Engineering Applications of Artificial Intelligence 137 (2024) 109212

Contents lists available at ScienceDirect

Engineering Applications of Artificial Intelligence

journal homepage: www.elsevier.com/locate/engappai

A comprehensive survey of weapon target assignment problem: Model, algorithm, and application

Jinrui Li [a], Guohua Wu [b,*], Ling Wang [c]

[a] Central South University, School of Traffic and Transportation Engineering, Changsha Hunan, 410075, PR China
[b] Central South University, School of Automation, Changsha Hunan, 410083, PR China
[c] Tsinghua University, Department of Automation, Beijing, 100084, PR China

ARTICLE INFO

Keywords:
Weapon target assignment problem
Combination optimization
Exact algorithm
Heuristic algorithm
Meta-heuristic algorithm
Machine learning

ABSTRACT

This paper provides an overview of the weapon target assignment problem, which aims to optimize the assignment of weapons to targets in order to maximize weapon damage to targets. The weapon target assignment problem can be viewed as a specialized instance of the optimal resource assignment problem. With the advancement of weapons technology, high-speed and high-lethality missiles have become more advanced, and their tactical applications more diverse. These missiles can strike targets with greater accuracy and improved concealment, posing a significant threat to both attackers and defenders. Consequently, the weapon target assignment problem has become a pressing concern in the field of military offense and defense. Subsequently, researchers worldwide are devoting significant efforts to address the weapon target assignment problem through the utilization of exact algorithms, heuristic algorithms, meta-heuristic algorithms, and artificial intelligence methods. This paper provides a brief review of the weapon target assignment problem development history, formula, solution techniques, and applications. We categorize weapon target assignment problems into four different formulas, considering the complexity of combat scenarios, and summarize various solution methods for each category. Furthermore, we also emphasize the relevance of weapon target assignment problems in national defense applications. Lastly, we conclude by discussing potential avenues for future research in addressing the weapon target assignment problem.

1. Introduction

The rapid advancement of military equipment and combat technology has led to the increasing complexity of modern warfare. Adversaries are adopting evolving strategies such as fake attacks, electronic interference, joint attacks, and deceptive operations, further intensifying the challenges faced in military operations. Since the introduction of the V-1 flying bomb, missile technology has undergone unprecedented advancements, now possessing the capability to carry various types of warheads, such as high-explosive bombs, incendiary bombs, nuclear weapons, as well as chemical, biological, or radioactive payloads (Yanushevsky, 2018). Consequently, missiles and kinetic energy penetrators have emerged as primary weapons for long-range strikes in many nations, motivating the emergence of air defense and anti-missile systems. The weapon target assignment (WTA) problem can be analyzed from both offensive and defensive perspectives. The attacker's primary aim during a strike mission is to inflict maximum damage upon the defender's protected assets through the utilization of aerial weaponry. Conversely, defenders focus on executing air defense and anti-missile missions to intercept diverse airborne threats such as fighter aircraft, missiles, and swarms (Tuncer and Cirpan, 2023). Among these challenges, the efficient assignment of weapons to targets stands out as a key issue of research and a formidable obstacle to overcome.

The WTA problem describes how to assign a group of weapons to a group of targets, considering various constraints, such as weapon quantities, weapon types, weapon effective radius, weapon/target damage probability, and launch & interception time cost (Menq et al., 2007). The air defense and anti-missile scenario of the WTA problem is shown in Fig. 1. Based on our investigation, it was Manne (1958) who first introduced and formulated the WTA problem, establishing it as a classic optimization problem. Inspired by the "personnel-assignment" problem, Manne applied the concept to the combat field, providing a solution and laying the foundation for subsequent research in this field. Fig. 1 presents a schematic diagram of the air defense and anti-missile

* Corresponding author.
E-mail addresses: reznovlee@csu.edu.cn (J. Li), guohuawu@csu.edu.cn (G. Wu), wangling@tsinghua.edu.cn (L. Wang).

https://doi.org/10.1016/j.engappai.2024.109212
Received 8 December 2023; Received in revised form 9 August 2024; Accepted 26 August 2024
Available online 5 September 2024

scenario in the WTA problem.

Currently, WTA problems are classified into two categories: Static-WTA (SWTA) problems and Dynamic-WTA (DWTA) problems, depending on whether the time dimension is involved. SWTA focuses on scenarios where a set of targets and a set of weapons are given, and the commander aims to destroy the targets by assigning weapons effectively, without considering time constraints. DWTA builds upon SWTA by adding the time constraint. As time goes by, many aspects of the scenario change, such as weapon positions, the number of weapons, target positions, target threat levels, target attack directions, and enemy attack intentions. Owing to the complexity of DWTA, it is further subdivided into two categories. The first is the *m-stage* problem, which breaks down DWTA into a multi-stage SWTA problem. The other category is the *Shoot-Look-Shoot* (SLS) problem, in which the WTA scenario is entirely dynamic, and cannot be easily decomposed into multi-stage SWTA problems.

The small-scale WTA problem can be effectively addressed through the utilization of exact algorithms. However, as Lloyd (1986) demonstrated, the WTA problem falls under the category of NP-complete problems, so the large-scale WTA cannot be easily solved by exact algorithms. Consequently, the solution methods for WTA primarily focus on obtaining approximate solutions (Brown et al., 2005) or finding feasible solutions within restricted timeframes to meet combat constraints (Rosenberg et al., 2005). As a result, the current research endeavors in the field of WTA predominantly revolve around the application of both exact algorithms, meta-heuristic algorithms, and heuristic algorithms. With the continuous advancement of machine learning, particularly the rapid progress in neural networks (Su et al., 2012) and reinforcement learning (Shokoohi et al., 2022), an increasing number of researchers are incorporating machine learning methods to solve WTA problems, signifying a significant milestone in the field of WTA problem-solving.

Since the inception of the WTA problem in 1958, numerous researchers have conducted extensive research on it, employing combinatorial optimization tools. Some researchers have conducted comprehensive reviews of the WTA problem. For example, Matlin (1970) provided an extensive review of WTA models, segmenting WTA literature into allocation models, game models, and special feature models. Naseem (2017) provided a detailed introduction to Threat Evaluation and Target Assignment (TEWA) in the "decision support system". Pryluk et al. (2016) focused on reviewing the *shoot-look-shoot* problem in DWTA, exploring the impact of missile geometry, guidance systems, and kill probability on WTA. Cai (2006) conducted an extensive examination of the DWTA problem, presenting it through various lenses including basic concepts, fundamental models, solution algorithms, and research shortages. Kline (2019) provided a thorough overview of WTA issues, covering WTA models, algorithms, recent developments, and literature influence. In Kline's paper (2019), the WTA problem was divided into two distinct problems, SWTA and DWTA, and at the same time, the solution methods are were divided into exact algorithms and

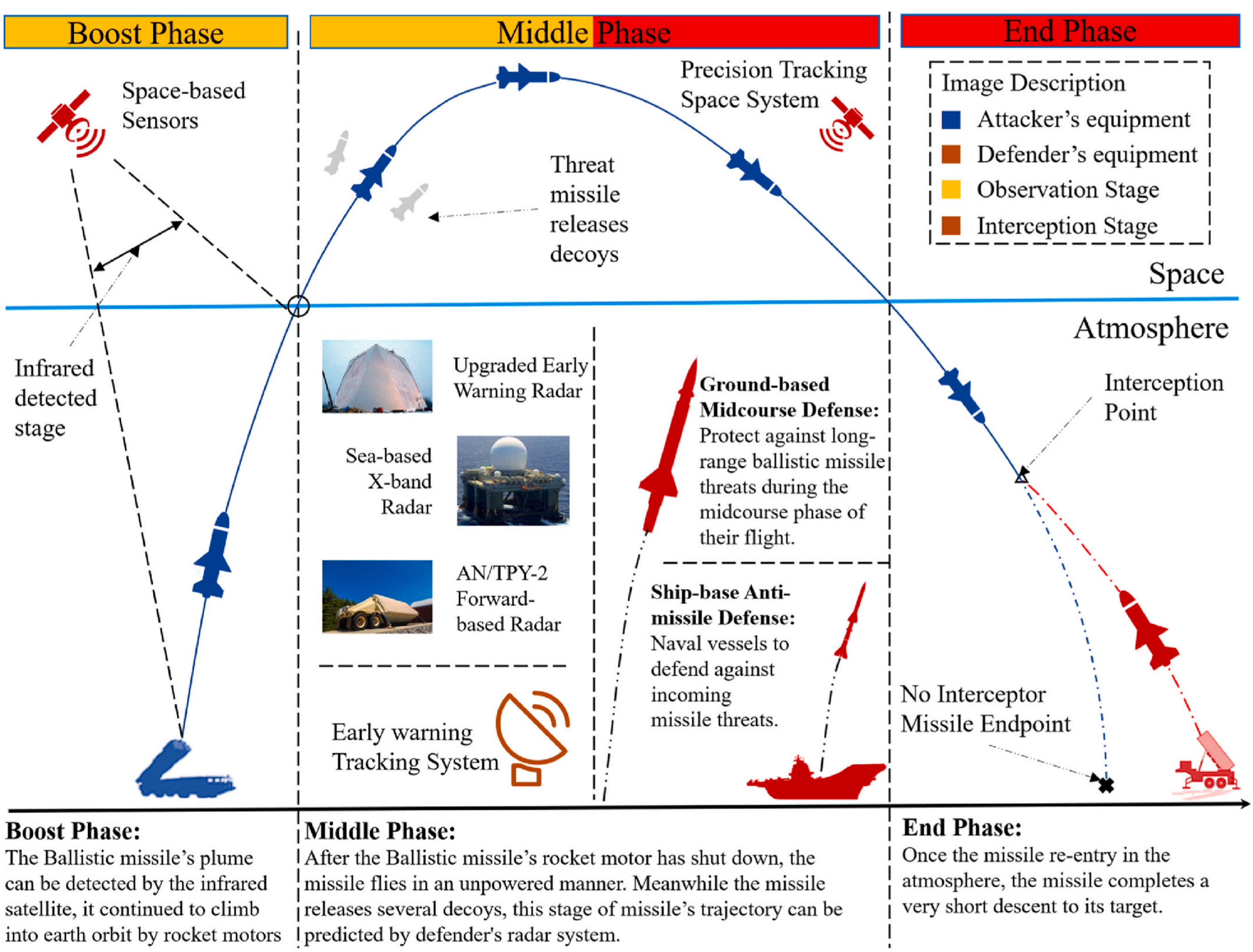

Fig. 1. The air defense and anti-missile scenario of the WTA problem.

approximate algorithms, for review. In recent years, the rapid advancement of artificial intelligence technology has yielded significant breakthroughs in solving combinatorial optimization problems, including machine learning solving the WTA problem.

Currently, research on WTA problems is experiencing a surge in interest, and existing review articles fail to encompass the characteristics of new methods. Moreover, the scenarios described by WTA problems are becoming increasingly complex, making it difficult for current classification methods in review literature to clearly describe new models. Additionally, although WTA problems are directly related to national defense challenges, few articles explore their practical military applications. Given these deficiencies in existing surveys regarding models, algorithms, applications, and future directions for WTA problems, there is an urgent need for a comprehensive and detailed overview of this subject.

This paper aims to present a thorough review of existing literature on WTA, and the subsequent sections are structured as follows. In Section 2, we analyze the literature trends, outlining the developmental roadmap of the research on WTA, and tracing the evolution of models and algorithms. In Section 3, we focus on a comprehensive analysis of the varieties of WTA models, offering a coarse classification and a detailed categorization of WTA models. In Section 4, we concentrate on an in-depth analysis of WTA solution methods, providing an extensive overview and classification of WTA problems according to four types of methods: exact algorithms, heuristic algorithms, meta-heuristic algorithms, and machine learning algorithms. In Section 5, we delve into the practical utilization of WTA in the field of national defense. Finally, in Section 6, we present potential future directions for the development of the WTA problem, drawing insights from the content covered in the literature review. The structure of this paper is shown in Fig. 2.

2. History of WTA

The origins of WTA research can be traced back to the end of World War II. The first documented model of the problem and solution method was introduced by Manne (1958). Since then, research on WTA has been based on Manne's hypothesis. This section provides a summary of the history of WTA models and development of solution methods.

2.1. Development of the WTA model

Manne (1958) initially proposed the WTA problem with the objective of minimizing the expected value of the surviving target, which was later expanded upon by denBroeder (1959), Perkins (1961), Lemus (1963), Burr (1985) and others. Bellman et al. (1959) proposed a new WTA model based on Manne's research, which added time elements to Rand's report. Soland (1973) proposed the canonical WTA model, which omits the damage probability of the weapon, effectively reducing the difficulty of solving. Furman (1973) and Murphey (2000) proposed variants of Soland's model , while Chang et al. (1987) proposed the more compact model that one weapon can only shoot one target based on Soland's model. O'Meara (1990) proposed WTA model with the minimum expectation of target survival, while Hosein et al. (1990) divided the WTA process into two stages and proposed a WTA model considering time. Then, various models have been proposed subsequently by considering maximal target destruction (Murphey, 2000), maximal weapon damage with global killing probability (Lee et al., 2002), maximal defender's protected assets' (Paradis et al., 2005), and minimal weapon launch cost (Song et al., 2009). These models do not have significant differences in their structure, only in the distinctions in the constraints and objective functions. Patrick (1990) proposed a WTA model considering time cost which divides the WTA into multiple combat phases. Subsequently, several researchers devoted themselves to the more adversarial WTA problem. Hohzaki and Nagashima (2009) proposed a WTA model with Stackelberg equilibrium. Lötter et al. (2013) introduced threat assessment in WTA, establishing a dual-objective optimization model that considers threat and target survival probability for the first time. Pan et al. (2019) designed the WTA game model considering the confrontation between red team and blue team. Karasakal (2021) proposed dual optimization objective and multi-layer optimization objective, respectively. As weapon systems undergo continuous and iterative updates, the diversity of aircraft and missile types, the range of threats they face, and the complexities of interception are all experiencing varying degrees of growth.

Furthermore, the methods of operational collaboration with these weapons are also becoming increasingly intricate and varied. Hence, the WTA model must also undergo updates and iterations to effectively align with emerging operational demands. In general, the development of formulas for WTA problems involves a gradual transition from simple to complex. The initial version of the adversarial WTA model was constrained by technical limitations and failed to yield satisfactory solutions. However, advancements in computing and algorithmic technologies have enabled the effective resolution of most complex problems.

2.2. Development of solving methods for WTA problems

After Manne (1958) proposed the WTA problem, initial attempts at solving it were constrained by limited computing power, leading to the use of restrictive conditions and small-scale data. At this stage, the WTA problem could typically be transformed into a linear programming problem, enabling the application of dynamic programming, branch and bound, linear programming, and other mathematical programming techniques for solutions (Ahner and Parson, 2015; Bellman et al., 1959;

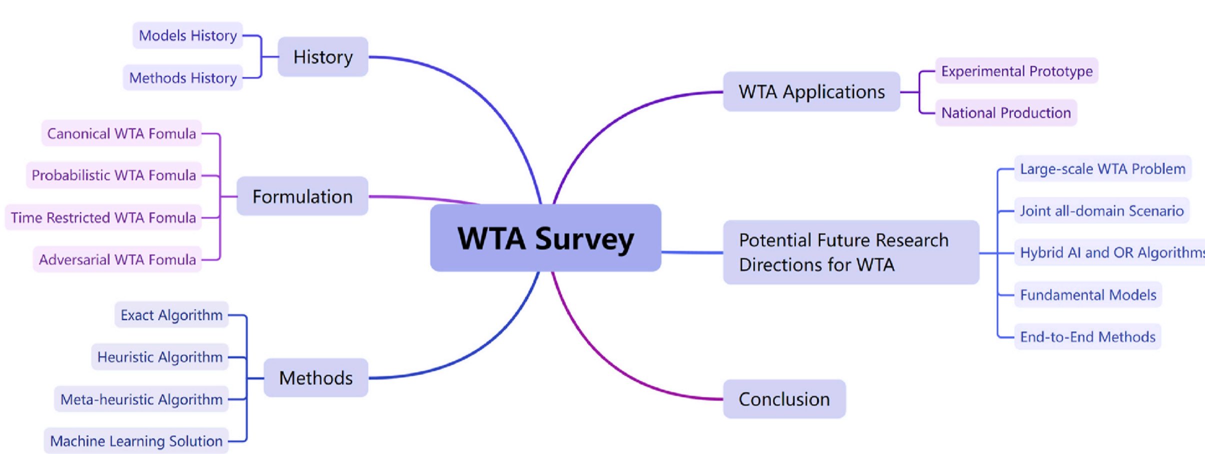

Fig. 2. The Structure of this paper.

Davis et al., 2017; Hohzaki and Nagashima, 2009). Subsequently, researchers began expanding the scale of the problem by integrating sequential optimization and nonlinear programming in staged solutions (Day, 1966), aiming to approximate solutions within acceptable computing power constraints. Despite efforts to apply heuristic algorithms, optimization remained focused on non-dynamic WTA problems initially (Chang et al., 1987; Orlin, 1987). Even attempts at solving dynamic WTA problems in multi-stage approaches were hindered by computational limitations (Hosein et al., 1988). As military demands increased, scholars sought simplified methods to streamline models, leading to the exploration of rule-based approximate optimization algorithms (Xin et al., 2011). However, these approaches still demanded considerable computational effort (Hosein and Athans, 1990). The emergence of metaheuristic algorithms such as genetic algorithms (Pepyne et al., 1997), particle swarm optimization (Yang et al., 2018; Zhou et al., 2016), differential evolution, NSGA-II, and ant colony optimization enriched the solution methods for WTA problems. Presently, meta-heuristic algorithms and various heuristic methods dominate the landscape of WTA problem-solving techniques. However, with the advancement of artificial intelligence technology, researchers have begun employing machine learning methods to tackle WTA problems. Techniques such as multi-agent systems (Sun et al., 2021), neural networks (Karasakal et al., 2021), reinforcement learning (Shokoohi et al., 2022), and expert systems (Sahin and Leblebicioglu, 2009) have been explored. Reinforcement learning, in particular, has gained traction due to its dynamic observation learning ability and the capacity to continuously optimize policy networks during gameplay, making it a popular direction for solving WTA problems.

It's evident that with advancements in algorithms, the methodology for solving the WTA problem has become increasingly applicable to real combat scenarios. There has been a notable shift towards leveraging artificial intelligence to tackle more intricate military optimization challenges. Furthermore, as operational demands escalate, the efficacy of algorithms in solving WTA problems may readily extend to other combat scenarios, including aircraft pursuit and sensor assignment problem (Lirov et al., 1989).

