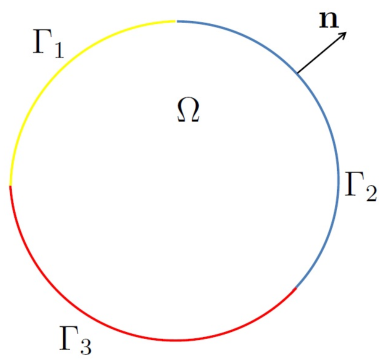
Nonlinear initial-boundary value problem:

$$u_t - \nabla \cdot (a\nabla u) = u^2 + 1, \quad x \in \Omega \tag{1}$$

$$u|_{\Gamma_1} = g, \qquad \frac{\partial u}{\partial n}\Big|_{\Gamma_2} = 0, \qquad a\frac{\partial u}{\partial n} + b(u - u_b)\Big|_{\Gamma_3} = 0. \tag{2}$$

$$u(x, 0) = u_0. \tag{3}$$



Time discretisation: $t_k = k\tau,\ \ k = 0, 1, .., N; \quad u_k = u(k\tau)$.

$$u_k \in H^1(\Omega), \qquad v \in H_0^1(\Omega) := \{v \in H^1(\Omega),\ \ v|_{\Gamma_1} = 0\},$$

For each time moment $(k = 1, 2, ..., N)$, we use the following iterative procedure:

Step 1.

Initialize the counter: $m = 1$;
$\widetilde{u}_k = 0$.

Step 2.

Solve the problem:

$$(u_k, v) - (u_{k-1}, v) + \tau(a\nabla u_k, \nabla v) + \tau \int\limits_{\Gamma_3} b(u_k - u_b)v d\Gamma = \tau(\widetilde{u}_k^2 + 1, v), \qquad u|_{\Gamma_1} = g,$$

or

$$(u_k, v) - (u_{k-1}, v) + \tau(a\nabla u_k, \nabla v) + \tau \int\limits_{\Gamma_3} b(u_k - u_b)v d\Gamma = \tau(u_k\widetilde{u}_k + 1, v), \qquad u|_{\Gamma_1} = g,$$

where $\widetilde{u}_k$ is an approximation of the solution at last iteration.

Step 3.

$m = m + 1$;
if $(m < M)$ $\{\widetilde{u}_k = u_k;$ goto step 2$\}$ else $\{$procedure finish$\}$