400108547 تمرین سوم مدارهای الکتریکی و الکترونیکی پرهام رضایی

①

1. $f(t) = e^{\lambda t}(A\sin\omega t + B\cos\omega t)$

حل:

$$\mathcal{L}(\sin\omega t) = \mathcal{L}\left(\frac{e^{i\omega t} - e^{-i\omega t}}{2i}\right) = \frac{1}{2i}\mathcal{L}(e^{i\omega t}) - \frac{1}{2i}\mathcal{L}(e^{-i\omega t})$$

$$= \frac{1}{2i}\,\frac{1}{S-i\omega} - \frac{1}{2i}\,\frac{1}{S+i\omega} = \frac{1}{2i}\,\frac{S+i\omega-S+i\omega}{s^2+\omega^2} = \frac{\omega}{s^2+\omega^2}$$

$$\mathcal{L}(\cos\omega t) = \mathcal{L}\left(\frac{e^{i\omega t}+e^{-i\omega t}}{2}\right) = \frac{1}{2}\left(\mathcal{L}(e^{i\omega t}) + \mathcal{L}(e^{-i\omega t})\right) =$$

$$\frac{1}{2}\left(\frac{1}{S-i\omega} + \frac{1}{S+i\omega}\right) = \frac{S}{s^2+\omega^2}$$

$$\Rightarrow \mathcal{L}(f(t)) = \mathcal{L}(Ae^{\lambda t}\sin\omega t) + \mathcal{L}(Be^{\lambda t}\cos\omega t)$$

$$= A\mathcal{L}(e^{\lambda t}\sin\omega t) + B\mathcal{L}(e^{\lambda t}\cos\omega t) = A\frac{\omega}{(S-\lambda)^2+\omega^2} + B\frac{(S-\lambda)}{(S-\lambda)^2+\omega^2}$$

$$= \frac{A\omega + Bs - B\lambda}{(S-\lambda)^2+\omega^2} = \frac{BS + A\omega - B\lambda}{(S-\lambda)^2+\omega^2}$$

2.

$$f(t) = \sum_{k=0}^{\infty}\delta(t-k) = \delta(t) + \delta(t-1) + \delta(t-2) + \dots$$

حل: $f(t) \rightarrow F(s) \qquad f(t-\alpha) \rightarrow e^{-\alpha s}F(s) \qquad t<0 \Rightarrow f(t)=0$

$F(s) = \int_{0^-}^{\infty} f(t)e^{-st}dt$

اثبات: $\int_{0^-}^{\infty} f(t-\alpha)e^{-st}dt = \int_{-\alpha^-}^{\infty} f(r)e^{-s(r+\alpha)}dr$

$= \int_{0^-}^{\infty} f(r)e^{-s(r+\alpha)}dr = e^{-s\alpha}\int_{0^-}^{\infty} f(r)e^{-sr}dr = e^{-s\alpha}F(s)$

$f_1(t) = \delta(t)$ $\quad$ $f_1(t-k) = \delta(t-k)$ $\quad$ حال داریم که

$F_1(s) = 1$ $\quad$ $e^{-sk} F_1(s) = e^{-sk}$

$$\Rightarrow \quad F(s) = \sum_{k=0}^{\infty} e^{-sk}$$

دنباله هندسی $a = e^{-s}$ ← با فرض همگرایی (e بتوان منفی > ۱)

$$A = \sum_{k=0}^{\infty} a^k = 1 + a^2 + a^3 + \dots = \frac{1}{1-a}$$

$$aA = A - 1 \Rightarrow (a-1)A = -1 \Rightarrow A = \frac{1}{1-a}$$

$$\rightarrow \quad F(s) = \frac{1}{1-e^{-s}}$$

②

$$F(s)=\frac{2s^4+4s^3+9s^2+3s}{(s-1)(s^2+1)(s+2)^2}$$

مخرج درجه 5 پس صورت درجهش کمتره.

$$F(s)=\frac{A}{s-1}+\frac{Bs+C}{s^2+1}+\frac{D}{(s+2)^2}+\frac{E}{s+2}$$

$$A=(s-1)F(s)\Big|_{s=1}=\frac{2+4+9+3}{2\times3^2}=\frac{18}{18}=1$$

$$D=(s+2)^2F(s)\Big|_{s=-2}=\frac{2(-2)^4+4(-2)^3+9(-2)^2+3(-2)}{-3\times5}=\frac{32-32+36-6}{-15}=-2$$

$$E=\left[\frac{d}{ds}(s+2)^2F(s)\right]\Big|_{s=-2}\Rightarrow\frac{d}{ds}\left(\frac{2s^4+4s^3+9s^2+3s}{(s-1)(s^2+1)}\right)=$$

