محاسبات عددی

تمرین تئوری ۲

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① تا ★ خود روند spline مکعبی رو طی کردم. از ★ محاسبات مسئله است.

روند ساخت spline مکعبی:

$$S(x) = \begin{cases} S_0(x) & x \in [t_0, t_1) \\ \vdots & \\ S_{n-1}(x) & x \in [t_{n-1}, t_n] \end{cases}$$

$$s(t_i) = y_i$$

$$\Rightarrow S_{i-1}(t_i) = y_i = S_i(t_i) \qquad 1 \le i \le n-1$$

$$S_i'(t_{i+1}) = S_{i+1}'(t_{i+1}) \qquad 0 \le i \le n-2$$

$$S_i''(t_{i+1}) = S_{i+1}''(t_{i+1}) \qquad 0 \le i \le n-2$$

قرار می‌دهیم $\Delta t_i = t_{i+1} - t_i$, $z_i = S''(t_i)$

درجه ۳ است ← مشتق دوم خطی

$$S_i''(x) = \frac{x - t_i}{\Delta t_i} z_{i+1} - \frac{x - t_{i+1}}{\Delta t_i} z_i$$ فرمول خط

$$\Rightarrow S_i'(x) = \frac{(x-t_i)^2 z_{i+1}}{2\Delta t_i} - \frac{(x-t_{i+1})^2 z_i}{2\Delta t_i} + \tilde{C}_i$$

درجه ۲ است ← ثابت جدید هم $\tilde{C}_i$ ، $C_i - D_i = \tilde{C}_i$ رو می‌دهد.

$$\Rightarrow S_i(x) = \frac{z_{i+1}}{6\Delta t_i}(x-t_i)^3 + \frac{z_i}{6\Delta t_i}(t_{i+1}-x)^3 + C_i(x-t_i) + D_i(t_{i+1}-x)$$

$$S_i(t_i) = y_i \Rightarrow y_i = \frac{z_i}{6\Delta t_i}(\Delta t_i)^3 + D_i \Delta t_i \Rightarrow D_i = \frac{y_i}{\Delta t_i} - \frac{z_i \Delta t_i}{6}$$

$$S_i(t_{i+1}) = y_{i+1} \Rightarrow y_{i+1} = \frac{z_{i+1}}{6\Delta t_i}(\Delta t_i)^3 + C_i \Delta t_i \Rightarrow C_i = \frac{y_{i+1}}{\Delta t_i} - \frac{z_{i+1}\Delta t_i}{6}$$

$$\Rightarrow S_i(x) = \frac{z_{i+1}}{6\Delta t_i}(x-t_i)^3 + \frac{z_i}{6\Delta t_i}(t_{i+1}-x)^3 + \left(\frac{y_{i+1}}{\Delta t_i} - \frac{z_{i+1}}{6}\Delta t_i\right)(x-t_i) + \left(\frac{y_i}{\Delta t_i} - \frac{z_i}{6}\Delta t_i\right)(t_{i+1}-x)$$

حالا با برابری مشتق‌ها zi ها را بدست می‌آوریم.

$$S_i'(t_i) = S_{i-1}'(t_i) \qquad 1 \le i \le n$$

$$s_i'(x) = \frac{z_{i+1}}{2\Delta t_i}(x-t_i)^2 - \frac{z_i}{2\Delta t_i}(t_{i+1}-x)^2 + \frac{y_{i+1}}{\Delta t_i} - \frac{z_{i+1}}{6}\Delta t_i - \frac{y_i}{\Delta t_i} + \frac{z_i \Delta t_i}{6}$$

$$s_i'(t_i) = -\frac{\Delta t_i}{3} z_i - \frac{\Delta t_i}{6} z_{i+1} - \frac{y_i}{\Delta t_i} + \frac{y_{i+1}}{\Delta t_i}$$

$$s_{i-1}'(t_i) = \frac{\Delta t_{i-1}}{6} z_{i-1} + \frac{\Delta t_{i-1}}{3} z_i - \frac{y_{i-1}}{\Delta t_{i-1}} + \frac{y_i}{\Delta t_{i-1}}$$

