Supplementary Materials

Supplementary Table 1. Swarm performance metrics. The table describes the metrics used to assess the performance of the swarm passing-through process in a complex environment. The definitions of all performance metrics are described in detail in the paper. Among them, the *arrival rate* and *surpass rate* are the metric of swarm's passing-through safety, and the *arrival time* and *average speed* are the metrics of swarm's passing-through efficiency. A few typical scenes are chosen to generate the snaps of the swarm's passing-through process based on these four approaches. The correlations between the scenes and the corresponding figures are marked in the table for clarity.

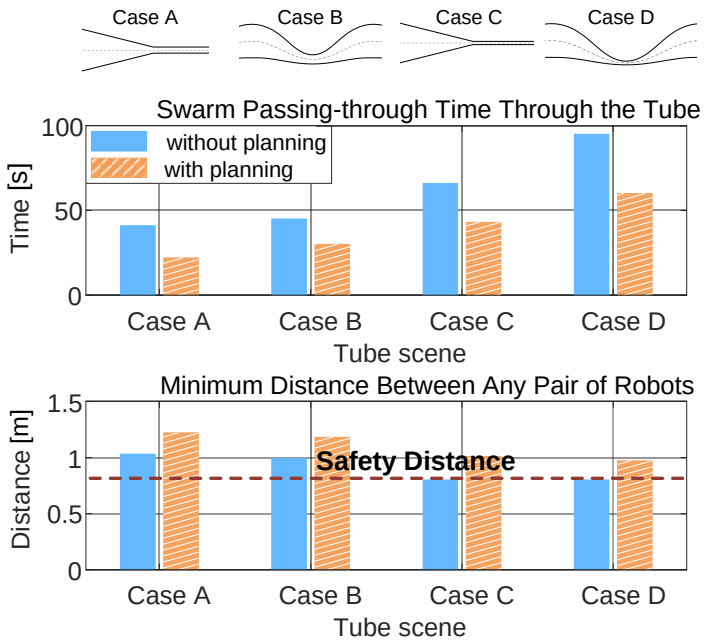
TABLE I: Performance metrics for different methods in twenty scenes.

Scene	Metrics	Method			
		Method Proposed	Basic Virtual Tube	Flocking Method	NMPC
Scene 1	Arrival time(s)	33.13	**29.08**	32.24	29.90
	Arrival rate(%)	**100**	90	70	50
	Surpass rate(%)	**0**	40	40	\
	Average speed(m/s)	2.2983	**2.3327**	2.0071	1.8511
Scene 2	Arrival time(s)	32.71	**28.13**	46.36	∞
	Arrival rate(%)	**100**	30	65	10
	Surpass rate(%)	**0**	20	65	\
	Average speed(m/s)	2.3269	**2.3734**	2.0769	0.3527
Scene 3	Arrival time(s)	34.87	**31.22**	∞	30.80
	Arrival rate(%)	**100**	**100**	95	60
	Surpass rate(%)	**0**	30	5	\
	Average speed(m/s)	**2.2631**	2.2328	1.9873	1.8641
Scene 4	Arrival time(s)	35.33	**27.36**	36.54	∞
	Arrival rate(%)	**100**	**100**	**100**	40
	Surpass rate(%)	**0**	50	40	\
	Average speed(m/s)	2.3324	**2.5029**	2.0617	1.3916
Scene 5	Arrival time(s)	32.89	31.01	31.83	**28.50**
	Arrival rate(%)	**100**	55	25	50
	Surpass rate(%)	**0**	35	30	\
	Average speed(m/s)	**2.2487**	1.8992	2.0690	1.7426
Scene 6 (Fig. 2, 3, 4, 5)	Arrival time(s)	**26.02**	29.51	∞	28.70
	Arrival rate(%)	**100**	20	15	60
	Surpass rate(%)	**0**	60	100	\
	Average speed(m/s)	**2.4711**	1.9336	2.1102	1.8365
Scene 7	Arrival time(s)	30.68	29.98	30.05	**29.40**
	Arrival rate(%)	**100**	70	**100**	25
	Surpass rate(%)	**0**	**0**	30	\
	Average speed(m/s)	**2.2494**	1.9392	1.9874	1.6600
Scene 8	Arrival time(s)	34.67	36.32	34.44	**32.20**
	Arrival rate(%)	**100**	70	85	30
	Surpass rate(%)	**0**	40	70	\
	Average speed(m/s)	**2.3510**	1.9190	2.0895	1.8414
Scene 9	Arrival time(s)	**27.20**	27.86	27.81	29.10
	Arrival rate(%)	**80**	75	30	50
	Surpass rate(%)	**25**	35	**25**	\
	Average speed(m/s)	**2.3096**	1.9274	1.9918	1.6426
Scene 10 (Fig. 6, 7, 8, 9)	Arrival time(s)	34.74	**33.85**	35.44	∞
	Arrival rate(%)	**100**	25	30	50

