

کامیاب
میری

تقریب امتیازی

پرہیز صافی

400108547

دو تا تیلور ردیف فرم

$$(1) \quad f(x_i+h) = f(x_i) + h f'(x_i) + \frac{h^2}{2} f''(x_i) + \frac{h^3}{6} f'''(x_i) + O(h^4)$$

$$(2) \quad f(x_i-h) = f(x_i) - h f'(x_i) + \frac{h^2}{2} f''(x_i) - \frac{h^3}{6} f'''(x_i) + O(h^4)$$

$$f_{i-1} + f_{i+1} - 2f_i = (1) + (2) - 2f(x_i) = 2 \times \frac{h^2}{2} f''(x_i) + \underbrace{2O(h^4)}_{O(h^4)}$$

$$\Rightarrow f_{i-1} + f_{i+1} - 2f_i = h^2 f''(x_i) + O(h^4)$$

$$\Rightarrow \frac{f_{i-1} + f_{i+1} - 2f_i}{h^2} = f''(x_i) + O(h^2)$$

$$\Rightarrow f''(x_i) = \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2} + O(h^2)$$

خطا برشی
فواصل سه.

3) (الف) سب سے پہلے، x_i کے

$$1) f(x_i + 2h) = f(x_i) + 2hf'(x_i) + 2h^2 f''(x_i) + \frac{4h^3}{3} f'''(x_i) + \frac{2h^4}{3} f^{(4)}(x_i) + O(h^5)$$

$$2) f(x_i + h) = f(x_i) + hf'(x_i) + \frac{h^2}{2} f''(x_i) + \frac{h^3}{6} f'''(x_i) + \frac{h^4}{24} f^{(4)}(x_i) + O(h^5)$$

$$3) f(x_i - h) = f(x_i) - hf'(x_i) + \frac{h^2}{2} f''(x_i) - \frac{h^3}{6} f'''(x_i) + \frac{h^4}{24} f^{(4)}(x_i) + O(h^5)$$

$$4) f(x_i - 2h) = f(x_i) - 2hf'(x_i) + 2h^2 f''(x_i) - \frac{4h^3}{3} f'''(x_i) + \frac{2h^4}{3} f^{(4)}(x_i) + O(h^5)$$

حال آئیے $f'''(x_i)$ کے لیے، f'' ، f' ، f کے

تکرار دہیں۔

| جملہ | 1 | 2 | 3 | 4 |
|-------------|---|----|---|----|
| تکرار دہیں۔ | 1 | -2 | 2 | -1 |

$$f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2} = (1 - 2 + 2 - 1)f(x_i) + (2h - 2h - 2h + 2h)f'(x_i)$$

$$+ (2h^2 - 2 \frac{h^2}{2} + 2 \frac{h^2}{2} - 2h^2)f''(x_i) + (\frac{4h^3}{3} - 2 \frac{h^3}{6} - 2 \frac{h^3}{6} + \frac{4h^3}{3})f'''(x_i)$$

$$+ (2 \frac{h^4}{3} - 2 \frac{h^4}{24} + 2 \frac{h^4}{24} - \frac{2h^4}{3}) + O(h^5)$$

$$\Rightarrow f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2} = 2h^3 f'''(x_i) + O(h^5)$$

↓ $\div 2h^3$

$$\Rightarrow f^3(x_i) = \frac{f_{i+2} - 2f_{i+1} + 2f_{i-1} - f_{i-2}}{2h^3} + O(h^2)$$

(د)

$$1) f(x_{i+2h}) = f(x_i) + 2hf'(x_i) + 2h^2f''(x_i) + \frac{4h^3}{3}f^3(x_i) + \frac{2h^4}{3}f^4(x_i) + \frac{4h^5}{15}f^5(x_i) + O(h^6)$$

$$2) f(x_{i+h}) = f(x_i) + hf'(x_i) + \frac{h^2}{2}f''(x_i) + \frac{h^3}{6}f^3(x_i) + \frac{h^4}{24}f^4(x_i) + \frac{h^5}{120}f^5(x_i) + O(h^6)$$

$$3) f(x_{i-h}) = f(x_i) - hf'(x_i) + \frac{h^2}{2}f''(x_i) - \frac{h^3}{6}f^3(x_i) + \frac{h^4}{24}f^4(x_i) - \frac{h^5}{120}f^5(x_i) + O(h^6)$$

$$4) f(x_{i-2h}) = f(x_i) - 2hf'(x_i) + 2h^2f''(x_i) - \frac{4h^3}{3}f^3(x_i) + \frac{2h^4}{3}f^4(x_i) - \frac{4h^5}{15}f^5(x_i) + O(h^6)$$

