An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area

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Table of Contents

Table of Contents

Motivation : Why this study ?

Key Objectives

- Establish and estimate a **new DSGE** (Dynamic stochastic general equilibrium) model for the Euro Area with both price and **wage stickiness** as well as additional features such as partial **indexation** and external **habit formation**.
- **Compare** this model to benchmarks (VAR), **estimate** impulse responses for a large panel of **structural shocks** (> 11 shocks) and investigate the main contributors of **variance** in output and inflation through a variance and a historical **decomposition**.

Methods: How do they obtain their results?

A Bayesian Model

Bayesian approach to find the posterior distribution of endogenous variables using **empirical data from the Euro Area**.

Estimation/Calibration Method

The model is estimated using the Bayesian estimation/calibration method instead of more traditional methods like GMM for better **model fit**.

Decomposition Method

Variance : Studies the contribution of structural shocks to the variance in the **predicted endogenous variables** by looking at **forecast errors.**

Results: What did the Authors Find?

- **Model Comparison:**
  - The model performs **at least as well as most existing models (VARs and BVARs)**.

- **Impulse Responses:**
  - Adding the 11 proposed structural shocks enables the model to **match data** much **more effectively** than models incorporating a limited variety of shocks.

- **Variance Decomposition:**
  - **Output** fluctuations are primarily explained by: **labor & monetary** shocks + fiscal shocks (in the **short run**) and **not by productivity shocks** as proposed in earlier studies.
  - **Inflation** variations are driven by: **price markup** shocks + monetary policy (in the **medium/long run**).

Table of Contents

New Model Features

New Model Features

Model Feature	Implementation
External Habit in Consumption	Utility depends on $C_t - h\,C_{t-1}$
Investment Adjustment Cost	Adjustment cost $\propto \left(\frac{I_t}{I_{t-1}} - 1\right)^2$
Variable Capacity Utilization	Firms choose utilization u_t; $\hat{u}_t = \psi\,\hat{r}_t^K$, with capital services $u_t K_{t-1}$.
Calvo Wage Stickiness	A fraction ξ_w of households do not adjust wages.

Structural Shock Processes

Shock Name	Explanation
TFP Shock	ϵ_t^a (AR(1), ρ_a) in production function
Preference Shock	ϵ_t^b (AR(1), ρ_b) in consumption Euler
Inv. Cost Shock	ϵ_t^I (AR(1), ρ_I) in investment equation
Labor Supply Shock	ϵ_t^L (AR(1), ρ_L) in wage eq.
Gov. Spending Shock	ϵ_t^G (AR(1), ρ_G) in resource constraint
Price Markup Shock	η_t^p (i.i.d.) in inflation equation
Wage Markup Shock	η_t^w (i.i.d.) in wage equation
Eq. Premium Shock	ν_t^Q (i.i.d.) in Q-equation
Mon. Policy Shock	η_t^R (i.i.d.) in interest rule
Infl. Target Shock	π_t^* (AR(1)) in interest rule

Real wage equation

- We assume that when a household cannot adjust their wage, they set their wage according to

$$W_t^{\tau} = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_w} W_{t-1}^{\tau}$$

- This yields the following **real wage equation**

$$\hat{w}_t = \frac{\beta}{1+\beta} E_t\left[\hat{w}_{t+1}\right] + \frac{1}{1+\beta}\hat{w}_{t-1} + \frac{1}{1+\beta} E_t\left[\hat{\pi}_{t+1}\right] - \frac{1+\beta\,\gamma_w}{1+\beta}\hat{\pi}_t$$

$$+ \frac{\gamma_w}{1+\beta}\hat{\pi}_{t-1} + \frac{(1-\beta\,\xi_w)\,(1-\xi_w)}{(1+\beta)\,\xi_w}\,\frac{(1+\lambda_w)\,\sigma_L}{(1+\lambda_w)\,\lambda_w}$$

$$\times\left[\hat{w}_t - \sigma_L\,\hat{L}_t - \frac{\sigma_c}{1-h}\left(\hat{C}_t - h\,\hat{C}_{t-1}\right) - \epsilon_t^L - \eta_t^w\right]$$

