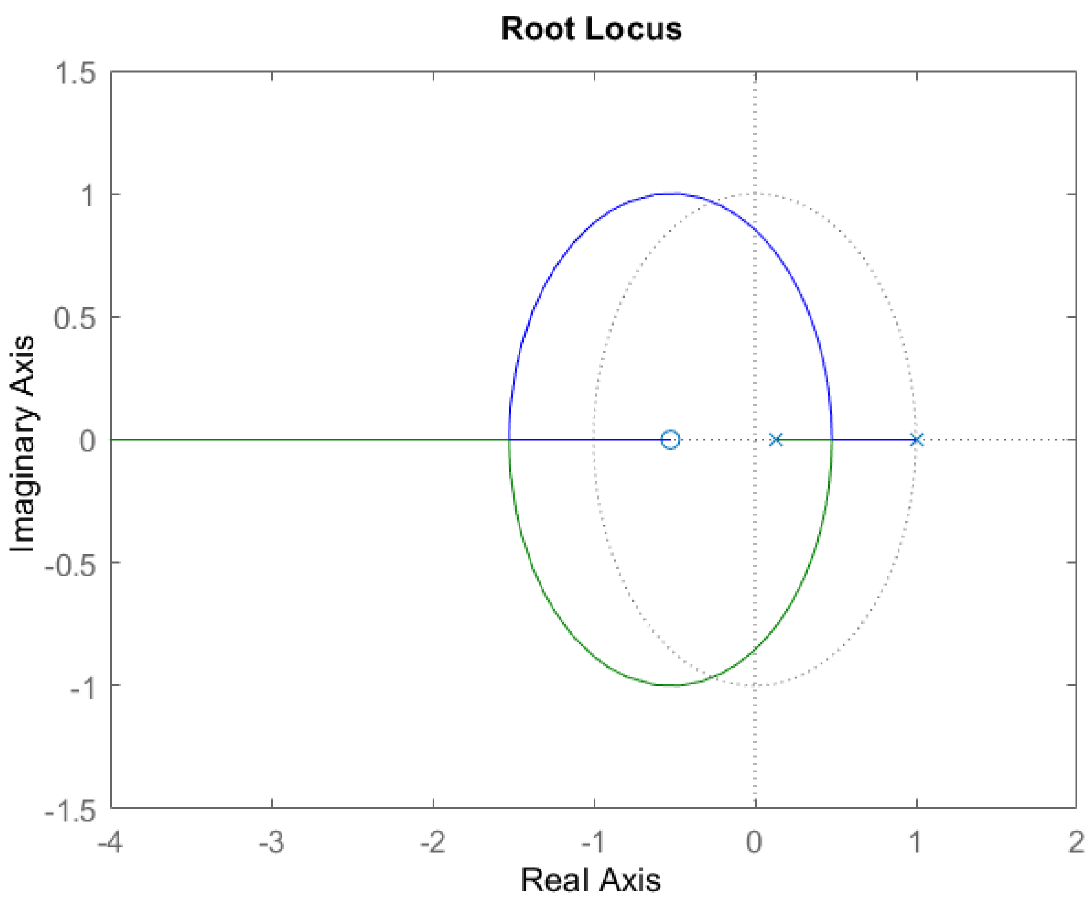
$b_0 = \begin{vmatrix} -0.35 & 1 \\ 1 & -0.35 \end{vmatrix} = -0.8775$
$b_2 = \begin{vmatrix} -0.35 & 1.55 \\ 1 & -2.2 \end{vmatrix} = -0.78$
$|b_0| > |b_2|$
The final analysis of stability requires a resize of the unit circle.
First let's make the unit circle bigger:
$z^n \to (1 + n\epsilon)z^n$
$D(z)_{new} = (1 + 3\epsilon)z^3 + -2.2(1 + 2\epsilon)z^2 + 1.55(1 + \epsilon)z - 0.35$
Evaluate the Jury's necessary conditions:
$D(1)_{new} = 1 + 3\epsilon - 2.2 - 4.4\epsilon + 1.55 + 1.55\epsilon - 0.35$
$D(1)_{new} = 0.15\epsilon$
since ϵ is positive $D(1)_{new}$ is positive. So the necessary conditions of Jury is satisfied for the "slightly bigger unit circle"
Now let us do the same thing with the "slightly smaller unit circle"
$z^n \to (1 - n\epsilon)z^n$
$D(z)_{new} = (1 - 3\epsilon)z^3 + -2.2(1 - 2\epsilon)z^2 + 1.55(1 - \epsilon)z - 0.35$
$D(1)_{new} = 1 - 3\epsilon - 2.2 + 4.4\epsilon + 1.55 - 1.55\epsilon - 0.35$
$D(1)_{new} = -0.15\epsilon$
since ϵ is positive $D(1)_{new}$ is negative. So the necessary conditions of Jury is not satisfied for the "slightly smaller unit circle", hence we have a pole on the unit circle.