$1 \le i \le n-1$

$$\Rightarrow \frac{\Delta t_{i-1}}{6} z_{i-1} + \frac{\Delta t_i + \Delta t_{i-1}}{3} z_i + \frac{\Delta t_i}{6} z_{i+1} = \frac{y_{i+1}-y_i}{\Delta t_i} - \frac{y_i - y_{i-1}}{\Delta t_{i-1}}$$

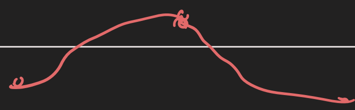
برای حالت طبیعی داریم که $Z_0 = Z_n = 0$ و

$$\Rightarrow A = \begin{bmatrix} \frac{\Delta t_0 + \Delta t_1}{3} & \frac{\Delta t_1}{6} & & & \\ \frac{\Delta t_1}{6} & \frac{\Delta t_1 + \Delta t_2}{3} & \frac{\Delta t_2}{6} & & \\ & \frac{\Delta t_2}{6} & \ddots & \ddots & \\ & & \ddots & \ddots & \frac{\Delta t_{n-2}}{6} \\ & & & \frac{\Delta t_{n-2}}{6} & \frac{\Delta t_{n-2} + \Delta t_{n-1}}{3} \end{bmatrix}$$

$$A \begin{bmatrix} z_1 \\ \vdots \\ z_{n-1} \end{bmatrix} = \begin{bmatrix} \frac{y_2 - y_1}{\Delta t_1} - \frac{y_1 - y_0}{\Delta t_0} \\ \vdots \\ \frac{y_n - y_{n-1}}{\Delta t_{n-1}} - \frac{y_{n-1} - y_{n-2}}{\Delta t_{n-2}} \end{bmatrix}$$

حال با این محاسبه z_i ها هم درمیاید.

به مسئله بازمی گردیم.



الف)

$n=2$

t_0	$\frac{1}{2}$	y_0	2
t_1	1	y_1	1
t_2	2	y_2	$\frac{1}{2}$

طبق برابری تستی دوم که بالا نوشتیم

$$\frac{\Delta t_0}{6} z_0 + \frac{\Delta t_1 + \Delta t_0}{3} z_1 + \frac{\Delta t_1}{6} z_2 = \frac{y_2 - y_1}{\Delta t_1} - \frac{y_1 - y_0}{\Delta t_0}$$

$$\Delta t_0 = 1 - \frac{1}{2} = \frac{1}{2} \qquad z_0 = z_2 = 0$$

$$\Delta t_1 = 2 - 1 = 1$$

$$\Rightarrow \frac{\frac{3}{2}}{3} z_1 = \frac{\frac{1}{2} - 1}{1} - \frac{1-2}{\frac{1}{2}}$$

$$\Rightarrow \frac{z_1}{2} = -\frac{1}{2} + 2 = \frac{3}{2} \Rightarrow \boxed{z_1 = 3}$$

داشتیم

$$\Rightarrow S_i(x) = \frac{z_{i+1}}{6\Delta t_i}(x - t_i)^3 + \frac{z_i}{6\Delta t_i}(t_{i+1} - x)^3 + \left(\frac{y_{i+1}}{\Delta t_i} - \frac{z_{i+1}}{6}\Delta t_i\right)(x - t_i)$$

$$+ \left(\frac{y_i}{\Delta t_i} - \frac{z_i}{6}\Delta t_i\right)(t_{i+1} - x)$$

$$S_0(x) = \frac{3}{6 \times \frac{1}{2}}\left(x - \frac{1}{2}\right)^3 + 0 + \left(\frac{1}{\frac{1}{2}} - \frac{3}{6}\,\frac{1}{2}\right)\left(x - \frac{1}{2}\right) +$$

$$\left(\frac{2}{\frac{1}{2}} - 0\right)(1 - x) =$$

$$S_0(x) = \left(x - \frac{1}{2}\right)^3 + \frac{7}{4}\left(x - \frac{1}{2}\right) - 4(x - 1)$$

$$S_1(x) = 0 + \frac{3}{6}(2 - x)^3 + \left(\frac{1}{2} - 0\right)(x - 1) + \left(1 - \frac{3}{6}\right)(2 - x)$$

$$= -\frac{1}{2}(x - 2)^3 + \frac{1}{2}(x - 1) - \frac{1}{2}(x - 2)$$

$$\Rightarrow \quad S(x)=\begin{cases} \left(x-\frac{1}{2}\right)^3+\left(\frac{-9}{4}x+\frac{25}{8}\right) & x<1 \\ -\frac{1}{2}(x-2)^3+\frac{1}{2} & x\geq 1 \end{cases}$$

