```
restart;
```

$eq := diff(X(t), t\$1) = A.X(t) + b;$

$$\frac{\mathrm{d}}{\mathrm{d}t} X(t) = A.X(t) + b \tag{1}$$

$\mathrm{X} := \mathrm{Vector}(4, symbol = x) :$
$print(x = X);$
$cond := Vector([0.1, 0.1, 0.1, 0.1]) :$
$print(x0 = cond);$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$x0 = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} \tag{2}$$

$tmp := Matrix([[lambda1, 0], [0, lambda2]]) :$
$print(R = tmp);$
$R := tmp :$

$$R = \begin{bmatrix} \lambda 1 & 0 \\ 0 & \lambda 2 \end{bmatrix} \tag{3}$$

$tmp := Matrix(2, shape = identity);$
$print(E = tmp);$
$E := tmp :$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{4}$$

$with(LinearAlgebra) :$
$tmp := DiagonalMatrix([R, -E]) :$
$print(M = tmp);$
$M := tmp :$

$$M = \begin{bmatrix} \lambda 1 & 0 & 0 & 0 \\ 0 & \lambda 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \tag{5}$$

$m := [seq(\text{alpha}, i = 1 ..4)]:$
$m[1] := \text{alpha} - \text{beta}:$
$s := [seq(\text{beta}, i = 1 ..3)]:$
$tmp := BandMatrix([s, m, s]):$
$print(V = tmp);$
$V := tmp:$

$$V = \begin{bmatrix} \alpha - \beta & \beta & 0 & 0 \\ \beta & \alpha & \beta & 0 \\ 0 & \beta & \alpha & \beta \\ 0 & 0 & \beta & \alpha \end{bmatrix} \tag{6}$$

$with(LinearAlgebra):$
$tmp := V.M.V^{-1}:$
$\#print(A = tmp);$
$A := tmp:$

$tmp := 3:$
$print(\text{alpha} = tmp);$
$\text{alpha} := tmp:$

$$\alpha = 3 \tag{7}$$

$tmp := 1:$
$print(\text{beta} = tmp);$
$\text{beta} := tmp:$

$$\beta = 1 \tag{8}$$

$tmp := -2:$
$print(lambda1 = tmp);$
$lambda1 := tmp:$

$$\lambda 1 = -2 \tag{9}$$

$tmp := -5:$
$print(lambda2 = tmp);$
$lambda2 := tmp:$

$$\lambda 2 = -5 \tag{10}$$

$tmp := Vector([1, 1, 5, 10]):$
$print(b = tmp);$
$b := tmp:$

$$b = \begin{bmatrix} 1 \\ 1 \\ 5 \\ 10 \end{bmatrix}$$

(12)

(13)

(14)

```
#built-in solution
z := A.X + b :
sys := seq(diff(X[i](t), t) = z[i](t), i = 1 ..4) :
init := seq(X[i](0) = cond[i], i = 1 ..4) :
out := {seq(X[i](t), i = 1 ..4)} :
res := dsolve({sys, init}, out);
```

$$\left\{x_1(t) = -\frac{15}{34}\,e^{-2t} + \frac{2}{85}\,e^{-5t} + \frac{44}{85}, x_2(t) = \frac{149}{170} - \frac{213}{340}\,e^{-t} - \frac{15}{68}\,e^{-2t} + \frac{6}{85}\,e^{-5t}, x_3(t) = \frac{2}{85}\,e^{-5t} + \frac{429}{85} - \frac{169}{34}\,e^{-t}, x_4(t) = 10 - \frac{99}{10}\,e^{-t}\right\} \tag{15}$$

```
#analytic solution
with(LinearAlgebra) :
eq := MatrixExponential(A, t).cond + (MatrixInverse(A).(MatrixExponential(A, t)
    - IdentityMatrix(4)).b);
```

$$\begin{bmatrix} 0.02352941177\,e^{-5t} - 0.4411764705\,e^{-2t} + \frac{44}{85} \\ -0.2205882353\,e^{-2t} + 0.07058823535\,e^{-5t} - 0.6264705882\,e^{-t} + \frac{149}{170} \\ 0.02352941177\,e^{-5t} - 4.970588235\,e^{-t} + \frac{429}{85} \\ -9.9\,e^{-t} + 10 \end{bmatrix} \tag{16}$$

```
#calculate difference
dx1 := (out[1] = eq[1]) - res[1];
dx2 := (out[2] = eq[2]) - res[2];
dx3 := (out[3] = eq[3]) - res[3];
dx4 := (out[4] = eq[4]) - res[4];
```