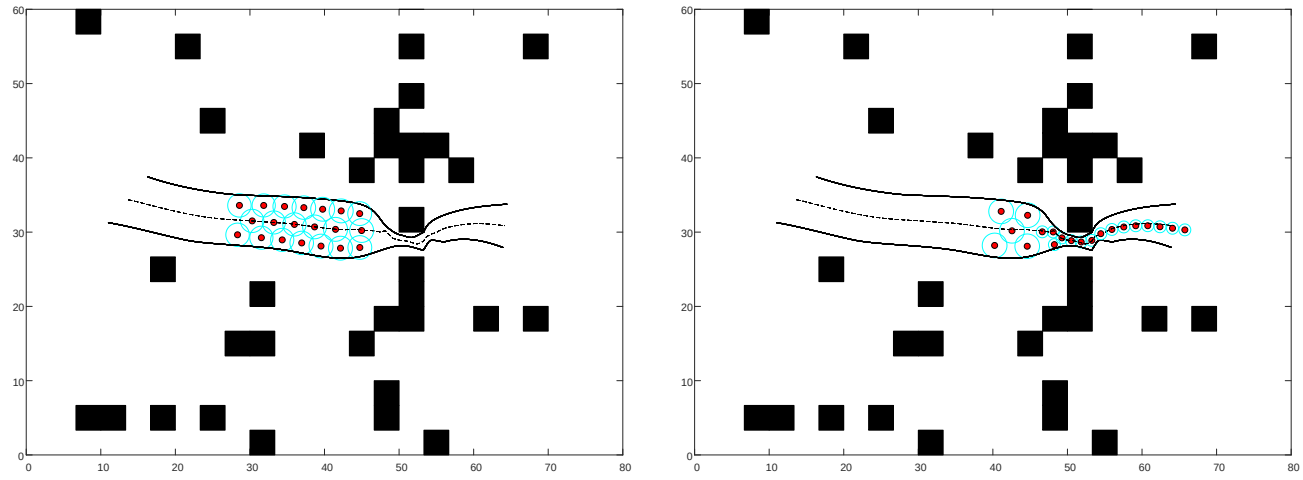
	Surpass rate(%)	**0**	75	60	\
	Average speed(m/s)	**2.0534**	1.9455	2.0333	1.4430
Scene 11	Arrival time(s)	**26.04**	30.67	35.25	∞
	Arrival rate(%)	**60**	30	45	25
	Surpass rate(%)	**40**	65	50	\
	Average speed(m/s)	**2.5446**	1.9568	2.0235	0.8503
Scene 12	Arrival time(s)	34.09	**32.48**	40.28	∞
	Arrival rate(%)	**90**	5	5	5
	Surpass rate(%)	**0**	45	100	\
	Average speed(m/s)	**2.3083**	1.9249	2.0748	0.5072
Scene 13	Arrival time(s)	**31.56**	32.21	∞	∞
	Arrival rate(%)	**100**	80	25	25
	Surpass rate(%)	**0**	50	80	\
	Average speed(m/s)	**2.3180**	1.9213	2.1107	1.7255
Scene 14	Arrival time(s)	33.18	**32.47**	34.62	36.40
	Arrival rate(%)	**100**	90	55	35
	Surpass rate(%)	35	40	**20**	\
	Average speed(m/s)	**2.2647**	1.9461	2.0205	1.6509
Scene 15	Arrival time(s)	34.80	32.05	**31.03**	∞
	Arrival rate(%)	**100**	55	**100**	35
	Surpass rate(%)	**0**	**0**	10	\
	Average speed(m/s)	**2.2638**	1.9536	2.0157	1.3100
Scene 16	Arrival time(s)	**25.93**	28.09	26.95	27.40
	Arrival rate(%)	**100**	90	90	50
	Surpass rate(%)	**0**	**0**	10	\
	Average speed(m/s)	**2.3946**	1.9368	2.0008	1.8432
Scene 17 (Fig. 10, 11, 12, 13)	Arrival time(s)	**27.40**	30.18	29.58	∞
	Arrival rate(%)	**100**	75	70	55
	Surpass rate(%)	**0**	**0**	10	\
	Average speed(m/s)	**2.4241**	1.9570	2.0009	1.3753
Scene 18	Arrival time(s)	**27.09**	31.59	31.69	30.70
	Arrival rate(%)	**100**	55	**100**	30
	Surpass rate(%)	**0**	25	**0**	\
	Average speed(m/s)	**2.5089**	1.9460	2.0511	1.8355
Scene 19 (Fig. 14, 15, 16, 17)	Arrival time(s)	**32.64**	33.93	34.88	32.70
	Arrival rate(%)	**100**	70	85	50
	Surpass rate(%)	**0**	10	55	\
	Average speed(m/s)	**2.3623**	1.9574	2.0239	1.8135
Scene 20 (Fig. 18, 19, 20, 21)	Arrival time(s)	29.77	**26.23**	27.53	∞
	Arrival rate(%)	**90**	45	15	75
	Surpass rate(%)	**25**	60	55	\
	Average speed(m/s)	**2.1649**	1.9022	2.0274	1.6478

TABLE II: Actual time for solving the speed and density planning problem.