3. Formula

Manne (1958) first modeled the WTA problem as a nonlinear programming formula. In the subsequent development process, several researchers (Bracken et al., 1987; Karasakal, 2008; Kwon et al., 1999, 2007; Ma et al., 2015b; Song et al., 2009) attempted to transform the nonlinear formula into linear formulas and use exact algorithms to solve them. With the development of solving technologies and intelligent optimization algorithms, the nonlinear model of the WTA problem can be solved by designing appropriate optimization algorithms. At the same time, depending on the progress of computing power, the scenarios considered are more complex, and the decision space has expanded. Large-scale WTA problems with more constraints then can be effectively solved without linearization of the formula (Bellman et al., 1959; Chang et al., 1987; Chen et al., 2012; Gallagher and Kelly, 1991; Li et al., 2018; Rezende et al., 2018; Song et al., 2009; Xin et al., 2011; Yanxia et al., 2008).

There have been numerous variants of the WTA problem, in which different scenarios addressed in the literature. However, merely categorizing the WTA model into the SWTA and the DWTA model may not fully capture the complexity of this research topic clearly. According to the classification principle of Matlin's literature (1970), this paper divides WTA problems into four categories: canonical formulas, probabilistic formulas, time restricted formulas, and adversarial formulas. In order to make the expression of each formula formal and clear, we give the following basic symbol definitions in Table 1.

Table 1
Main symbol of this paper.

Symbol	Meaning of symbol
$m_{i,k}$	The k-th missile of weapon type i, if $k=1$, $m_{i,k}$ simplified as m_i
$w_{i,k}$	The number of weapon types, if $k=1$, $w_{i,k}$ simplified as w_i
M_i	The total number of weapon type i
W	The total number of weapons available to the defender, where $W=\sum_{i=0}^{I} M_i$
$n_{j,l}$	The l-th entity of target type j, if $l=1$, $n_{j,l}$ simplified as n_j
$t_{j,l}$	The number of target types, if $l=1$, $t_{j,l}$ simplified as t_j
N_j	The total number of target type j
T	The total number of targets, where $T=\sum_{j=0} N_j$
v_j	Indicates the destroyed value of target type j
c_{ij}:	The cost of assigning weapon type i to target type j
p_{ij}:	Probability of weapon type i destroying target type j
$u_{i,jl}$	A non-negative integer variable representing the number of weapons of types i assigned to the l-th entity in the type j target, if $l=1$, $u_{i,jl}$ simplified as u_{ij}
$x_{ik,jl}$	A binary variable indicating whether the k-th missile of the type i is assigned to the l-th target of the type j, if $k=1$ and $l=1$, $x_{ik,jl}$ simplified as x_{ij}
A	The set of feasible assignments of weapons to targets, that is, $A=[x_{ik,jl}]^{W\times T}$
$s^a_{ik,jl}$	The safety margin of the asset a, if $k=1$ and $l=1$, $s^a_{ik,jl}$ simplified as s^a_{ij}
$EDV(j)$	The damage value of the target with type j to the protected asset a
l_j	The threat level of the target with type j
E_{ij}	The shooting advantage of the weapon system i attacking the target j.
$q_{jl,a}$	Damage probability of the l-th entity of the type j target to asset a, if $l=1$, $q_{jl,a}$ simplified as q_{ja}
w_j	The threat value of the target of type j
S_{jl}	A binary variable, denoting whether the l-th entity of the type j target exists or not

3.1. Canonical WTA formula (P1)

According to our investigation, most papers set the number of entities per type of weapon and target to 1, so therefore, we will use the simplified symbols in this paper and provide a detailed explanation where the formula does not in the above case. The canonical WTA formula considers such a scenario, the number of weapons owned by the defender is w_i, $i=1,2,\cdots,m_i$, the number of targets owned by the attacker is $t_j, j=1,2,\cdots,n_j$, there is a cost c_{ij} when weapon i is assigned to target j, each target j has a static destruction value. The decision variable x_{ij} indicates whether weapon type i is assigned to target type j, and the decision variable u_{ij} indicates the amount of ammunition used if weapon type i is assigned to target type j (Soland, 1973). Then the canonical WTA formula can be expressed as follows.

$$\text{P1}-1 \ \min \sum_{j=1}^{n_j} c_{ij}x_{ij} \tag{3-1}$$

$$\text{s.t.} \sum_{j=1}^{n_j} x_{ij} \le 1, for\ j=1,\cdots,n_j \tag{3-2}$$

$$x_{ij}\in\{0,1\}, for\ i=1,\cdots,m_i, j=1,\cdots,n_j \tag{3-3}$$

The objective function (3-1) of formula P1-1 represents the minimum shooting cost, and the constraint in (3-2) represent that each type of weapon can only fire one round of ammunition at a target. In actual combat, all targets can only be intercepted under ideal conditions, so the objective function of the canonical WTA can be modified to maximize target destruction, leading to the formula P1-2 (Murphey, 2000).

$$\text{P1}-2 \ \max \sum_{j=1}^{n_j} v_j x_{ij} \tag{3–4}$$

$$\text{s.t.} \sum_{j=1}^{n_j} x_{ij} \le w_i, for\ j = 1, \cdots, n_j \quad (3–5)$$

$$x_{ij} \in \{0, 1\}, for\ i = 1, \cdots, m_i, j = 1, \cdots, n_j \quad (3–6)$$

The objective function and constraint conditions of formula P1-2 are relatively simple, and obtaining the exact solution is relatively easy. The objective (3-4) means to maximize the target destruction value, and the constraint in (3-5) means that the amount of missile allocated by weapon i to target j cannot exceed the upper limit of the missile inventory of weapon i, and by default one weapon only fires one missile. Similarly, the ultimate goal of the defender intercepting the targets is to ensure that the defender's assets' damage value is minimized (that is, the safety margin of the protected assets is maximized), so we get the following formula.

$$\text{P1} - 3 \max \sum_{i=1}^{m_i} \sum_{a}^{A} s_{ij}^{a} x_{ij} \quad (3–7)$$

$$\text{s.t.} \sum_{j=1}^{n_j} x_{ij} \le w_i, for\ j = 1, \cdots, n_j \quad (3–8)$$

$$x_{ij} \in \{0, 1\}, for\ i = 1, \cdots, m_i, j = 1, \cdots, n_j, \quad (3–9)$$

where s_{ij}^{a} in formula (3-7) represents the safety margin of the protected assets, the formula is a simplified version based on the works of these literatures (Han et al., 2016; Miercort and Soland, 1971; Paradis et al., 2005), and the objective function is to maximize the safety margin of the protected assets under given the current weapon assignment results.

Formulas P1-1 and P1-2 are used by several researchers (Bogdanowicz and Coleman, 2007; Bogdanowicz, 2009, 2012; Karasakal, 2008; Kwon et al., 2007; Murphey, 2000) as initial formulas to describe WTA problem. However, the canonical formula is a simplified version of WTA problem, that does not capture the complexities of real combat scenarios. Since then, subsequent formulations have expanded and extended these models to incorporate more complex objectives and constraints.

3.2. Probabilistic WTA formula (P2)

On the basis of the canonical WTA problem, some researchers introduced the killing probability of the weapon i to the target j in the objective function as p_{ij}, and at the same time considered the limitation of the quantity of weapons in the constraints. Lee et al. (2003) proposed a WTA formula considering the global killing probability and the expected damage value of the target to the defender's assets. the global killing rate is defined in formula (3-10).

$$PK(j) = 1 - \prod_{i=1}^{m_i} \left(1 - p_{ij}\right)^{x_{ij}} \quad (3–10)$$

Then we get the formula P2-1:

$$\text{P2} - 1 \min \sum_{j=1}^{n_j} EDV(j) * PK(j) \quad (3–11)$$

$$\text{s.t.} \sum_{j=1}^{n_j} x_{ij} = 1, i = 1, 2, \cdots, m_i \quad (3–12)$$

$$x_{ij} \in \{0, 1\}, for\ i = 1, \cdots, m_i, j = 1, \cdots, n_{j,} \quad (3–13)$$

where the definition of $EDV(i)$ in formula (3-11) is given in Table 1. In addition, the selection of the objective function of the WTA problem can vary. Ni et al. (2011) proposed a formula to determine the objective by comprehensively considering the target destruction probability and its threat value. This formula (3-12) sets the minimum number of weapons as t_j, which is not necessarily been 1. The formula is as follows.

$$\text{P2} - 2 \max \sum_{j=1}^{n_j} v_j \left(1 - \prod_{j=1}^{n_j} \left(1 - p_{ij}\right)^{x_{ij}}\right) \quad (3–14)$$

$$\text{s.t.} \sum_{j=1}^{n_j} x_{ij} \le w_i, i = 1, 2, \cdots, m_i \quad (3–15)$$

$$\sum_{i=1}^{m_i} x_{ij} \ge t_j, j = 1, 2, \cdots, n_j \quad (3–16)$$

$$x_{ij} \ge 0, integral, i = 1, 2, \cdots, m_i, j = 1, 2, \cdots, n_j, \quad (3–17)$$

the objective function (3-14) in formula P2-2 represents the maximum value of the weapon's interception rate against incoming targets. The constraints in (3-15) indicate that the number of all weapons does not exceed the maximum quantity. The constraints in (3-16) indicate that total number of weapons used should exceed the minimum number of weapons required by target j and make sure that the number of weapons assigned to target j is non-negative and discrete. P2-2 is a WTA formula originally proposed by Manne (1958), after which denBroeder (1959), Lu et al. (2021), Sahin and Leblebicioglu (2009), Liang (2016), Wu et al. (2008; 2008), Li (2012), Xin et al. (2019), Lu (2006), Li (2015), Shalumov (2017) and other researchers all use the prototype of this formula to model the WTA problem.

In addition, the optimization objective of WTA problem is to minimizing weapon launch costs (Kwon et al., 2007), achieving the best weapon-target match (Gao et al., 2010), maximizing target interception probability (Manne, 1958), and minimize the defender's protected asset damage (Bin and Jie, 2012), etc., so there are several variants of the probabilistic WTA problem as follows.

$$\text{P2} - 3 \min \sum_{(i,j) \in A} c_{ij} x_{ij} \quad (3–18)$$

$$\text{s.t.} \sum_{\{j \in n_j | \{i,j\} \in A\}} u_{ij} \le \overline{u}_{ij}, \forall\ i \in m_i \quad (3–19)$$

$$\sum_{\{j \in n_j | \{i,j\} \in A\}} x_{ij} \le \overline{x}_{ij}, \forall\ i \in m_i \quad (3–20)$$

$$u_{ij} - \overline{u}_{ij} x_{ij} \le 0, \forall\ (i,j) \in A \quad (3–21)$$

$$1 - \prod_{i \in m_i | (i,j) \in A} \left(1 - p_{ij}\right)^{x_{ij}} \ge d_j, \forall\ j \in n_j \quad (3–22)$$

$$u_{ij} \ge 0, integer\ for\ all\ (i,j) \in A \quad (3–23)$$

$$x_{ij} \in \{0, 1\}, \forall\ (i,j) \in A, \quad (3–24)$$

where A in formula P2-3 is a decision matrix of x_{ij} with dimension $m \times n$, $\overline{x}_{ij}$ in formula (3-20) indicates the upper limit of the target that weapon system i can shoot. $\overline{u}_{ij}$ in (3-19) and (3-21) represents the upper limit of the ammunition launched by weapon i to target j. In the case discussed in this section, $\overline{x}_{ij}$ and $\overline{u}_{ij}$ are both set to 1. If for all $i \in m, \overline{x}_{ij} \ge |m|$, then this problem becomes the one considered in Kwon's paper (1999). These nonlinear constraints can be transformed into linear inequalities by taking the logarithm of both sides.

$$\text{P2} - 4 \max \sum_{j=1}^{n_j} l_j \prod_{i=1}^{m_i} \left(E_{ij} \cdot x_{ij}\right) \quad (3–25)$$

$$\text{s.t.} \sum_{j=1}^{n_j} x_{ij} = 1, \forall\ i \in m_i \quad (3–26)$$

$$\sum_{i=1}^{m_i}\sum_{j=1}^{n_j} x_{ij}\left(1-x_{ij}\right)=0 \tag{3–27}$$

Formula P2-4 is presented by O'Meara (1990), and its objective function in formula (3-25) represents the survival expectation of the minimization target. The core part of the formula proposed by Gao et al. (2010) can also be expressed by formula P2-4. In formula (3-25), l_j represents the threat level of the j-th target, and E_{ij} represents the shooting effectiveness of the weapon system i attacking the target j.

$$\text{P2}-5 \ \min \sum_{j=1}^{n_j} v_j \prod_{i=1}^{m_i}\left(1-p_{ij}x_{ij}\right) \tag{3–28}$$

$$\text{s.t.} \ \sum_{j=1}^{n_j} x_{ij}=1, i=1,2,\cdots,m_i \tag{3–29}$$

$$x_{ij}\geq 0 \tag{3–30}$$

Formula P2-5 was originally proposed by Chang (1987), and its constraints are more compact than formula P1-1, which the weapon i can only launch missile to one target, which is shown in formula (3-29). Such strict constraints are not common in WTA problems. Manne (1958) performed a linear transformation when dealing with this formula. P2-5 is one of the earliest mathematical models of the WTA problem. Readers interested in this topic may refer to Song (2009), Lu (2021), Zhu (2011), Sahin (2009), Wu (2008, 2008), Durgut (2017), Ma et al. (2015), Şahin (2014), Lee (2010), Hughes (2022) and other researchers proposed the improved WTA formulas on the basis of P2-5.

$$\text{P2}-6 \ \max \sum_{a=1}^{K} v_j \prod_{j\in n_j}\left[1-q_{ja}\prod_{i\in m_i}\left(1-p_{ij}\right)^{x_{ij}}\right] \tag{3–31}$$

$$\text{s.t.} \ \sum_{j=1}^{n_j} x_{ij}\leq w_i, \forall\, i\in 1,2,\cdots,m_i \tag{3–32}$$

$$\sum_{i=1}^{m_i} x_{ij}\leq t_j, \forall\, j\in 1,2,\cdots,n_j \tag{3–33}$$

$$\sum_{j=1}^{n_j} x_{ij}\leq \overline{u}_{ij}, \forall\, i\in 1,2,\cdots,m_i, \tag{3–34}$$

where K is the number of threatened assets, and q_{ja} is the lethality probability of target j destroying asset a, which can be evaluated based on the actual combat situation. In addition, further formulas for minimizing the damage to protected assets of the defender can be found in Lee (2002, 2003), Xin (2010), Khosla (2006), Keith (2019) and Mekawey et al. (2009) et al.

Song et al. (2009) proposed a variant based on the P1-1 formula. They developed a strategy to balance the number of weapons and targets, ensuring that no matter whether the number of weapons or the number of targets are consistent, they are added by means of virtual targets/weapons. For the virtual target, the weapon's kill probability is 1. For virtual weapons, the probability of killing the target is 0. The resulting formula is as follows.

$$\text{P2}-7 \ \min \sum_{i=1}^{Num}\sum_{j=1}^{Num} w_j\left(1-p_{ij}\right)x_{ij} \tag{3–35}$$

$$\text{s.t.} \ \sum_{j=1}^{n} x_{ij}\leq 1 \tag{3–36}$$

$$\sum_{i=1}^{m} x_{ij}\leq 1 \tag{3–37}$$

$$x_{ij}\in\{0,1\}, \tag{3–38}$$

where w_j represents the threat value of the target j. The concept of target threat value reflects variations in target speed, lethality, and guidance laws. Therefore, researchers such as Oxenham (2006), Karasakal (2021), Lötter et al. (2013), Azak and Bayrak (2008), Naeem (2009) and other researchers introduced the target threat value judgment into the model when studying the WTA problem to increase the authenticity of the model.

For the probabilistic WTA Problem, the constraints of most researchers are significantly different. In addition to the most basic ammunition constraints, that all targets must be assigned, and basic parameter constraints (Kwon et al., 2007), some researchers also proposed a variety of different constraints: (1) The weapon system has the ability to shoot multiple targets at the same time. (2) Weapons can be purchased when facing saturation attacks (additional weapon purchase costs) (Koleszar et al., 1999). (3) There is a channel occupation when weapons are launched (Peng et al., 2016). (4) The space constraint generated by the trajectory of the intercepting missile when it travels (Gao et al., 2010). (5) The capability of the weapon to the target and the combat feasibility constraint (Bin and Jie, 2012; Xin et al., 2019). (6) The non-negative integer constraints, along with upper and lower bound constraints of the decision variable (Karasakal, 2008; Ma et al., 2015a). (7) The firepower transfer constraint (Li et al., 2015b) when the target distance is too short to intercept the weapon conflict. Readers are referred to the related literature for more details on WTA problem constraints.

In addition, due to the difficulty of air missile interception in actual operations, the number of interceptor missiles used for different targets is not the same (Li et al., 2004), this variability is not considered in the models discussed. A preferable approach is to regularly scan the airspace information, which has the advantage of leaving the missing target for the second batch to intercept when the first batch of secondary targets are not intercepted.

3.3. Time restricted WTA formula (P3)

In actual combat, the state of incoming targets tends to change over time. The WTA formula, as it considers the time restricted, introduces a time dimension to the probabilistic WTA Problem, rendering the problem solution time-sensitive. Therefore, the traditional WTA solution method may encounter difficulties in being applied (Randleff and Clausen, 2007). The most simplified time restricted WTA Problem formula divides the entire combat process into two stages. This type of problem, as referred to in Hosein's literature (1990), is termed the 2-*stage* problem, and the specific formula is as follows.

$$\text{P3}-1 \ min_{\{x_{ij}\}} F_1=\sum_{\overrightarrow{W}\in\{0,1\}^N} \Pr[\overrightarrow{x'}=\overrightarrow{w}]F_2^{*}(\overrightarrow{x'},\overrightarrow{w}) \tag{3–39}$$

$$\text{s.t.} \ x_{ij}\in\{0,1\}, i=1,2,\cdots,m_i, j=1,2,\cdots,n_j, with\ w_j=1-\sum_{i=1}^{m_i}x_{ij}, \tag{3–40}$$

where x_{ij} in formula (3-40) represents the decision variable in the first stage, s_j represent the observation of targets in first stage. The weapons in the second stage are defined as the set of weapons available after the first stage. The target state at the beginning of the second phase is an n-dimensional random vector. The probability that s_j is 1 is the probability that target j survives in the first stage. Otherwise, the target j was destroyed in first stage. Therefore, the distribution of the random variable s_j is:

$$\Pr[s_j=\beta]=\beta\prod_{i=1}^{m_i}\left(1-p_{ij}(1)\right)^{x_{ij}}+[1-\beta]\left\{1-\prod_{i=1}^{m_i}\left(1-p_{ij}(1)\right)^{x_{ij}}\right\} \tag{3–41}$$

for $\beta=1,2,i=1,2,\cdots m_i$

The distribution formula of s_j is also called the target state evolution equation of the system. Weapon stats also evolve over time. This evolution is deterministic and depends on the assignment's decision in the first phase. Its evolution is given by the following equation.

$$x_i=1-\sum_{j=1}^{n_j}x_{ij},i=1,2,\cdots,n_j \tag{3–42}$$

It means weapon i appears in the second stage if and only if it was not used in the first stage. The iteration of x_i is also called the weapon state evolution of the system. Let $F_2^*(\vec{s},\vec{x})$ denote the optimal cost for the first stage problem of $\vec{x}$ with initial generation target state $\vec{s}$ and initial weapon state, which is defined as follows.

$$F_2^*(\vec{s},\vec{x})=\sum_{i=1}^{m_i}v_js_j \tag{3–43}$$

The objective function in formula (3-39) of P3-1 is the sum of the probabilities of all possible second-stage target states multiplied by the optimal value of the given state. Note that the distribution of the targets state of second stage and the weapons states of second stage are both dependent on the first stage. This 2-stage formula can also be extended to multi-stage situations, and readers can refer to Patrick's literature (1990) for more details.