NKPC

- Similarly, when a firm cannot adjust their price, they set the price to

$$P_t^{\tau} = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} P_{t-1}^{\tau}$$

- This yields the following **New Keynesian Phillips Curve**

$$\hat{\pi}_t = \frac{\beta}{1+\beta\gamma_p} E_t[\hat{\pi}_{t+1}] + \frac{\gamma_p}{1+\beta\gamma_p}\hat{\pi}_{t-1} + \frac{1}{1+\beta\gamma_p}\frac{(1-\beta\,\xi_p)(1-\xi_p)}{\xi_p}\left[\alpha\,\hat{r}_t^k + (1-\alpha)\,\hat{w}_t - \epsilon_t^a + \eta_t^p\right]$$

Table of Contents

What is the Bayesian Approach ? (1/3)

Bayesian vs Frequentist

- **Frequentist approach:** Assume data-generating process, assume that data is a realization of an unobserved population, maximize likelihood function to get a point estimate, and estimate standard errors. **Cannot make probability statements.**
- **Bayesian approach:** Assume that parameter is a random variable with a given distribution (prior), assume a conditional distribution for the data given parameters (likelihood function), use Bayes rule to calculate posterior (note that this can now be done fully computationally, no further assumptions needed). **Can make probability statements.**

What is the Bayesian Approach ? (2/3)

Reasoning in Distributions

Prior Distribution - $P(\theta)$

- The researcher chooses a distribution for the model parameters. The prior quantifies the uncertainty regarding the parameters before accounting for the model structure and data.

Likelihood function - $P(Data|\theta)$

- Measures how likely the realization of data is given the parameters.

Posterior Distribution

- Updated probability distribution of the parameters

$$P(\theta|Data) = \frac{P(Data|\theta)P(\theta)}{P(Data)}$$

What is the Bayesian Approach ? (3/3)

Reading Results :

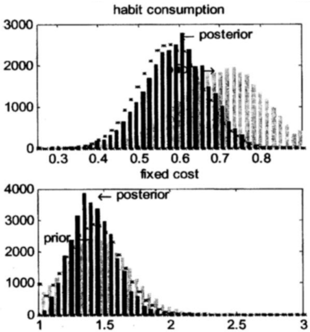

Figure: Prior/Posterior distributions for some Parameters

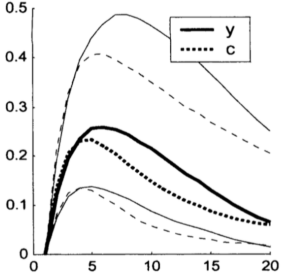

Figure: Model estimations of endogenous variables

Table of Contents

Calibrating the Model : Traditional Approach

The Traditional Method : Use the General Method of Moment (GMM) to minimize a distance function between theoretical and empirical moments

How does it work in practice ?

1. Theoretical impulse response functions are generated using the model
2. Empirical impulse response functions are estimated from the data
3. Model parameters are adjusted to minimize the difference between both response functions

Calibrating the Model : The Author's Approach (1/2)

Bayesian Estimation

Calibration is done through **likelihood maximization** using the likelihood function we introduced earlier.

Key features

- Parameters are adjusted until the calculated likelihood function is as high as possible, meaning model prediction are close to real data
- No "one" best fit as range of likely parameter values are explored

Calibrating the Model: The Author's Approach (2/2)

1. Simplify the model based on past data for the Euro Area
2. Split the data between hidden factors and real data
3. Use the Kalman filter to match the observed data
4. Adjust the model's parameters by exploring a range of possibilities

The Kalman Filter

Used to compare theoretical to empirical forecasts for hidden factors based on an observation equation.

$$x_t = \hat{x}_t + K_t \left(y_t - HFx_{t-1} \right)$$

with K_t the Kalman Gain (reliability), H the observation matrix, and F the state transition matrix.