Scene	1	2	3	4	5	6	7	8	9	10
Time (ms)	127.29	214.70	251.38	234.10	266.06	294.31	319.33	313.09	251.14	263.31
Scene	11	12	13	14	15	16	17	18	19	20
Time (ms)	243.63	287.24	255.29	279.28	271.31	292.85	312.51	160.73	223.55	279.08



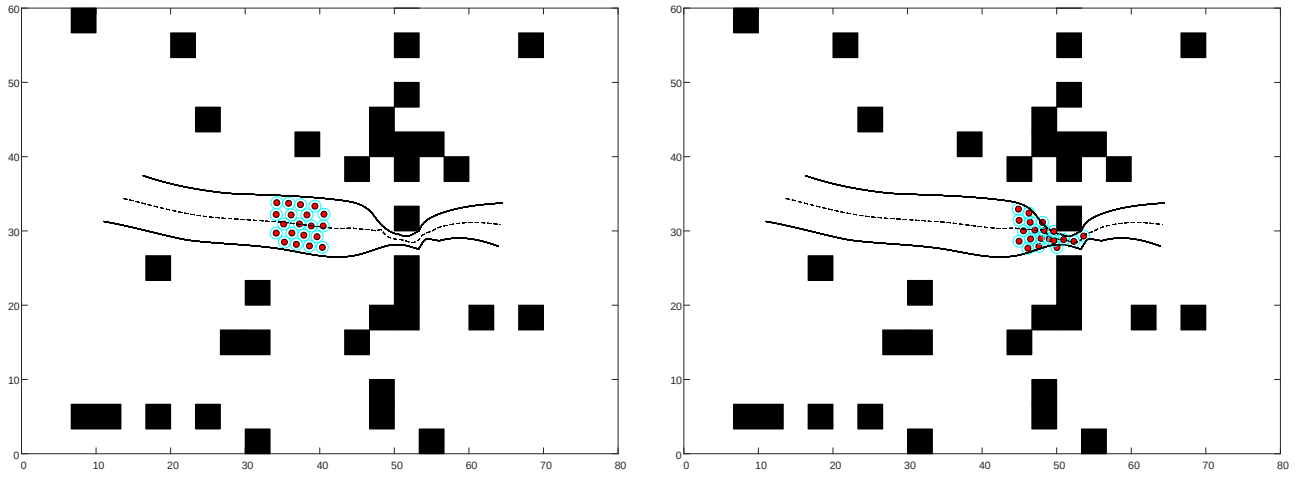
Supplementary Fig.1. Swarm passing-through time and the minimum distance between any pair of robots in four different virtual tube scenes without planning and with planning. The passing-through time and the minimum distance are evaluation indicators of safety and efficiency respectively. Case A, B, C, D are the four scenes manually set in the paper, including a normally narrowing trapezoidal virtual tube, a normally narrowing curved virtual tube, a rapidly narrowing trapezoidal virtual tube, and a rapidly narrowing curved virtual tube.



(a) Before passing through the narrow space

(b) Passing through the narrow space

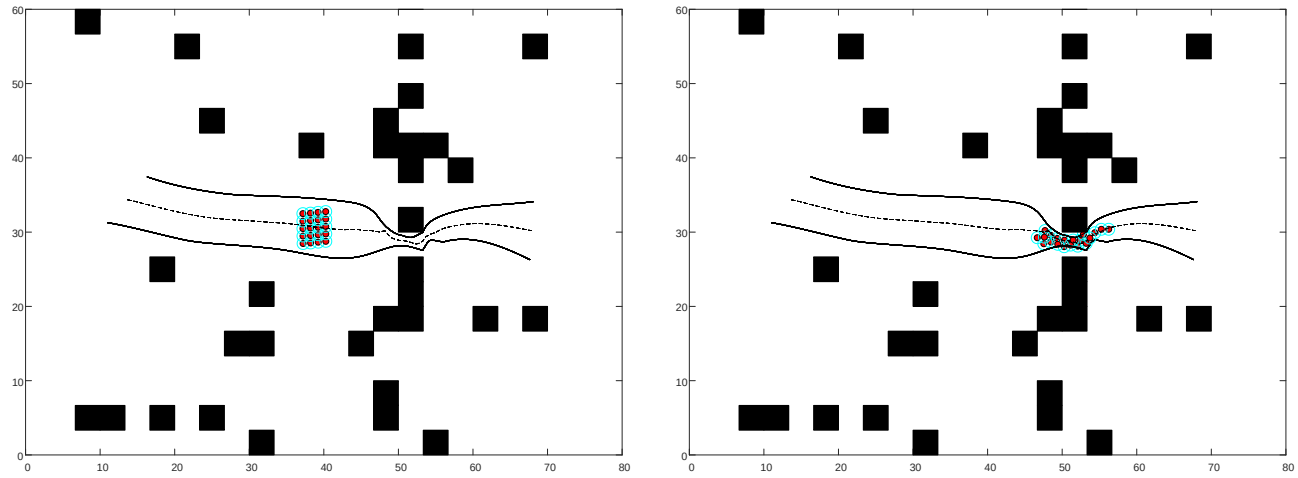
Supplementary Fig.2. The passing-through process of the swarm in scene 6 according to the method proposed. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