ب) ش بہ قبل داریم کہ

$$\frac{\Delta t_0}{6}z_0+\frac{\Delta t_1+\Delta t_0}{3}z_1+\frac{\Delta t_1}{6}z_2=\frac{y_2-y_1}{\Delta t_1}-\frac{y_1-y_0}{\Delta t_0}$$

ولی در شرط دقیق داریم و $z_0=z_2=0$ دقیق نیست!

از قبل داشتیم کہ در درونہ محاسبہ

$$s_i'(x)=\frac{1}{2}(x-t_i)^2\frac{z_{i+1}}{\Delta t_i}-\frac{1}{2}(x-t_{i+1})^2\frac{z_i}{\Delta t_i}+\underbrace{\frac{y_{i+1}}{\Delta t_i}-\frac{z_{i+1}}{6}\Delta t_i-\frac{y_i}{\Delta t_i}+\frac{z_i\Delta t_i}{6}}_{C_i-D_i}$$

در نقطه ۶:

$$S_0'(x)=z_1\left(x-\frac{1}{2}\right)^2-z_0(x-1)^2-2+\frac{z_0-z_1}{12}$$

$$S_0'\left(\frac{1}{2}\right)=-4 \Rightarrow -4=-z_0\left(\frac{1}{4}\right)-2+\frac{z_0-z_1}{12}$$

$$\Rightarrow \frac{z_0}{4}+\frac{z_1-z_0}{12}=2 \qquad (1)$$

$$S_1'(x) = \frac{1}{2}(x-1)^2 z_2 - \frac{1}{2}(x-2)^2 z_1 + \frac{1}{2} - \frac{z_2}{6} - 1 + \frac{z_1}{6}$$

$$S_1'(2) = -\frac{1}{4} \Rightarrow -\frac{1}{4} = \frac{z_2}{2} - \frac{1}{2} + \frac{z_1 - z_2}{6}$$

$$\Rightarrow \frac{z_2}{2} + \frac{z_1 - z_2}{6} = \frac{1}{4} \quad (2)$$

$$\frac{\Delta t_0}{6} z_0 + \frac{\Delta t_1 + \Delta t_0}{3} z_1 + \frac{\Delta t_1}{6} z_2 = \frac{y_2 - y_1}{\Delta t_1} - \frac{y_1 - y_0}{\Delta t_0}$$

$$\frac{1}{12} z_0 + \frac{1}{2} z_1 + \frac{1}{6} z_2 = -\frac{1}{2} + 2 = \frac{3}{2} \quad (3)$$

$$\begin{bmatrix} \frac{1}{6} & \frac{1}{12} & 0 \\ 0 & \frac{1}{6} & \frac{1}{3} \\ \frac{1}{12} & \frac{1}{2} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{1}{4} \\ \frac{3}{2} \end{bmatrix}$$

$\searrow A^{-1}$

$$\hookrightarrow \begin{bmatrix} z_0 \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \frac{20}{3} & \frac{2}{3} & -\frac{4}{3} \\ -\frac{4}{3} & -\frac{4}{3} & \frac{8}{3} \\ \frac{2}{3} & \frac{11}{3} & -\frac{4}{3} \end{bmatrix} \begin{bmatrix} 2 \\ \frac{1}{4} \\ \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 11.5 \\ 1 \\ 0.25 \end{bmatrix}$$

$$\Rightarrow S_0(x) = \frac{1}{3}\left(x - \frac{1}{2}\right)^3 + \frac{11.5}{3}(1-x)^3 + \left(2 - \frac{1}{12}\right)\left(x - \frac{1}{2}\right) + \left(4 - \frac{11.5}{12}\right)(1-x)$$

