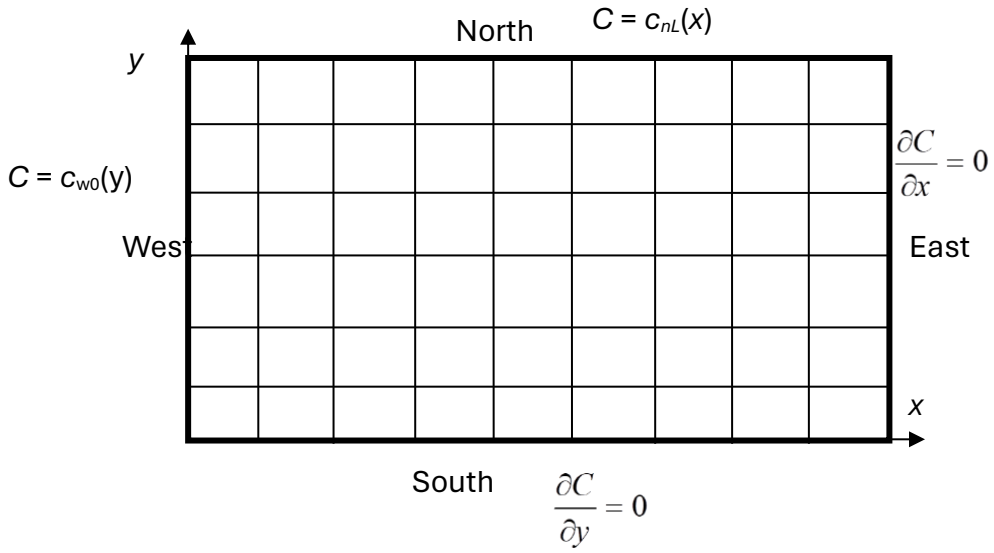
EX2010 Mathematics – Computing Coursework



In a rectangular domain illustrated above, the concentration C satisfies the following advection-diffusive equation,

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \Gamma\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right)$$

in which u and v are velocity in x and y directions; Γ is a diffusion coefficient. The length L_x and width L_y of the domain are 50 m and 1 m, respectively. The corresponding boundary conditions are

- On the west boundary $x = 0$: $C = C_{w0}(y) = C_{ya}\left(\frac{(L_y - y)y_a}{(L_y - y_a)y}\right)^{\alpha}$ where C_{ya} is the centration at $y = y_a$ and α is a coefficient. In this problem, $y_a = 0.1$, $C_{ya} = 1.0$ and $\alpha = 1/8$;
- On the north boundary $y = L_y$: $C = C_{nL}(x) = 0$;
- On the east boundary $x = L_x$: $\frac{\partial C}{\partial x} = 0$
- On the south boundary $y = 0$: $\frac{\partial C}{\partial y} = 0$

At the initial time, $C = 0$. You are asked to calculate the concentration distribution at $t = 100$s using the finite difference method. For simplicity, both the velocity and the diffusion coefficient are specified as constant values, i.e. $u = 1.5$ m/s, $v = 0$ and $\Gamma = 0.001$. However, in the Matlab your group will produce, these shall be kept as input arguments /parameters, together with the mesh sizes and time step sizes, allowing flexibility of using other values in the future.

Task 1: using the mesh illustrated above, where the x- and y- directional mesh sizes are Δx and Δy, respectively, and a uniform time step Δt, discretise the governing equation at all internal nodes. You shall use implicit Euler's method to discretise the time derivative, and central finite difference scheme to deal with the diffusive term (right hand side of the equation). For the

convective term $u\frac{\partial C}{\partial x}, v\frac{\partial C}{\partial y}$, you can use either central or upwind schemes, but the hybrid scheme is preferred, i.e. using the central scheme when |Pe| < 2 and the upwind scheme otherwise.

Task 2: discretise and implement the boundary conditions. You need to implicitly apply the Neumann boundary condition (east and south boundaries) into the governing equations when dealing with corresponding nodes on the east and south boundaries.

Task 3: Write a Matlab code to solve the problem based on the derivation in Task 1 and Task 2. You must use your own solver for discretised linear system of equations (Jacobi, Gauss-Seidel or SOR). You are suggested to write a Matlab function with input of Δx, Δy, Δt, u, v and Γ. This will benefit the discussion in Task 4.

Task 4: analyse the results

- Using $\Delta x = 0.5$m, Δy= 0.05m and $\Delta t = 0.5$s, illustrate the distribution of *C* at different time instants, e.g. t = 1s, 5s, 10s, 25s, 50s, 100s.
- Discuss the sensitivity of the results at t = 50s to mesh sizes
- Discuss the sensitivity of the results at t = 50s to the time step size

You need to provide both your report to describe Task 1, 2 and 4, and the Matlab code. You need to submit a zipped folder that contains all the functions that you have implemented in addition to a main script which is the interface that runs the whole simulation (you must load the variables directly in the main script, do not allow the user to fix them on the fly). All coding needs to be explained by appropriate comments. The zipped folder shall also contain the written report in PDF or MS Word.

After submitting your work, you will be asked to prepare the make a presentation and the coursework marker will ask any individual of your group to answer questions.

The marking scheme and weighting

- Task 1 and Task 2: 30%
- Task 3: 40% (appropriate comments are essential. Without them, at least 10% out of 40% will be deduced)
- Task 4: 30%

For submission, only one member shall submit the zipped folder, but everyone needs to fill the peer assessment form to evaluate your peers' participation. The presentation/interview is another way to assess the participation from.