(b) Passing through the narrow space

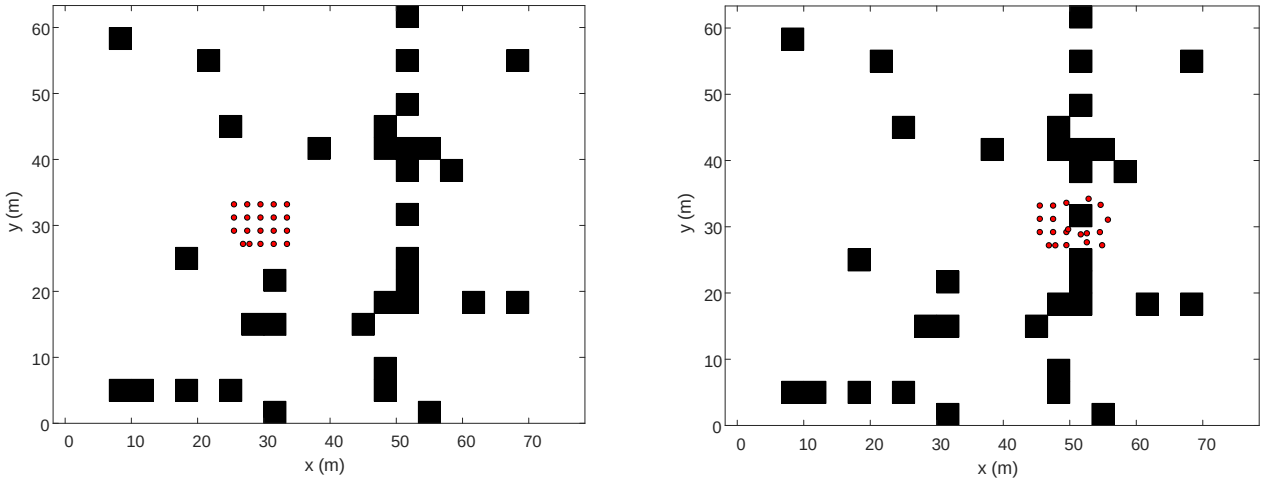
Supplementary Fig.3. The passing-through process of the swarm in scene 6 according to the basic virtual tube control method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

(b) Passing through the narrow space

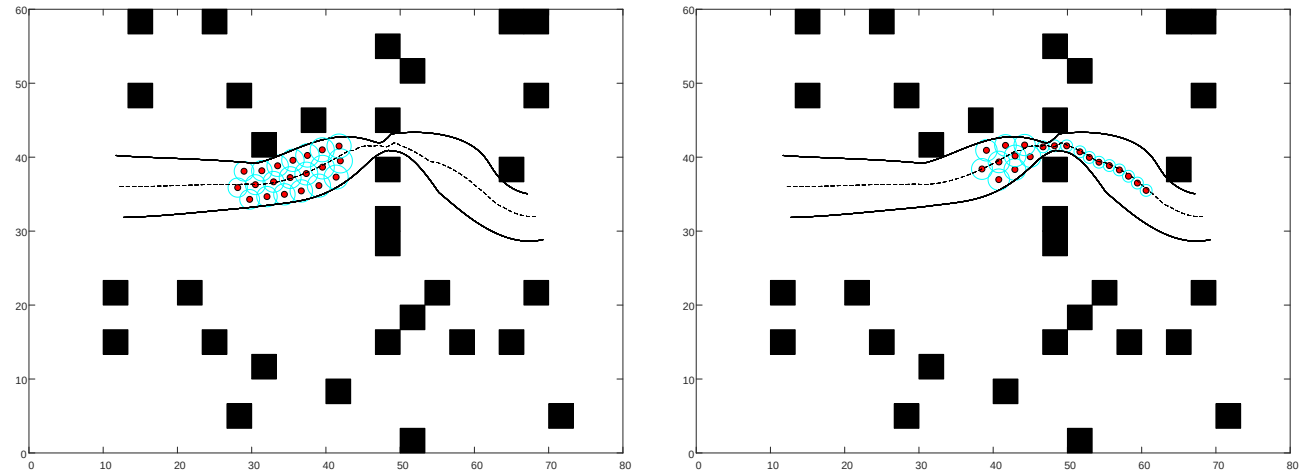
Supplementary Fig.4. The passing-through process of the swarm in scene 6 according to the flocking method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