Another time restricted WTA model was given by Xin (2011). According to the time dimension, it is divided into several stages of equal duration, and the situation information of the battlefield is obtained at a stationary time stage. Time restricted WTA Problem is transformed into solving the multi-stage probabilistic WTA Problem, which can be understood as a timed version of the formula P2-6, the formula is as follows.

$$\text{P3}-2\ \max J_t(X^t)=\sum_{a=1}^{K(t)}v_j\prod_{j=1}^{n_j(t)}\left[1-q_{ja}\prod_{h=t}^{S}\prod_{i=1}^{m_i(t)}\left(1-p_{ij}(h)\right)^{x_{IJ}(h)}\right] \tag{3–44}$$

$$\text{s.t.}\ \sum_{j=1}^{n_j}x_{ij}(t)\le m_i,\forall\, t\in\{1,2,\cdots,S\},\forall\, i=1,2,\cdots,m_i \tag{3–45}$$

$$\sum_{i=1}^{m_i}x_{ij}(t)\le n_j,\forall\, t\in 1,2,\cdots,S,\forall\, j=1,2,\cdots,n_j \tag{3–46}$$

$$\sum_{i=1}^{m_i}\sum_{j=1}^{n_j}x_{ij}(t)\le N_i,\forall\, i=1,2,\cdots,m_i \tag{3–47}$$

$$x_{ij}(t)\le f_{ij}(t),\forall\, t\in 1,2,\cdots,S,\forall\, i=1,2,\cdots,m_i,\forall\, j=1,2,\cdots,n_j, \tag{3–48}$$

where $J_t(\cdot)$ in formula (3-44) is the objective function, which means to maximize the target damage, $X^t=[X_t,X_{t+1},\cdots,X_S]$, $X_t=[x_{ij}]_{W\times T}$ is decision variable matrix at time t. h is the index of the stage, a represents the protected assets of the defender, and q_{ja} represents the lethality of the target j on the protected assets a. N_i represents the weapons available at the current stage. $m_i(t)$ and $n_j(t)$ represent the number of observations of weapons and targets at stage t, respectively. In addition, it increases the complexity of the time restricted WTA problem and the difficulty of generating feasible solutions.

Peng et al. (2016) proposed a dual-objective time restricted WTA formula considering the maximization of target damage and the minimization of weapon ammunition cost. The method of time restricted is consistent with formula P3-2, and the target damage probability threshold constraint and time are added window constraint, the specific formula is as follows.

$$\text{P3}-3\ \max f(t)=\sum_{j=1}^{n_j(t)}v_j(t)\left[1-\prod_{i=1}^{m_i(t)}\left(1-p_{ij}(t)\right)^{x_{ij}(t)}\right] \tag{3–49}$$

$$\min g(t)=\sum_{j=1}^{n_j(t)}\sum_{i=1}^{m_i(t)}x_{ij}(t) \tag{3–50}$$

$$\text{s.t.}\ P_j(t)\ge P_{dj}(t),\forall\, t\in\{1,2,\cdots,S\},\forall\, j\in\{1,2,\cdots,n\} \tag{3–51}$$

$$\sum_{i=1}^{n_j(t)}x_{ij}(t)\le 1,\forall\, t\in\{1,2,\cdots,S\},\forall\, j\in\{1,2,\cdots,n\} \tag{3–52}$$

$$t_j^T\ge t^A,\forall\, t\in\{1,2,\cdots,S\},\forall\, j\in\{1,2,\cdots,n\}, \tag{3–53}$$

where t^A in formula (3-53) represents the algorithm time window of the weapon, t^T represents the target time window, $P_j(t)$ in formula (3-51) represents the damage probability of the target in the t-th stage, and $P_{dj}(t)$ represents the damage probability threshold of the target in the t-th stage. In the experimental stage, the author set the $P_{dj}(t)$ as 0.9. It considers the time frame of the target and the algorithm, guaranteeing that the weapon's assignment stage consumes less time than the target's time window and ensuring that the target can be intercepted by anti-missile weapons.

The objective function selection for the dual-objective time restricted WTA problem optimization model offers a range of combinations. Li et al. (2017), Shalumov (2017), and Shi (2021) have applied an objective function consistent with P3-3. Meanwhile, in Lötter's paper (2013), a model was developed for minimizing costs and the cumulative survival probability of the target threat. Additionally, in Silav's paper (2021), a bi-objective optimization model was proposed to maximize target non-leakage probability and weapon system engagement order stability. Further, other researchers have explored more optimization objectives in the WTA problem.

For instance, in Zhao's paper (2022), a double-layer dual-objective optimization model was proposed, taking into account the expected value of the importance of the target being damaged and the expected value of combat consumption. In this model, the upper layer seeks to optimize the interception effect for the entire combat stage and locally optimize the weapon assignment scheme for each stage. The lower layer, on the other hand, aims to optimize the weapon assignment scheme globally, taking into account the expected value of the importance of the target being disrupted and the combat consumption, and the formula is as follows.

P3-4 Upper Level:

$$\max Z_1=\sum_{j=1}^{n_j(t)}\left(1-\prod_{i=1}^{m_i(t)}\left(\sum_{t=1}^{S}\left(1-p_{ij}(t)\right)^{x_{ij}(t)}\right)\right) \tag{3–54}$$

$$\text{s.t.}\ 1-\prod_{i=1}^{m_i(t)}\left(1-p_{ij}(t)\right)^{x_{ij}(t)}\ge K_a \tag{3–55}$$

$$\sum_{i=1}^{m_i(t)}x_{ij}(t)\le 1,\forall j,t \tag{3–56}$$

$$\sum_{j=1}^{n_j(t)}x_{ij}(t)\le a,\forall i,t \tag{3–57}$$

$$x_{ij}(t)=\{0,1\} \tag{3–58}$$

Lower Level:

$$\max Z_2 = \sum_{t=1}^{S-1}\left[\sum_{j=1}^{n_j(t)}\left(\alpha_j(t)\cdot\beta_j\cdot P_{rj}(\beta_j(t)=k)\right)+\sum_{j=1}^{n_j(t)}\left(\alpha_j(t)\cdot\beta_j\cdot P_{rj}(\beta_j(t)=0)\right)\right] \tag{3–59}$$

$$\min Z_3 = \sum_{i=1}^{m_i(t)}\sum_{t=1}^{S}\left(C_i(t)\cdot\sum_{j=1}^{n_j(t)}x_{ij}(t)\right) \tag{3–60}$$

$$\text{s.t. } x_{ij}(t)\le C_{ij}(t) \tag{3–61}$$

$$\beta_j(t), x'_{ij}(t)=\{0,1\}, \tag{3–62}$$

where K_a in the upper level Z_1 formula represents the effective damage coefficient, and k represents the status factor of system. In the Lower level Z_2 formula, $\alpha_j(t)$ is the expected value coefficient of damage importance of the j-th target in the t-stage, $\beta_j(t)$ represents the working status coefficient of the j-th target in the t-stage, $P_{rj}\left(\beta_j(t)=k\right)$ is used to represent the probability of effectively hitting the j-th target in the case of $\beta_j(t)=k$, the formula of $P_{rj}\left(\beta_j(t)=k\right)$ is defined as follows.

$$P_{rj}(\beta_j(t)=k)=k\cdot\prod_{i=1}^{m_i(t)}\left(1-p_{ij}(t)\right)^{x_{ij}(0)}\cdot\prod_{t=1}^{S}\prod_{i=1}^{m_i(t)}\left(1-p_{ij}(t)\right)^{x'_{ij}(t)}+(1-k)\cdot\left(1-\prod_{i=1}^{m_i(t)}\left(1-p_{ij}(t)\right)^{x_{ij}(0)}\right)\cdot\left(1-\prod_{t=1}^{S}\prod_{i=1}^{m_i(t)}\left(1-p_{ij}(t)\right)^{x'_{ij}(t)}\right) \tag{3–63}$$

In the lower level formula, $C_i(t)$ represents the consumption cost of the weapon at stage t, and $C_{ij}(t)$ represents the overall damage index of weapon i to target j. If it is reasonable to assign weapon i to target j at stage t, then $C_{ij}(t)=1$. $x'_{ij}(t)$ indicates whether the distribution of weapons in the $t+1$ stage changes. If $x'_{ij}(t)=1$, the distribution target of the i-th weapon does not change. More details about this formula can be found in reference (Shi et al., 2021).

$$\text{P3}-5\ \min Z_{ND}=\sum_{i\in m_i}\sum_{j\in n_j}\sum_{k\in K}\left|Y_{ijk}-x_{ijk}\right| \tag{3–64}$$

$$\max Z_{PNL}=\prod_{i\in m_i}\left(1-\prod_{k\in K,j\in n_j}\left(1-p_{ijk}\right)^{Y_{ijk}}\right) \tag{3–65}$$

$$\text{s.t. }\sum_{k\in K,i\in m_i}Y_{ijk}\le d_j-f_i,\forall j\in n_j \tag{3–66}$$

$$\sum_{(i,j)\in Sep_{ij}}Y_{ijk}\le 1 \tag{3–67}$$

$$\sum_{k\in Seq_{ij}}Y_{ijk}\le\overline{u}_{ij},\forall (i,j)\in\left(m_i,n_j\right) \tag{3–68}$$

$$Y_{ijk}\in\{0,1\} \tag{3–69}$$

The objective function Z_{ND} of formula (3-64) and (3-65) are very interesting formulas. The author uses the absolute value loss function least absolute deviations (LAD) that minimizes the initial assignment and the actual assignment as the objective function. This kind of literature that uses the loss function as the objective function less common. Silav (2019) uses the same objective function as P3-5, and Hocaoğlu (2019) uses the loss function of the target cumulative actual damage probability and cumulative expected damage probability as the optimization objective.

3.4. Adversarial WTA formula (P4)

This section presents formulas for the adversarial WTA Problem that are more complex and closer to the actual combat scenarios than the formulas mentioned earlier. In actual combat situations, commanders have to take into account not only task planning and firepower assignment but also the opponent's intentions, patterns, and tactics. They need to be able to discern if the enemy's true attack direction and combat style are concealed. Hence, some researchers have incorporated game theory while constructing the WTA formula.

Game theory refers to the discipline that studies the strategy of related parties in the game between multiple individuals or teams under specific conditions, and the fact corresponds to the strategy (Ho et al., 2022). Han et al. (2016), Golany (2015), Keith (2021) and Hohzaki (2009) used game theory in combat issues such as missile planning. Pan et al. (2019) proposed a WTA problem formula considering red-blue confrontation, which not only considered the assignment of missiles, but also added the option of electronic interference. In this scenario, the red team is referred to as $R=\{r_1,r_2,\cdots,r_m\}$, and the blue team is denoted as $B=\{b_1,b_2,\cdots,b_n\}$. f^R means the red team's damage to the enemy. f^B means the blue team's damage to the enemy. M^R is the type of recognition results of the opponent team on the R team, based on the target type identification of team B. The relative killing probability of r_i members of group R. t_I^R is the interference factor of r_i in team R, and the fighter or weapon chooses an action at least one between electronic interference and missile fire attack in a decision step. k_{ij}^x and l_{ij}^x are the action labels of missile fire attack and electronic interference of type x when r_i selects b_j as the action object. The authors obtain $F^R=max_{s_*^{R_U}}\left(f^R-f^B\right)$ and $F^B=max_{s_*^{R_U}}\left(f^B-f^R\right)$. When transforming the model into a non-cooperative zero-sum game theory model, the non-cooperative Nash Equilibrium is taken as the optimal strategy, namely $s_*=\left[s_*^{R_U},s_*^{B_U}\right]$, the final WTA formula is as follows.

$$\text{P4}-1\ F^R=\max_{s^{R_U}}\left(\min_{s^{B_U}}\left(f^R-f^B\right)\right),\forall s^{R_U},s^{B_U} \tag{3–70}$$

$$F^B=\max_{s^{B_U}}\left(\min_{s^{R_U}}\left(f^B-f^R\right)\right),\forall s^{R_U},s^{B_U} \tag{3–71}$$

$$\text{s.t.} f^R=\sum_{j=1}^{n_j}\left[1-\prod_{i=1}^{m_i}\left(1-\left(\prod_{j=1}^{n_j}\left(t_j^B\left(M^B\right)\right)^{l_{ji}^B}\right)p_{ij}^B\left(M^B\right)\right)^{k_{ij}^R}\right] \tag{3–72}$$

$$f^B=\sum_{i=1}^{m_i}\left[1-\prod_{j=1}^{n_j}\left(1-\left(\prod_{i=1}^{m_i}\left(t_j^R\left(M^R\right)\right)^{l_{ij}^{RT}}\right)p_{ji}^B\left(M^R\right)\right)^{k_{ji}^B}\right] \tag{3–73}$$

$$\sum_{j=1}^{n_j}\left(k_{ij}^x+l_{ij}^x\right)=1,i=1,2,\cdots,m_i \tag{3–74}$$

$$\sum_{i=1}^{m_i}\left(k_{ji}^y+l_{ji}^y\right)=1,j=1,2,\cdots,n_j \tag{3–75}$$

Formula P4-1 considers the situation of red-blue confrontation. It is a WTA formula considering non-zero-sum game conditions. What sets this formula apart is that it considers not only the defensive side but also incorporates the offensive side's strategy. The author utilizes the Cooperative Nash Equilibrium to determine the optimal strategy. Readers are referred to Cruz, Golany (2015) and Galati's (2007) paper for more details on the use of non-cooperative games in WTA.

In Pan's literature (2019), the attacker and defender are placed in the same position within the game scenario. However, in actual situations, the defender's response plan is likely to be based on the behavior of the attacker as well as the resources available to them, resulting in an asymmetrical game (Atkinson and Kress, 2023). This means that the defender's plan of action is likely to be different depending on the unique circumstances they face. The Stackelberg game formula of the

WTA problem was proposed by Hohzaki and Nagashima (2009), the attacker B is the game leader, and the defender B is the game follower. Assuming that player A's strategy space is D, player B's strategy space is F, and the overall cost function of the game is $C(d,f)$, the Stackelberg game formula for the WTA problem is as follows.

$$\text{P4}-2\underset{f_j\in F}{\text{Min}}\underset{x_{ij}\in D}{\text{max}}C(d,f) \tag{3–76}$$

Formula (3-76) is a simplified version of Hohzaki's formula (2009). The author provided comprehensive descriptions of the game strategy space, feasible domain, and expected loss value of Player A. The transformation and processing of game theory problems are also thoroughly presented. Readers can refer to the original text for more details. Apart from non-cooperative game (Pan et al., 2019) and Stackelberg game (Brown et al., 2005; Haywood et al., 2022), some researchers have employed swarm intelligence and evolutionary game (Leboucher et al., 2013), zero-sum game (Brown et al., 2005; Golany et al., 2015; Han et al., 2016; Hohzaki and Nagashima, 2009) and other game theory methods to describe the WTA problem.

Another WTA problem that is more suitable for real-world scenarios takes into consideration the threat detection of the target. This type of problem is called the TEWA problem, which stands for adversarial WTA. The TEWA problem requires a thorough threat assessment and ranking of incoming targets, which is a critical component of the air and missile defense process. To learn more about the specifics of threat assessment and ranking, readers can refer to the literature listed in Azak and Bayrak (2008), Naeem (2009, 2009), Ma et al. (2015), Li et al. (2017), and Naseem's (2017) references. A basic formula of the TEWA problem can be found in the proposal by Lötter et al. (2013), which is similar to formula P2-5.

$$\text{P4}-3\ \min\sum_{j=1}^{n_j}w_j\prod_{i=1}^{m_i}\left(1-p_{ij}\right)^{x_{ij}} \tag{3–77}$$

$$\min\sum_{i=1}^{m_i}c_{ij}\sum_{j=1}^{n_j}x_{ij} \tag{3–78}$$

$$\text{s.t.}\sum_{j=1}^{n_j}x_{ij}\le 1, i=1,2,\cdots,m \tag{3–79}$$

$$x_{ij}\in\{0,1\}, i=1,2,\cdots,m_i, j=1,2,\cdots,n_j \tag{3–80}$$

In real combat situations, solving the WTA problem requires not only identifying optimal weapon-to-target assignments, but also taking into account the dynamic model of weapons to prevent missile interference during travel. To address this, a mixed integer nonlinear programming formula has been proposed, which integrates the missile dynamics model and the WTA problem. The formula is as follows.

$$\text{P4}-4\ \min_{X,\Gamma^0,\Psi^0} miss\left(\Gamma^0,\Psi^0\right)+\sum_{i=1}^{m_i}x_{ij}r \tag{3–81}$$

$$\text{s.t.}\dot{R_n}=f_n(R_n) \tag{3–82}$$

$$\dot{R_m^i}=f_m\left(R_m^i\right), for\ x_{ij}=1 \tag{3–83}$$

$$\dot{R_m^i}=0, for\ x_{ij}=0 \tag{3–84}$$

In formula (3-82), where $R_n=[p_n,\dot{p_n}]\in R^6$ represents the state vector of the target, $R_m^i=\left[p_m^i,\dot{p_m^i}\right]\in R^6$, $i\in\{1,2,\cdots,m\}$ represents the state vector of the surface-to-air missile, where p represents the position, and $\dot{p}$ represents the velocity. $\Gamma^0=\{\gamma^i(0), i=1,2,\cdots,m\}$ indicates the initial launch pitch angle of the interceptor missile, $\Psi^0=\{\psi^i(0), i=1,2,\cdots,m\}$ represent the heading angle of the intercepting missile, let t_f be the end time of the scene, and define the $miss(\Gamma^0,\Psi^0)=min_{i=1,\cdots,n,t\in[0,t_f]}\left\|p_n(t)-p_m^i(t)\right\|$. The author assumes that the interceptor missile silo is stationary, the target is detected at the radar's end, and the anti-missile system utilizes a closed-loop guidance and control algorithm. As a result, the defender is only required to determine the initial launch conditions.