$$S_1(x) = \frac{0.25}{6}(x-1)^3 + \frac{1}{6}(2-x)^3 + \left(\frac{1}{2} - \frac{0.25}{6}\right)(x-1) + \left(1 - \frac{1}{6}\right)(2-x)$$

$$\Rightarrow S(x) = \begin{cases} -3.5x^3 + 11x^2 - 12.375x + 5.875 & \frac{1}{2} \leq x < 1 \\ -0.125x^3 + 0.875x^2 - 2.25x + 2.5 & 1 \leq x \leq 2 \end{cases}$$

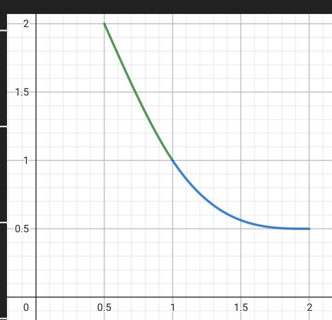
ج)

طبق تابع کنش الف

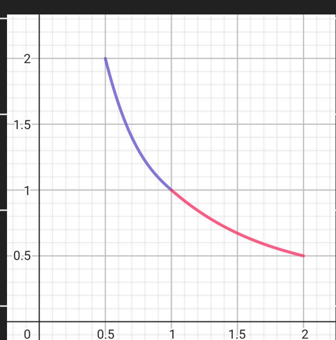
$$f_1(1.5) = \left(-\frac{1}{2}(x-2)^3 + \frac{1}{2}\right)\Big|_{x=1.5} = \frac{1}{16} + \frac{1}{2} = \frac{9}{16} \simeq 0.5625$$

طبق تابع کنش ب :

$$f_2(1.5) = \left(-0.125x^3 + 0.875x^2 - 2.25x + 2.5\right)\Big|_{x=1.5} \simeq 0.671875$$



تصویر spline الف :



تصویر spline ب :

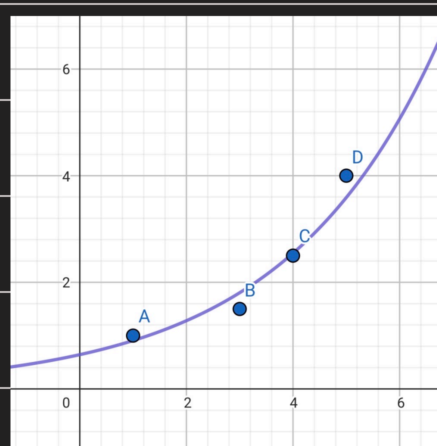
2

$$y = ae^{bx}$$

$$\Rightarrow \ln(y) = \underbrace{\ln(a)}_{\alpha} + \underbrace{b}_{\beta}x$$

پس تقریب خطی $\ln(y)$ است.

x	1	3	4	5
y	1	1.5	2.5	4
$\ln y = m$	$\ln 1$	$\ln 1.5$	$\ln 2.5$	$\ln 4$
x^2	1	9	16	25
xm	$\ln 1$	$9\ln 1.5$	$16\ln 2.5$	$25\ln 4$



ae^{bx}

$$E(\alpha,\beta) = \sum_{i=1}^{n} (m_i - \beta x_i - \alpha)^2$$

$$\min \Rightarrow \frac{\partial E}{\partial \alpha} = \frac{\partial E}{\partial \beta} = 0$$

$$\Rightarrow 0 = \frac{\partial E}{\partial \alpha} = -2\sum_{i=1}^{n}(m_i - \beta x_i - \alpha) \qquad 0 = \frac{\partial E}{\partial \beta} = -2\sum_{i=1}^{n} x_i(m_i - \beta x_i - \alpha)$$

$$\Rightarrow \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \sum m_i \\ \sum x_i m_i \end{bmatrix}$$