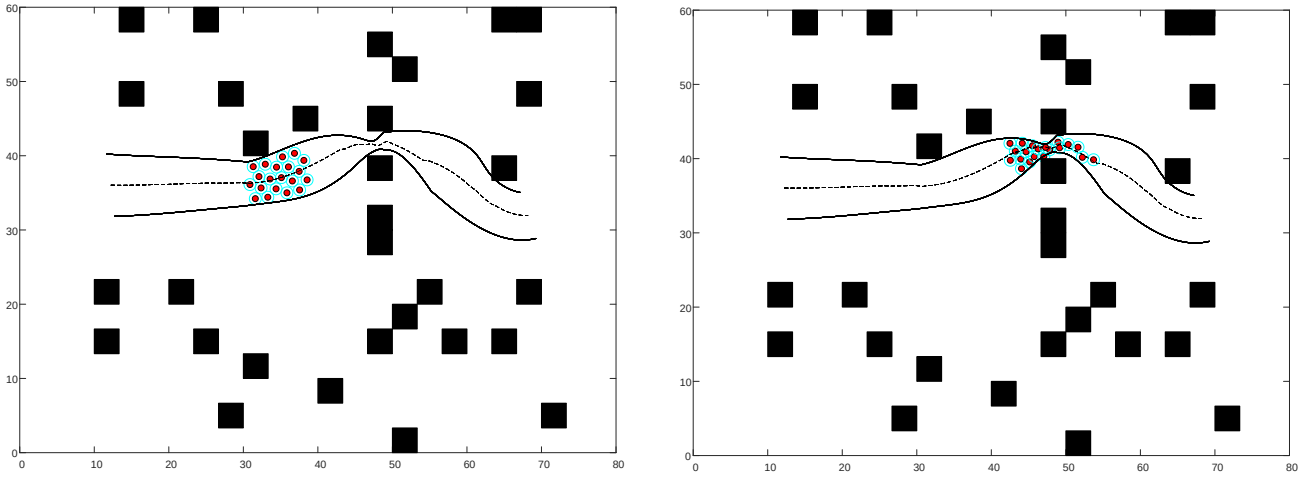
(b) Passing through the narrow space

Supplementary Fig.5. The passing-through process of the swarm in scene 6 according to the NMPC method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



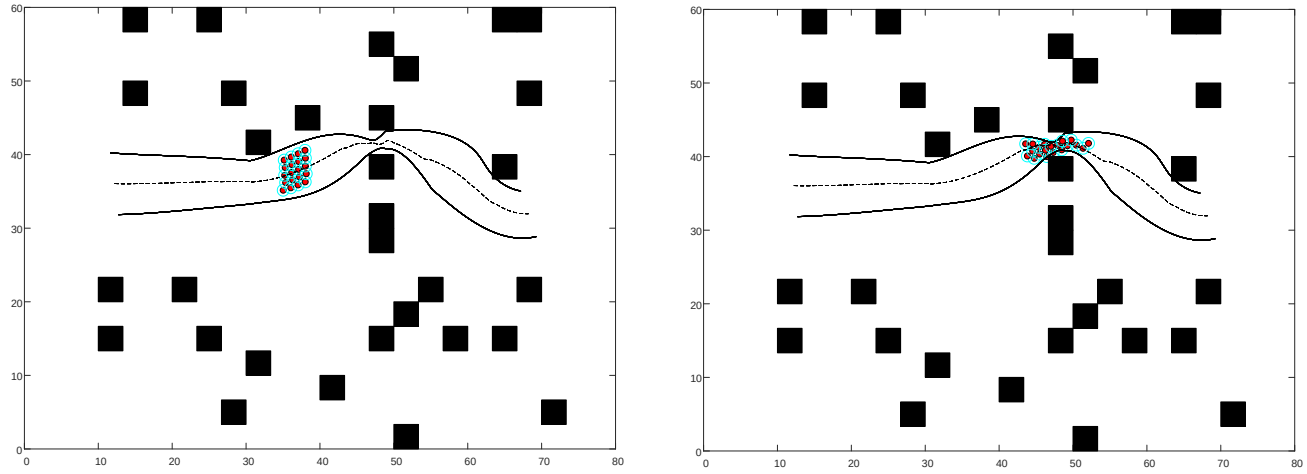
(a) Before passing through the narrow space (b) Passing through the narrow space

Supplementary Fig.6. The passing-through process of the swarm in scene 10 according to the method proposed. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space (b) Passing through the narrow space