حل باير فدايب f', f^2, f^3, f^5 ، $\frac{1}{3}$ كنيم . تاراج دهيم بترتيب فدايب $1, -4, -4, 1$ ،
دريم كړي :

$$\begin{aligned} f_{i+2} - 4f_{i+1} - 4f_{i-1} + f_{i-2} &= \underbrace{(1-4-4+1)}_{-6} f(x_i) + \underbrace{(2h-4h+4h-2h)}_0 f'(x_i) \\ &+ \underbrace{(2h^2 - 4\frac{h^2}{2} - 4\frac{h^2}{2} + 2h^2)}_0 f''(x_i) + \underbrace{(\frac{4h^3}{3} - \frac{4h^3}{6} + \frac{4h^3}{6} - \frac{4h^3}{3})}_0 f^3(x_i) \\ &+ \underbrace{(\frac{2h^4}{3} - \frac{4h^4}{24} - \frac{4h^4}{24} + \frac{2h^4}{3})}_{\frac{4h^4}{3} - \frac{8h^4}{24} = \frac{4h^4}{3} - \frac{h^4}{3} = h^4} f^4(x_i) + \underbrace{(\frac{4h^5}{15} - \frac{4h^5}{120} + \frac{4h^5}{120} - \frac{4h^5}{15})}_0 f^5(x_i) + O(h^5) \end{aligned}$$

$$\Rightarrow f_{i+2} - 4f_{i+1} - 4f_{i-1} + f_{i-2} + 6f_i = h^4 f^4(x_i) + O(h^6)$$

$$\Rightarrow f^4(x_i) = \frac{f_{i+2} - 4f_{i+1} + 6f_i - 4f_{i-1} + f_{i-2}}{h^4} + O(h^2)$$

(4)

$$1) f(x_i + 2h) = f(x_i) + 2hf'(x_i) + 2h^2 f''(x_i) + \frac{4h^3}{3} f'''(x_i) + \frac{2h^4}{3} f^{(4)}(x_i) + O(h^5)$$

$$2) f(x_i + h) = f(x_i) + hf'(x_i) + \frac{h^2}{2} f''(x_i) + \frac{h^3}{6} f'''(x_i) + \frac{h^4}{24} f^{(4)}(x_i) + O(h^5)$$

$$3) f(x_i - h) = f(x_i) - hf'(x_i) + \frac{h^2}{2} f''(x_i) - \frac{h^3}{6} f'''(x_i) + \frac{h^4}{24} f^{(4)}(x_i) + O(h^5)$$

$$4) f(x_i - 2h) = f(x_i) - 2hf'(x_i) + 2h^2 f''(x_i) - \frac{4h^3}{3} f'''(x_i) + \frac{2h^4}{3} f^{(4)}(x_i) + O(h^5)$$

| | | | | | | |
|----------------|---|----|---------------|----------------|----------------|------|
| | 0 | 1 | 2 | 3 | 4 | مشتق |
| رابع سطر ضرایب | 1 | 2 | 2 | $\frac{4}{3}$ | $\frac{2}{3}$ | |
| | 1 | 1 | $\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{1}{24}$ | |
| | 1 | -1 | $\frac{1}{2}$ | $-\frac{1}{6}$ | $\frac{1}{24}$ | |
| | 1 | -2 | 2 | $-\frac{4}{3}$ | $\frac{2}{3}$ | |

