HW3 Solutions

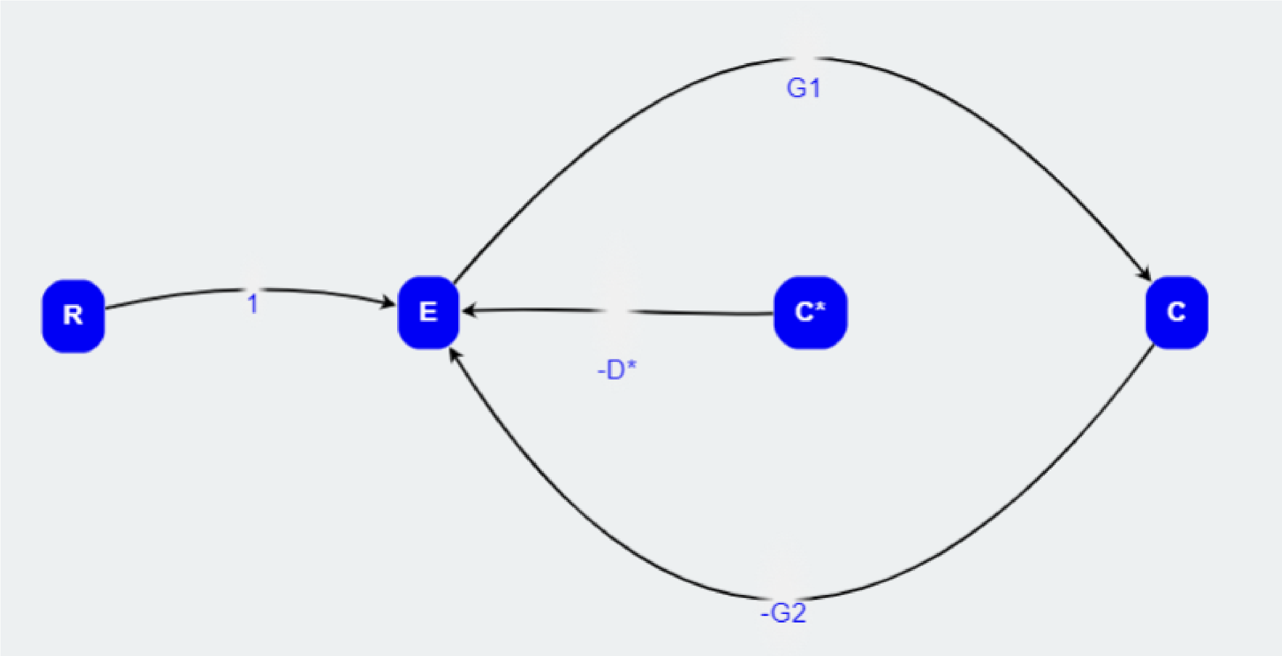
Written by Reza Shahriari

۱ بخش اجباری

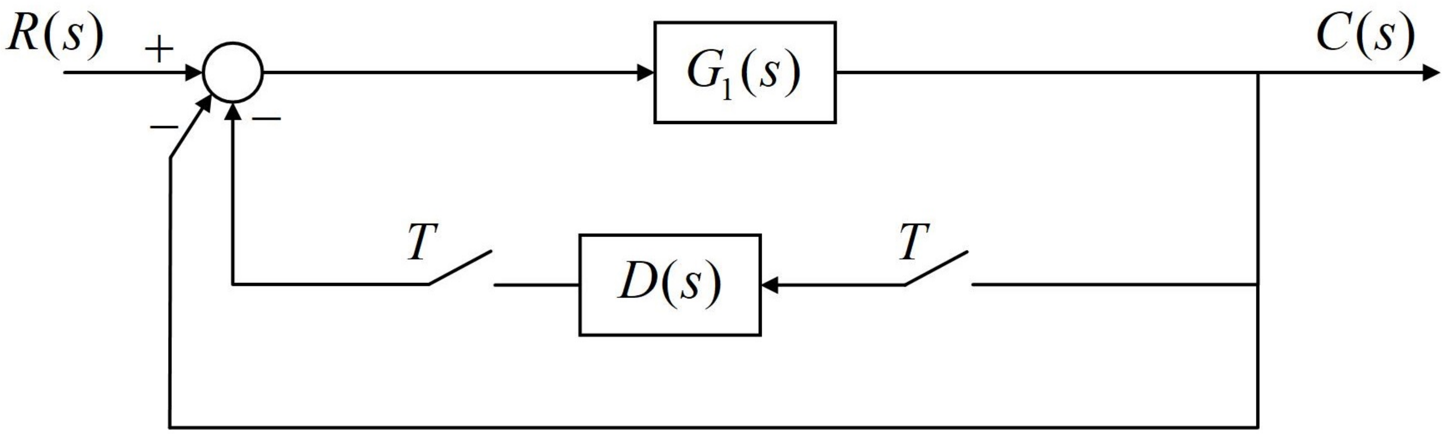
Disclaimer:
This is the solution manual to the homework assigned to students of Digital Control - Dr.Talebi.
We do not guarantee that this solution is precise and thorough so please contact your TA to propose your innovative solutions and/or any probable mistakes.