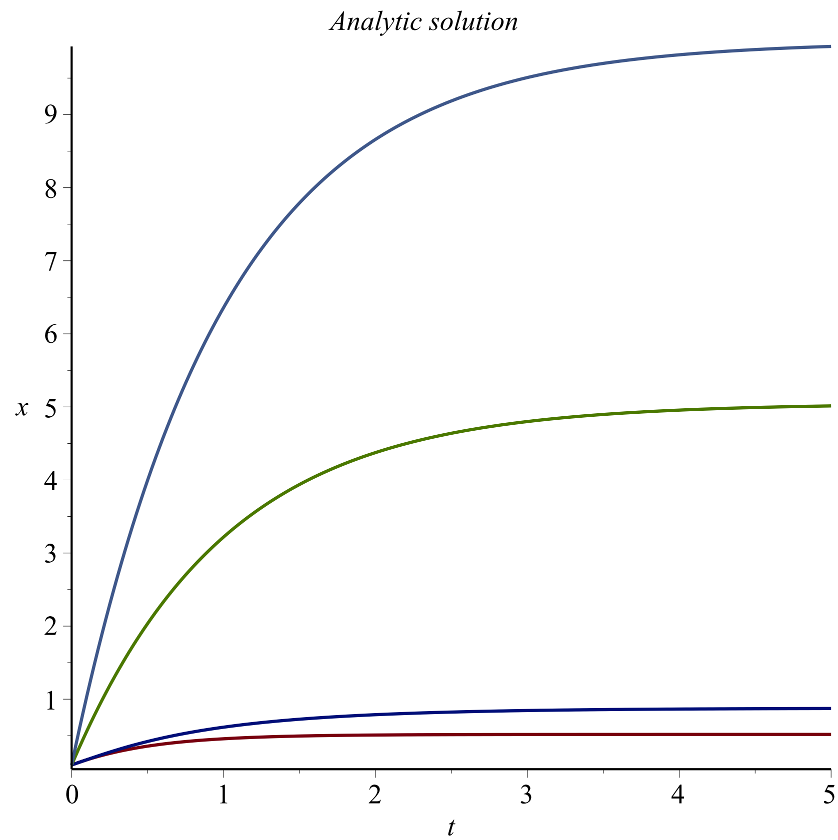
$$0 = 1.\ 10^{-11}\,e^{-5t} + 1.\ 10^{-10}\,e^{-2t}$$