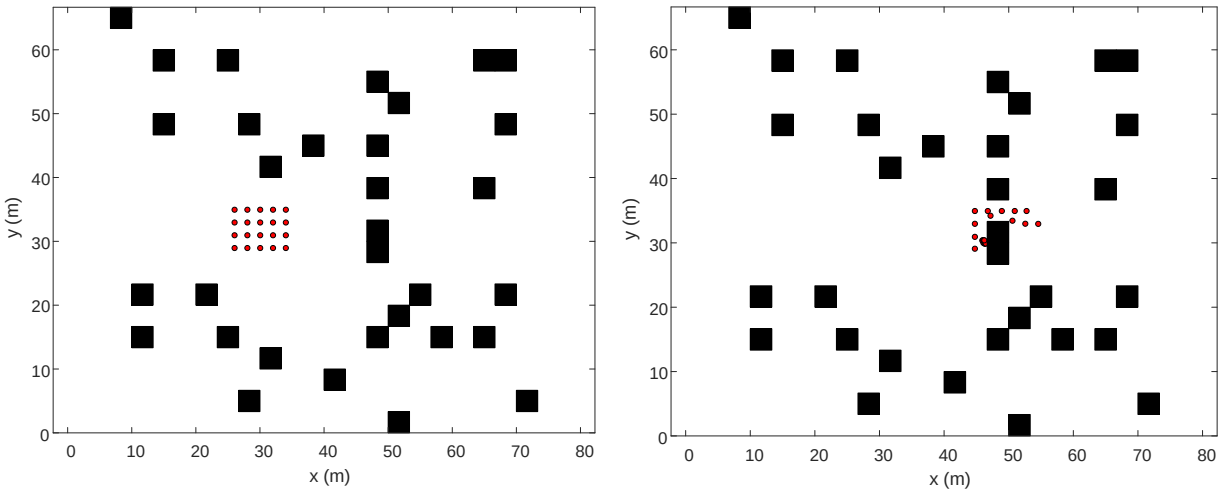
Supplementary Fig.7. The passing-through process of the swarm in scene 10 according to the basic virtual tube control method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

(b) Passing through the narrow space

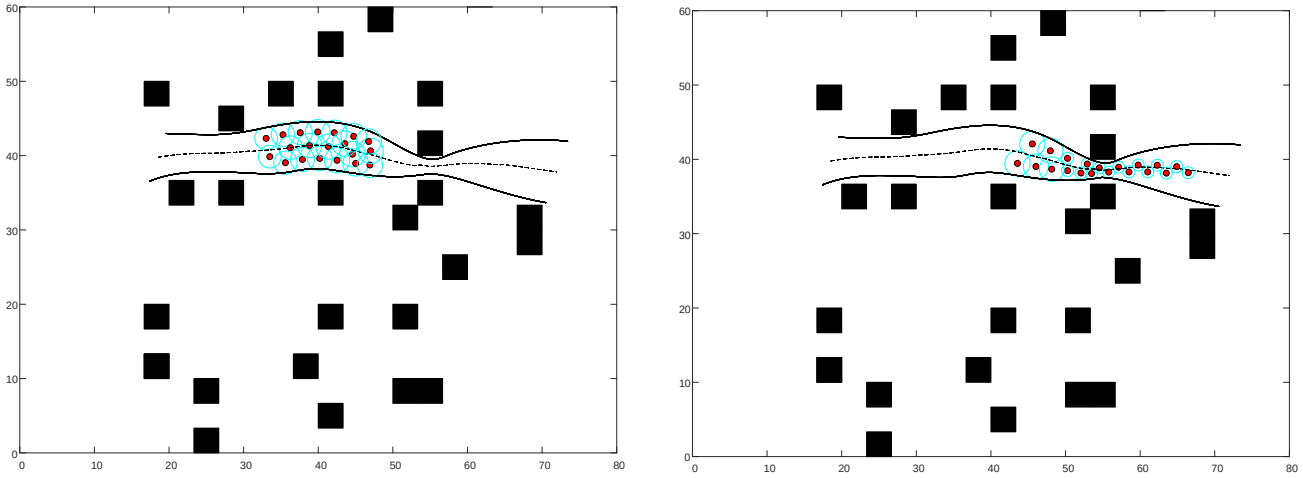
Supplementary Fig.8. The passing-through process of the swarm in scene 10 according to the flocking method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

(b) Passing through the narrow space

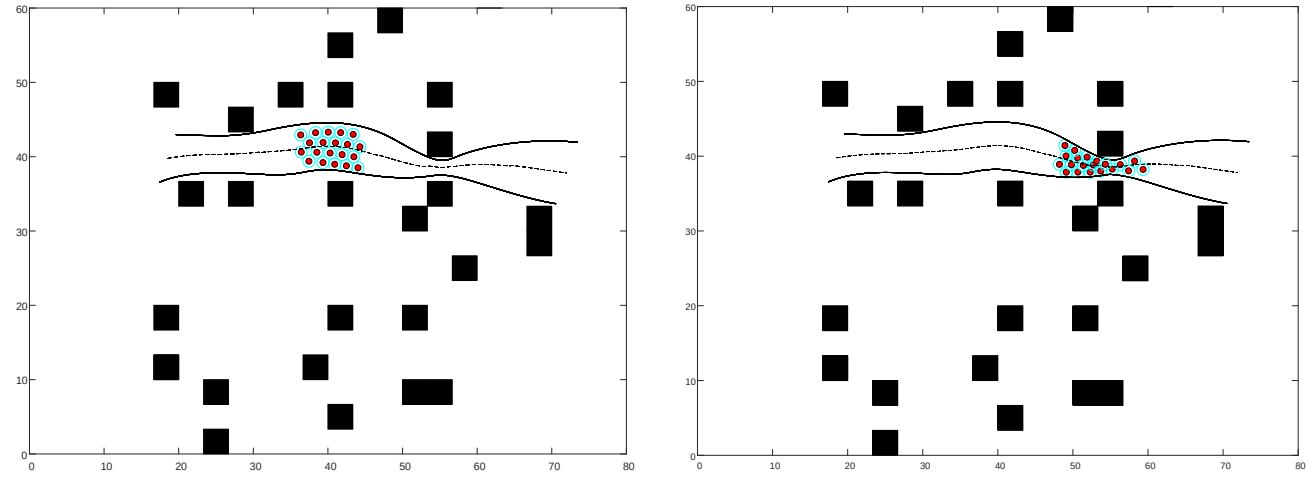
Supplementary Fig.9. The passing-through process of the swarm in scene 10 according to the NMPC method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