The objective function described in formula P4-4 aims to minimize both the miss distance and cost of the missile to reach the target. The first constraint ensures that the state vector of the missile adheres to the dynamic model. Meanwhile, the second and third constraints indicate that the dynamic state vector of a target is determined by the assigned weapon. If weapon i is not assigned to target j, the dynamic state vector of target j will be 0. To avoid redundancy, this section summarizes the optimization model of the WTA problem without delving into the details of the dynamic model of the weapon system.

4. Methods

Fundamentally, the WTA problem can be represented as an optimal model, and various solution methods exist, such as exact algorithms, Meta-heuristic algorithms, heuristic algorithms, and machine learning algorithms. In this section, we will review the algorithms utilized for each type of formula, leveraging the formula classification outlined in the previous section.

4.1. Exact algorithm

Lloyd (1986) firstly proved that the WTA problem is NP-complete, making it challenging to solve WTA problem using exact algorithms in large-scale and complex scenarios. As the WTA problem requires timely solutions in real scenarios, the number of exact algorithms available for solving the WTA problem is limited. In particular, the research on exact algorithms for the WTA problem is primarily concentrated on formulas P1 and P2. For the canonical WTA problem, where there are m weapons, n targets, and the launch cost of each weapon is c_{ij}, the optimization goal is typically to minimize the cost of weapon launch. However, with n^m weapon assignment schemes in total, the WTA problem becomes increasingly complex as m and n increase, with the number of distribution schemes growing exponentially (Jin et al., 2023). To address this complexity and enable researchers to solve the WTA problem more smoothly using exact algorithms. Several methods are currently used, including small-scale WTA problems, simplifying constraint complexity, and converting nonlinear formulas into linear ones.

4.1.1. Exact algorithm of P1 formula

Although Formula P1 is relatively simple, it does not accurately reflect real combat situations, which makes it challenging to design precise algorithms that meet practical requirements, resulting in limited research in this area. In response, Bogdanowicz (2009) proposed an accurate algorithm named Swt-opt based on the auction algorithm, which is designed to solve the P1-1 formula. The author assumes that the number of weapons and targets are integers, each weapon is allocated only once, and each target is allocated only one weapon. The author also proved that Swt-opt converges to an optimal solution within a finite number of steps, with an average time complexity of $O\left(n_{max}^{4c}\right)$. Furman (1973) demonstrated how the generalized Lagrangian multiplier method can effectively utilize specific graph structures to solve the WTA problem when minimizing the Lagrangian quantity, with calculation results showing that the gap between the result and the upper bound is within 3%. Li (2004) used linear programming to relax the WTA problem, obtaining an approximate optimal solution of the canonical problem by solving the relaxed formula and verifying its credibility through simulation results. The author's experiments show that the relaxation of the formula produces an optimal solution that is very close to the optimal solution of the canonical problem.

For the solution of formula P1-2, Day (1966) proposed a three-stage modeling solution method. The author decomposes the canonical large problem into a group of smaller orientation problems and a larger orientation problem, forming a nonlinear programming problem for solving. The former uses the sequential optimization method to solve and provides information for the nonlinear programming solution of the latter. Ahuja et al. (2007) proposed an integer programming and a lower bound method based on network flow. They use a branch and bound algorithm to solve the WTA problem. Calculation results show that the algorithm can optimally solve medium-sized WTA problems (up to 80 weapons and 80 targets) and obtain almost optimal solutions for fairly large instances (up to 200 weapons and 200 targets). Soland (1973) assumed a 0–1 implicit enumeration format combined with branch and bound to solve. Miercort (1971) devised a branch-and-bound algorithm for the solution. In addition, Miercort expresses the P1-2 formula as a "subtractive" defense through analysis, which is similar to the Stackelberg game model proposed by Hohzaki and Nagashima (2009). Unfortunately, the author did not provide a more precise description of this analysis. Li (2011) combined the consensus algorithm with the Swt-opt algorithm to ensure the validity of the distribution on a variety of dynamic network topologies. The improved algorithm is also robust in terms of communication, computing, and inconsistent situational awareness.

4.1.2. Exact algorithm of P2 formula

Formula P2 is currently the most commonly used modeling method in the study of WTA problems, and therefore, the exact algorithm plays a significant role in its solution. To this end, Ni et al. (2011) performed Lagrangian relaxation on the P2-2 formula. The author increases the number of targets to make the probability expectation of the target being destroyed discrete, thus eliminating the nonlinearity of the objective function. To deal with this nonlinear conversion method, the author guides the search probability expectation by finding the mapping relationship between the decision variable x_{ij} and the probability expectation that the target is destroyed. The canonical problem is then transformed through Lagrangian relaxation and solved using the simplex method. After analysis, Brown (2005) concluded that since the decision variables belong to the integer field, a new mixed-integer programming model can be created by using the dual of its internal maximization and solved using CPLEX.

Kwon (2007) simplified P2-3 by first transforming the canonical problem's nonlinear integer programming formula into a linear integer programming formula. To further reduce the complexity of the formula, the upper limit of weapon i ammunition is set to 1, the number of ammunitions launched by weapon i in the direction of target j is set to 1, and the large number θ is set to 100. The constraints are then relaxed, and the linear programming formula is converted into a special case of the knapsack problem. A branch pricing algorithm is designed to solve it, and the initial column is generated by a greedy algorithm, allowing for the optimal solution to be obtained on a certain scale of WTA problems. In Kwon's situation (1999), after transforming nonlinear constraints into linear constraints, the Lagrangian relaxation method and branch and bound method were applied to solve the problem. Additionally, an efficient primitive heuristic is proposed to find a feasible solution, which facilitates the solution process.

Formula P2-4, as described by Ma et al. (2015), was expressed as a nonlinear integer programming model. It is proven that the model can be equivalently transformed into a linear integer problem, and the optimal solution is obtained by solving the linear integer problem by CPLEX. Cha and Kim (2010) proposed a branch-and-bound algorithm with the objective of minimizing the total threat to the target. The algorithm expresses the total threat as a function of the target's destruction probability and designs a branch-and-bound (B&B) algorithm for solving the attack scheduling problem of active targets. Both dominance and lower bounds and heuristic algorithms are developed to obtain the initial upper bound of the B&B algorithm and to obtain a good solution in a short time. Computational experimental results demonstrate that the proposed B&B algorithm can find the optimal solution for medium-sized problems in a reasonable CPU time, while the heuristic algorithm can obtain a good solution in a very short time.

Manne (1958) was one of the earliest researchers to model and solve the WTA problem, and he designed an algorithm based on the linear version of formula P2-5 to solve it. Lu (2021) converted P2-5 into an integer linear programming model with two columns and used column enumeration and branch-and-bound techniques to solve it. To reduce the number of columns needing enumeration, they proposed a new approach to border and weapon domination. Their algorithm was able to solve all instances, and it was particularly efficient for accurately solving 80 weapon and 80 target instances in only 0.40s. Moreover, it could solve larger problem instances than previous methods, with the maximum problem size solved being 400 weapons and 400 targets, with an average execution time of 4.68s. Martijn (2020) built on Gould's (1984) multiple salvo formula, time exact algorithms using dynamic programming. And they also show that a related problem is NP-complete. Time exact algorithms using dynamic programming. And they also show that a related problem is NP-complete. Additionally, a nonlinear branch-and-bound algorithm was proposed for solving non-transformed problems by Kline (2019), providing optimal solutions for smaller-scale untransformed SWTA problems. Feghhi (2021) proposed a real-time search algorithm to decompose the WTA problem and provided a real-time exhaustive search algorithm by reducing the solution space and deleting impossible solutions. Lastly, a new exact solution method called the branch adjustment method was proposed by Andersen (2022), involving a compact piecewise linear convex underapproximation of the WTA objective function. The algorithm builds on any existing branch-cut or branch-and-bound algorithm and can handle large-scale problem instances with up to 1500 weapons and 1000 targets.

4.1.3. Exact algorithm of P3 formula

The P3 formula, which takes into account the time restricted, poses significant challenges in terms of timeliness, making it difficult to accurately solve most models. As a result, there are only a few precise algorithms available for solving the P3-2 formula (Karasakal, 2008). To overcome this challenge, the canonical problem is decomposed into two integer linear programming models, which are solved separately. The first model focuses on minimizing the expected probability deviation of the minimum weapon assignment, while the second model introduces an artificial weapon system that can select targets not correctly assigned and aims to minimize its use. The author employs the OSL solver to solve the problem and suggests that designing an algorithm could lead to a more accurate solution.

Lee (2020) proposed an approach to formulate the WTA problem with physical and seeker interference constraints which is solvable in Mixed Integer Linear Programming (MILP). The author discretizes the time window and generate predicted intercept point (PIP) set. To handle interference constraints to the MILP formula, the interference tables that explain which pairs of assignments make interference, are made prematurely before the solver executed.

Hosein and Athans (1990) made the assumption that all targets have equal value, and that the weapon-to-target engagement depends only on the number of phases. According to their model, the number of weapons used at each phase is linearly related to the number of targets. The authors demonstrated that this stochastic problem can be solved by solving a related deterministic problem when the number of targets approaches infinity. To simplify the calculations involved in the transition between stages, the authors simplified the probability of killing, the number of weapons used, and the evolution distribution of weapons and target states at different stages. The author utilized the dynamic programming method to solve the problem.

Through our research on the precise algorithm of the WTA problem, we can clearly see that so far the research on WTA has mainly focused on the solution of the P2 formula. For formula P1, due to its simple problem, the optimal solution of the problem can be easily obtained by using nonlinear transformation, but it is precisely because of the simplification of the problem that the practical value of the formula P1 in actual scenarios is very low. For the formula P3, due to the increase of the time element, the precise solution of the P3 formula has become a huge challenge, and the P3 formula can only be solved by reducing the problem scale or simplifying constraints. The overview of the exact algorithm for solving the WTA problem is shown in Table 2.

4.2. Heuristic algorithm

A heuristic algorithm is a problem-solving approach that uses rules of thumb, intuition, or practical experience to guide the search for a solution. Heuristic algorithms aim to find a reasonably good solution in a more efficient manner, especially for complex or large-scale problems where finding an optimal solution is computationally expensive or impractical. Therefore, many researchers use heuristic algorithms to solve the WTA problem (Gallagher and Kelly, 1991).

In addition to meta-heuristic algorithms, researchers have designed corresponding heuristic algorithms tailored to the unique characteristics of WTA problems. For instance, Xin (2019) established a mathematical model for cooperative sensor and weapon configuration and designed an edge-return construction heuristic algorithm (MRBCH). Ma et al. (2021) proposed a two-stage hybrid heuristic search algorithm for uncertain WTA problems based on prior knowledge.

Zhu (2022) proposed a two-level dynamic anti-missile WTA model based on rolling horizon optimization and marginal benefit re-planning, along with an improved two-level recursive BBO algorithm based on hybrid migration and variation to analyze static and dynamic battlefield environments quickly. Hosein et al. (1988; 2002, 1990) and Kwon (1997) designed suboptimal solution heuristics for WTA formulas, while Li (2008) proposed an optimal matching method between sensors and weapon systems based on geometric programming. Ma et al. (2015) defined a new approximate subproblem based on P2-5 and designed a feasible descending iterative scheme for finding a suboptimal solution. Pryluk et al. (2013; 2016) proposed an iterative algorithm based on *Shoot-Look-Shoot*. Zhang (2020) established a A-DWTA model based on a receding horizon decomposition strategy (A-DWTA/RH), and

Table 2
Exact algorithm of WTA.

Researchers	Year	Method & Tools	Formula
Furman (1973)	1973	Lagrangian multiplier	P1-1
Li (2004)	2004	Linear programming	P1-1
Day (1966)	1966	Sequential optimization	P1-2
Ahuja et al. (2007)	2007	Branch and bound algorithm	P1-2
Soland (1973)	1973	Branch and bound algorithm	P1-2
Miercort (1971)	1971	Branch and bound algorithm	P1-2
Bogdanowicz (2009)	2009	Swt-opt algorithm	P1-2
Ni et al. (2011)	2011	Simplex method	P2-2
Brown (2005)	2005	CPLEX solver	P2-2
Kwon (2007)	2007	Branch and bound algorithm	P2-3
Kwon (1999)	1999	Lagrangian relaxation & branch and bound algorithm	P2-3
Ma et al. (2015)	2015	CPLEX solver	P2-4
Cha and Kim (2010)	2010	Branch and bound algorithm	P2-4
Manne (1958)	1958	Linear programming with variables transformation	P2-5
Lu (2021)	2021	Column enumeration and branch-and-bound algorithm	P2-5
Kline (2019)	2019	Branch and bound algorithm	P2-5
Feghhi (2021)	2021	Real-time exhaustive search algorithm	P2-5
Martijn (2020)	2020	Time exact algorithms using dynamic programming	P2-5
Andersen (2022)	2022	Branch adjustment algorithm	P2-5
Hosein and Athans (1990)	1990	Dynamic programming algorithm	P3-1
Karasakal (2008)	2006	OSL solver	P3-2
Lee (2020)	2020	MILP solver	P3-3
Chang (2023)	2023	CPLEX solver	P3-3

simultaneously designed a heuristic algorithm based on statistical marginal returns (HA-SMR). Han et al. (2016) constructed a three-level nonlinear integer formula of "defender-attacker-defender" and designed a self-heuristic game tree search to solve it. Naeem (2009) proposed a many-to-many stable combination algorithm for weighted proposals, while Metler (1990) developed the multi-constrained Weapon Assignment and Resource Management (WARM) algorithm. Liles (2023) developed a Markov Decision process (MDP) model of a stochastic dynamic assignment problem, similar to the WTA problem, in which a UAV is assigned to an incoming cruise missile, approximate policy iteration with least squares temporal differences (API-LSTD) is used to learn to find high-quality solutions. Summers et al. (2020) formulated the WTA problem as a Markov decision process and utilized a simulation-based Approximate Dynamic Programming (ADP) method to solve problem instances based on typical scenarios. Ni et al. (2011) also proposed an efficient rule-based heuristic algorithm for solving the WTA problem. Finally, Silav (2019) proposed the NRH (new and replace heuristic) algorithm, the CEH (Change and exchange heuristic) algorithm, and the interference redistribution algorithm to solve the dual-objective WTA problem, while Karasakal (2011) and Burr (1985) developed heuristic algorithms based on greedy construction.

Various heuristic algorithms have been developed for solving the WTA problem, each designed with unique characteristics in mind. While Griffiths (1991) did not explicitly model the WTA optimization problem, the author considers the damage probability of the weapon to the target as a probability distribution, rather than a fixed value. Consequently, the author used simulation methods and historical combat data to obtain partial analytical solutions. Lirov et al. (1989) developed an algorithm based on the discretization of differential games, which formed an assignment game tree to choose the best assignment. Golany et al. (2015) designed a zero-sum game model and solved its Nash equilibrium. Murphey (2000) proposed an algorithm for approximately solving the dynamic WTA problem under the unified weapon assumption. Acar (2023) proposed a quantum algorithm for WTA problem, which effectively accelerates the search of the solution space and improves the solution speed. Liu (2021) proposed the modified Estimate of Distribution Algorithm (EDA) to solve the target assignment problem. Şahin (2014) designed a fuzzy network decision-making method to assist commanders in making weapons assignment decisions. Bogdanowicz (2012, 2013) proposed algorithms based on depth-first search and intelligent pruning assignment. Mekawey et al. (2009) proposed the fusion objective optimization method with the Hungarian algorithm to solve the WTA problem. Nguyen (1997) designed a generating function algorithm by analyzing the combat model. Bin and Jie (2012) proposed a distribution estimation algorithm (EDA) based on binary codes. Leboucher et al. (2013) used the binomial distribution to simulate various probabilities in the WTA problem, and used the continuous Hungarian algorithm to solve it. Dionne (2008) proposed the application of a sequential optimization technique to the problem of dynamic assignment of weapons targets, reducing the search space and inherently providing elegant degradation. Naeem and Masood (2010) developed a Stable Marriage Algorithm (SMA)-based algorithm for dynamic multi-air threat assessment and weapon assignment optimization. Newman (2011) proposed a heuristic algorithm for multi-objective optimization in which queues are supplemented and reordered at allocation time, replacing locally small increments with larger allocation scheme changes to improve the solution.

According to our survey, heuristics are relatively less used than meta-heuristics in solving the WTA problem. It can be seen from the literature analysis that the use frequency of heuristic algorithms in solving WTA problems is increasing year by year, and different from meta-heuristic algorithms, researchers will design corresponding heuristic algorithms according to the characteristics of WTA problems when designing heuristic algorithms. strategy, so it is more suitable for solving WTA problems. The literature overview on the heuristic algorithm for solving the WTA problem is shown in Table 3.

Table 3
Heuristic algorithm of WTA.

Researchers	Year	Algorithm
Arslan (2024)	2024	2-Stage heuristic algorithm
Bai (2024)	2024	Pareto-Optimal-Matching
Liles (2023)	2023	Approximate dynamic programming
Hendrickson (2023)	2023	Distributed primal-dual optimization algorithm
Zhu (2022)	2022	BBO algorithm
Ma et al. (2021)	2021	Two-stage hybrid heuristic search algorithm
Liu (2021)	2021	Modified estimate of distribution algorithm
Shi et al. (2021)	2021	Sparse Evolutionary algorithm
Zhang (2020)	2020	Statistical marginal returns algorithm
Summers et al. (2020)	2020	Simulation-based approximate dynamic programming
Xin (2019)	2019	Edge-return construction heuristic algorithm
Silav (2019)	2019	New and replace heuristic algorithm
Han et al. (2016)	2016	Self-heuristic game tree search algorithm
Pryluk et al. (2013, 2016)	2016, 2013	Iterative heuristic algorithm
Golany et al. (2015)	2015	Heuristic algorithm based on Nash equilibrium
; ,)Hosein et al. (1988; 1990; 2002a), Kwon (1997), and Ma et al. (2015a)	1990, 2002, 1988, 1997, 2015	Suboptimal solution heuristics
Şahin (2014)	2014	Fuzzy network decision-making method
Leboucher et al. (2013)	2013	Continuous Hungarian algorithm
Bogdanowicz (2012, 2013)	2012, 2013	Depth-first search and intelligent pruning algorithm
Bin and Jie (2012)	2012	Distribution estimation algorithm (EDA) based on binary codes
Ni et al. (2011)	2011	Rule-based heuristic algorithm
Karasakal et al. (2011) and Burr (1985)	2011, 1985	Greedy construction heuristic algorithm
Naeem (2010)	2010	Stable marriage algorithm
Newman (2011)	2011	Reprogramming heuristic algorithm
Mekawey et al. (2009)	2009	Hungarian algorithm
Naeem (2009)	2009	Many-to-many stable combination algorithm
Li (2008)	2008	Geometric programming
Dionne (2008)	2008	Sequential optimization algorithm
Murphey (2000)	2000	Minimum marginal return algorithm
Nguyen (1997)	1997	Generating function algorithm
Metler (1990)	1990	Multi-constrained heuristic algorithm
Lirov et al. (1989)	1989	Heuristic algorithm based on Differential game

4.3. *Meta-heuristic algorithm*

Meta-heuristic algorithm is a higher-level algorithmic framework that can be used to solve optimization problems, such as Vehicle Routing Problems (VRP), Knapsack Problems, and WTA problems (Larson and Kent, 1994). It is a general algorithmic approach that guides the search for solutions in a problem space. Unlike specific algorithms designed for particular problem domains, meta-heuristic algorithms are versatile and can be applied to a wide range of problems. This section will introduce meta-heuristic techniques for solving the WTA problem from the perspective of meta-heuristic algorithms, which including genetic algorithm, particle swarm optimization algorithm, ant colony algorithm, and others.