پیش حساب. (☹) → 1

$$\frac{(8s^3+12s^2+18s+3)(s-1)(s^2+1)-(2s^4+4s^3+9s^2+3s)(3s^2-2s+1)}{(s-1)^2(s^2+1)^2}\Bigg|_{s=-2}=\frac{15}{15}=1$$

$$F(s)=\frac{1}{s-1}+\frac{Bs+C}{s^2+1}-\frac{2}{(s+2)^2}+\frac{1}{s+2}$$

دو مقدار غیر pole انتخاب می‌کنیم 0 و 1- :

$$F(0)=\frac{0+0+0+0}{\bigcirc}=0=-1+\frac{C}{1}-\frac{2}{4}+\frac{1}{2}=-1+C\Rightarrow C=1$$

$$F(-1)=\frac{2-4+9-3}{-2\times2\times1}=\frac{4}{-4}=-1=\frac{-1}{2}+\frac{1-B}{2}-2+1=-1-\frac{B}{2}$$

$$\Rightarrow\frac{B}{2}=0\Rightarrow B=0$$

$$\Rightarrow F(s)=\frac{1}{s-1}+\frac{1}{s^2+1}-\frac{2}{(s+2)^2}+\frac{1}{s+2}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s-1}\right)=e^{t}u(t) \qquad \mathcal{L}(\sin(t)u(t))=\frac{1}{s^2+1}\Rightarrow\mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right)=\sin(t)u(t)$$

↳ بخاطر پله

$\mathcal{L}^{-1}\left(\frac{1}{S+2}\right) = e^{-2t}u(t)$ $\mathcal{L}(-t\,f(t)) = \frac{dF(s)}{ds} = \left(\frac{1}{S+2}\right)' = -\frac{1}{(S+2)^2}$

$\Rightarrow \mathcal{L}^{-1}\left(\frac{-2}{(S+2)^2}\right) = 2(-t\,e^{-2t}u(t)) = -2te^{-2t}u(t)$

$\Rightarrow f(t) = (e^{t} + \sin(t) - 2te^{-2t} + e^{-2t})\,u(t)$

③

full response = $\overset{\text{zir}}{\text{zero}}$ response + zero response
input ... state — zir, zsr

$$\text{full response} = \underset{\text{input}}{\overset{\text{zir}}{\text{zero}}}\ \text{response} + \underset{\text{state}}{\overset{\text{zsr}}{\text{zero}}}\ \text{response}$$

$x(t) = e^{-t}u(t)$ $\qquad$ $\underset{\text{full response}}{y(t)} = \left(-\frac{1}{2}\sin t + \frac{7}{10}e^{-t} + \frac{1}{2}te^{-t}\right)u(t)$

$$\Rightarrow \text{zir}(X(t)) = y(t) - \text{zsr} = \left(-\frac{1}{2}\sin t + \frac{7}{10}e^{-t} + \frac{1}{2}te^{-t}\right)u(t) - \frac{1}{5}e^{-t}u(t)$$

$$= \left(-\frac{1}{2}\sin t + \left(\frac{7}{10} - \frac{2}{10}\right)e^{-t} + \frac{1}{2}te^{-t}\right)u(t) = -\frac{1}{2}\sin t\ u(t) + \frac{1}{2}e^{-t}u(t) + \frac{1}{2}te^{-t}u(t)$$

$$\mathcal{L}\left(-\frac{1}{2}\sin t\ u(t)\right) = \frac{-1}{2}\mathcal{L}(\sin t\ u(t)) = \frac{-1}{2}\ \frac{1}{s^2+1} = \frac{-1}{2s^2+2}$$

$$\mathcal{L}\left(\frac{1}{2}e^{-t}u(t)\right) = \frac{1}{2}\mathcal{L}\left(e^{-t}u(t)\right) = \frac{1}{2}\ \frac{1}{s+1} = \frac{1}{2s+2}$$