با محاسبه امل تیلور مرتبه 5

گرفته یعنی در محاد است با تا h^4

و بقیه $O(h^5)$ شود پس دنبال

$$[a \ b \ c \ d] \begin{bmatrix} 2 & 2 & \frac{4}{3} & \frac{2}{3} \\ 1 & \frac{1}{2} & \frac{1}{6} & \frac{1}{24} \\ -1 & \frac{1}{2} & -\frac{1}{6} & \frac{1}{24} \\ -2 & 2 & -\frac{4}{3} & \frac{2}{3} \end{bmatrix} = [1 \ 0 \ 0 \ 0]$$

$$\Rightarrow [a \ b \ c \ d] = \left[-\frac{1}{12}, \frac{8}{12}, -\frac{8}{12}, \frac{1}{12}\right]$$

$$\Rightarrow -\frac{1}{12} f_{i+2} + \frac{8}{12} f_{i+1} - \frac{8}{12} f_{i-1} + \frac{1}{12} f_{i-2} = \overset{\substack{\uparrow \\ \text{Sum} \\ 0}}{-\frac{1}{12} + \frac{8}{12} - \frac{8}{12} + \frac{1}{12} = 0} f(x_i) + hf'(x_i) + O(h^5)$$

$$\Rightarrow f'(x_i) = \frac{-f_{i+2} + 8f_{i+1} - 8f_{i-1} + f_{i-2}}{12h} + O(h^4)$$

$$1) f(x_i + 2h) = f(x_i) + 2hf'(x_i) + 2h^2 f''(x_i) + \frac{4h^3}{3} f'''(x_i) + \frac{2h^4}{3} f^{(4)}(x_i) + \frac{4h^5}{15} f^{(5)}(x_i) + O(h^6) \quad (\text{مع})$$

$$2) f(x_i + h) = f(x_i) + hf'(x_i) + \frac{h^2}{2} f''(x_i) + \frac{h^3}{6} f'''(x_i) + \frac{h^4}{24} f^{(4)}(x_i) + \frac{h^5}{120} f^{(5)}(x_i) + O(h^6)$$

$$3) f(x_i - h) = f(x_i) - hf'(x_i) + \frac{h^2}{2} f''(x_i) - \frac{h^3}{6} f'''(x_i) + \frac{h^4}{24} f^{(4)}(x_i) - \frac{h^5}{120} f^{(5)}(x_i) + O(h^6)$$

$$4) f(x_i - 2h) = f(x_i) - 2hf'(x_i) + 2h^2 f''(x_i) - \frac{4h^3}{3} f'''(x_i) + \frac{2h^4}{3} f^{(4)}(x_i) - \frac{4h^5}{15} f^{(5)}(x_i) + O(h^6)$$

مستقيم

فترات

$$A = \begin{bmatrix} 2 & 2 & \frac{4}{3} & \frac{2}{3} & \frac{4}{15} \\ 1 & \frac{1}{2} & \frac{1}{6} & \frac{1}{24} & \frac{1}{120} \\ -1 & \frac{1}{2} & -\frac{1}{6} & \frac{1}{24} & -\frac{1}{120} \\ -2 & 2 & -\frac{4}{3} & \frac{2}{3} & -\frac{4}{15} \end{bmatrix}$$

$$[a \ b \ c \ d] A = [0 \ 1 \ 0 \ 0 \ 0]$$

$$\Rightarrow [a \ b \ c \ d] = \left[-\frac{1}{12} \quad \frac{16}{12} \quad \frac{16}{12} \quad -\frac{1}{12} \right]$$

$$\Rightarrow -\frac{1}{12} f_{i+2} + \frac{16}{12} f_{i+1} + \frac{16}{12} f_{i-1} - \frac{1}{12} f_{i-2} = \left(-\frac{1}{12} + \frac{16}{12} + \frac{16}{12} - \frac{1}{12} \right) f(x_i)$$

$$+ h^2 f''(x_i) + O(h^6)$$

$$\Rightarrow -\frac{1}{12} f_{i+2} + \frac{16}{12} f_{i+1} + \frac{16}{12} f_{i-1} - \frac{1}{12} f_{i-2} = \frac{30}{12} f(x_i) + h^2 f''(x_i) + O(h^6)$$

$$\Rightarrow h^2 f''(x_i) = -\frac{1}{12} f_{i+2} + \frac{16}{12} f_{i+1} - \frac{30}{12} f_i + \frac{16}{12} f_{i-1} - \frac{1}{12} f_{i-2} + O(h^6)$$