4.3.1. *Genetic algorithm*

Given the unique characteristics of the WTA problem, meta-heuristic algorithms play a crucial role in its successful resolution (Pepyne et al., 1997). Among these algorithms, Genetic Algorithm (GA) has emerged as the most widely used method for addressing various WTA problems. Pepyne et al. (1997) utilized GA to solve WTA problems in theater missile defense. Song et al. (2009) transformed the nonlinear integer programming formula of the WTA problem into a linear formula and employed GA to solve it. Khosla (2006) used a fusion of GA and simulated annealing algorithms to tackle the WTA problem. Li (2015), Lötter et al. (2013), and Li (2018) designed a non-dominated sorting GA for the WTA problem in ground air defense. Standard GA has been used by Julstrom (2009), Lu (2013), Li (2010), Malhotra (2001), Bayrak (2013), Dou (2009), Lee (2003), Lu (2006), Erdem (2003), Johansson (2010a), Wu (2008, 2008), Luo (2005), Li (2009) and others to address the WTA problem.

4.3.2. *Particle swarm optimization algorithm*

The WTA problem can be tackled well using the particle swarm optimization algorithm, as demonstrated by several researchers. Zhou et al. (2016) and utilized the discrete particle swarm optimization algorithm, while Peng et al. (2016) employed a hybrid multi-objective discrete particle swarm optimization algorithm to solve the problem in air combat. Yang et al. (2018) applied the multi-information particle swarm optimization algorithm to the multiple kill Vehicle weapon assignment problem, and Kong (2021) utilized an improved multi-objective particle swarm optimization algorithm to solve formulas P2 and P3. Bo (2011) used a hybrid particle swarm optimization algorithm to solve the WTA problem, while Wang (2012) adopted a fusion particle swarm optimization method that combines local search and global search, while He (2022) solved the WTA problem of Spatial Crowdsourcing Mode using the heuristic variable weight nonlinear learning factor particle swarm optimization algorithm. Liu (2009) devised a novel approach by combining a heuristic particle swarm optimization algorithm with a decentralized cooperative auction algorithm. Finally, Teng (2008) applied the traditional particle swarm optimization algorithm to solve the WTA problem.

4.3.3. *Ant colony algorithm*

The ant colony algorithm plays a significant role in solving the WTA problem (Gao et al., 2010). Researchers like Rezende et al. (2018), Wang (2008), Gao et al. (2010), and Lee (2002) have used traditional ant colony algorithms to tackle this problem. Additionally, Li (2017) improved upon the traditional ant colony optimization algorithm by using a Pareto ant colony optimization algorithm to solve the dual-objective WTA problem. Rezende et al. (2018) also combined the ant colony algorithm with the greedy algorithm, adopting a multi-ant colony parallel strategy to improve optimization results.

4.3.4. *Other meta-heuristic algorithm*

In addition to the three aforementioned algorithms, numerous meta-heuristic algorithms have been utilized to tackle the WTA problem. Shi et al. (2021) devised an evolutionary algorithm to solve large-scale sparse WTA problems, while Li (2017) employed a parallel differential evolution algorithm to resolve the WTA decision in C2. Lai (2019) created a population optimization algorithm for the P3 formula of WTA. Li (2018), Chang (2018), and Wang (2023) utilized a factorized multi-objective evolutionary algorithm (MOEA/D) to solve the WTA problem. In comparison, Li et al. (2015) contrasted the performance of adaptive NSGA-II (ANSGA-II) and adaptive MOEA/D (AMOEA/D) algorithms in multi-objective WTA problems, demonstrating that the ANSGA-II algorithm surpasses the AMOEA/D algorithm. Durgut (2017), Chang (2018), Kong (2019) and Wang (2023) employed the artificial bee colony algorithm to solve the WTA problem. Lee (2010) and Chang (2023) developed an improved very large-scale neighborhood search algorithm to solve the constrained WTA problem, while Pan et al. (2019)

proposed a new uncertainty adversarial game WTA model, which was resolved by a decomposition coevolutionary algorithm. Xin (2010) employed the tabu search algorithm to solve WTA problems with various constraints. Liang (2016) created an adaptive chaotic parallel clone selection algorithm for resolving the air defense WTA problem, and Li (2012) designed an arbitrary-time algorithm based on decentralized cooperative auctions. Xu (2020) established a dual-objective multi-stage task allocation model for sensor weapon task allocation, and used the improved MOEA/D framework to solve it. Kline (2020) proposed a mode of switching between greedy and ED heuristics and selecting the algorithm with better results during the optimization stage. Additionally, Mu (2017) introduced a multi-scale quantum harmonic oscillator algorithm to solve the WTA problem, whereas Rosenberger (2005) and Bogdanowicz and Coleman (2007) devised an auction algorithm for the nonlinear WTA problem. Galati (2007) proposed a neighborhood search algorithm for unit-level team resource assignment (ULTRA), and Orlin (1987) transformed the canonical nonlinear consensus of WTA into a linear formula, which was then transformed into a network flow with minimum cost, and resolved by the network flow-attrition algorithm.

4.3.5. *Hybrid meta-heuristic algorithm*

Several researchers have proposed integrating multiple meta-heuristic algorithms to solve the WTA problem. For instance, Lee (2003) proposed an improved genetic ant colony optimization algorithm. Johansson (2010; 2022), and Wang (2012) proposed a particle swarm optimization algorithm based on genetic operators. Liu (2022) proposed an adaptive simulated annealing-particle swarm optimization (SA-PSO) algorithm by introducing the simulated annealing algorithm into the adaptive PSO algorithm. Li (2016) suggested a multi-objective evolutionary algorithm integrating DMOEA-εC, NSGA-II, and MOEA/D-AWA (Juan et al., 2017) adaptive weight adjustment (MOEA/D-AWA).

Ghanbari (2021) employed the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) and Strength Pareto Evolutionary Algorithms (SPEA-II) to solve a multi-objective multi-stage Resource Allocation problem.

It can be seen from the literatur e survey that the meta-heuristic method occupies a considerable proportion in solving WTA problems and can solve the four types of WTA formulas given in this paper. Among them, GA, PSO and AC algorithm are commonly used meta-heuristic methods to solve WTA, and the combination of GA, PSO and AC algorithm may obtain better solution results. In addition, the performance of other meta-heuristic algorithms is also outstanding, and it can be seen from literature research that the combination of multiple meta-heuristic algorithms will make up for the defects of their respective algorithms, and may obtain unexpected high-quality solutions. An overview of meta-heuristic algorithms for solving the WTA problem is shown in Table 4.

Table 4
Meta-heuristic algorithm of WTA.

Algorithm	Researchers	Number
GA algorithm	Pepyne et al. (1997), Song et al. (2009), Khosla (2006), Li (2018), Julstrom (2009), Lu (2013), Dou (2009), Malhotra (2001), Bayrak (2013), Lee (2003), Lu (2006), Erdem (2003), Johansson (2010), Wu (2008, 2008), Luo (2005) and Li (2009)	16
PSO algorithm	Zhou et al. (2016), Peng et al. (2016), Yang et al. (2018), Kong (2021), Bo (2011), Wang (2012), He et al. (2022), Liu (2009) and Teng (2008)	9
AC algorithm	Rezende et al. (2018), Gao et al. (2010), Wang (2008), Lee (2002) and Li (2017)	5
NSGA-II	Zhao (2023), Li et al. (2015), Li et al. (2015) and Lötter (2013),	4
MOEA/D	Yi (2024), Li et al. (2018) and Xu (2020)	3
Bee colony algorithm	Durgut (2017), Chang (2018) and Wang (2023)	3
Auctions algorithm	Li et al. (2012), Rosenberger (2005), and Bogdanowicz (2007)	3
DE algorithm	Li et al. (2017) and Kline (2020)	2
Evolutionary algorithm	Shi et al. (2021)	1
Population optimization algorithm	Lai (2019)	1
NSGA-III	Li (2010)	1
VLNS	Chang (2023) and Lee (2010)	1
Decomposition coevolutionary algorithm	Pan et al. (2019)	1
Clone selection algorithm	Liang (2016)	1
Multi-scale quantum harmonic oscillator algorithm	Mu (2017)	1
GA + PSO	Johansson (2010b, 2022) and Wang (2012)	3
DMOEA-εC + NSGA-II + MOEA/D-AWA	Li (2016, 2017)	2
MOPSO + NSGA II	Tunga (2024)	1
NS algorithm	Galati (2007)	1
Network flow-attrition algorithm	Orlin (1987)	1
GA + AC	Lee (2003)	1
SA + PSO	Liu (2022)	1
NSGA-II + SPEA-II	Ghanbari (2021)	1

4.4. *Machine learning solution*

Besides traditional heuristic and meta-heuristic algorithms, artificial intelligence methods have proven to be effective in solving WTA problems. Sahin and Leblebicioglu (2009) proposed a standard expert system for a specific WTA problem, which uses meshing on the training set to build an expert system.

Chang et al. (1987) developed an iterative linear network programming algorithm for solving the WTA problem. Wei (2008) designed a multi-objective optimal game-theoretic resource management method that combines Pareto game theory and machine learning. Lee (2010) and Dağdeviren (2009) combined Analytic Hierarchy Process (AHP) and Principal Component Analysis (PCA) in a mixed approach to weapon system selection, which determined the assignment of decision weights.

As computer computing power has improved, machine learning techniques - especially deep neural networks - have gained attention as effective solutions for combinatorial optimization problems, especially on large-scale problems (Wang et al., 2022). Researchers such as Karasakal (2021), Su et al. (2012), Xie (2006), Bertsekas et al. (2000) and Altinoz (2021) have all utilized artificial neural network methods in their respective algorithms. Additionally, due to the WTA problem's characteristics, it can be modeled as a Markov decision process (Summers et al., 2020), and as such, researchers like Azak and Bayrak (2008), Luo et al. (2016), Guo et al. (2022), Shokoohi et al. (2022) and Liu (2022) have employed the reinforcement learning Q-Learning algorithm to solve the WTA problem. Luo et al. (2021) proposed a data-driven policy optimization with deep reinforcement learning (PODRL) for solving WTA problem. Liu (2023) proposed a multi-agent based deep reinforcement learning method for solving WTA problems.

From the literature survey, there are relatively few machine learning methods to solve WTA problems. Nevertheless, a promising trend observed in the literature indicates a growing interest in machine learning approaches for addressing WTA problems. Among the current machine learning methods, neural networks and reinforcement learning techniques have emerged as primary strategies, effectively handling the time constraints outlined in formulas P3 and P4. Table 5 presents a comprehensive overview of the literature regarding machine learning solutions for the WTA problem.

By reviewing the literature on WTA solving methods, we have obtained the advantages and limitations of four types of methods for solving WTA problems. The literature overview is shown in Table 6. It

Table 5
Machine learning method of WTA.

Researchers	Year	Algorithm & Method	Classification
Merkulov (2024)	2024	Reinforcement Learning as well as a Greedy Search Algorithm	Reinforcement learning
Heon (2024)	2024	Reinforcement Learning and Graph Neural Networks	Reinforcement learning
Kong (2024)	2024	Twin-delayed Deep Deterministic Policy Gradient (TD3) Algorithm	Reinforcement learning
Zhang (2024)	2024	Deep Q-network with Graph Neural Network	Reinforcement learning
Dabholkar (2024)	2024	Asynchronous advantage actor-critic and proximal policy optimization	Reinforcement learning
Zuo (2024)	2024	Multi-agent deep deterministic policy gradient	Reinforcement learning
Wang (2023), Meng (2021)	2023, 2021	Deep Q-Network	Reinforcement learning
Gaudet (2023), Liu (2023)	2023	Proximal policy optimization	Reinforcement learning
Li (2023)	2023	Deep Deterministic Policy Gradient algorithm	Reinforcement learning
Byun (2023)	2023	Recurrent Neural Network-Based Actor-Critic	Reinforcement learning
Na (2023)	2023	Reinforcement Learning with Pointer Network	Reinforcement learning
Li (2023)	2023	Deep Deterministic Policy Gradient algorithm with dual noise and prioritized experience replay	Reinforcement learning
Liu (2023)	2023	Proximal policy optimization for task assignment of general and narrow agents (PPO-TAGNA)	Reinforcement learning
Shokoohi et al. (2022)	2022	MA-DDCOP method	Reinforcement learning
Liu (2022), Lai (2023) and Fu (2023)	2022, 2023	Hierarchical reinforcement learning	Reinforcement learning
Guo et al. (2022)	2022	Epsilon-Greedy Q-learning	Reinforcement learning
Altinoz (2021)	2021	Artificial neural network method	Neural networks
Karasakal et al. (2021)	2021	Artificial neural network method	Neural networks
Wang (2023), Zhang (2021)	2021	Multi-agent reinforcement learning	Reinforcement learning
Luo et al. (2021)	2021	Data-driven policy optimization with deep reinforcement learning	Reinforcement learning
Luo et al. (2016)	2016	Q-Learning algorithm	Reinforcement learning
Su et al. (2012)	2012	SMO algorithm with Artificial neural network method	Neural networks
Lee (2010) and Dağdeviren (2009)	2010, 2009	Analytic Hierarchy Process & Principal Component Analysis	Machine learning
Sahin (2009)	2009	Expert system	Artificial intelligence
Carling (2009)	2009	Knowledge base	Artificial intelligence
Azak and Bayrak (2008)	2008	Q-Learning algorithm	Reinforcement learning
Wei (2008)	2008	Pareto game theory and machine learning	Machine learning
Xie (2006)	2006	Preponderant-function with neural network method	Neural networks
Bertsekas et al. (2000)	2000	neuro-dynamic programming	Reinforcement learning
Chang et al. (1987)	1987	Network programming algorithm	Neural networks

can be seen from Table 5 that the popular methods for solving the WTA problem with precise algorithms are mainly branch and bound algorithms and solvers. Although the exact algorithm can give the optimal solution or near-optimal solution of the WTA problem, there are problems of too long solution time and small scale. The meta-heuristic algorithm can give a feasible solution to the problem within an acceptable time, but when designing the algorithm, it is necessary to further modify the strategy according to the model to deal with the specific WTA problem. At present, the heuristic algorithms commonly used on WTA issues are mainly search-based and rule-based heuristics. Compared with the meta-heuristic algorithm, the heuristic algorithm is more suitable for the WTA model, so the generality of the algorithm is poor. The machine learning method for solving WTA has appeared very early, but it has developed rapidly in recent years, and the trained model has a good effect on solving timing problems. Since machine learning methods are sensitive to data, massive amounts of data are required for algorithm model training, and the time cost is high.

5. WTA applications

WTA problems are widespread in actual combat situations, especially in defense scenarios. In response to the threat posed by enemy missiles, numerous countries, research institutions, and researchers have developed missile defense systems, weapon distribution kits, and combat resource planning software for intercepting enemy missiles for various purposes (Anderson and Hong, 2008). The missile defense system is designed to detect, track, intercept, and destroy incoming missile. Although it was initially developed as a defense against nuclear-armed intercontinental ballistic missiles (ICBMs), it has since expanded its application to include short-range non-nuclear tactical and theater missiles.

5.1. WTA products developed by countries or regions

In order to ensure their own territorial security, many countries have developed their own air defense and anti-missile weapon systems. The United States renamed the National Missile Defense (NMD) to the Ground-Based Midcourse Defense (GMD) in 2002. This system is used to intercept incoming warheads in space during the mid-course of ballistic flight. The GMD system includes the Exoatmospheric Kill Vehicle (EKV) developed by Raytheon, the Ground-Based Interceptor (GBI) developed by Orbital Sciences and other equipment. Israel has a wealth of anti-aircraft and anti-missile weapon systems capable of dealing with a wide range of missiles and air threats, including the Arrow series, David's Sling, Barak 8 Iron Dome, and Iron Beam etc. Russia is one of the first countries to develop air defense and anti-missile weapon systems. The A-135 developed by Russia is currently deployed near the national capital, Moscow, to protect major cities in Russia.

WTA is a crucial component of combat mission planning and decision-making. Over the years, many applications and systems have integrated WTA into comprehensive decision support systems. One of the earliest examples is the Semiautomatic Ground Environment system (SAGE) proposed by the U.S. Air Force and the Massachusetts Institute of Technology in 1957 for air defense operations. Since then, decision-making systems for air defense and anti-missile operations have gradually adopted the ideas of SAGE (Everett et al., 1957). In 1979, J. R. Simpson described the Office of Naval Research's Operational Decision Aid Program, which integrated the Air Strike Timing Decision Aid, Route Planning, Electronic Warfare aid, and Strike Outcome Calculator to form a comprehensive combat action plan decision support system (Glenn and Zachary, 1979). American President Lines (APL) and several other agencies developed the Air-Directed Surface-to-Air Missile (ADSAM) System in 1995 for the System Analysis and Simulation of Air and Missile Defense.

In 1996, Wilkins developed an AI Planner called SOCAP for joint military action plans, capable of generating large-scale military

Table 6
The method's Advance and limitation of WTA in this paper.

Method	Number	Popular method	Superiority	Limitation
Exact Algorithm	21	B&B (Ahuja et al., 2007; Soland, 1973; Miercort and Soland, 1971; Kwon et al., 2007; Cha and Kim, 2010; Kline et al., 2019), solver (Brown et al., 2005; Ma et al., 2015a,b; Karasakal, 2008; Lee et al., 2020)	Optimal/near optimal solution can be found	Only small scale or simple scenario can be solved
Meta-heuristic Algorithm	96	GA (Pepyne et al., 1997), (Song et al., 2009; Lötter et al., 2013; Lee et al., 2003), PSO (Zhou et al., 2016; Peng et al., 2016)	Methods can efficiently explore the search space and are not problem-specific	Methods have different performance in different conditions, and require further modification for specific problem
Heuristic Algorithm	29	Rule-base (Ni et al., 2011; Golany et al., 2015; Ma et al., 2021; Li et al., 2008), search-base (Pryluk et al., 2013, 2016; Ma et al., 2021; Summers et al., 2020; Han et al., 2016), match-base (Leboucher et al., 2013; Naeem and Masood, 2010; Mekawey et al., 2009; Naeem et al., 2009a,b)	Methods are easily implemented, and have relatively short computational time	Methods are specifically designed and no guarantee for solution quality
Machine Learning	25	Neural networks (Altinoz, 2021; Karasakal et al., 2021; Su et al., 2012; Qi-feng et al., 2006; Bertsekas et al., 2000), Reinforcement learning (Shokoohi et al., 2022; Liu et al., 2022; Guo et al., 2022; Luo et al., 2016, 2021)	Learning from data without human interference	Limited interpretability and requiring sufficient data

operations including WTA tasks for joint operations (Wilkins and Desimone, 1996). Liao integrated case-based reasoning (CBR) with a decision support system and designed the Standard Operation Procedure (SOP) in 1999 for military operations and training (Liao, 2000).