$$t\,f(t) \rightarrow -\frac{d\,F(s)}{ds}$$

$$\mathcal{L}\left(\frac{1}{2}te^{-t}u(t)\right) = \frac{1}{2}\mathcal{L}\left(te^{-t}u(t)\right) = -\frac{1}{2}\left(\frac{1}{s+1}\right)' = \frac{1}{2}\ \frac{1}{(s+1)^2}$$

$$\Rightarrow Y(s) = \frac{-1}{2s^2+2} + \frac{1}{2s+2} + \frac{1}{2(s+1)^2}$$

$$x(t) = e^{-t}u(t) \Rightarrow X(s) = \frac{1}{s+1}$$

tranfer

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{-1}{2s^2+2} + \frac{1}{2s+2} + \frac{1}{2(s+1)^2}}{\frac{1}{s+1}} = \frac{-s-1}{2s^2+2} + \frac{1}{2} + \frac{1}{2(s+1)}$$

$$= \frac{1}{2} + \frac{-(s+1)^2}{2(s^2+1)(s+1)} + \frac{s^2+1}{2(s+1)(s^2+1)} = \frac{1}{2} + \frac{-s^2-2s-1+s^2+1}{2(s^2+1)(s+1)} = \frac{1}{2} - \frac{s}{(s^2+1)(s+1)}$$

$x(t) = 10\cos(t-45^{\circ})u(t)$ $\frac{45}{180} \times \pi = \frac{\pi}{4} \leftarrow$ رادیان 45 درجه

$\downarrow$

$x(t) = 10\cos(t - \frac{\pi}{4})\, u(t)$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\circledast\ \cos(-x) = \cos x$

$\sin(-x) = -\sin x$

$\Rightarrow x(t) = 10(\cos t\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\sin t)\, u(t)$

$\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\Rightarrow x(t) = 20\frac{\sqrt{2}}{2}(\cos t + \sin t)\, u(t) =$

$5\sqrt{2}\,(\cos t + \sin t)\, u(t)$

$X(s) = \mathcal{L}(x(t)) = 5\sqrt{2}\,(\mathcal{L}(\cos(t)\,u(t)) + \mathcal{L}(\sin(t)\,u(t))) =$

$5\sqrt{2}\,(\frac{s}{s^2+1} + \frac{1}{s^2+1}) = \frac{5\sqrt{2}\,(s+1)}{s^2+1}$

پاسخ حالت صفر می‌خواهیم پس صرفاً در H باید ضرب کنیم.

$Y(s) = H(s)X(s) = (\frac{1}{2} - \frac{s}{(s^2+1)(s+1)})\frac{5\sqrt{2}\,(s+1)}{s^2+1} =$

$\frac{5\sqrt{2}}{2}\frac{s+1}{s^2+1} - \frac{5\sqrt{2}\, s}{(s^2+1)^2}$

$\mathcal{L}(-tf(t)) = \frac{dF(s)}{ds}$ $\Rightarrow \mathcal{L}(-t\sin(t)\,u(t)) = (\frac{1}{s^2+1})' = \frac{-2s}{(s^2+1)^2}$

$\Rightarrow y(t) = \mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}(\frac{5\sqrt{2}}{2}\frac{s}{s^2+1} + \frac{5\sqrt{2}}{2}\frac{1}{s^2+1} - \frac{5\sqrt{2}\, s}{(s^2+1)^2})$

$$= \frac{5\sqrt{2}}{2} \mathcal{L}^{-1}\left(\frac{s}{s^2+1}\right) + \frac{5\sqrt{2}}{2} \mathcal{L}^{-1}\left(\frac{1}{s^2+1}\right) + \frac{5\sqrt{2}}{2} \mathcal{L}^{-1}\left(\frac{-2s}{(s^2+1)^2}\right)$$

$$= \frac{5\sqrt{2}}{2} \cos(t)\, u(t) + \frac{5\sqrt{2}}{2} \sin(t)\, u(t) + \frac{5\sqrt{2}}{2} \left(-t \sin(t)\, u(t)\right)$$

$$\Rightarrow y(t) = \frac{5\sqrt{2}}{2} \left(\cos(t) + \sin(t) - t \sin(t)\right) u(t)$$

جواب حالت صفر:

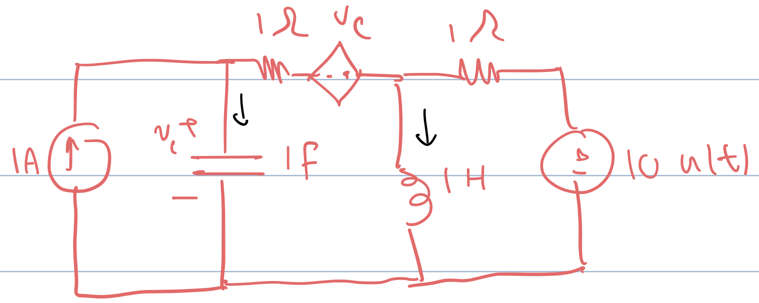
$$\boxed{\frac{5\sqrt{2}}{2} \left(\cos(t) + \sin(t) - t \sin(t)\right) u(t)}$$

جواب کلی را اگر می‌خواستیم

با جواب ورودی صفر جمع می‌شد:

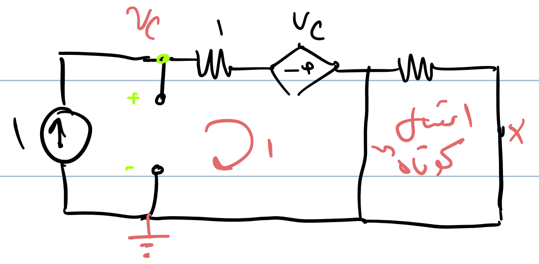
$$\left[\frac{5\sqrt{2}}{2} \left(\cos(t) + \sin(t) - t \sin(t)\right) + \frac{1}{5} e^{-t}\right] u(t)$$

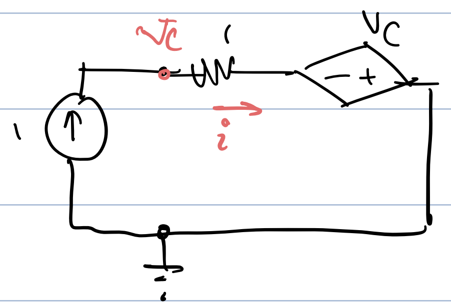
④



روش ۱: بدون لاپلاس :

در $t=0^-$ داریم که





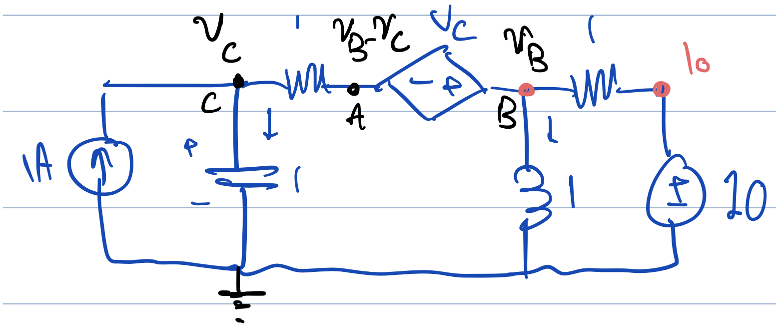
افتش ز
۰ و v_c

$i=1$

$$\Rightarrow v_c - i\times 1 + v_c = 0 \Rightarrow 2v_c = 1$$

$$\Rightarrow v_c = \frac{1}{2}$$

$\Rightarrow v_c(0^+) = v_c(0^-) = \frac{1}{2}\, v$ $\quad i_\ell(0^+) = i_\ell(0^-) = i = 1A$



$t > 0$

حال داریم که :

$$\text{KCL@C}: \; -1 + i_c + \frac{v_c - (v_B - v_c)}{1} = 0 \Rightarrow \quad C\frac{dv_c(t)}{dt} + 2v_c - v_B = 1$$

$$\text{KCL@AUB}: \; \frac{v_A - v_c}{1} + \frac{v_B - 10}{1} + i_\ell = 0 \Rightarrow v_B - 2v_c + v_B - 10 + i_\ell(t) = 0$$