$\downarrow \div h^2$

$$\Rightarrow f^2(x_i) = \frac{-f_{i+2} + 16f_{i+1} - 30f_i + 16f_{i-1} - f_{i-2}}{12h^2} + O(h^4)$$

$$1) f(x_i+3h) = f(x_i) + 3hf'(x_i) + \frac{9h^2}{2}f''(x_i) + \frac{9h^3}{2}f'''(x_i) + \frac{27h^4}{8}f^{(4)}(x_i) + \frac{81h^5}{40}f^{(5)}(x_i) + \frac{81h^6}{80}f^{(6)}(x_i) + O(h^7)$$

$$2) f(x_i+2h) = f(x_i) + 2hf'(x_i) + 2h^2f''(x_i) + \frac{4h^3}{3}f'''(x_i) + \frac{2h^4}{3}f^{(4)}(x_i) + \frac{4h^5}{15}f^{(5)}(x_i) + \frac{4h^6}{45}f^{(6)}(x_i) + O(h^7)$$

$$3) f(x_i+h) = f(x_i) + hf'(x_i) + \frac{h^2}{2}f''(x_i) + \frac{h^3}{6}f'''(x_i) + \frac{h^4}{24}f^{(4)}(x_i) + \frac{h^5}{120}f^{(5)}(x_i) + \frac{h^6}{720}f^{(6)}(x_i) + O(h^7)$$

$$4) f(x_i-h) = f(x_i) - hf'(x_i) + \frac{h^2}{2}f''(x_i) - \frac{h^3}{6}f'''(x_i) + \frac{h^4}{24}f^{(4)}(x_i) - \frac{h^5}{120}f^{(5)}(x_i) + \frac{h^6}{720}f^{(6)}(x_i) + O(h^7)$$

$$5) f(x_i-2h) = f(x_i) - 2hf'(x_i) + 2h^2f''(x_i) - \frac{4h^3}{3}f'''(x_i) + \frac{2h^4}{3}f^{(4)}(x_i) - \frac{4h^5}{15}f^{(5)}(x_i) + \frac{4h^6}{45}f^{(6)}(x_i) + O(h^7)$$

$$6) f(x_i-3h) = f(x_i) - 3hf'(x_i) + \frac{9h^2}{2}f''(x_i) - \frac{9h^3}{2}f'''(x_i) + \frac{27h^4}{8}f^{(4)}(x_i) - \frac{81h^5}{40}f^{(5)}(x_i) + \frac{81h^6}{80}f^{(6)}(x_i) + O(h^7)$$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & \frac{9}{2} & \frac{9}{2} & \frac{27}{8} & \frac{81}{40} & \frac{81}{80} \\ 2 & 2 & \frac{4}{3} & \frac{2}{3} & \frac{4}{15} & \frac{4}{45} \\ 1 & \frac{1}{2} & \frac{1}{6} & \frac{1}{24} & \frac{1}{120} & \frac{1}{720} \\ -1 & \frac{1}{2} & -\frac{1}{6} & \frac{1}{24} & -\frac{1}{120} & \frac{1}{720} \\ -2 & 2 & -\frac{4}{3} & \frac{2}{3} & -\frac{4}{15} & \frac{4}{45} \\ -3 & \frac{9}{2} & -\frac{9}{2} & \frac{27}{8} & -\frac{81}{40} & \frac{81}{80} \end{bmatrix}$$

$$[a \ b \ c \ d \ e \ f] A = [0 \ 0 \ 1 \ 0 \ 0 \ 0]$$

$$\Rightarrow [a \ b \ c \ d \ e \ f] = \left[-\frac{1}{8}, 1, -\frac{13}{8}, \frac{13}{8}, -1, \frac{1}{8} \right]$$

$$\Rightarrow -\frac{1}{8}f_{i+3} + f_{i+2} - \frac{13}{8}f_{i+1} + \frac{13}{8}f_{i-1} - f_{i-2} + \frac{1}{8}f_{i-3} =$$

$$\left(-\frac{1}{8} + 1 - \frac{13}{8} + \frac{13}{8} - 1 + \frac{1}{8} \right) f_i + 1 \times h^3 f'''(x_i) + O(h^7)$$

$$\Rightarrow h^3 f'''(x_i) = -\frac{1}{8} f_{i+3} + \frac{3}{8} f_{i+2} - \frac{13}{8} f_{i+1} + \frac{13}{8} f_i - \frac{3}{8} f_{i-1} + \frac{1}{8} f_{i-2} + O(h^7)$$

$$\{ \div h^3$$

$$\Rightarrow f'''(x_i) = \frac{-f_{i+3} + 3f_{i+2} - 13f_{i+1} + 13f_i - 3f_{i-1} + f_{i-2}}{8h^3} + O(h^4)$$