In 2008, Tenenbaum proposed Spatially Produced Airspace Routes from Tactical Evolved Networks (SPARTEN), a multi-objective coordinated mission planning tool in operations. In 2010, Benaskeur (2010) designed the COmbat Resource Assignment Support (CORALS), which is used to support the command team of the defending force to optimally plan operations against a range of threats using centralized planning to rationally plan combat missions. In 2024, Ahmad proposed an Intelligent Decision Support System for Ground-Based Air Defense (GBAD) environments, which consist of Defended Assets (DA) on the ground that require protection from enemy aerial threats (Ahmad et al., 2024).

5.2. WTA products developed by institutions and researchers

Apart from the research and development of air defense and anti-missile systems by various government agencies, many research institutions and researchers have also designed corresponding systems. For instance, Mustafa proposed the Threat Evaluation Weapon Assignment System (TEWAS), which uses artificial neural network and reinforcement learning technology to perform threat assessment and WTA (Azak and Bayrak, 2008). Sahin and Leblebicioglu (2009) designed a standard expert system for Turkey's Advanced Technologies Research Institute (ILTAREN), which solves the WTA problem by constructing meshing on the training set. AT&T Bell Laboratories developed the Weapon Assignment and Resource Management (WARD) system for the Naval Research Laboratory to solve WTA problems in conventional combat, swarm combat, and non-swarm scenarios (Johansson and Falkman, 2010a,b,c). Sumanta (2019) developed a WTA module based on the Belief-Desire-Content (BDI) structure for the Integrated Air Defenses system of the Defense Research and Development Organization of India, which can automatically detect and track targets and distribute weapons in an integrated mode to eliminate threats.

In 2010, Johansson and Falkman (2010a,c, 2022) developed the System for Weapon Assignment Research & Development (SWARD) system, which is used in a simulation environment for verifying WTA algorithms. Based on the SWARD system, Johansson designed various algorithms, including the greedy maximum marginal revenue, genetic algorithm, particle swarm algorithm, and other versions, to solve WTA problems.

In 2003, Cruz proposed Strategies for Human-Automaton Resource Entity Deployment (SHARED) and Team Dynamics and Tactics (TDT) for mission planning of military operations (Cruz et al., 2003). Gonsalves (2004) proposed the Software Toolkit for Optimizing Mission Plans (STOMP) for mission planning in 2004, which rapidly generates, analyzes, and visualizes mission plans and provides necessary interfaces and connections for an air force C2 system and a synthetic battlespace environment.

In 2008, Tenenbaum proposed Spatially Produced Airspace Routes from Tactical Evolved Networks (SPARTEN), a multi-objective coordinated mission planning tool for multi-objective planning in operations. Benaskeur (2010) designed the Combat Resource Assignment Support (CORALS) in 2010, which is used to support the command team of the defending force in optimally planning operations against a range of threats using centralized planning to rationally plan combat missions. In 2024, Ahmad proposed an Intelligent Decision Support System for Ground-Based Air Defense (GBAD) environments, which consist of Defended Assets (DA) on the ground that require protection from enemy aerial threats (Ahmad et al., 2024).

6. Potential future research directions for WTA

Although research on the WTA problem has reached an impressive state over the last few decades, research on WTA problem has reached an impressive state, there are still many open problems and new application areas are continually emerging for the problem. Here, we outline a few of potential future research directions for the WTA problem.

6.1. The WTA problem model considering the large-scale uncertain scenarios

At present, the scope of academic research is still relatively limited, typically addressing regional conflicts or local skirmishes. In the 2022 conflict between Russia and Ukraine, and the 2024 conflict between Iran and Israel, cruise missiles and rocket missiles have become the main offensive weapon, and both sides used thousand of missiles to penetrate and intercept to reach their goals. As for the possibility of a larger bilateral war in the future, it is clear that the use of missiles could be the key to victory. Both offensive and defensive systems must have the ability to bear large-scale WTA in order to maintain an advantage in the war. Secondly, with the continuous maturity of electronic jamming, cyber-attacks and other means, it is difficult for both sides to fully perceive the battlefield situation. The existence of the fog of war has greatly interfered with the WTA's decision-making direction. If decisions are made without considering the enemy's intentions, it is likely to result in a poor combat plan. Therefore, it is necessary to analyze and speculate on uncertain scenarios in order to make decisions on the optimal combat plan under restricted conditions. Future research on WTA modeling will need to focus on handling large-scale uncertainty.

6.2. Model construction and machine learning methods in complex confrontation scenarios

Considering the limitations of the current WTA model, even the P4 formula mentioned in this article is difficult to describe the weapon target assignment in actual combat accurately. Therefore, in order to build a real WTA model more accurately, more relevant constraints need to be added. The battlefield environment is exceptionally intricate, characterized by the fog of war and game confrontations. It operates in real-time and is highly confrontational. With the situation evolving unpredictably over time. In such scenarios, traditional optimization algorithms may struggle to cope, leading to exceedingly complex models that are difficult to solve. Fortunately, model-free reinforcement learning offers an effective solution to this problem. This approach doesn't necessitate modeling the environment instead, it requires the agent to interact with the environment to gather information and make decisions. Therefore, as the complexity of the model increases, machine learning may become one of the methods that can effectively solve complex WTA problems in the future. AI technology finds extensive applications in the realm of US military decision-making. Companies like Anduril, through their development of Lattice systems, focus on integrating decisions for complex combat scenarios. They utilize deep learning models to offer decision points and recommend solutions for combat missions. Concurrently, the US Air Force has been progressively enhancing the ABMS system, by introducing the Cloud Base Command and Control (CBC2) system. This integration of artificial intelligence enables AI-driven battlefield intelligence analysis, real-time data feeding, early warning, and threat detection. These advancements showcase AI's capability to handle autonomous decision-making in intricate, dynamic scenarios, presenting a broader solution for the complex models of WTA problem.

6.3. The WTA problem model considering the game confrontation between two sides

To prevent adversaries from anticipating their actions, both sides of the war employ various methods of deception, elicitation, and camouflage. Therefore, game theory is one of the key factors that must be considered in the confrontation between the two sides. The performance of the WTA problem is mainly reflected in the real and false targets and information asymmetry. The attacker needs to predict and evaluate the opponent's defense strategy and formulate real and false battle plans. The defender needs to implement camouflage and defense strategies to achieve the effect of deceiving the attacker as much as possible. In addition, given the complexity of the game confrontation of the WTA problem, researchers may need to utilize combat simulation such as Extended Air Defense Simulation (EADSIM), Modern Air Combat Environment (MACE), and Joint Theater Level Simulation - Global Operations (JTLS-GO) to simulate real battlefield scenarios to compensate for the impact of simplified models on combat space, accuracy and reliability, and further promote the engineering application of WTA models and algorithms. Therefore, WTA solutions need to have faster response capabilities and stronger game countermeasure ability.

6.4. A distributed decision-making framework for WTA problems in joint all-domain operations (JADO)

Joint all-domain operations effectively combine the combat forces of various domains, so that the combat forces can carry out joint operations across domains. Joint all-domain operations mainly includes sea, underwater, land, air, network, space, electromagnetic and other fields. Due to the complexity of the joint all-domain system, if the traditional centralized decision-making method is used, traditional centralized decision-making methods require extremely high data processing volumes and computing power at the command-and-control core, and the risk of complete loss of command-and-control ability is accompanied by the destruction of the command entity. Therefore, to alleviate the data and calculation pressure on command-and-control center, and enhance the robustness of command-and-control level, future all-domain operational planning may evolve into a distributed decision-making framework. So that each domain or node completes task planning and resource scheduling independently without the intervention of command-and-control center. The distributed operational framework also improves the performance of the solution methods for joint all-domain operational task planning and resource scheduling, which makes the solution process do not need to understand the global situation to solve the planning and scheduling problem.

6.5. The solution technology of artificial intelligence and traditional optimization algorithm fusion for WTA problem

With the rapid development of artificial intelligence technology in recent decades, AI has played a significant role in the field of combat. However, machine learning methods have high requirements for input samples and training, and in some scenarios, they may need to be completely reconstructed to meet new combat requirements. Traditional optimization methods, especially meta-heuristic algorithms, can solve combat problems in the form of a general framework, but there are still challenges such as falling into local optimal solutions and difficult balancing solution accuracy and speed. At the same time, the meta-heuristic algorithm requires a discretized description of the problem. It is very difficult to make phased decisions at what scale when observing the battlefield situation, but this is also one of the issues worthies of study. Therefore, by combining the advantages and disadvantages of machine learning and traditional optimization algorithms, machine learning algorithms can be integrated with traditional optimization algorithms or even mathematical programming algorithms in a hierarchical or partitioned way, this integration can better address various optimization problems in combat.

6.6. Human-machine hybrid decision intelligent solution technique for WTA problem

In combat, the advantage of commander decision-making over machine decision-making lies in the ability to analyze the global situation more comprehensively and compensate for areas that are challenging for machines to interpret, leading to more explanatory decisions. However, at the same time, because the computational capability of the human brain in processing massive amounts of data is significantly lower than that of the machine, it is difficult to quickly extract useful features from massive data and make accurate operational decisions. Therefore, a hybrid decision-making method that combines the strengths of both commander and machine intelligence can synthesize the advantages, resulting in more reasonable and effective operational decision-making.

6.7. End-to-end intelligence solution techniques for WTA problem

In an automated engagement management system, the entire decision-making process is automated from the time a target appears to the time it is destroyed or driven away. The automated decision-making process can significantly reduce the deviations of in human planning caused by human intervention, and also greatly improve the data flow in all aspects of combat. In actual combat, where battlefield conditions change in real-time, it is difficult for existing multi-stage or probabilistic models to accurately model and solve WTA problems in real scenarios. Hence, more intelligent methods, such as large language models, large-scale decision-making models, and other advanced artificial intelligence methods, are required for effective modeling and problem-solving to achieve precise and integrated decision-making. In the future combat decision-making research, the end-to-end decision-making process will substantially improve combat efficiency. An automatic decision-making system gathers target information through situation awareness. Through data processing, target recognition, intention perception, mission planning, and resource scheduling, the final decision results are output to guide each combat platform to execute combat tasks. Taking the end-to-end reinforcement learning method as an example, by training the reinforcement learning model corresponding to the scenario, the model can obtain decision-making results immediately in the current scenario after deployment, while traditional optimization algorithms may struggle to provide high-quality solutions efficiently within the time constraints of dynamic combat environments. Therefore, with the continuous advancement of machine learning technology, WTA models that consider more complex combat situation can be effectively solved.

7. Conclusion

As research on the WTA problem has progressed since its introduction and initial modeling by Manne (1958), it has significantly contributed to the development of this problem and the theory and technology of combinatorial optimization in general. In this paper, we have sorted and summarized the current models of WTA problems. Through a comprehensive literature survey, we provide an overview of the development history of the WTA problem, and by analyzing the differences in formulas in these literatures in terms of objective functions and constraints, we classify a total of 19 formulas into four categories, from simple to complex. We have also summarized current exact, approximate optimization, and other algorithms for solving the WTA problem. Additionally, we have investigated the related works of WTA, focusing on WTA problems that consider air combat, threat assessment, and dynamic models. Finally, we review the application of WTA in actual combat systems.

While this survey provides a fundamental understanding of WTA and its related issues, advancements in missile systems from their range to their accuracy and anti-jamming capabilities have exponentially increased the pressure on homeland defense. As regional conflicts continue to escalate, researchers around the world are actively working on developing new solutions to tackle WTA challenges.

CRediT authorship contribution statement

Jinrui Li: Writing – review & editing, Writing – original draft, Investigation. **Guohua Wu:** Writing – review & editing, Supervision. **Ling Wang:** Writing – review & editing, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgement

This work was supported by the National Natural Science Foundation of China under Grant 62373380.

References

Acar, E., Hatipoğlu, S., Yılmaz, İ., 2023. A quantum algorithm for solving weapon target assignment problem. Eng. Appl. Artif. Intell. 125, 106668.

Ahmad, A., Amjad, R., Basharat, A., Farhan, A.A., Abbas, A.E., 2024. Fuzzy knowledge based intelligent decision support system for ground based air defence. J. Ambient Intell. Hum. Comput. 15, 2317–2340.

Ahner, D.K., Parson, C.R., 2015. Optimal multi-stage allocation of weapons to targets using adaptive dynamic programming. Optimization Letters 9, 1689–1701.

Ahuja, R.K., Kumar, A., Jha, K.C., Orlin, J.B., 2007. Exact and heuristic algorithms for the weapon-target assignment problem. Oper. Res. 55, 1136–1146.

Altinoz, O.T., 2021. Evolving model for synchronous weapon target assignment problem. In: 2021 International Conference on INnovations in Intelligent SysTems and Applications (INISTA). IEEE, pp. 1–8.

Andersen, A.C., Pavlikov, K., Toffolo, T.A.M., 2022. Weapon-target assignment problem: exact and approximate solution algorithms. Ann. Oper. Res. 312, 581–606.

Anderson, J., Hong, L., 2008. Sensor resource management driven by threat projection and priorities. Inf. Sci. 178, 2007–2021.

Arslan, C., Karasakal, O., Kırca, Ö., 2024. Naval air defense planning problem: a novel formulation and heuristics. Nav. Res. Logist. 1–25.

Atkinson, M.P., Kress, M., 2023. Resource allocation in two-layered cyber-defense. Nav. Res. Logist. 70, 574–583.

Azak, M., Bayrak, A.E., 2008. A new approach for threat evaluation and weapon assignment problem, hybrid learning with multi-agent coordination. In: 2008 23rd International Symposium on Computer and Information Sciences, pp. 1–6.

Bai, Z., Zhou, H., Shi, J., Xing, L., Wang, J., 2024. A hybrid multi-objective evolutionary algorithm with high solving efficiency for UAV defense programming. Swarm Evol. Comput. 87, 101572.

Bayrak, A.E., Polat, F., 2013. Employment of an evolutionary heuristic to solve the target allocation problem efficiently. Inf. Sci. 222, 675–695.

Bellman, R.E., Dreyfus, S.E., Gross, O.A., Johnson, S.M., 1959. On the Computational Solution of Dynamic-Programming Processes-XIV: Missile-Allocation Problems. RAND Corporation, Santa Monica, CA.

Benaskeur, A., Kabanza, F., Beaudry, É., 2010. CORALS: a real-time planner for anti-air defense operations. ACM Transactions on Intelligent Systems and Technology 1, 13.

Bertsekas, D.P., Homer, M.L., Logan, D.A., Patek, S.D., Sandell, N.R., 2000. Missile defense and interceptor allocation by neuro-dynamic programming. IEEE Trans. Syst. Man Cybern. Syst. Hum. 30, 42–51.

Bin, X., Jie, C., 2012. An estimation of distribution algorithm with efficient constructive repair/improvement operator for the dynamic weapon-target assignment. In: Proceedings of the 31st Chinese Control Conference, pp. 2346–2351.

Bo, L., Zheng, Q., Rui, W., Gao, Y.b., Li-ping, S., 2009. A hybrid heuristic particle swarm optimization for coordinated multi-target assignment. In: 2009 4th IEEE Conference on Industrial Electronics and Applications, pp. 1929–1934.

Bogdanowicz, Z.R., 2009. A new efficient algorithm for optimal assignment of smart weapons to targets. Comput. Math. Appl. 58, 1965–1969.

Bogdanowicz, Z.R., 2012. Advanced input generating algorithm for effect-based weapon–target pairing optimization. IEEE Trans. Syst. Man Cybern. Syst. Hum. 42, 276–280.

Bogdanowicz, Z., Coleman, N., 2007. Sensor-target and weapon-target pairings based on auction algorithm. In: Proceedings of the 11th WSEAS International Conference on Applied Mathematics, pp. 92–96.

Bogdanowicz, Z.R., Tolano, A., Patel, K., Coleman, N.P., 2013. Optimization of weapon–target pairings based on kill probabilities. IEEE Trans. Cybern. 43, 1835–1844.

Bracken, J., Falk, J.E., Tai, A.J.A., 1987. Robustness of preallocated preferential defense with assumed attack size and perfect attacking and defending weapons. Nav. Res. Logist. 34, 23–41.

Brown, G., Carlyle, M., Diehl, D., Kline, J., Wood, K., 2005. A two-sided optimization for theater ballistic missile defense. Oper. Res. 53, 745–763.

Brown, G.G., Carlyle, W.M., Salmerón, J., Wood, K., 2005. Analyzing the vulnerability of critical infrastructure to attack and planning defenses, Emerging Theory. Methods, and Applications 102–123.

Burr, S.A., Falk, J.E., Karr, A.F., 1985. Integer prim-read solutions to a class of target defense problems. Oper. Res. 33, 726–745.

Byun, M., Na, H., Moon, I.C., 2023. Time-efficient weapon-target assignment by actor-critic reinforcement. In: 2023 IEEE International Conference on Systems, Man, and Cybernetics (SMC), pp. 1419–1424.

Cai, H., Liu, J., Chen, Y., Wang, H., 2006. Survey of the research on dynamic weapon-target assignment problem. J. Syst. Eng. Electron. 17, 559–565.

Carling, R., 2009. A knowledge-base system for the threat evaluation and weapon assignment process. Nav. Eng. J. 105, 31–41.

Cha, Y.-H., Kim, Y.-D., 2010. Fire scheduling for planned artillery attack operations under time-dependent destruction probabilities. Omega 38, 383–392.

Chang, S.c., James, R.M., Shaw, J.J., 1987. Assignment algorithm for kinetic energy weapons in boost phase defence. In: 26th IEEE Conference on Decision and Control, pp. 1678–1683.

Chang, T., Kong, D., Hao, N., Xu, K., Yang, G., 2018. Solving the dynamic weapon target assignment problem by an improved artificial bee colony algorithm with heuristic factor initialization. Appl. Soft Comput. 70, 845–863.

Chang, X., Luo, Z., Shi, J., Liu, Y., Dang, S., Jin, X., 2023a. Bi-objective multi-stage weapon target assignment problem with limited ammunition. In: 2023 9th International Conference on Big Data and Information Analytics (BigDIA), pp. 773–780.

