Distribution System Reliability Cost/Worth Analysis Using Analytical and Sequential Simulation Techniques

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***Abstract*–The requirements for extensive justification of new facilities and emphasis on the optimization of system cost and reliability are steadily increasing. Customer interruption cost analysis provides valuable input to electric power supply reliability worth assessment. This paper presents both a generalized analytical approach and a time sequential Monte Carlo simulation technique for evaluating the customer interruption cost in complex radial distribution systems. Studies are conducted on two practical distribution systems. The results obtained using the analytical technique incorporating average restoration times are compared with those obtained using the time sequential simulation method and random restoration times. The effects on customer interruption cost indices associated with alternate supply and protection devices are also considered.**

INTRODUCTION

The basic function of a modern electric power system is to provide electric power to its customers at the lowest possible cost with acceptable reliability levels. The two aspects of economics and reliability often conflict and present power system managers, planners and operators with a wide range of challenging problems. The price that a customer is willing to pay for higher reliability is directly connected to the interruption costs created by power failures. If the price that a customer pays for increased reliability is less than the decrease in interruption costs, the customer could be expected to react favorable to the increased charge. Some customers therefore may be willing to pay more to receive higher reliability and others may be willing to pay less for lower reliability. Utilities may also be willing to provide higher reliability of power supply at no increased customer cost because of competition. Decision-making depends on many aspects such as social, economic, environmental and government considerations etc. and is a difficult task. System customer interruption cost analysis provides the opportunity to incorporate cost analysis and quantitative reliability assessment into a common structured framework, which can assist the decision making process.

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Considerable research has been done on generation and transmission system reliability evaluation, and service interruption cost applications. Both analytical and Monte Carlo simulation methods are used in these areas [1-6]. Relatively little work has been done in the area of distribution systems. An analytical technique using the contingency numeration approach is presented and used to evaluate customer interruption cost indices of a radial distribution system in Reference [7]. An approximate simulation technique is also used in [7] to evaluate a small distribution system.

This paper presents a generalized analytical technique and a time sequential Monte Carlo simulation technique to evaluate the reliability cost/worth indices of a complex distribution system. Different switch, fuse, breaker, alternate supply models are included in the analysis. This paper shows that the approximate analytical technique is fast and provides comparable results to the Monte Carlo simulation approach.

RELIABILITY COST/WORTH INDICES

A distribution system is the segment of an overall power system which links the bulk system to the individual customers. The proliferation of equipment and the basic configuration structure results in a relatively high proportion of customer outages being associated with the distribution system [8].

The basic distribution system reliability indices are the three load point indices of average failure rate λ, the average outage duration r and the annual outage duration U. These three basic indices are important individual load point parameters. The system indices of *SAIFI, SAIDI, CAIDI, ASAI* and *ASUI* can be calculated from the three basic load point indices [9]. The reliability cost/worth indices of expected energy not supply (*EENS*), expected interruption cost (*ECOST*) and interrupted energy assessment rate (*IEAR*) can also be calculated using the three basic load point indices. The equations used to calculate these indices are given in the following sections.

MODELS USED IN THE ANALYSIS

A number of different models are required in order to evaluate customer interruption cost indices. These include the equipment operating models for lines, breakers, fuses, disconnect switches and alternate supplies, the load models and the customer sector interruption cost models. The equipment element model uses a two state up/down representation to model the operation/repair cycle. A probability model is used to represent the operation of fuses, breakers and alternate supplies in which probabilities represent the likelihood of successful device or facility operation. The average load at each load point is used as the load model.

A Standard Industrial Classification (SIC) can be used to divide customers into large user, industrial, commercial, agriculture, residential, government & institutions and office & buildings categories. Postal surveys conducted by the University of Saskatchewan. have been utilized to estimate the customer interruption losses for the seven different customer sectors [10, 11]. The surveys show that the cost of an interruption depends on the type of customer interrupted, and on the magnitude and the duration of the interruption. The survey data has been analyzed to give the sector customer damage functions (SCDF) which are used as the customer interruption cost models. The SCDF are shown in Table 1.