(b) Passing through the narrow space

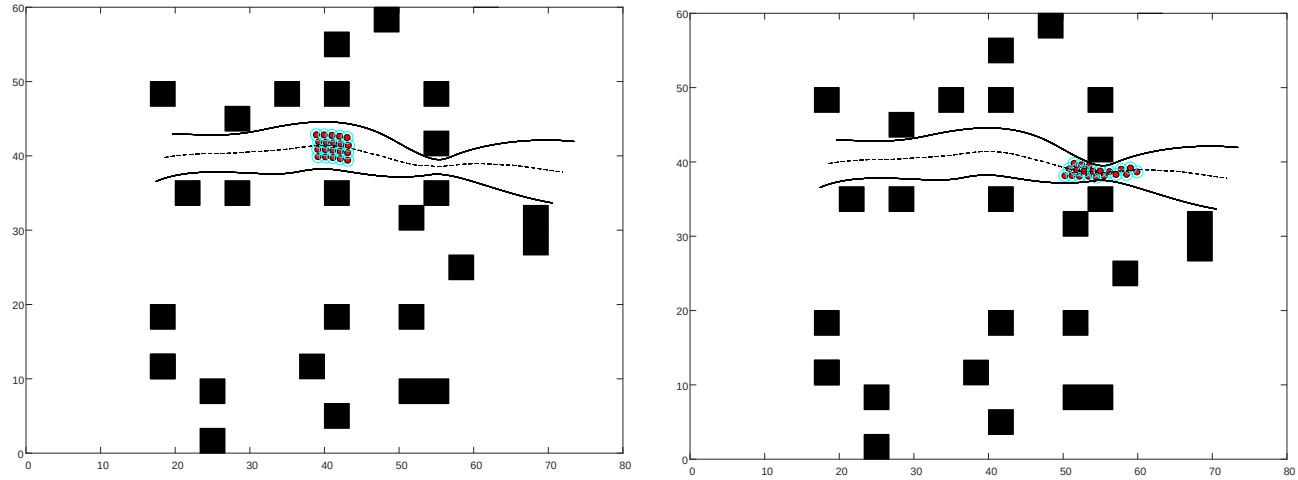
Supplementary Fig.10. The passing-through process of the swarm in scene 17 according to the method proposed. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

(b) Passing through the narrow space

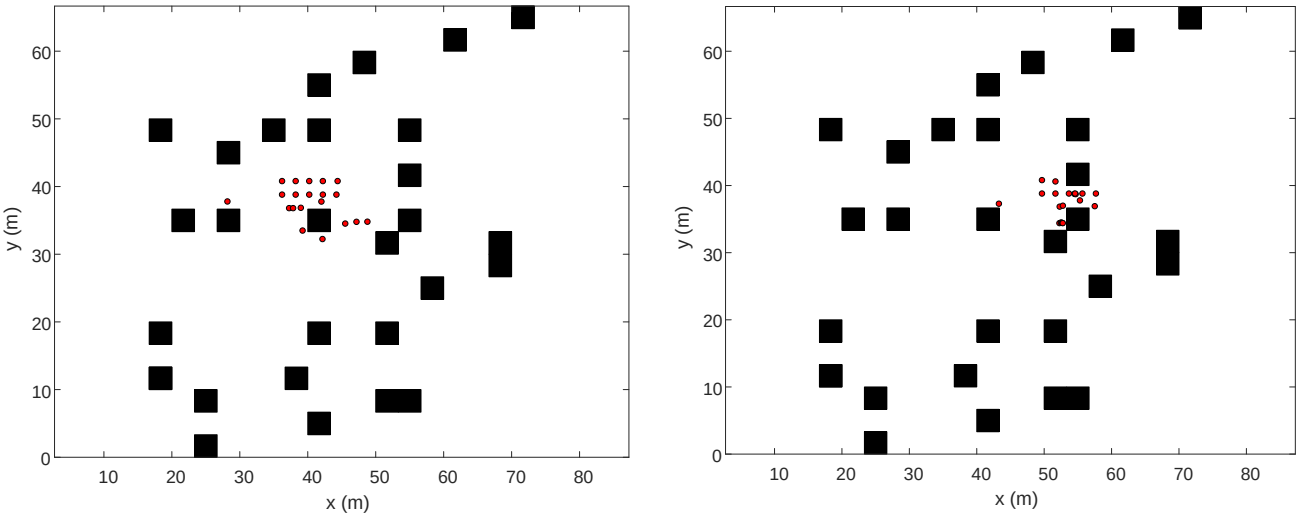
Supplementary Fig.11. The passing-through process of the swarm in scene 17 according to the basic virtual tube control method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