Chang, X., Shi, J., Luo, Z., Liu, Y., 2023b. Adaptive large neighborhood search algorithm for multi-stage weapon target assignment problem. Comput. Ind. Eng. 181, 109303.

Chen, H., Liu, Z., Sun, Y., Li, Y., 2012. Particle swarm optimization based on genetic operators for sensor-weapon-target assignment. In: 2012 Fifth International Symposium on Computational Intelligence and Design, pp. 170–173.

Cruz, J.B., Genshe, C., Garagic, D., Xiaohuan, T., Dongxu, L., Dan, S., Mo, W., Xu, W., 2003. Team dynamics and tactics for mission planning. In: 42nd IEEE International Conference on Decision and Control, vol. 3574. IEEE Cat. No.03CH37475), pp. 3579–3584.

Dabholkar, A., Hare, J.Z., Mittrick, M., Richardson, J., Waytowich, N., Narayanan, P., Bagchi, S., 2024. Adversarial attacks on reinforcement learning agents for command and control. arXiv preprint arXiv:2405.01693 1–11.

Dağdeviren, M., Yavuz, S., Kılınç, N., 2009. Weapon selection using the AHP and TOPSIS methods under fuzzy environment. Expert Syst. Appl. 36, 8143–8151.
Davis, M.T., Robbins, M.J., Lunday, B.J., 2017. Approximate dynamic programming for missile defense interceptor fire control. Eur. J. Oper. Res. 259, 873–886.
Day, R.H., 1966. Allocating weapons to target complexes by means of nonlinear programming. Oper. Res. 14, 992–1013.
De-Lin, L., Chun-Lin, S., Biao, W., Wen-Hai, W., 2005. Air combat decision-making for cooperative multiple target attack using heuristic adaptive genetic algorithm. In: 2005 International Conference on Machine Learning and Cybernetics, pp. 473–478.
denBroeder J, G.G., Ellison, R.E., Emerling, L., 1959. On optimum target assignments. Oper. Res. 7, 322–326.
Dionne, D., Pogossian, E., Grigoryan, A., Couture, J., Shahbazian, E., 2008. An optimal sequential optimization approach in application to dynamic weapon allocation in naval warfare. In: 2008 11th International Conference on Information Fusion, pp. 1–6.
Durgut, R., Kutucu, H., Akleylek, S., 2017. An artificial bee colony algorithm for solving the weapon target assignment problem. In: Proceedings of the 7th International Conference on Information Communication and Management. Association for Computing Machinery, Moscow, Russian Federation, pp. 28–31.
Erdem, E., Ozdemirel, N., 2003. An evolutionary approach for the target allocation problem. J. Oper. Res. Soc. 54, 958–969.
Everett, R.R., Zraket, C.A., Benington, H.D., 1957. SAGE: a data-processing system for air defense. In: Papers and Discussions Presented at the December 9-13, 1957, Eastern Joint Computer Conference: Computers with Deadlines to Meet. Association for Computing Machinery, Washington, D.C., pp. 148–155
Feghhi, N., Kosari, A.R., Atashgah, M.A.A., 2021. A real-time exhaustive search algorithm for the weapon-target assignment problem. Sci. Iran. 28, 1539–1551.
Fu, Q., Fan, C., Heng, Y., 2023. Air defense intelligent weapon target assignment method based on deep reinforcement learning. In: Proceedings of the 7th International Conference on Control Engineering and Artificial Intelligence. Association for Computing Machinery, Sanya, China, pp. 157–161.
Furman, G.G., Greenberg, H.J., 1973. Optimal weapon allocation with overlapping area defenses. Oper. Res. 21, 1291–1308.
Galati, D.G., Simaan, M.A., 2007. Effectiveness of the Nash strategies in competitive multi-team target assignment problems. IEEE Trans. Aero. Electron. Syst. 43, 126–134.
Gallagher, M.A., Kelly, E.J., 1991. A new methodology for military force structure analysis. Oper. Res. 39, 877–885.
Gao, X.-g., Zhu, Y.-w., Wang, H., 2010. A solving algorithm for target assignment optimization model based on ACO. In: 2010 Sixth International Conference on Natural Computation. IEEE, pp. 3753–3757.
Das, S.K., Mukherjee, S., 2019. Agent based decision making for integrated air defense system. arXiv preprint arXiv:1905.01358 1–8.
Ghanbari, A.A., Alaei, H., 2021. Meta-heuristic algorithms for resource management in crisis based on OWA approach. Appl. Intell. 51, 646–657.
Glenn, F., Zachary, W., 1979. Integration of Decision Aids for Strike Campaign Planning. ANALYTICS INC WILLOW GROVE PA.
Golany, B., Goldberg, N., Rothblum, U.G., 2015. Allocating multiple defensive resources in a zero-sum game setting. Ann. Oper. Res. 225, 91–109.
Gonsalves, P., Burge, J., 2004. Software toolkit for optimizing mission plans (STOMP). In: AIAA 1st Intelligent Systems Technical Conference, p. 6279.
Gould, J., 1984. On efficient salvo policies. Nav. Res. Logist. Q. 31, 159–162.
Griffiths, J.D., Marshall, S.A., Weale, T.G., Williams, J.E., 1991. A simple stochastic model of air–naval combat. J. Oper. Res. Soc. 42, 289–300.
Guo, J., Hu, G., Guo, Z., Zhou, M., 2022. Evaluation ,model, intelligent assignment, and cooperative interception in multimissile and multitarget engagement. IEEE Trans. Aero. Electron. Syst. 58, 3104–3115.
Han, C.Y., Lunday, B.J., Robbins, M.J., 2016. A game theoretic model for the optimal location of integrated air defense system missile batteries. Inf. J. Comput. 28, 405–416.
Haywood, A.B., Lunday, B.J., Robbins, M.J., 2022. Intruder detection and interdiction modeling: a bilevel programming approach for ballistic missile defense asset location. Omega 110, 102640.
He, S., Yue, S.H., Wang, G., Wang, S.Y., Liu, J.Y., Liu, W., Guo, X.K., 2022. Target assignment algorithm for joint air defense operation based on spatial crowdsourcing mode. Electronics 11.
Hendrickson, K., Ganesh, P., Volle, K., Buzaud, P., Brink, K., Hale, M., 2023. Decentralized weapon–target assignment under asynchronous communications. J. Guid. Control Dynam. 46, 312–324.
Heon, O.S., Byueon, G.W., Cho, Y.I., Kwon, S., Woo, J.H., 2024. Artificial intelligence in combat decision-making: weapon target assignment via reinforcement learning and graph neural networks. TechRxiv 1–13.
Ho, E., Rajagopalan, A., Skvortsov, A., Arulampalam, S., Piraveenan, M., 2022. Game theory in defence applications: a review. Sensors 22, 1032.
Hocaoğlu, M.F., 2019. Weapon target assignment optimization for land based multi-air defense systems: a goal programming approach. Comput. Ind. Eng. 128, 681–689.
Hohzaki, R., Nagashima, S., 2009. A Stackelberg equilibrium for a missile procurement problem. Eur. J. Oper. Res. 193, 238–249.
Hongtao, L., Fengju, K., 2016. Adaptive chaos parallel clonal selection algorithm for objective optimization in WTA application. Optik 127, 3459–3465.
Hosein, P.A., Athans, M., 1990. An asymptotic result for the multi-stage weapon-target allocation problem. In: 29th IEEE Conference on Decision and Control, vol. 241, pp. 240–245.
Hosein, P.A., Walton, J.T., Athans, M., 1988. Dynamic weapon-target assignment problems with vulnerable C2 nodes. Massachusetts Inst of Tech Cambridge Lab for Information and Decision Systems. Report LIDS-P.1786, 1–10.
Hoseint, P.A., Athanst, M., 2002. Preferential Defense Strategies. Part I: the Static Case.
Hughes, M.S., Lunday, B.J., 2022. The weapon target assignment problem: rational inference of adversary target utility valuations from observed solutions. Omega 107, 102562.
Jiandong, Z., Qiming, Y., Guoqing, S., Yi, L., Yong, W., 2021. UAV cooperative air combat maneuver decision based on multi-agent reinforcement learning. J. Syst. Eng. Electron. 32, 1421–1438.
Jihua, D., Xingbao, Y., Yonghong, L., 2009. Improved genetic algorithm for multichannel ship-to-air missile weapon system WTA problem. In: 2009 2nd IEEE International Conference on Computer Science and Information Technology, pp. 210–214.
Jin, T., He, S., Li, H., Zheng, X., 2023. Evaluation model and exact optimization algorithm in missile–target assignment. J. Guid. Control Dynam. 46, 1834–1841.
Johansson, F., Falkman, G., 2010a. Real-time allocation of defensive resources to rockets, artillery, and mortars. In: 2010 13th International Conference on Information Fusion, pp. 1–8.
Johansson, F., Falkman, G., 2010b. A suite of metaheuristic algorithms for static weapon-target allocation. In: Proceedings of the 2010 International Conference on Genetic and Evolutionary Methods. Las Vegas Nevada, USA, pp. 132–138.
Johansson, F., Falkman, G., 2010c. SWARD: system for weapon allocation research & development. In: 2010 13th International Conference on Information Fusion, pp. 1–7.
Johansson, F., Falkman, G., 2022. Real-time allocation of firing units to hostile targets. J. Adv. Infor. Fusion 6, 187–199.
Juan, L., Chen, J., Xin, B., Lu, C., 2017. Efficient multi-objective evolutionary algorithms for solving the multi-stage weapon target assignment problem: a comparison study, 2017. IEEE Congress Evoluti. Comput. 435–442.
Julstrom, B.A., 2009. String-and permutation-coded genetic algorithms for the static weapon-target assignment problem. In: Proceedings of the 11th Annual Conference Companion on Genetic and Evolutionary Computation Conference: Late Breaking Papers. Association for Computing Machinery, Montreal, Québec, Canada, pp. 2553–2558.
Karasakal, O., 2008. Air defense missile-target allocation models for a naval task group. Comput. Oper. Res. 35, 1759–1770.
Karasakal, O., Özdemirel, N.E., Kandiller, L., 2011. Anti-ship missile defense for a naval task group. Nav. Res. Logist. 58, 304–321.
Karasakal, O., Karasakal, E., Silav, A., 2021. A multi-objective approach for dynamic missile allocation using artificial neural networks for time sensitive decisions. Soft Comput. 25, 10153–10166.
Keith, A.J., 2019. Operational decision making under uncertainty: inferential, sequential, and adversarial approaches. Theses and Dissertations. 2464, 134–172.
Keith, A., Ahner, D., 2021. Counterfactual regret minimization for integrated cyber and air defense resource allocation. Eur. J. Oper. Res. 292, 95–107.
Khosla, D., Nichols, T., 2006. Hybrid evolutionary algorithms for network-centric command and control. In: SPIE Defense + Commercial Sensing. SPIE, 2006.
Kline, A., Ahner, D., Hill, R., 2019. The weapon-target assignment problem. Comput. Oper. Res. 105, 226–236.
Kline, A.G., Ahner, D.K., Lunday, B.J., 2019. Real-time heuristic algorithms for the static weapon target assignment problem. J. Heuristics 25, 377–397.
Kline, A.G., Ahner, D.K., Lunday, B.J., 2020. A heuristic and metaheuristic approach to the static weapon target assignment problem. J. Global Optim. 78, 791–812.
Koleszar, G.E., Bexfield, J.N., Miercort, F.A., 1999. A Description of the Weapon Optimization and Resource Requirements Model (WORRM). INSTITUTE FOR DEFENSE ANALYSES ALEXANDRIA VA.
Kong, D., Chang, T., Pan, J., Hao, N., Xu, K., Zhang, L., Yang, G., 2019. A decision variable-based combinatorial optimization approach for interval-valued intuitionistic fuzzy MAGDM. Inf. Sci. 484, 197–218.
Kong, L.R., Wang, J.Z., Zhao, P., 2021. Solving the dynamic weapon target assignment problem by an improved multiobjective particle swarm optimization algorithm. Applied Sciences-Basel 11.
Kong, X., Zhou, Y., Li, Z., Wang, S., 2024. Multi-UAV simultaneous target assignment and path planning based on deep reinforcement learning in dynamic multiple obstacles environments. Front. Neurorob. 17, 1302898.
Kwon, O., Lee, K., Park, S., 1997. Targeting and scheduling problem for field artillery. Comput. Ind. Eng. 33, 693–696.
Kwon, O., Kang, D., Lee, K., Park, S., 1999. Lagrangian relaxation approach to the targeting problem. Nav. Res. Logist. 46, 640–653.
Kwon, O., Lee, K., Kang, D., Park, S., 2007. A branch-and-price algorithm for a targeting problem. Nav. Res. Logist. 54, 732–741.
Lai, C.-M., Wu, T.-H., 2019. Simplified swarm optimization with initialization scheme for dynamic weapon–target assignment problem. Appl. Soft Comput. 82, 105542.
Lai, X., Shu, J., Meng, H., Huang, H., Huang, L., Yang, X., 2023. Research on intelligent missile target assignment method based on target system value and deep reinforcement learning. In: 2023 IEEE International Conference on Sensors, Electronics and Computer Engineering (ICSECE), pp. 664–675.
Larson, E.V., Kent, G.A., 1994. A New Methodology for Assessing Multilayer Missile Defense Options. RAND CORP SANTA MONICA, CA.
Leboucher, C., Shin, H.-S., Siarry, P., Chelouah, R., Le Ménec, S., Tsourdos, A., 2013. A two-step optimisation method for dynamic weapon target assignment problem. Recent Adv. Meta-Heur. Their Appli. Real Scen. 109–129.
Lee, M.Z., 2010. Constrained weapon–target assignment: enhanced very large scale neighborhood search algorithm. IEEE Trans. Syst. Man Cybern. Syst. Hum. 40, 198–204.
Lee, Z.-J., Lee, W.-L., 2003. A hybrid search algorithm of ant colony optimization and genetic algorithm applied to weapon-target assignment problems. In: Liu, J., Cheung, Y.-m., Yin, H. (Eds.), Intelligent Data Engineering and Automated Learning. Springer Berlin Heidelberg, Berlin, Heidelberg, pp. 278–285.