Table 1: Sector Interruption Cost (\$/kW)

User Sector	Interruption Duration (Min.) & Cost (\$/kW)				
	1min.	20 min	60 min.	240 min.	480 min.
Larger user	1.005	1.508	2.225	3.968	8.240
Industrial	1.625	3.868	9.085	25.16	55.81
Commercial	0.381	2.969	8.552	31.32	83.01
Agricultural	0.060	0.343	0.649	2.064	4.120
Residential	0.001	0.093	0.482	4.914	15.69
Govt.&Inst.	0.044	0.369	1.492	6.558	26.04
Office&bldg.	4.778	9.878	21.06	68.83	119.2

Table 1 gives the interruption costs for five discrete outage durations. A log-log interpretation of the cost data is used where the interruption duration lies between two separate times,. In the case of durations greater than 8 hours, a linear extrapolation with the same slope as that between the 4 and 8 hour values was used to calculate the interruption cost.

A GENERALIZED ANALYTICAL APPROACH

The basic procedure used in the generalized analytical method to evaluate the customer interruption cost indices can be summarized in the following steps:

Step 1: Find the average failure rate λ_j, the average repair time r_j and the average switching time s_j for a failed element j.

Step 2: Find the affected load points using a direct search technique according to the network configuration. The failure rate λ_{ij} and the failure duration r_{ij} for an affected load point i can be calculated using Equations (1) and (2).

$$\lambda_{ij} = \lambda_j \prod_{k=1}^{Npr} (1 - p_k) \tag{1}$$

where p_k is the probability that fuse (or breaker) k operates successfully. N_{pr} is the total number of breakers and fuses between the load point i and the failed element j.

$$r_{ij} = p_a s_j + (1 - p_a) r_j \tag{2}$$

where p_a is the probability of being able to transfer load for a load point that can be isolated from the failed element. p_a is zero for load points that cannot be isolated by disconnect switches from the failed element j.

Step 3: Using the outage time r_{ij} and the customer type at load point i, determine the per unit (kW) interruption cost c_{ij} using the corresponding sector customer damage function (SCDF).

$$c_{ij} = f(r_{ij}) \tag{3}$$

where $f(r_{ij})$ is the SCDF.

Step 4: Evaluate the expected energy not supply $EENS_{ij}$ and expected interruption cost $ECOST_{ij}$ of the load point i caused by failures of element j.

$$EENS_{ij} = L_i r_{ij} \lambda_{ij} \tag{4}$$

$$ECOST_{ij} = c_{ij} L_i \lambda_{ij} \tag{5}$$

where L_i is the average load of load point i.

Step 5: Repeat 1-4 for all elements in order to calculate the total load point $EENS_i$, $ECOST_i$ and $IEAR_i$ for load point i using the following equations:

$$EENS_i = \sum_{j=1}^{N_e} L_i r_{ij} \lambda_{ij} = L_i \sum_{j=1}^{N_e} r_{ij} \lambda_{ij} \tag{6}$$

$$ECOST_i = \sum_{j=1}^{N_e} c_{ij} L_i \lambda_{ij} = L_i \sum_{j=1}^{N_e} c_{ij} \lambda_{ij} \tag{7}$$

$$IEAR_i = \frac{ECOST_i}{EENS_i} = \frac{\sum_{j=1}^{N_e} c_{ij} \lambda_{ij}}{\sum_{j=1}^{N_e} r_{ij} \lambda_{ij}} \tag{8}$$

where N_e is the total number of elements in the distribution system.

Step 6: Repeat 5 until the $EENS_i$, $ECOST_i$ and $IEAR_i$ of all the load points are evaluated.

Step 7: Evaluate the total system $EENS, ECOST$ and $IEAR$ using the following equations:

$$EENS = \sum_{i=1}^{N_p} EENS_i = \sum_{i=1}^{N_p} L_i \sum_{j=1}^{N_e} r_{ij} \lambda_{ij} \tag{9}$$

$$ECOST = \sum_{i=1}^{N_p} ECOST_i = \sum_{i=1}^{N_p} L_i \sum_{j=1}^{N_e} c_{ij} \lambda_{ij} \tag{10}$$

$$IEAR = \frac{ECOST}{EENS} \tag{11}$$

where N_p is the total number of load points in the system.

It can be seen from Equation (8) that the load point interrupted energy assessment rate is independent of the average load.