سوال اول

First draw the original SFG :



$C = G_1 E$
$E = R - G_2 C - D^* C^*$
$C = \frac{G_1 R}{G_1 G_2 + 1} - \frac{D^* C^*}{1 + G_1 G_2}$
Derive C^* as follows:
$C^* = (\frac{G_1 R}{1 + G_1 G_2})^* - (\frac{1}{1 + G_1 G_2})^* D^* C^*$
$C^* = \frac{(\frac{G_1 R}{1 + G_1 G_2})^*}{(1 + (\frac{1}{G_1 G_2})^* D^*)}$
$C = \frac{G_1 R}{G_1 G_2 + 1} - \frac{D^* (\frac{(\frac{G_1 R}{1 + G_1 G_2})^*}{(1 + (\frac{1}{G_1 G_2})^* D^*)})}{1 + G_1 G_2}$



شکل ۱: شکل سوال اول

سوال دوم

Recall the equivalent for forward discretization :
$s \rightarrow \frac{z-1}{T}$
we must substitute it in the transfer function to get the following discrete transfer function:
$H(z)_{forward} = 10\frac{z-0.75}{z+1.5}$
from the original transfer function we have:
$\frac{s+1}{0.1s+1} \xrightarrow{s\rightarrow j\omega} \frac{j\omega+1}{0.1j\omega+1}\frac{j\omega+1}{0.1j\omega+1} = \frac{\sqrt{\omega^2+1}}{\sqrt{(0.1\omega)^2+1}}\angle\tan^{-1}\omega - \tan^{-1}0.1\omega$
$\omega = 3 \rightarrow \varphi = \tan^{-1}3 - \tan^{-1}0.3 \simeq 55°$
$H(z)_{forward} = 10\frac{z-0.75}{z+1.5} \xrightarrow{z=e^{j\omega T}} 10\frac{e^{j\omega T}-0.75}{e^{j\omega T}+1.5}$
$10\frac{\cos\omega T-0.75+j\sin\omega T}{\cos\omega T+1.5+j\sin\omega T}$
$\varphi = \tan^{-1}\frac{\sin\omega T}{\cos\omega T-0.75} - \tan^{-1}\frac{\sin\omega T}{\cos\omega T+1.5}$
$\varphi = \tan^{-1}\frac{\sin 0.75}{\cos 0.75-0.75} - \tan^{-1}\frac{\sin 0.75}{\cos 0.75+1.5}$
$\varphi = \tan^{-1}\frac{.68}{.0183} - \tan^{-1}\frac{0.68}{0.23168} = 88.46 - 16.94 = 71.52$
Discretize using the Backward rule to get :
$s \rightarrow \frac{z-1}{Tz}$
$H(z)_{Backward} = 3.57\frac{z-0.8}{z-0.286}$
$H(z) = 3.57\frac{z-0.8}{z-0.286} \xrightarrow{z\rightarrow e^{j\omega T}} 3.57\frac{e^{j\omega T}-0.8}{e^{j\omega T}-0.286}$
$3.57\frac{\cos\omega T-0.8+j\sin\omega T}{\cos\omega T-0.286+j\sin\omega T}$
$\varphi = \tan^{-1}\frac{\sin\omega T}{\cos\omega T-0.8} - \tan^{-1}\frac{\sin\omega T}{\cos\omega T-0.286}$
$\varphi = \tan^{-1}\frac{\sin 0.75}{\cos 0.75-0.8} - \tan^{-1}\frac{\sin 0.75}{\cos 0.75-0.286}$
$\varphi = \tan^{-1}\frac{0.68}{0.0683} - \tan^{-1}\frac{0.68}{0.4457}$
$\varphi \simeq 40$

