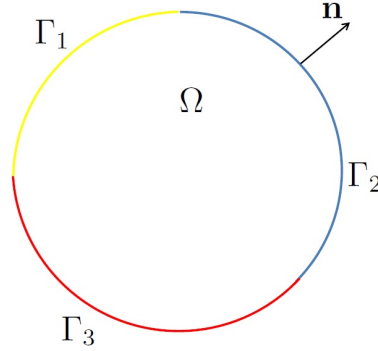


Boundary-value problem for the Poisson equation:

$$-\nabla \cdot (a \nabla u) = f, \quad x \in \Omega \quad (1)$$

$$u|_{\Gamma_1} = g, \quad \frac{\partial u}{\partial n} \Big|_{\Gamma_2} = 0, \quad a \frac{\partial u}{\partial n} + b(u - u_b) \Big|_{\Gamma_3} = 0. \quad (2)$$



Reducing to the weak form:

$$\begin{aligned} u &\in H^1(\Omega), \quad v \in H_0^1(\Omega) := \{v \in H^1(\Omega), \quad v|_{\Gamma_1} = 0\}, \\ - \int_{\Omega} \nabla \cdot (a \nabla u) v dx &= \int_{\Omega} f v dx, \\ \int_{\Omega} a \nabla u \nabla v dx - \int_{\Gamma_1} a \frac{\partial u}{\partial n} v d\Gamma - \int_{\Gamma_2} a \frac{\partial u}{\partial n} v d\Gamma - \int_{\Gamma_3} a \frac{\partial u}{\partial n} v d\Gamma &= \int_{\Omega} f v dx, \\ \int_{\Omega} a \nabla u \nabla v dx + \int_{\Gamma_3} b(u - u_b) v d\Gamma &= \int_{\Omega} f v dx, \end{aligned}$$

Weak form:

$$(a \nabla u, \nabla v) + \int_{\Gamma_3} b(u - u_b) v d\Gamma = (f, v), \quad u|_{\Gamma_1} = g.$$