TIME SEQUENTIAL SIMULATION TECHNIQUE

The process used to assess the customer interruption cost of a distribution system using the time sequential simulation technique consists of the following steps: This process also recognizes the overlap that can occur in the different equipment failure/repair cycles.

Step 1: Generate a random number for each element in the system and convert these random numbers into time to failure (TTF) values using the appropriate element failure probability distributions.

Step 2: Comparing the TTF of all elements, find the failure event j that has the minimum TTF and the location of the element that caused the event j.

Step 3: Generate two random numbers for the element with minimum TTF and convert them into times to repair (TTR) and times to switch (TTS) using the appropriate probability distributions for the element repair and switching times.

Step 4: Find the load points that are affected by failed event j

Step 5: Determine the failure duration r_{ij} for the load point i in the system configuration.

$$r_{ij} = ks_j + (1-k)r_j \qquad (12)$$

where k is a control constant depending on the probability p_a of load being transferred to an alternate supply. A random number is generated and is converted into a uniformly distributed random value. If the random value is less than p_a, $k=0$, otherwise, $k=1$.

Step 6: If the restoration time caused by the new failure event is overlapped by the old restoration time caused by the old failure event, the overlapping time is deducted from the new restoration time.

Step 7: Evaluate the per unit interruption cost c_{ij} of load point i using r_{ij} and the load point SCDF $f(r_{ij})$.

$$c_{ij} = f(r_{ij}) \qquad (13)$$

Step 8: Evaluate the energy not supply ENS_{ij} and the interruption cost $COST_{ij}$ of the load point i due to the failure event j.

$$ENS_{ij} = L_i r_{ij} \qquad (14)$$

$$COST_{ij} = c_{ij} L_i \qquad (15)$$

Step 9: Add the ENS_{ij} and the $COST_{ij}$ to their total values respectively.

Step 10: Repeat Step 5-9 for all load points.

Step 11: If the total simulation time is less than the specified simulation time, go to Step 12, otherwise, go to Step 13.

Step 12: Generate a new random number for the repaired element and convert it into the new TTF and go to Step 2.

Step 13: The total energy not supply ENS_i and the interruption cost $COST_i$ of the load point i for the total simulation years are:

$$ENS_i = \sum_{j=1}^{N_s} L_i r_{ij} = L_i \sum_{j=1}^{N_s} r_{ij} \qquad (16)$$

$$COST_i = \sum_{j=1}^{N_s} c_{ij} L_i = L_i \sum_{j=1}^{N_s} c_{ij} \qquad (17)$$

where N_s is the total number of failure events in the specified simulation period. The expected energy not supply $EENS_i$, the expected interruption cost $ECOST_i$ and $IEAR_i$ can be calculated using the following equations:

$$EENS_i = \frac{ENS_i}{TST} \qquad (18)$$

$$ECOST_i = \frac{COST_i}{TST} \qquad (19)$$

$$IEAR_i = \frac{ECOST_i}{EENS_i} = \frac{COST_i}{ENS_i} = \frac{\sum_{j=1}^{N_s} c_{ij}}{\sum_{j=1}^{N_s} r_{ij}} \qquad (20)$$

Where TST is the total specified simulation period in years.

The system $EENS$, $ECOST$ and $IEAR$ can be calculated using equations similar to Equations (9), (10) and (11).

SYSTEM ANALYSIS

Two computer programs designated as DISRE1 and DISRE2 which use the described generalized analytical technique and the time sequential simulation approach respectively have been developed. These programs can be used to assess a range of distribution systems. The set of reliability indices provided by the two programs include:

- the three basic load point indices,
- the probability distributions of three basic load point indices,
- the load point cost/worth indices (*EENS, ECOST, IEAR*),
- the probability distributions of the cost/worth indices,
- the basic system indices (*SAIFI, SAIDI,...*),
- the probability distributions of the basic system indices,
- the system cost/worth indices,
- and the probability distributions of the system cost/worth indices.

These indices can be used to evaluate the reliability of an existing distribution system and to provide useful planning information regarding improvements to existing systems and the design of new distribution systems. This paper is focused on the development and utilization of the cost/worth indices for individual load points and for the system. The following illustrates applications to two distribution systems from a test system designated as the RBTS developed by the Power System Research Group at the University of Saskatchewan [12]. The basic reliability parameters required in the analysis are presented in [12]. The restoration times of the elements are assumed to be lognormally distributed with standard deviations of half their average values.