$$0 = 6.\ 10^{-11}\,e^{-5t}$$

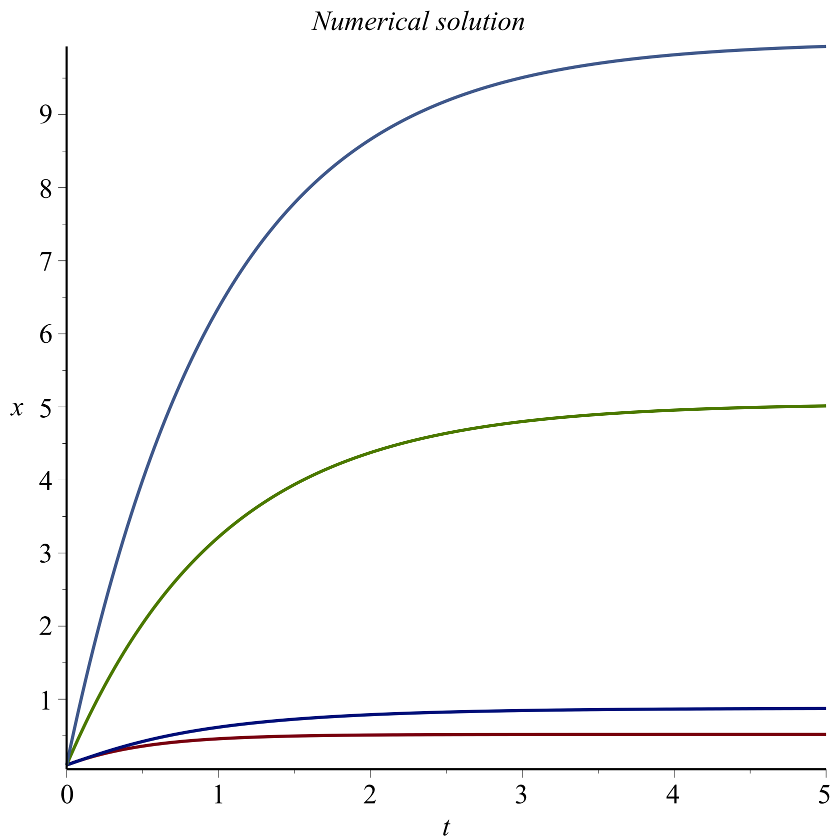
$$0 = 1.\ 10^{-11}\,e^{-5t}$$

$$0 = 0. \tag{17}$$

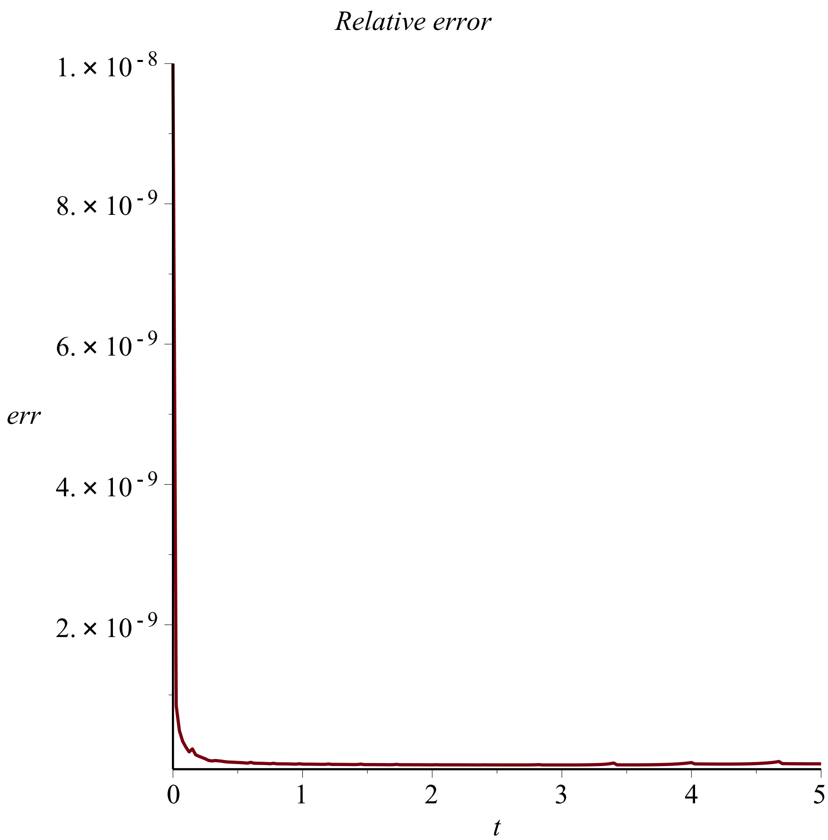
```
#plot solution
T := 5 :
#analytic
plot(eq[ ], t = 0 ..T, title = `Analytic solution`, labels = [t, x]);
```



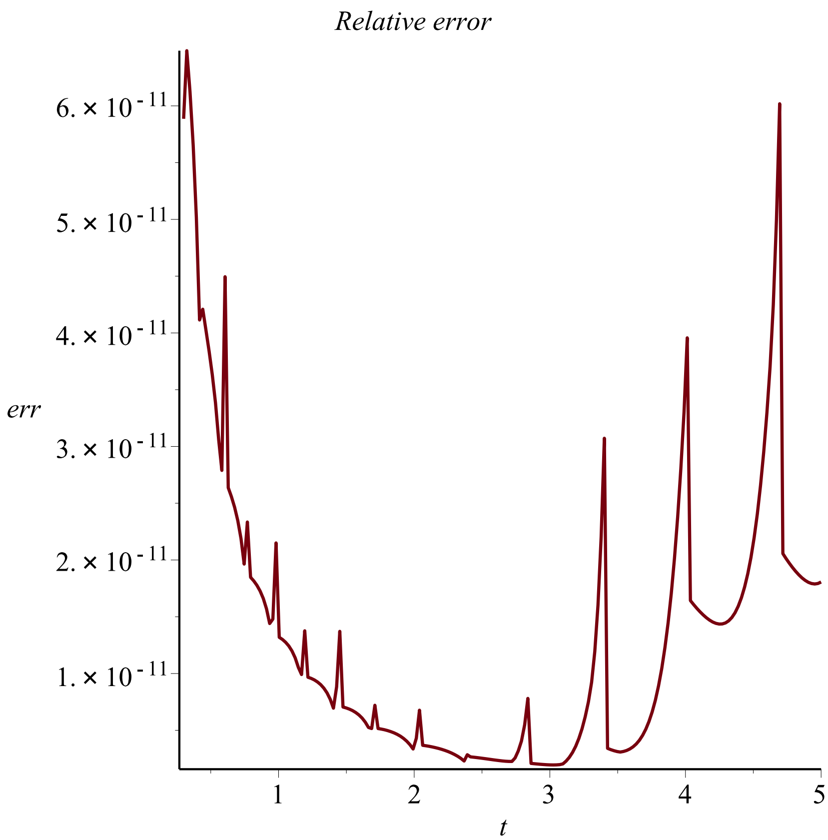

```
res := dsolve( {sys, init}, out, numeric, method = dverk78) :
with(plots) :
odeplot( res, [seq( [ t, x[i](t) ], i = 1 ..4) ], 0 ..T,  labels = [t, x], title = `Numerical solution`);
```




```
#residual
odeplot( res, [ t, Norm(eq - Vector([ seq( x[i](t), i=1 ..4 ) ])) / Norm(eq) ], 0 ..T, title
    = `Relative error`, labels = [t, err] );
```



$odeplot(\ res, [\ t, Norm(eq - Vector([\ seq(\ x[i](t), i=1..4\ )])) / Norm(eq)\ ], 0.3..T, title$
$= \text{`Relative error`}, labels = [t, err]\ );$



Relative error
err
6. × 10^-11
5. × 10^-11
4. × 10^-11
3. × 10^-11
2. × 10^-11
1. × 10^-11
1
2
3
4
5
t