$$\begin{bmatrix} 4 & 13 \\ 13 & 51 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \ln 1 + \ln 1.5 + \ln 2.5 + \ln 4 \\ \ln 1 + 3\ln 1.5 + 4\ln 2.5 + 5\ln 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 13 \\ 13 & 51 \end{bmatrix}^{-1} = \frac{1}{35}\begin{bmatrix} 51 & -13 \\ -13 & 4 \end{bmatrix} \Rightarrow \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 51 & -13 \\ -13 & 4 \end{bmatrix}\begin{bmatrix} 2.708 \\ 11.813 \end{bmatrix}$$

$$= \frac{1}{35}\begin{bmatrix} -15.461 \\ 12.048 \end{bmatrix} = \begin{bmatrix} -0.441 \\ 0.344 \end{bmatrix} \xRightarrow{a = e^{\alpha}} a \simeq 0.643$$

$$b \simeq 0.344$$

الف

x	1	3	5
y	2	y_1	y_2
x^2	1	9	25
xy	2	$3y_1$	$5y_2$

$$E(a,b) = \sum_{i=1}^{3} (y_i - ax_i - b)^2 \xrightarrow{\min} \frac{\partial E(a,b)}{\partial a} = \frac{\partial E(a,b)}{\partial b} = 0$$

$$\Rightarrow \begin{cases} \frac{\partial E}{\partial b} = -2 \sum_{i=1}^{3} (y_i - b - ax_i) = 0 \\ \frac{\partial E}{\partial a} = -2 \sum_{i=1}^{3} x_i (y_i - b - ax_i) = 0 \end{cases}$$

$$\Rightarrow \begin{bmatrix} \sum 1 & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 9 \\ 9 & 35 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 2 + y_1 + y_2 \\ 2 + 3y_1 + 5y_2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 9 \\ 9 & 35 \end{bmatrix}^{-1} = \frac{1}{24} \begin{bmatrix} 35 & -9 \\ -9 & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} b \\ a \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 35 & -9 \\ -9 & 3 \end{bmatrix} \begin{bmatrix} 2 + y_1 + y_2 \\ 2 + 3y_1 + 5y_2 \end{bmatrix}$$

$$\begin{bmatrix} b \\ a \end{bmatrix} = \frac{1}{24} \begin{bmatrix} 70 + 35y_1 + 35y_2 - 18 - 27y_1 - 45y_2 \\ -18 - 9y_1 - 9y_2 + 6 + 9y_1 + 15y_2 \end{bmatrix} =$$

$$\frac{1}{24} \begin{bmatrix} 52 + 8y_1 - 10y_2 \\ -12 + 6y_2 \end{bmatrix} \Rightarrow b = \frac{1}{24}(52 + 8y_1 - 10y_2)$$

$$a = -\frac{1}{2} + \frac{y_2}{4}$$

ب)

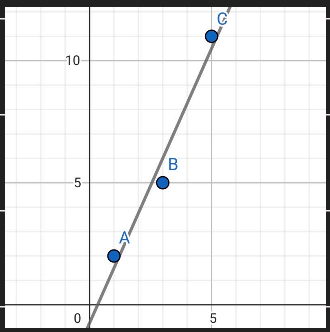
حالت ۱)

$y_1 = 5$
$y_2 = 11$

$$b = \frac{1}{24}(52 + 8\times 5 - 10\times 11) = \frac{1}{24}(52+40-110) = \frac{-18}{24} = \frac{-3}{4}$$

$$a = -\frac{1}{2} + \frac{11}{4} = \frac{9}{4}$$

$$\rightsquigarrow \quad ax+b = \frac{9}{4}x - \frac{3}{4}$$



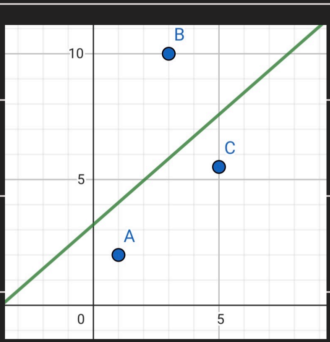
حالت ۲)

$y_1 = 10$
$y_2 = 5.5$

$$b = \frac{1}{24}(52 + 8.10 - 10\times 5.5) = \frac{77}{24}$$

$$a = -\frac{1}{2} + \frac{5.5}{4} = -\frac{4}{8} + \frac{11}{8} = \frac{7}{8}$$

$$\rightarrow \quad ax+b = \frac{7}{8}x + \frac{77}{24}$$



زاویه خط ها:

بردار جهت ها داریم که

$a = (1, \frac{9}{4}) \quad b = (1, \frac{7}{8})$

$$\cos\phi = \frac{\langle a,b\rangle}{\|a\|\|b\|} = \frac{1.1 + \frac{9}{4}\cdot\frac{7}{8}}{\sqrt{1^2+(\frac{9}{4})^2}\sqrt{1^2+(\frac{7}{8})^2}} = \frac{1+\frac{63}{32}}{\sqrt{1+\frac{81}{16}}\sqrt{1+\frac{49}{64}}} \approx \frac{2.96875}{\sqrt{10.7041015625}}$$

$$\approx 0.907399$$

$$\xrightarrow{\cos^{-1}} \quad \phi \approx 24.851^\circ$$

الف) چندجمله‌ای $f(x)=1$ را در نقاط x_i درونیابی می‌کنیم.

$f(x_i)=1$

چند جمله‌ای درجه حداکثر n که در n نقطه ۱ است $\Leftarrow P(x)=1$

$$\Rightarrow \sum_{k=1}^{n}\Big(\prod_{i\neq k}\frac{x-x_i}{x_k-x_i}\Big)\underbrace{f(x_k)}_{=1}=1 \Rightarrow \sum_{k=1}^{n}\Big(\prod_{i\neq k}\frac{x-x_i}{x_k-x_i}\Big)=1$$

حکم ثابت شد.

ب)
$$\sum\prod\frac{x-x_i}{x_k-x_i}P(x_k)=P(x)$$

تیلین

$$\circledast\quad \sum_{k=0}^{n}\frac{P(x_k)}{\prod_{\substack{i=0\\ i\neq k}}^{n}(x_k-x_i)}=1 \Longrightarrow \sum_{k=0}^{n}\frac{|P(x_k)|}{\Big|\prod_{\substack{i=0\\ i\neq k}}^{n}(x_k-x_i)\Big|}\geq\Big|\sum_{k=0}^{n}\frac{P(x_k)}{\prod_{\substack{i=0\\ i\neq k}}^{n}(x_k-x_i)}\Big|=1$$

بدون از دست دادن کلیت فرض کنید $x_0>\dots>x_n$

حداقل ۱ تا $k-i$ حداقل ۱ تا $n-k$

$$(\Sigma)\quad \Big|\prod_{\substack{i=0\\ i\neq k}}^{n}(x_k-x_i)\Big|=\prod_{i=0}^{k-1}|x_k-x_i|\prod_{i=k+1}^{n}|x_k-x_i|\geq k!\,(n-k)!=\frac{n!}{\binom{n}{k}}$$

$$\binom{n}{k}=\frac{n!}{k!(n-k)!}$$

ماکسیمم $|P(x_i)|$ ها فرض کنیم تو $i=k$ باشد. طبق $\circledast$

$$\circledast\quad 1\leq\sum_{i=0}^{n}\frac{|P(x_i)|}{\Big|\prod_{j\neq i}(x_i-x_j)\Big|}\leq |P(x_k)|\sum_{i=0}^{n}\frac{1}{\Big|\prod_{j\neq i}(x_i-x_j)\Big|}\overset{(\Sigma)}{\leq}|P(x_k)|\sum_{k=0}^{n}\frac{1}{\frac{n!}{\binom{n}{k}}}$$

$$= \frac{|P(X_k)|}{n!} \underbrace{\sum_{k=0}^{n} \binom{n}{k}} = \frac{|P(X_k)|}{n!} 2^n \Rightarrow \frac{|P(X_k)|\, 2^n}{n!} \geq 1$$

تعداد حالات زیرمجموعه k عضوی ← $\binom{n}{k} =$

برابر تعداد زیرمجموعه‌های مجموعه n عضوی $= 2^n$

هر عنصر یا دارد یا نه

$$\Rightarrow |P(X_k)| \geq \frac{n!}{2^n}$$

حکم ثابت شد.

$$P(k) = \frac{k!\,(n+1-k)!}{(n+1)!}$$ P درجه n (5)

$k = 0, 1, \ldots, n$

$P(n+1) = ?$ ← $P(k)$ ها را درنظر بگیرید.