1. Application to a typical urban distribution system

The system in Fig. 1 is a typical urban distribution system connected to bus 2 of the RBTS. There are four types of customer: residential, commercial, small industrial and government/institutional. Transformer repair times are used in the evaluation. The techniques described earlier were used to evaluate the reliability of this system.

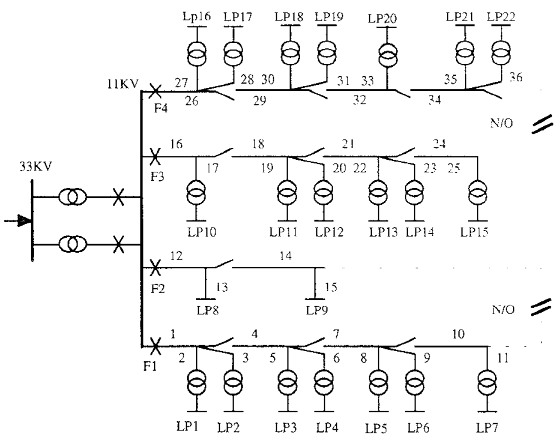

Fig. 1 A typical urban distribution system

The *EENS*, *ECOST* and *IEAR* indices for all the load points, the feeders and the system using the generalized analytical method and the time sequential simulation technique with and without considering overlapping time have been evaluated. The load point *ECOST* obtained using the three different approaches are shown in Table 2. In Table 2, (A), (SO) and (SN) represent the analytical method and the simulation approach with and without considering overlapping time respectively. It can be seen from Table 2 that the results obtained using the three different techniques are very close and that overlapping time has little effect on the cost indices for this system. The simulation was performed for a period of 13000 years.

Table 2: Interruption costs of load points

Load (i)	ECOST (A) (k$/yr)	ECOST (SO) (k$/yr)	ECOST (SN) (k$/yr)
1	4.20918	4.19809	4.19826
2	4.25362	4.15691	4.15717
3	4.25362	4.30068	4.30088
4	7.99919	8.56027	8.56038
5	8.05653	8.73097	8.73156
6	17.91651	16.1819	16.25811
7	17.8365	17.68862	17.68878
8	4.92633	4.81804	4.81804
9	5.30873	5.40358	5.40358
10	4.2099	4.54642	4.54645
11	4.25362	4.07148	4.07177
12	3.58793	3.67408	3.67429
13	8.00932	8.56978	8.56983
14	8.02366	7.48112	7.48114
15	17.75671	16.97079	16.97199
16	17.97356	18.94897	18.94946
17	3.55054	3.79771	3.80975
18	3.54180	3.24912	3.24959
19	3.57919	3.4662	3.46651
20	8.05906	8.68713	8.68817
21	8.00932	8.0136	8.01391
22	17.84787	17.23425	17.23496

Tables 3 and 4 show the system cost/worth indices obtained using the analytical and simulation techniques respectively. The difference in the ECOST between the analytical value and the simulation value is 0.2% based on the analytical result. The relative difference in EENS and IEAR is 0.4% and 0.6% respectively.

Table 3 System cost indices using the analytical approach

FEEDER	EENS (MWh/yr)	ECOST (k$/yr)	IEAR ($/kWh)
1	11.978	64.52515	5.38697
2	1.52209	10.23506	6.72435
3	10.20291	45.84114	4.49295
4	11.1413	62.56136	5.61527
Total	34.84431	183.1627	5.2566

Table 4 System cost indices using the simulation method

FEEDER	EENS	ECOST	IEAR
1	12.04926	63.89	5.3024
2	1.51298	10.22162	6.75594
3	10.21642	45.3085	4.43487
4	11.22663	63.40747	5.64795
Total	35.00529	182.8276	5.22286

The probability distributions of the load point, feeder and system indices can be obtained using the time sequential simulation technique. Fig. 2 shows the probability distributions of the interruption cost and the energy not supplied for Feeder 1. It can be seen from the figure that the energy not supplied is between 2 and 3 kWh for over 30 percent of the failures and interruption costs are larger than $9,000 for over 60 percent of the failures.