Calibrating the model : Pros & Cons of this approach

Advantages

+ **Incorporating uncertainty:** Can use full prior information coming from previous macro/microeconometric studies
+ **Numerical Stability:** Using priors helps reduce instability linked to data scarcity

Limitations

– **Prior sensitive:** It is not clear how priors affect the results
– **Practical use:** Reasoning in distribution sometimes leads to results that are hard to use for policy makers as they are not precise enough

Estimation results : Impulse response functions

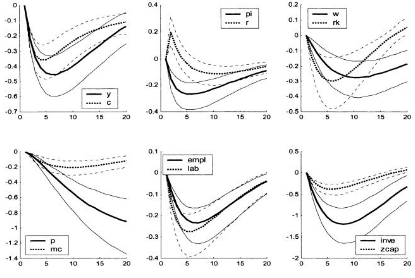

FIGURE 11. Monetary Policy Shock

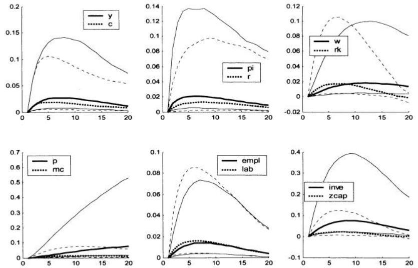

FIGURE 12. Inflation Objective Shock

Figure: Temporary Monetary Policy Shock

Figure: Persistent Monetary Policy (inflation objective) Shock

Table of Contents

VAR(p)

- A bivariate (i.e., a two-variable) first-order VAR process (i.e., a VAR(1) process) is given as

$$y_{1t} = a_{11}\, y_{1,t-1} \;+\; a_{12}\, y_{2,t-1} \;+\; \epsilon_{1t},$$
$$y_{2t} = a_{21}\, y_{1,t-1} \;+\; a_{22}\, y_{2,t-1} \;+\; \epsilon_{2t},$$

  where ϵ_{1t} and ϵ_{2t} are two error terms (independent of the history of y_{1t} and y_{2t}) that may be correlated.

- The idea of the VAR(p) model is to regress each component of $\mathbf{y}_t$ on its own lags and on the lags of the other components.
- The model can describe lagged or dynamic dependencies among variables. For example, one may ask how y_{2t} affects the future path of y_{1t}, and vice versa.

Comparison Table

TABLE 2. ESTIMATION STATISTICS

Summary of the model statistics: VAR—BVAR—DSGE

	VAR(3)	VAR(2)	VAR(1)	DSGE-model
In sample RMSE (80:2–99:4)				
Y	0.42	0.44	0.50	0.54
π	0.20	0.21	0.23	0.21
R	0.12	0.12	0.13	0.12
E	0.19	0.20	0.22	0.21
w	0.48	0.51	0.54	0.57
C	0.42	0.44	0.48	0.60
I	1.03	1.08	1.17	1.26

Posterior probability approximation (80:2–99:4)

	VAR(3)	VAR(2)	VAR(1)	DSGE-model
Prediction error decomposition[1]	−303.42	−269.11	−269.18	
Laplace approximation	−315.65	−279.77	−273.55	−269.59
Modified harmonic mean[2]	−305.92	−270.28	−268.41	−269.20
Bayes factor rel. to DSGE model	0.00	0.34	2.20	1.00
Prior probabilities	0.25	0.25	0.25	0.25
Posterior odds	0.00	0.10	0.62	0.28

Table of Contents

Variance Decomposition : Method

- **Goal :** Quantify the **average expected contribution** of each structural shock to the forecast error variance[1] of endogenous variables at **different time horizons**
- **Main attributes :** forward-looking, horizon-dependent

$$VD_{i,h} = \frac{\text{Variance due to shock i}}{\text{Total forecast error variance at horizon h}} \times 100\%$$

- **Simple interpretation :** the **larger** the % of variance explained, the **bigger the impact** of a shock on the variable considered

[1] expected squared difference between the actual value and the predicted value of an endogenous variable ($E[(y_t - \hat{y}_t)^2]$)