سوال سوم

Recall the formula for tustin with prewarping:
$s \rightarrow \frac{1}{\tan(\frac{\omega_1 T}{2})}\frac{z-1}{z+1}$
we must substitute it in the transfer function to get the following discrete transfer function:
$H(z) = 4.89\frac{z-0.768}{z-0.161}$
$H(z) = 4.89\frac{z-0.768}{z+0.161} \xrightarrow{z \rightarrow e^{j\omega T}} 4.89\frac{e^{j\omega T}-0.768}{e^{j\omega T}+0.161}$
$4.89\frac{\cos\omega T-0.768+j\sin\omega T}{\cos\omega T+0.161+j\sin\omega T}$
$\varphi = \tan^{-1}\frac{\sin\omega T}{\cos\omega T-0.768} - \tan^{-1}\frac{\sin\omega T}{\cos\omega T+0.161}$
$\varphi = \tan^{-1}\frac{0.68}{.0363} - \tan^{-1}\frac{0.68}{0.8926}$
$\varphi = -86.944 - 37.3$
$\varphi = 55.756$

سوال چهارم

Recall the equivalent formula for zero pole matching. Substitute it to get the discrete transfer function as bellow:
$H_{PZ} = 4.150\frac{z-0.779}{z-0.082}$
$H_{PZ} = 4.150\frac{z-0.779}{z-0.082} \xrightarrow{z \rightarrow e^{j\omega T}} 4.150\frac{e^{j\omega T}-0.779}{e^{j\omega T}-0.082}$
$4.150\frac{\cos\omega T-0.779+j\sin\omega T}{\cos\omega T-0.082+j\sin\omega T}$
$\varphi = \tan^{-1}\frac{\sin\omega T}{\cos\omega T-0.779} + \tan^{-1}\frac{\sin\omega T}{\cos\omega T-0.082}$
$\varphi = \tan^{-1}\frac{0.68}{-0.047} - \tan^{-1}\frac{0.68}{0.6496}$
$\varphi = -86.046 - 46.3097$
$\varphi = 47.64$

سوال پنجم

Looking back at the calculations of section 4.3.3 of Digital Control of Dynamic Systems by G.Franklin:
Let's choose T=1

$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k) \\ y(k) &= Hx(k) \end{aligned} \tag{۱}$$

$$\Phi = e^{AT}$$
$$\Gamma = \int_0^T e^{A\eta} d\eta B$$
$$\Phi = L^{-1}\left\{(sI - AT)^{-1}\right\}$$
$$AT = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$$
$$\Phi = L^{-1}\left\{\begin{pmatrix} \frac{s+1}{s^2+s+1} & \frac{-1}{s^2+s+1} \\ \frac{1}{s^2+s+1} & \frac{s+1.57\times10^{-16}}{s^2+s+1} \end{pmatrix}\right\}$$

The rest is routine and left as an exercise to students.

سوال ششم

Follow section 6.1 , equations 6.27 through 6.31 of Digital Control of dynamic systems by G.Franklin for the solution.

۲ بخش امتیازی

سوال هفتم

Solution:

To obtain V_o, we will replace the R-$2R$ network with its Thevenin equivalent circuit, as shown in Fig. 2. The computation of the Thevenin resistance and Thevenin voltage is shown in Figs. 3 and 4, respectively, where repeated use of Thevenin's theorem has been made to simplify the circuit systematically.

