$$A = \lim_{n\to\infty} (x_1 \cdots x_n)^{\frac{1}{n}} \qquad x_i > 0$$

$$\Rightarrow \ln(A) = \ln\left(\lim_{n\to\infty}(X_1 \cdots X_n)^{\frac{1}{n}}\right) = \lim_{n\to\infty} \frac{1}{n} \ln(X_1 \cdots X_n)$$

$$= \lim_{n\to\infty} \frac{1}{n} \left(\ln(X_1) + \dots + \ln(X_n)\right)$$

حال توزیع $\ln(X_1)$ را می‌یابیم .

$$f_X(x) = g(\lambda) \frac{1}{x^2 \ln(x)} \qquad \lambda \geq x \geq 2 \qquad Y = \ln(X)$$

$$f_Y(x) = f_X(x) \left|\frac{\partial x}{\partial Y}\right| \Rightarrow f_Y(\ln x) = f_X(x)\,|x| \qquad \ln(\lambda) \geq x \geq \ln(2) > 0$$

$$\frac{\partial Y}{\partial x} = \frac{\partial}{\partial x} \ln(x) = \frac{1}{x}$$

$$\Rightarrow f_Y(x) = g(\lambda) \frac{1}{e^{2x} x} e^{x} = \frac{g(\lambda)}{x e^{x}}$$

$$\frac{1}{n}(Y_1 + \dots + Y_n) \to \mu = \mu_{Y_1}$$

حال

$$Y_1 : \frac{g(\lambda)}{xe^x} \rightsquigarrow E(Y) = \int_{\ln(2)}^{\ln(\lambda)} \frac{x\,g(\lambda)}{x e^x}\, dx =$$

$$g(\lambda) \int_{\ln(2)}^{\ln(\lambda)} \frac{x}{x\, e^x}\, dx = g(\lambda) \int_{\ln(2)}^{\ln(\lambda)} e^{-x}\, dx =$$

$$g(\lambda) \left(-e^{-x}\right) \Big|_{\ln(2)}^{\ln(\lambda)} =$$

$$g(\lambda) \left(e^{-\ln(2)} - e^{-\ln(\lambda)} \right) = g(\lambda) \left(\frac{1}{2} - \frac{1}{\lambda} \right)$$

$$\overset{\lambda=8}{\Rightarrow} g(8)\left(\frac{1}{2} - \frac{1}{8} \right) = 3\left(\frac{4-1}{8} \right) = \frac{9}{8}$$

$$\Rightarrow \ln(A) = \frac{9}{8} \Rightarrow \boxed{A = e^{\frac{9}{8}}}$$