Variance Decomposition : Results

Endogenous Variable	Time Horizon	Main Driving Shock(contribution to variation in%)
Output	Short to Long Run	Labor supply($\sim$ 30%)/ monetary policy($\sim$ 28%)
Output	Very short Run	Government Spending(25%)/Preference (19%)
Output	Long run	Investment($\sim$15%)/Productivity($\sim$10%)
Inflation	Very short to Long Run	Price markup (90-45%)
Inflation	Medium to long Run	Monetary policy (20-40%)

Figure: Variance Decomposition Summary Table

Main Observations

- **Output :** Supply, productivity and Labor contribute to less than 40% of the variance in output in the long-run compared to 45% for labor alone with VARs ([1]).
- **Inflation :** Monetary policy has a huge influence on the importance of the impact of other shocks on inflation.

Historical Decomposition : Method

- **Goal :** Rebuild the Shock paths that represent historical realizations
- **How does it Work ?**
  1. Start with the impulse response functions based on the model
  2. Infer the sequence of shocks that best explains movements in the observed data
- **Formula**[2]

$$y_t = (\sum_{k=0}^{t-1} a_1^k) a_0 + a_t' y_0 + \underbrace{\sum_{k=0}^{t-1} a_1^k S \epsilon_{t-k}}_{\text{Shock Contribution}}$$

With S the weight matrix for the different shocks and $y_t = a_0 + a_1 y_{t-1} + S\epsilon_t$

[2]Source : course on Structural VAR by Aurélien Poissonnier(ENSAE)

Historical Decomposition : Results

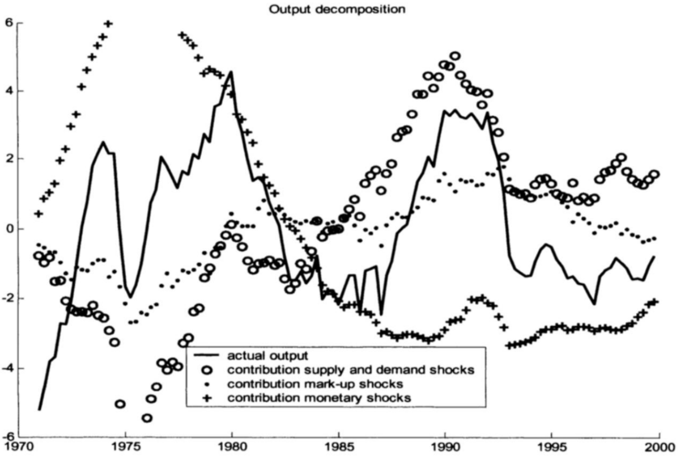

FIGURE 14. Output Decomposition

Table of Contents

The Prediction Problem(1/2)

Limitation n°1 : Wide prediction bands

Reasons:

1. **Short time frame:** The model is estimated on data over 1980:2-1999:4
2. **High number of estimated parameters and shocks considered**

The Prediction Problem(2/2)

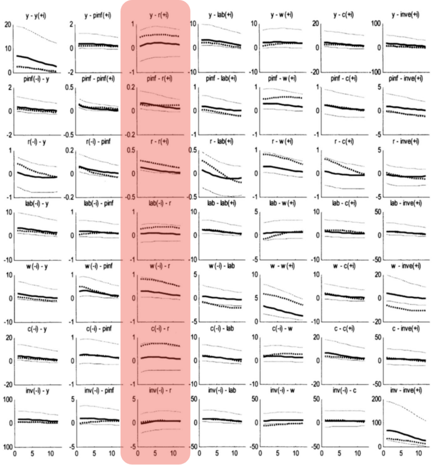

Figure: Comparison of Cross-Covariances: DSGE model VS data

Dimensionality Reduction Problem

Limitation n°2 : On identification issues

(Too) High dimensional parameter space : There are more shocks considered than endogenous variables ! (7 endogenous variable for **11** shocks)

Authors' Solution :

1. Categorization : shocks are split between temporary and persistent shocks
2. Independence: All shocks are assumed to be uncorrelated


Thank you

References

[1] Matthew D.Shapiro et al. "Sources of Business Cycle Fluctuations". In: *NBER Macroeconomic Annual* (1989), pp. 15–78.