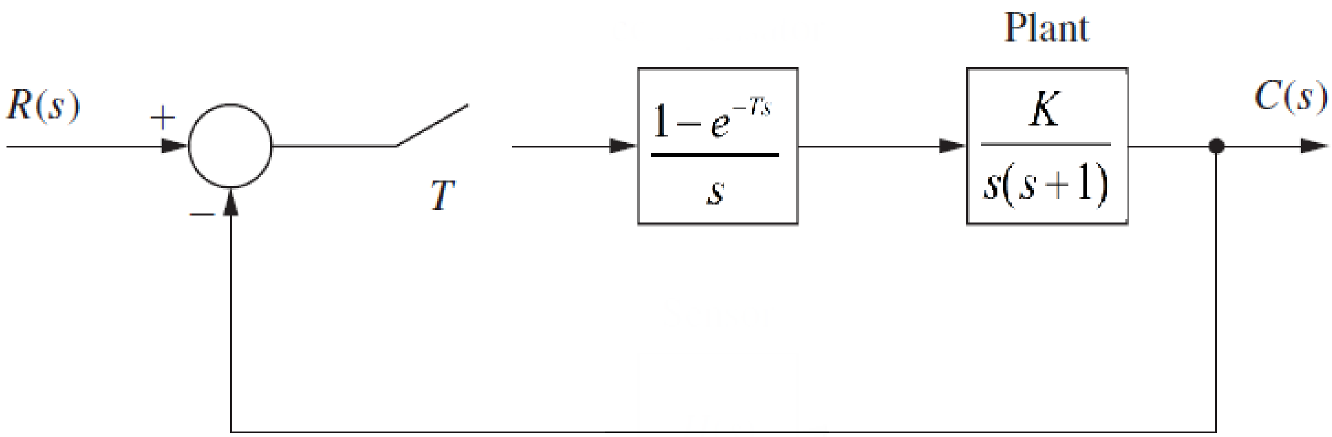
سوال سوم

First obtain the discrete transfer function for the given sampling period.

$$G(z) = Z\left\{\frac{1-e^{-Ts}}{s} \times \frac{K}{s(s+1)}\right\}$$
$$G(z) = (1-z^{-1})\left\{\frac{K}{s^2(s+1)}\right\} \xrightarrow{T=2}$$
$$G(z) = \frac{1.1353K(z+0.5232)}{(z-1)(z-0.1353)}$$

Which has one zero at z = 0.5232 and two poles at z = 1 and z = 0.1353
The two breaking points are 0.4783 and -1.52
the root locus can be as below:

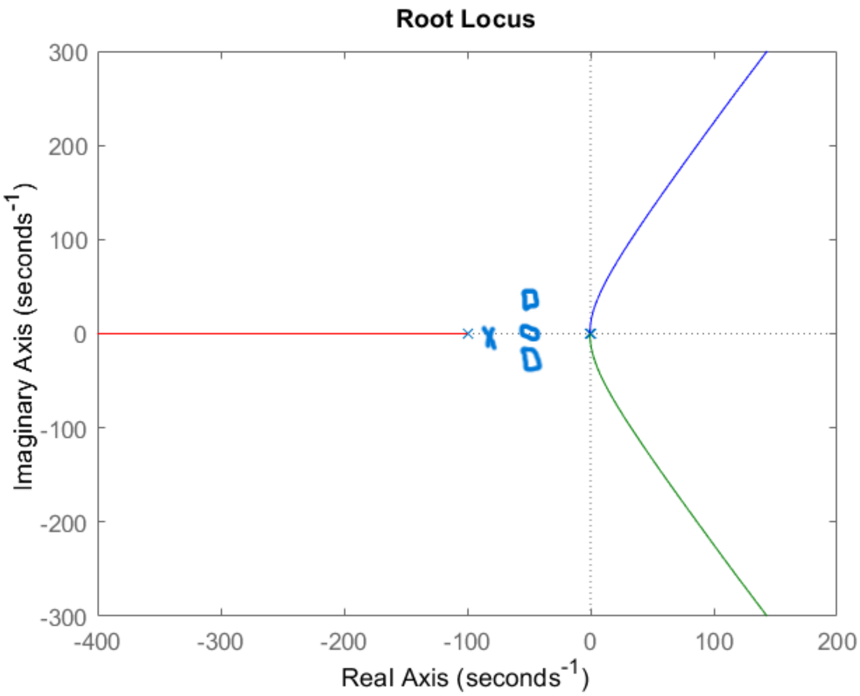


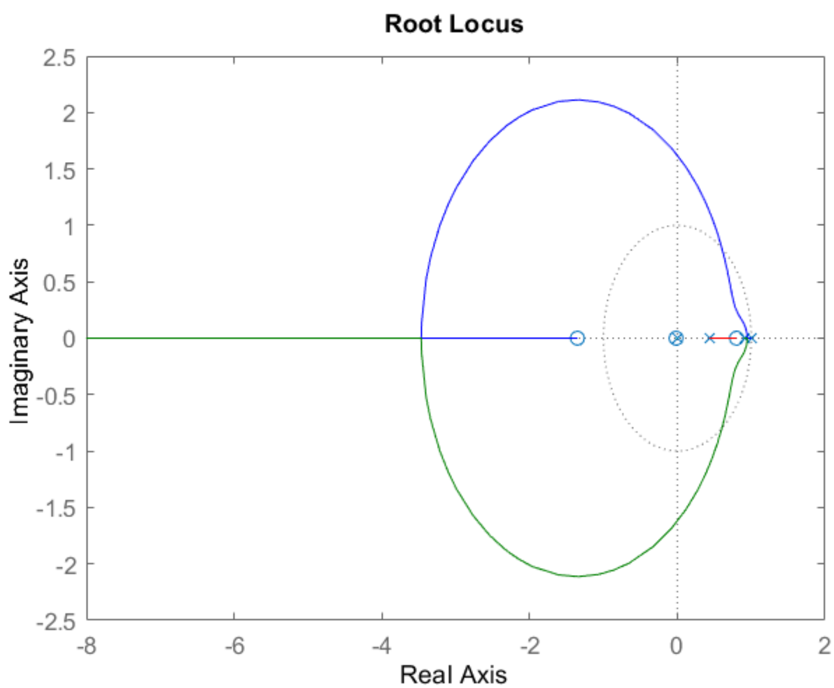


شکل ۱: شکل سوال سوم

سوال چهارم

First let's Design the controller:
the Desired poles are calculated as follows:
$\zeta = \sqrt{\frac{\ln(PO)^2}{\pi(\ln(PO))^2}} = 0.718$
$t_s = \frac{4}{\zeta\omega_n} => \omega_n = 3.95$
$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$
$s = -2.22 \pm j2.74$
We Define:
angle of the pole -100 : ϕ_1
angle of the pole -1 : ϕ_2
angle of the pole 0 : ϕ_3
it's 90 degrees : angle of the zero : θ_1
Angle Condition:
$-\phi_1 - \theta_1 + 90 - \phi_2 - \phi_3 = 180$
$-tan^{-1}(\frac{2.74}{97.78}) - \theta_1 + 90 - (180 - tan^{-1}(\frac{2.74}{1.22})) - (180 - tan^{-1}(\frac{2.74}{2.22})) = 180$
$\theta_1 = 24.34$
$G_c = K\frac{s+2.22}{s+8.3}$
$|GG_c(s = -2.22 + j2.74)| = 1$
$\frac{100K(2.74)}{\sqrt{2.22^2+2.74^2}\sqrt{1.22^2+2.74^2}\sqrt{97.78^2+2.74^2}\sqrt{6.08^2+2.74^2}} = 1$
$K = 25.181$
$G_c = 25.181\frac{s+2.22}{s+8.3}$
With sampling time of 0.1 we would have:
$G_{C,ZP} = 19.08\frac{z-0.8}{z-0.436}$


Root Locus
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
300
200
100
0
-100
-200
-300
-400
-300
-200
-100
0
100
200