$$\Rightarrow 2(v_B - v_c) = 10 - i_\ell(t)$$

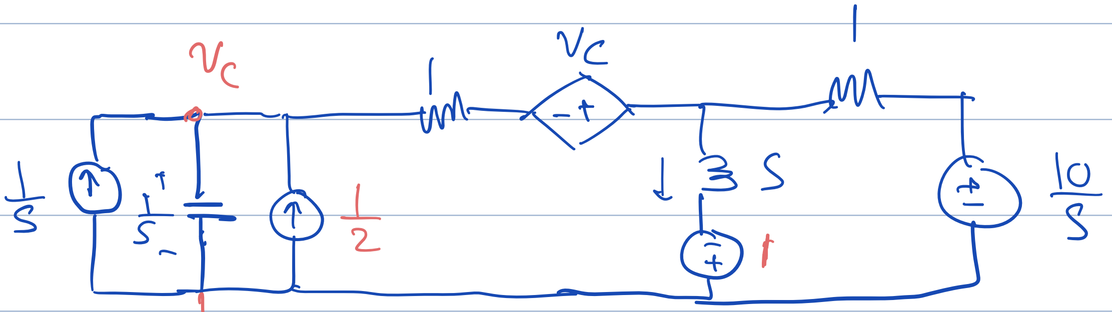
$v_c(0^+) = \frac{1}{2}$ $\quad i_\ell(0^+) = 1A$

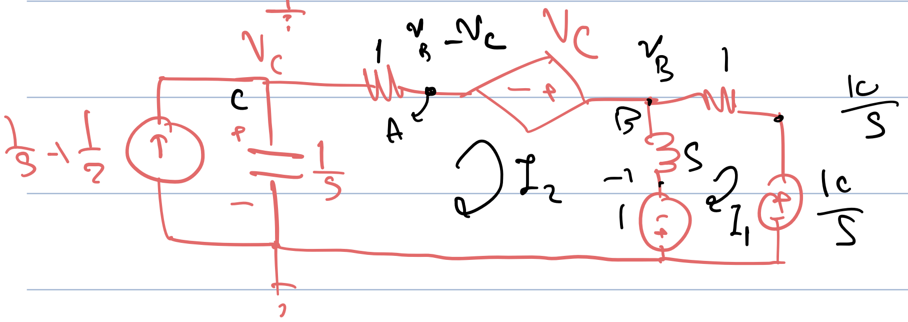
$$\Rightarrow \frac{1\,dV_C(0^+)}{dt} + 1 - V_B = 1 \quad \rightarrow \quad \frac{dV_C(0^+)}{dt} = V_B = 5$$

$$2(V_B - \frac{1}{2}) = 10 - 1 \Rightarrow 2V_B - 1 = 9 \Rightarrow V_B = 5$$

$$\Rightarrow \frac{dV_C(0^+)}{dt} = 5$$

با لاپلاس : مطابق مثل حالت اولیه را داریم $v_C(0^+) = \frac{1}{2}V$ $i_L(0^+) = 1A$





$$\text{kcl@B:} \; -\cancel{\frac{1}{s}} - \frac{1}{2} + sV_C + \cancel{\frac{V_B+1}{s}} + V_B - \frac{10}{s} = 0$$

$$\Rightarrow (\frac{1}{s}+1)V_B = \frac{1}{2} + \frac{10}{s} - sV_C \Rightarrow (s+1)V_B = \frac{s}{2} + 10 - s^2V_C$$

$$\Rightarrow V_B = \frac{\frac{s}{2} + 10 - s^2V_C}{s+1}$$

$$\text{kcl @ C:} \; -\frac{1}{s} - \frac{1}{2} + sV_C + \frac{V_C - V_B + V_C}{1} = 0$$

$$\Rightarrow -\frac{1}{s} - \frac{1}{2} + sV_C + 2V_C = V_B = \frac{\frac{s}{2} + 10 - s^2V_C}{s+1}$$

$$\Rightarrow -\frac{s+1}{s} - \frac{s+1}{2} + (s+2)(s+1)V_C = \frac{s}{2} + 10 - s^2V_C$$

$$\Rightarrow -1 + \frac{1}{s} - \frac{s}{2} - \frac{1}{2} + (2s^2 + 3s + 2)V_C = \frac{s}{2} + 10$$

$$\Rightarrow \quad (2s^2+3s+2)v_c = s + 10 + \frac{1}{s} + \frac{3}{2} \Rightarrow$$

$$\xrightarrow{\times 2s} v_c = \frac{2s^2+23s+2}{4s^3+6s^2+4s}$$

$$\mathcal{L}\left(\frac{df(t)}{dt}\right) = sF(s) - f(0^-) \Rightarrow \mathcal{L}\left(\frac{dV_c(0^+)}{dt}\right) = sV_c(s) - \frac{1}{2}$$