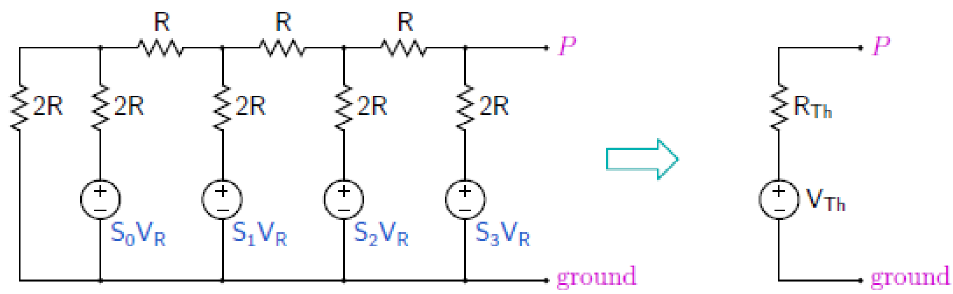
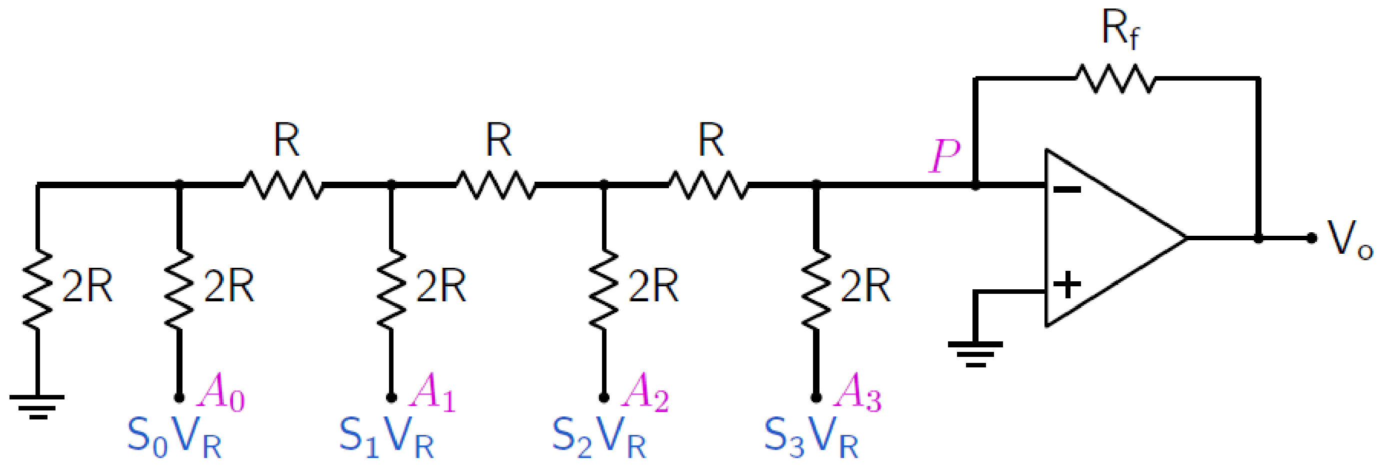


Figure 2: Representation of R-$2R$ ladder with Thevenin equivalent circuit.

Finally, we replace the R-$2R$ network in the original DAC circuit of Fig. 1 with its Thevenin equivalent circuit and obtain the circuit shown in Fig. 5. This circuit is simply an inverting amplifier, and the output voltage is given by

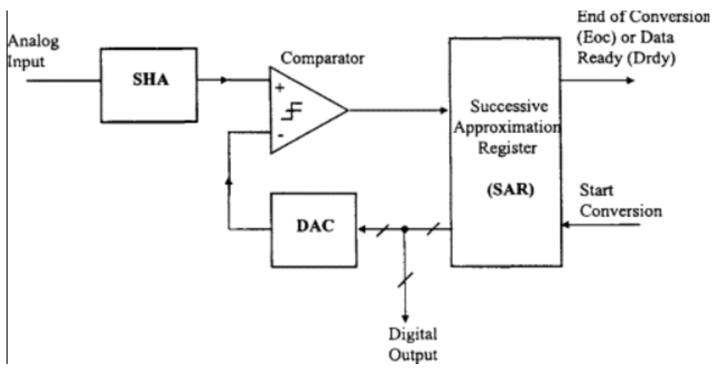
$$V_o = -\frac{R_f}{R} V_{\mathrm{Th}} = -\frac{V_R}{8} = -0.625\,\mathrm{V}, \tag{1}$$



شکل ۲: شکل سوال هفتم

سوال هشتم

This task must be done in multiple steps:
Firstly, compare 10 which is half of 20 with the input voltage:
$10 < 13.478$
$\text{MSB Gets Set we have}: 10000000$
$10 + 5 > 13.478$
$\text{the next bit remains 0}: 10000000$
$10 + 2.5 < 13.478$
$\text{the next bit gets set}: 1010000$
$10 + 2.5 + 1.25 > 13.478$
$\text{the next bit remains 0}: 10100000$
$10 + 2.5 + 0.625 < 13.478$
$\text{the next bit gets set}: 10101000$
$10 + 2.5 + 0.625 + 0.3125 < 13.478$
$\text{the next bit gets set}: 10101100$
$10 + 2.5 + 0.625 + 0.3125 + 0.15625$
$\text{the next bit remains 0}: 10101100$



شکل ۳: شکل سوال هشتم