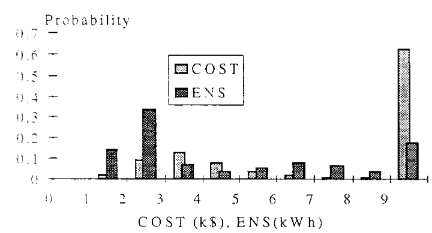

Fig 2. Probability distributions of COST and ENS for Feeder 1

Fig. 3 shows the probability distributions for load point failure duration, interruption cost and *ENS* for load point 1. The failure durations peak between 0 and 1 hour.

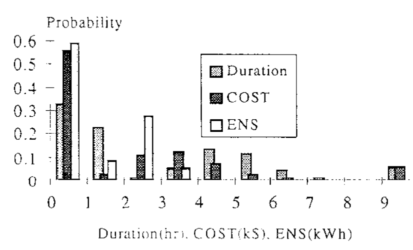

Fig 3 Probability distributions of COST and ENS for load point 1

The convergence of the simulation can be illustrated by convergence curves of the indices. Fig. 4 and 5 shows the convergence of the load point and feeder indices. It can be seen from these figures that both indices convergence reasonable quickly.

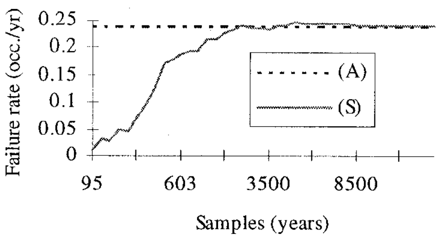

Fig 4 Convergence of the load point 1 failure rate

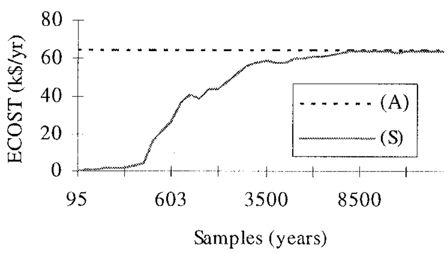

Fig 5 Convergence of ECOST for Feeder 1

2. Application to a mixed urban/rural distribution system

The time sequential simulation technique is used in the following example to illustrate the effect on the system interruption cost of different elements and parameters and provide input to the decision making process. Fig. 6 is a mixed urban/rural distribution system with residential, commercial small industrial and farm customers. Transformer replacement is used in this analysis rather than transformer

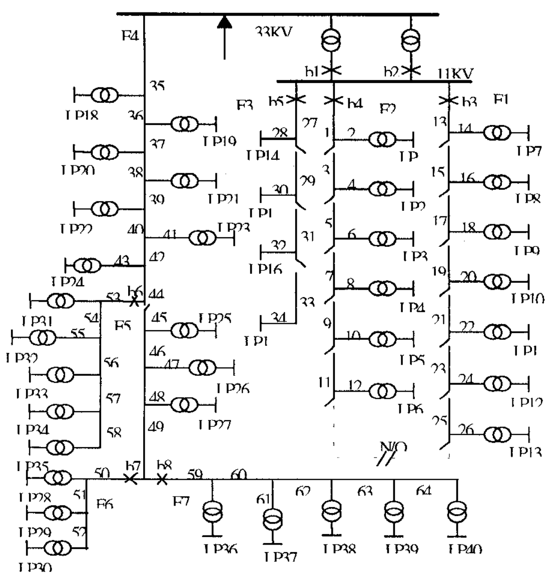

Fig. 6 A mixed urban/rural distribution system

repair as used in the previous system. The four different cases investigated are as follows. Case 1 is a basic case in which all breakers are assumed to be 100% reliable. The probabilities of the breakers in the three branches of Feeder 4 operating successfully are assumed to be 0.8 in Case 2. Based on Case 1, alternate supplies are installed in Feeders 3 and 4 in Case 3. Based on Case 3, one additional switch is installed in line section 40 in Case 4.

Breaker reliability effects

The reliability of the breakers installed in a distribution system affect the load point interruption costs. Figs. 7 shows the load point interruption costs for Cases 1 and 2. It can be seen from Fig. 7 that breaker reliability has a large effect on the load point interruption costs. When the breaker reliability decreases from 1 to 0.8, the interruption cost of Feeder 4 increases by 13.2% from 40,686$ to 46,065 $/yr and the total system cost increases by 8% from 66,968 $/yr to 72,347 $/yr.