$$= \frac{2s^2+23s+2}{4s^2+6s+4} - \frac{1}{2} = \frac{2s^2+23s+2-2s^2-3s-2}{4s^2+6s+4} = \frac{20s}{4s^2+6s+4}$$

$$\xrightarrow{\times \frac{2}{2}} \quad \frac{10s}{2s^2+3s+2}$$

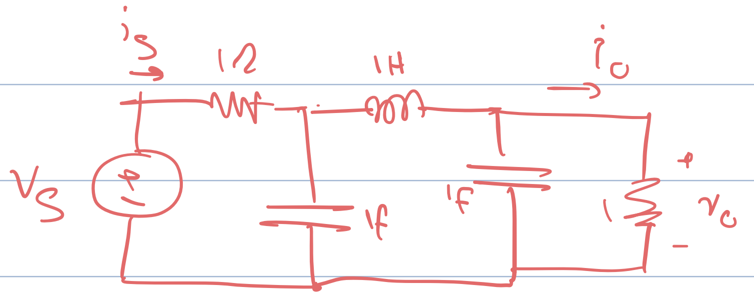
$$f(0^+) = \lim_{s\to\infty} sF(s) \Rightarrow$$

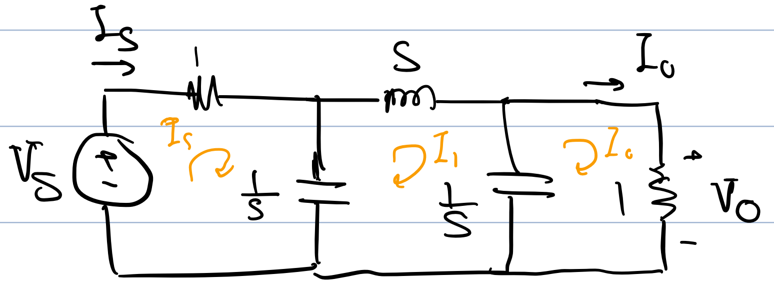
$$\frac{dv_c(0^+)}{dt} = \lim_{s\to\infty} \frac{s \times 10s}{2s^2+3s+2} = \lim_{s\to\infty} \frac{10}{2+\frac{3}{s}+\frac{2}{s^2}} = \lim_{s\to\infty} \frac{10}{2}$$

میل 0 میل 0

$$= 5$$

⑤





$I_o \cdot 1 = V_o$: دو سر مقاومت 1Ω

kVL @ I_s: $-V_s + I_s + \frac{1}{s}(I_s - I_1) = 0$ ①

kvl @ I_1: $\frac{1}{s}(I_1 - I_s) + sI_1 + \frac{1}{s}(I_1 - I_o) = 0$ ②

kvl @ I_o: $I_o + (I_o - I_1)\frac{1}{s} = 0$ ③

③ $(1 + \frac{1}{s}) I_o = \frac{I_1}{s} \Rightarrow \frac{s+1}{s} I_o = \frac{I_1}{s} \Rightarrow I_1 = (s+1) I_o$

② $\frac{1}{s}\left((s+1) I_o - I_s\right) + s(s+1) I_o + \frac{1}{s}\left((s+1)I_o - I_o\right) = 0$

$\Rightarrow \frac{s+1}{s} I_o - \frac{I_s}{s} + s(s+1) I_o + I_o = 0 \Rightarrow \frac{I_s}{s} = (s^2 + s + 1 + \frac{s+1}{s}) I_o \Rightarrow$

$I_s = (s^3 + s^2 + 2s + 1) I_o$

① $-V_s + (s^3 + s^2 + 2s + 1) I_o + \frac{1}{s}\left((s^3 + s^2 + 2s + 1) I_o - (s+1) I_o\right) = 0$

$\Rightarrow V_s = (s^3 + s^2 + 2s + 1 + s^2 + s + 1) I_o = (s^3 + 2s^2 + 3s + 2) I_o$

$$\Rightarrow \frac{I_o(s)}{I_s(s)} = \frac{I_o(s)}{(s^3 + s^2 + 2s + 1) I_o(s)} = \frac{1}{s^3 + s^2 + 2s + 1}$$

$$\frac{V_o(s)}{V_s(s)} = \frac{I_o(s)}{(s^3 + 2s^2 + 3s + 2) I_o(s)} = \frac{1}{s^3 + 2s^2 + 3s + 2}$$