Lee, Z.-J., Lee, C.-Y., Su, S.-F., 2002. An immunity-based ant colony optimization algorithm for solving weapon–target assignment problem. Appl. Soft Comput. 2, 39–47.
Lee, Z.-J., Su, S.-F., Lee, C.-Y., 2003. Efficiently solving general weapon-target assignment problem by genetic algorithms with greedy eugenics. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics) 33, 113–121.
Lee, J., Kang, S.-H., Rosenberger, J., Kim, S.B., 2010. A hybrid approach of goal programming for weapon systems selection. Comput. Ind. Eng. 58, 521–527.
Lee, D., Shin, M.K., Choi, H.-L., 2020. Weapon Target Assignment Problem with Interference Constraints. AIAA Scitech 2020 Forum, p. 388.
Lei, M., Xiaomei, Q., Peng, W., 2017. Application of sensor/weapon-target assignment based on multi-scale quantum harmonic oscillator algorithm. In: 2017 2nd International Conference on Image, Vision and Computing (ICIVC), pp. 1147–1151.
Lemus, F., David, K.H., 1963. An optimum allocation of different weapons to a target complex. Oper. Res. 11, 787–794.
Li, Z., Wang, J., 2010. Weapon-target assignment research based on genetic algorithm mixed with damage simulation. In: 2010 International Conference on Computer Application and System Modeling (ICCASM 2010). V15-460-V415-463.
Li, V.C., Curry, G.L., Boyd, E.A., 2004. Towards the real time solution of strike force asset allocation problems. Comput. Oper. Res. 31, 273–291.
Li, H., Wang, H., Sun, S., Qiu, Z., 2008. Optimization matching of sensor and weapon system based on geometric programming. In: 2008 3rd International Conference on Innovative Computing Information and Control, p. 97, 97.
Li, P., Wu, L., Lu, F., 2009. A mutation-based GA for weapon-target allocation problem subject to spatial constraints. In: 2009 International Workshop on Intelligent Systems and Applications, pp. 1–4.
Li, Z., Xiang-min, L., Jin-jin, D., Jin-zhu, C., Feng-xia, Z., 2011. Sensor-weapon-target assignment based on improved SWT-opt algorithm. In: 2011 IEEE 2nd International Conference on Computing, Control and Industrial Engineering, pp. 25–28.
Li, Z., Li, X., Yan, J., Dai, J., Kong, F., 2012. An anytime algorithm based on decentralized cooperative auction for dynamic joint fire distribution problem. In: 2012 IEEE International Conference on Mechatronics and Automation, pp. 2031–2036.
Li, J., Chen, J., Xin, B., Dou, L., 2015a. Solving multi-objective multi-stage weapon target assignment problem via adaptive NSGA-II and adaptive MOEA/D: a comparison study, 2015. IEEE Congress Evoluti. Comput. 3132–3139.
Li, J., Jie, C., Bin, X., 2015b. Efficiently solving multi-objective dynamic weapon-target assignment problems by NSGA-II. In: 2015 34th Chinese Control Conference (CCC), pp. 2556–2561.
Li, J., Chen, J., Xin, B., Dou, L., Peng, Z., 2016. Solving the uncertain multi-objective multi-stage weapon target assignment problem via MOEA/D-AWA, 2016. IEEE Congress Evoluti. Comput. 4934–4941.
Li, N., Huai, W., Wang, S., 2017. The solution of target assignment problem in command and control decision-making behaviour simulation. Enterprise Inf. Syst. 11, 1059–1077.
Li, Y., Kou, Y., Li, Z., Xu, A., Chang, Y., 2017. A modified pareto ant colony optimization approach to solve biobjective weapon-target assignment problem. Intern. J. Aerospace Eng. 2017, 1746124.
Li, X., Zhou, D., Pan, Q., Tang, Y., Huang, J., 2018. Weapon-target assignment problem by multiobjective evolutionary algorithm based on decomposition. Complexity 2018, 8623051.
Li, Y., Kou, Y., Li, Z., 2018. An improved nondominated sorting genetic algorithm III method for solving multiobjective weapon-target assignment part I: the value of fighter combat. Intern. J. Aerospace Eng. 2018, 8302324.
Li, T., Wang, G., Fu, Q., Guo, X., Zhao, M., Liu, X., 2023. An intelligent algorithm for solving weapon-target assignment problem: DDPG-DNPE algorithm. Comput. Mater. Continua (CMC) 76.
Liao, S.-h., 2000. Case-based decision support system: architecture for simulating military command and control. Eur. J. Oper. Res. 123, 558–567.
Liles, J.M., Robbins, M.J., Lunday, B.J., 2023. Improving defensive air battle management by solving a stochastic dynamic assignment problem via approximate dynamic programming. Eur. J. Oper. Res. 305, 1435–1449.
Lirov, Y., Rodin, E.Y., Ghosh, B.K., 1989. Automated learning by tactical decision systems in air combat. Comput. Math. Appl. 18, 151–160.
Liu, C., Sun, S., Tao, C., Shou, Y., Xu, B., 2021. Sliding mode control of multi-agent system with application to UAV air combat. Comput. Electr. Eng. 96, 107491.
Liu, J.Y., Wang, G., Guo, X.K., Wang, S.Y., Fu, Q., 2022. Intelligent air defense task assignment based on hierarchical reinforcement learning. Front. Neurorob. 16.
Liu, S., Liu, W., Huang, F., Yin, Y., Yan, B., Zhang, T., 2022. Multitarget allocation strategy based on adaptive SA-PSO algorithm. Aeronaut. J. 126, 1069–1081.
Liu, C., Li, J., Wang, Y., Yu, Y., Guo, L., Gao, Y., Chen, Y., Zhang, F., 2023. A time-driven dynamic weapon target assignment method. IEEE Access 11, 129623–129639.
Liu, J.-y., Wang, G., Fu, Q., Yue, S.-h., Wang, S.-y., 2023. Task assignment in ground-to-air confrontation based on multiagent deep reinforcement learning. Defence Technology 19, 210–219.
Lloyd, S.P., Witsenhausen, H.S., 1986. Weapons allocation is NP-complete. In: 1986 Summer Computer Simulation Conference, United States, pp. 1054–1058.
Lötter, D., Nieuwoudt, I., Van Vuuren, J., 2013. A multiobjective approach towards weapon assignment in a ground-based air defence environment. Orion 29, 31–54.
Lu, Y., Chen, D.Z., 2021. A new exact algorithm for the weapon-target assignment problem. Omega 98, 102138.
Lu, H., Zhang, H., Zhang, X., Han, R., 2006. An improved genetic algorithm for target assignment, optimization of naval fleet air defense. In: 2006 6th World Congress on Intelligent Control and Automation, pp. 3401–3405.
Lu, Y., Miao, W., Li, M., 2013. The air defense missile optimum target assignment based on the improved genetic algorithm. J. Theor. Appl. Inf. Technol. 48, 809–816.
Luo, P.c., Xie, J.j., Che, W.f., 2016. Q-learning based air combat target assignment algorithm. In: 2016 IEEE International Conference on Systems, Man, and Cybernetics (SMC), pp. 779–783.
Luo, W., Lü, J., Liu, K., Chen, L., 2021. Learning-based policy optimization for adversarial missile-target assignment. IEEE Transactions on Systems, Man, and Cybernetics: Systems 52, 4426–4437.
Ma, F., Ni, M., Gao, B., Yu, Z., 2015a. An efficient algorithm for the weapon target assignment problem. In: 2015 IEEE International Conference on Information and Automation, pp. 2093–2097.
Ma, F., Ni, M., Yu, Z., Tong, W., 2015b. An optimal assignment of multi-type weapons to single-target. In: 2015 IEEE Advanced Information Technology, Electronic and Automation Control Conference (IAEAC), pp. 44–47.
Ma, Y.Y., Wang, G.Q., Hu, X.X., Luo, H., 2021. Two-stage hybrid heuristic search algorithm for novel weapon target assignment problems. Comput. Ind. Eng. 162.
Malhotra, A., Jain, R., 2001. Genetic algorithm for optimal weapon allocation in multilayer defence scenario. Defence Sci. J. 51, 285–293.
Manne, A.S., 1958. A target-assignment problem. Oper. Res. 6, 346–351.
Martijn, v.E., 2020. On efficient algorithms for finding efficient salvo policies. Nav. Res. Logist. 67, 147–158.
Matlin, S., 1970. A review of the literature on the missile-allocation problem. Oper. Res. 18, 334–373.
Mekawey, H.I., El-Wahab, M.S.A., Hashem, M., 2009. Novel goal-based weapon target assignment doctrine. J. Aero. Comput. Inf. Commun. 6, 2–29.
Meng, F., Tian, K., Wu, C., 2021. Deep reinforcement learning-based radar network target assignment. IEEE Sensor. J. 21, 16315–16327.
Menq, J.-y., Tuan, P.-c., Liu, T.-s., 2007. Discrete Markov ballistic missile defense system modeling. Eur. J. Oper. Res. 178, 560–578.
Merkulov, G., Iceland, E., Michaeli, S., Riechkind, Y., Gal, O., Barel, A., Shima, T., 2024. Reinforcement Learning Based Decentralized Weapon-Target Assignment and Guidance. AIAA SCITECH 2024 Forum, p. 125.
Metler, W.A., Preston, F.L., Hofmann, J., 1990. A Suite of Weapon Assignment Algorithms for a SDI Mid-course Battle Manager. Naval Research Lab, Washington DC.
Miercort, F.A., Soland, R.M., 1971. Optimal allocation of missiles against area and point defenses. Oper. Res. 19, 605–617.
Murphey, R.A., 2000. An Approximate Algorithm for a Weapon Target Assignment Stochastic Program. Springer US, Boston, MA.
Na, H., Ahn, J., Moon, I.-C., 2023. Weapon–target assignment by reinforcement learning with pointer network. J. Aero. Inf. Syst. 20, 53–59.
Naeem, H., Masood, A., 2010. An optimal dynamic threat evaluation and weapon scheduling technique. Knowl. Base Syst. 23, 337–342.
Naeem, H., Masood, A., Hussain, M., Khan, S.A., 2009a. A novel two-staged decision support based threat evaluation and weapon assignment algorithm, asset-based dynamic weapon scheduling using artificial intelligence techinques. arXiv preprint arXiv:0907.0067 1–7.
Naeem, H., Masood, A., Hussain, M., Khan, S.A., 2009b. A semi-automated dynamic approach to threat evaluation and optimal defensive resource allocation, Emerging Intelligent Computing Technology and Applications. In: 5th International Conference on Intelligent Computing. Springer Berlin Heidelberg, Berlin, Heidelberg, pp. 8–17.
Naseem, A., Shah, S.T.H., Khan, S.A., Malik, A.W., 2017. Decision support system for optimum decision making process in threat evaluation and weapon assignment: current status, challenges and future directions. Annu. Rev. Control 43, 169–187.
Newman, A.M., Rosenthal, R.E., Salmerón, J., Brown, G.G., Price, W., Rowe, A., Fennemore, C.F., Taft, R.L., 2011. Optimizing assignment of Tomahawk cruise missile missions to firing units. Nav. Res. Logist. 58, 281–294.
Nguyen, B.U., Smith, P.A., Nguyen, D., 1997. An engagement model to optimize defense against a multiple attack assuming perfect kill assessment. Nav. Res. Logist. 44, 687–697.
Ni, M., Yu, Z., Ma, F., Wu, X., 2011. A Lagrange relaxation method for solving weapon-target assignment problem. Math. Probl Eng. 2011, 873292.
O'Meara, N., Soland, R., 1990. Optimal attack against an area defense protecting many targets. INFOR Inf. Syst. Oper. Res. 28, 40–52.
Orlin, D., 1987. Optimal weapons allocation against layered defenses. Nav. Res. Logist. 34, 605–617.
Oxenham, M.G., Cutler, P., 2006. Accommodating obstacle avoidance in the weapons allocation problem for tactical air defence. In: 2006 9th International Conference on Information Fusion, pp. 1–8.
Pan, Q., Zhou, D., Tang, Y., Li, X., 2019. A novel antagonistic weapon-target assignment model considering uncertainty and its solution using decomposition co-evolution algorithm. IEEE Access 7, 37498–37517.
Paradis, S., Benaskeur, A., Oxenham, M., Cutler, P., 2005. Threat evaluation and weapons allocation in network-centric warfare. In: 2005 7th International Conference on Information Fusion, pp. 1078–1085.
Patrick Hoseint, A., Athanst, M., 1990. LIDS-P-2003 preferential defense strategies. Part II : The Dynamic Case 1–27. LIDS-P-2003.
Peng, T., Huigang, L., Huang, J., Liang, S., 2008. Improved particle swarm optimization algorithm and its application in coordinated air combat missile-target assignment. In: 2008 7th World Congress on Intelligent Control and Automation, pp. 2833–2837.
Peng, G., Fang, Y., Chen, S., Peng, W., Yang, D., 2016. A hybrid multi-objective discrete particle swarm optimization algorithm for cooperative air combat DWTA. In: Gong, M., Pan, L., Song, T., Zhang, G. (Eds.), Bio-inspired Computing – Theories and Applications. Springer, Singapore, Singapore, pp. 114–119.
Pepyne, D.L., Looze, D.P., Logan, D.A., Sandell, N.R., LeBlanc, R., 1997. A decision aid for theater missile defense. In: Proceedings of 1997 IEEE International Conference on Evolutionary Computation (ICEC '97), pp. 563–568.

Perkins, F.M., 1961. Optimum weapon deployment for nuclear attack. Oper. Res. 9, 77–94.
Pryluk, R., Shima, T., Golan, O.M., 2013. Guidance-based missile allocation algorithm against a salvo attack. In: 52nd IEEE Conference on Decision and Control, pp. 5391–5396.
Pryluk, R., Shima, T., Golan, O.M., 2016. Shoot–shoot–look for an air defense system. IEEE Syst. J. 10, 151–161.
Qi-feng, X., Jin-fu, F., Yong-bing, Z., 2006. Decision of cooperative airfight and its realization by neural network. In: 2006 6th International Conference on ITS Telecommunications, pp. 298–301.
Gaudet, B., Drozd, K., Furfaro, R., 2023. Deep reinforcement learning for weapons to targets assignment in a hypersonic strike. arXiv preprint arXiv:2310.18509 1–13.
Randleff, L.R., Clausen, J., 2007. Decision support system for fighter pilots. IMM-PHD-2007-172.
Rezende, M.D., De Lima, B.S.L.P., Guimarães, S., 2018. A greedy ant colony system for defensive resource assignment problems. Appl. Artif. Intell. 32, 138–152.
Rosenberg, B., Burge, J., Gonsalves, P., 2005. Applying evolutionary multi-objective optimization to mission planning for time-sensitive targets. In: Proceedings of Genetic and Evolutionary Computation Conference (GECCO), pp. 1–8.
Rosenberger, J.M., Hwang, H.S., Pallerla, R.P., Yucel, A., Wilson, R.L., Brungardt, E.G., 2005. The Generalized Weapon Target Assignment Problem. Texas Univ at Arlington.
Sahin, M.A., Leblebicioglu, K., 2009. A standard expert system for weapon target assignment problem. In: 2009 International Symposium on Performance Evaluation of Computer & Telecommunication Systems, pp. 221–224.
Şahin, M.A., Leblebicioğlu, K., 2014. Approximating the optimal mapping for weapon target assignment by fuzzy reasoning. Inf. Sci. 255, 30–44.
Shalumov, V., Shima, T., 2017. Weapon–target-allocation strategies in multiagent target–missile–defender engagement. J. Guid. Control Dynam. 40, 2452–2464.
Shi, X.P., Zou, S.Q., Song, S.M., Guo, R., 2021. A multi-objective sparse evolutionary framework for large-scale weapon target assignment based on a reward strategy. J. Intell. Fuzzy Syst. 40, 10043–10061.
Shokoohi, M., Afsharchi, M., Shah-Hoseini, H., 2022. Dynamic distributed constraint optimization using multi-agent reinforcement learning. Soft Comput. 26, 3601–3629.
Silav, A., Karasakal, O., Karasakal, E., 2019. Bi-objective missile rescheduling for a naval task group with dynamic disruptions. Nav. Res. Logist. 66, 596–615.
Silav, A., Karasakal, E., Karasakal, O., 2021. Bi-objective dynamic weapon-target assignment problem with stability measure. Ann. Oper. Res. 2022, 1–19.
Soland, R.M., 1973. Optimal defensive missile allocation: a discrete min-max problem. Oper. Res. 21, 590–596.
Song, Z., Zhu, F., Zhang, D., 2009. A heuristic genetic algorithm for solving constrained weapon-target assignment problem. In: 2009 IEEE International Conference on Intelligent Computing and Intelligent Systems, pp. 336–341.
Su, M.-C., Lai, S.-C., Lin, S.-C., You, L.-F., 2012. A new approach to multi-aircraft air combat assignments. Swarm Evol. Comput. 6, 39–46.
Summers, D.S., Robbins, M.J., Lunday, B.J., 2020. An approximate dynamic programming approach for comparing firing policies in a networked air defense environment. Comput. Oper. Res. 117, 104890.
Sun, Z., Piao, H., Yang, Z., Zhao, Y., Zhan, G., Zhou, D., Meng, G., Chen, H., Chen, X., Qu, B., Lu, Y., 2021. Multi-agent hierarchical policy gradient for air combat tactics emergence via self-play. Eng. Appl. Artif. Intell. 98, 104112.
Tuncer, O., Cirpan, H.A., 2023. Adaptive fuzzy based threat evaluation method for air and missile defense systems. Inf. Sci. 643, 119191.
Tunga, H., Kar, S., Giri, D., Pan, I., 2024. Efficacy analysis of NSGAII and multi-objective particle swarm optimization (MOPSO) in agent based weapon target assignment (WTA) model. Int. J. Inf. Technol. 1–10.
Wang, S., Chen, W., 2012. Solving weapon-target assignment problems by cultural particle swarm optimization. In: 2012 4th International Conference on Intelligent Human-Machine Systems and Cybernetics, pp. 141–144.
Wang, F., Wang, X., Sun, S., 2022. A reinforcement learning level-based particle swarm optimization algorithm for large-scale optimization. Inf. Sci. 602, 298–312.
Wang, T., Fu, L., Wei, Z., Zhou, Y., Gao, S., 2023. Unmanned ground weapon target assignment based on deep Q-learning network with an improved multi-objective artificial bee colony algorithm. Eng. Appl. Artif. Intell. 117, 105612.
Wang, T., Fu, L.Y., Wei, Z.X., Zhou, Y.H., Gao, S., 2023. Unmanned ground weapon target assignment based on deep Q-learning network with an improved multi-objective artificial bee colony algorithm. Eng. Appl. Artif. Intell. 117.
Wang, Y., Du, G., Liu, Y., Wu, X., 2023. Anti-missile firepower allocation based on multi-agent reinforcement learning. In: International Conference on Autonomous Unmanned Systems. Springer, pp. 160–169.
Wei, M., Chen, G., Pham, K., Blasch, E., Wu, Y., 2008. Game theoretic target assignment approach in ballistic missile defense. In: Signal and Data Processing of Small Targets 2008. SPIE, pp. 193–201.
Wilkins, D., Desimone, R., 1996. Applying an AI planner to military operations planning. SRI International 1–30.
Wu, L., Wang, H.-y., Lu, F.-x., Jia, P., 2008a. An anytime algorithm based on modified GA for dynamic weapon-target allocation problem. In: 2008 IEEE Congress on Evolutionary Computation. IEEE World Congress on Computational Intelligence), pp. 2020–2025.
Wu, L., Xing, C., Lu, F., Jia, P., 2008b. An anytime algorithm applied to dynamic weapon-target allocation problem with decreasing weapons and targets. In: 2008 IEEE Congress on Evolutionary Computation. IEEE World Congress on Computational Intelligence), pp. 3755–3759.
Xin, B., Chen, J., Zhang, J., Dou, L., Peng, Z., 2010. Efficient decision makings for dynamic weapon-target assignment by virtual permutation and Tabu search heuristics. IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews) 40, 649–662.
Xin, B., Chen, J., Peng, Z., Dou, L., Zhang, J., 2011. An efficient rule-based constructive heuristic to solve dynamic weapon-target assignment problem. IEEE Trans. Syst. Man Cybern. Syst. Hum. 41, 598–606.
Xin, B., Wang, Y., Chen, J., 2019. An efficient marginal-return-based constructive heuristic to solve the sensor–weapon–target assignment problem. IEEE Transactions on Systems, Man, and Cybernetics: Systems 49, 2536–2547.
Xu, W., Chen, C., Ding, S., Pardalos, P.M., 2020. A bi-objective dynamic collaborative task assignment under uncertainty using modified MOEA/D with heuristic initialization. Expert Syst. Appl. 140, 112844.
Yang, L., Zhai, Z., Li, Y., Huang, Y., 2018. A multi-information particle swarm optimization algorithm for weapon target assignment of multiple kill vehicle. In: 2018 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM), pp. 1160–1165.
Yanushevsky, R., 2018. Modern Missile Guidance. CRC Press.
Yanxia, W., Longjun, Q., Zhi, G., Lifeng, M., 2008. Weapon target assignment problem satisfying expected damage probabilities based on ant colony algorithm. J. Syst. Eng. Electron. 19, 939–944.
Yi, X., Yu, H., Xu, T., 2024. Solving multi-objective weapon-target assignment considering reliability by improved MOEA/D-AM2M. Neurocomputing 563, 126906.
Zhang, K., Zhou, D., Yang, Z., Zhao, Y., Kong, W., 2020. Efficient decision approaches for asset-based dynamic weapon target assignment by a receding horizon and marginal return heuristic. Electronics 9, 1511.
Zhang, Y., Koch, S., Calvin, W.J.G., Srigrarom, S., Stütz, P., 2024. Interception of multiple drone targets by heterogeneous chasers using heuristic task allocations with DQN-GNN guidance model. In: 2024 SICE International Symposium on Control Systems (SICE ISCS), pp. 7–13.
Zhao, L.D., An, Z.X., Wang, B., Zhang, Y.Q., Hu, Y.H., 2022. A hybrid multi-objective bi-level interactive fuzzy programming method for solving ECM-DWTA problem. Complex Intelligent Sys. 8, 4811–4829.
Zhao, Y., Liu, J., Jiang, J., Zhen, Z., 2023. Shuffled frog leaping algorithm with non-dominated sorting for dynamic weapon-target assignment. J. Syst. Eng. Electron. 34, 1007–1019.
Zhou, Y., Li, X., Zhu, Y., Wang, W., 2016. A discrete particle swarm optimization algorithm applied in constrained static weapon-target assignment problem. In: 2016 12th World Congress on Intelligent Control and Automation (WCICA), pp. 3118–3123.
Zhu, B., Feng-xing, Z., Jia-hua, W., 2011. A novel approach to solving weapon-target assignment problem based on hybrid particle swarm optimization algorithm. In: Proceedings of 2011 International Conference on Electronic & Mechanical Engineering and Information Technology, pp. 1385–1387.
Zhu, X.W., Fan, C.L., Liu, S.L., Xing, H.X., Qi, C., 2022. Bi-level fuzzy expectation-based dynamic anti-missile weapon target allocation in rolling horizons. Electronics 11.
Zuo, X., Zhang, P., Li, H.-Y., Liu, Z.-G., 2024. Preference-based experience sharing scheme for multi-agent reinforcement learning in multi-target environments. Evolving Sys. 1–19.