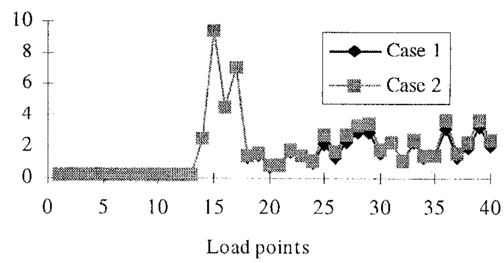

Fig. 7 Load point ECOST values for Case 1 and Case 2

Alternate supply effects

There is no alternate supply to Feeders 3 and 4 in the basic structure shown in Fig. 6. In order to investigate the effect of alternate supply on the customer interruption costs, two alternate supplies were installed in these two feeders. Fig. 8 shows the variations in load point interruption costs compared to the base case. It can be seen that alternate supply has a significant effect on the customer interruption costs. The customer interruption costs of Feeders 3 and 4 decrease by 8,314 $/yr and 7,212 $/yr respectively. The total system customer interruption costs decrease by 15,526 $/yr (23.2%) from 66,968 $/yr to 51,442 $/yr. These data can be compared with the investment cost of constructing two alternate supplies and used in the decision making process.

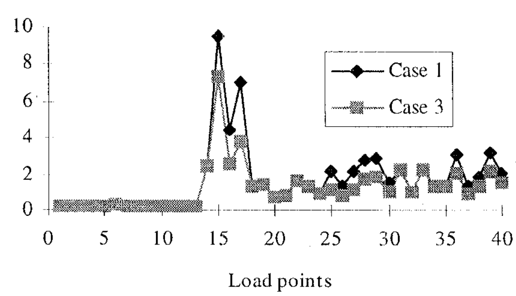

Fig. 8 Load point ECOST values for Case 1 and Case 3

Effect of disconnect switches

Switching devices affect the customer interruption durations and therefore affect the customer interruption costs. Fig. 9 shows the effect on the load point customer interruption costs of the additional switch in Case 4. The customer interruption costs of Feeder 4 decrease by 4,697 $/yr from 32,372 $/yr to 27,676 $/yr. This value is larger than the general annual investment cost of a switching device and therefore the additional switch is justified.

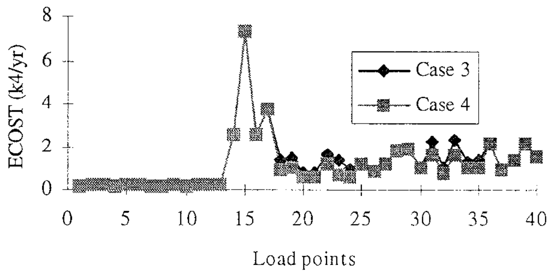

Fig. 9 Load point ECOST values for Case 3 and Case 4

The studies conducted shows that reliability cost/worth analysis can be a useful and efficient tool in distribution system planning and design. The studies conducted on the system shown in Fig. 6 used the time sequential approach. It should be appreciated that similar values could have been obtained using the analytical technique. If specific event parameters and the distributions associated with these parameters are required then the time sequential approach must be used.

CONCLUSION

This paper presents a generalized analytical technique and a time sequential simulation approach to evaluate load point and system customer interruption cost indices of complex radial distribution systems. These techniques have been used to evaluate the load point and system indices of EENS, ECOST and IEAR for two RBTS distribution systems. Overlapping time was considered in the simulation technique, and the results with and without considering overlapping time are compared. The results shows that overlapping time has little influence on the results when the system is small and the element restoration times are short and therefore can be ignored. When the distribution system includes many elements and the element repair times are relatively long, the effect of overlapping time should be considered. The paper also compares the results obtained by the simulation technique with those obtained using the analytical approach. The results show that the two techniques give comparable load point and system average cost estimates. The simulation technique can be used, however, to obtain both the average values of the interruption indices and their distributions. The paper also briefly illustrates how the two techniques and the cost data can be used in system planning and operation. The studies conducted show that the reliability cost/worth technique can be a useful and efficient tool in distribution system planning and design. It can also be concluded form the studies performed that the distributions of element restoration time have an effect on the cost indices, but these effects can generally be ignored when only average cost indices are required.

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