Root Locus
Imaginary Axis
Real Axis
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-8
-6
-4
-2
0
2

سوال پنجم

$D(1) = 1 + 0.05K - 1.2 + 0.07K + 0.2 + 0.005K^2 - 0.007K$
$0.005K^2 + 0.127K > 0$
$K(0.005K + 0.127) > 0$
$K < -25.4, K > 0$
$D(-1) = -1 + 0.05K - 1.2 - 0.07 - 0.2 + 0.005K^2 - 0.007K$
$0.005K^2$

سوال ششم

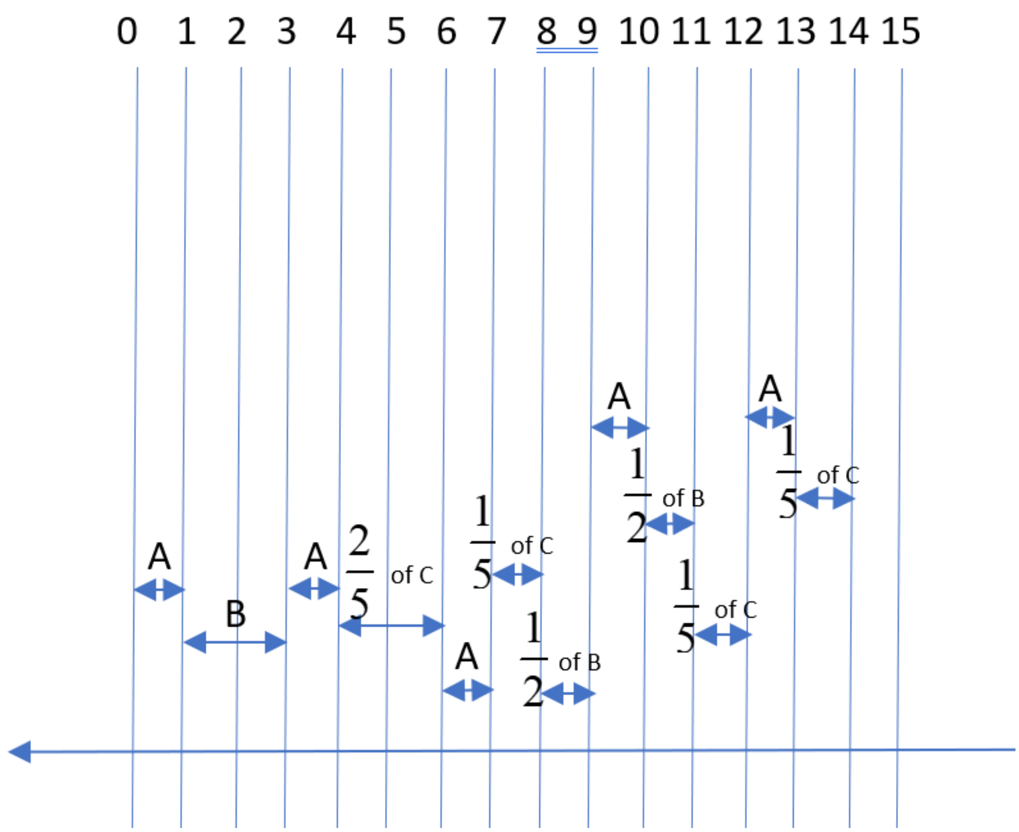
First Evaluate the sufficient conditions:

$$
\begin{aligned}
&\sum_{i=1}^{3} \frac{C_i}{T_i} \leq n(2^{\frac{1}{n}} - 1) \\
&\rightarrow (\frac{1}{3} + \frac{2}{8} + \frac{5}{20}) \leq 3(2^{\frac{1}{3}} - 1) \\
&0.83 \leq 0.7797
\end{aligned}
$$

It is clearly not satisfied.
Let's analyze other sufficient conditions:

$$
\begin{aligned}
&\prod_{i=1}^{3} (\frac{C_i}{T_i} + 1) \leq 2 \\
&\rightarrow (\frac{1}{3} + 1)(\frac{2}{8} + 1)(\frac{5}{20} + 1) \leq 2 \\
&2.08 \leq 2
\end{aligned}
$$

It is not satisfied either, so let's move to the necessary conditions.
$R_i \leq D_i$



Hence, the tasks are schedulable with the RM algorithm