درجه n ← درونیابی $n+1$ نقطه تعیین می‌کنند در $k = 0, 1, \ldots, n$

$$P(x) = \sum_{k=0}^{n} a_k(x)\,P(k) = \sum_{k=0}^{n} \prod_{\substack{i=0 \\ i\neq k}}^{n} \frac{x-i}{k-i} \; \frac{k!\,(n+1-k)!}{(n+1)!}$$

پس

$$P(n+1) = \sum_{k=0}^{n} \prod_{\substack{i=0 \\ i\neq k}}^{n} \frac{n+1-i}{k-i} \; \frac{k!\,(n+1-k)!}{(n+1)!} =$$

$$= \sum \frac{\cancel{(n+1)!}}{\prod(k-i)} \; \frac{k!\,\overbrace{(n+1-k)!}^{(n-k)!}}{\cancel{(n+1)!}} \; \frac{1}{n+1-k} =$$

$$\sum \frac{k!\,(n-k)!}{\prod\limits_{i\neq k}(k-i)} = \sum \frac{k!\,(n-k)!}{(-1)^{n-k}\,k!\,(n-k)!} = \sum_{k=0}^{n} (-1)^{n-k}$$

$$\underbrace{1 \times 2 \times \cdots \times k}_{0 \ldots} \quad k \quad \underbrace{1 \times 2 \times (n-k) \times (-1)^{n-k}}_{\ldots n}$$

n زوج: 1

n فرد: 0

⑥

$$f(x) = \frac{1}{x+c}$$

$$f[x_i, x_{i+1}] = \frac{\dfrac{1}{x_{i+1}+c} - \dfrac{1}{x_i+c}}{x_{i+1}-x_i} = \frac{-1}{(x_{i+1}+c)(x_i+c)}$$

$$f[x_i, x_{i+1}, x_{i+2}] = \frac{\dfrac{-1}{(x_{i+2}+c)(x_{i+1}+c)} - \dfrac{-1}{(x_i+c)(x_{i+1}+c)}}{x_{i+2}-x_i} = \frac{x_{i+2}+c-x_i-c}{(x_{i+2}-x_i)\prod_{j=0}^{2}(x_{i+j}+c)} = \frac{1}{\prod_{j=i}^{i+2}(x_j+c)}$$

با استقرا :

فرض استقرا :

$$f[x_i, \dots, x_{i+j}] = \frac{(-1)^j}{\prod_{k=i}^{i+j}(x_k+c)}$$

درصورت $(i,j) \to (i,j+1)$

برای $j-1$ درسته $\Rightarrow$

$$f[x_i, \dots, x_{i+j}] = \frac{f[x_{i+1}, \dots, x_{i+j}] - f[x_i, \dots, x_{i+j-1}]}{x_{i+j}-x_i}$$

مخرج مشترک

$$= \frac{1}{x_{i+j}-x_i}\left(\frac{(-1)^{j-1}}{\prod_{i+1}^{i+j}(x_k+c)} - \frac{(-1)^{j-1}}{\prod_{i}^{i+j-1}(x_k+c)}\right) =$$

$$\frac{1}{x_{i+j}-x_i}\left(\frac{(x_i+c)(-1)^{j-1} - (-1)^{j-1}(x_{i+j}+c)}{\prod_{i}^{i+j}(x_k+c)}\right) = \frac{(-1)^{j-1}\overbrace{(x_i+c-x_{i+j}-c)}^{-(x_{i+j}-x_i)}}{(x_{i+j}-x_i)\prod_{i}^{i+j}(x_k+c)}$$

$$= \frac{(-1)^j}{\prod_{i}^{i+j} x_k+c} \longrightarrow$$ حکم استقرا ثابت شد.

$$\Longrightarrow f[x_0, \dots, x_n] = \frac{(-1)^n}{(x_0+c)\cdots(x_n+c)}$$