(b) Passing through the narrow space

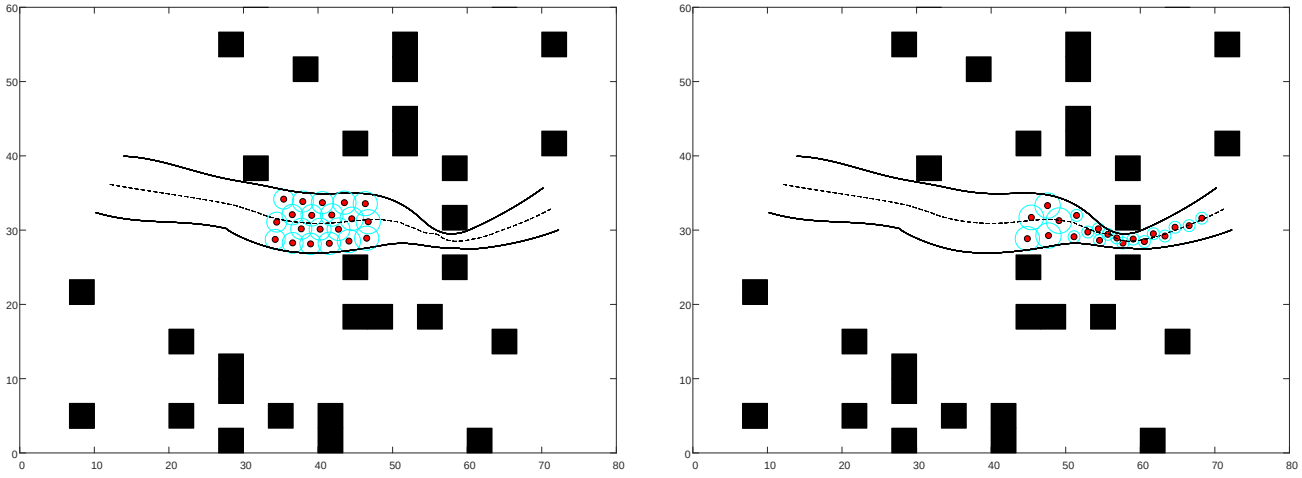
Supplementary Fig.12. The passing-through process of the swarm in scene 17 according to the flocking method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

(b) Passing through the narrow space

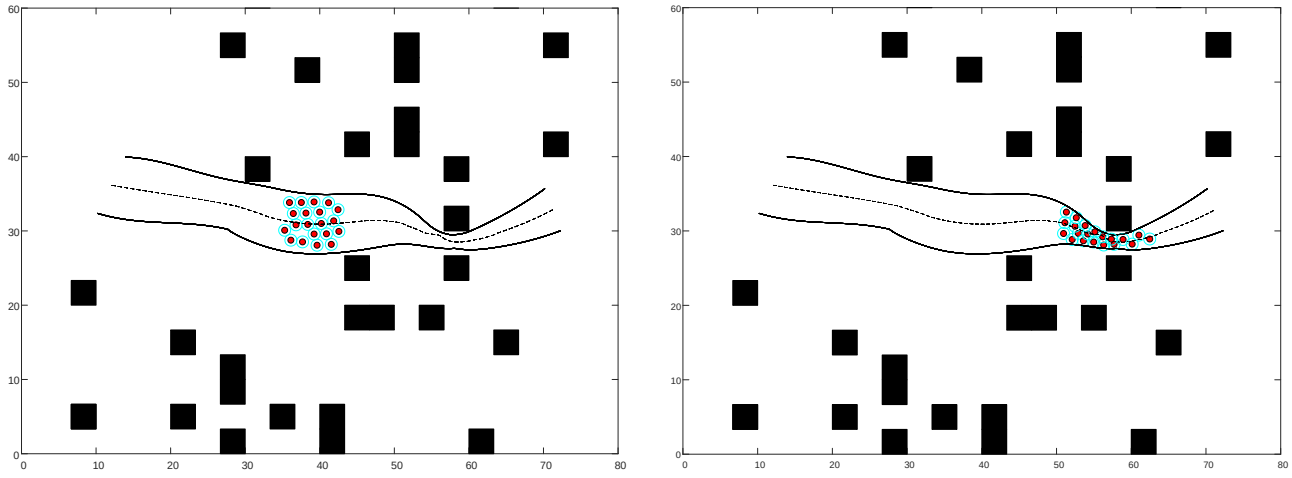
Supplementary Fig.13. The passing-through process of the swarm in scene 17 according to the NMPC method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

(b) Passing through the narrow space

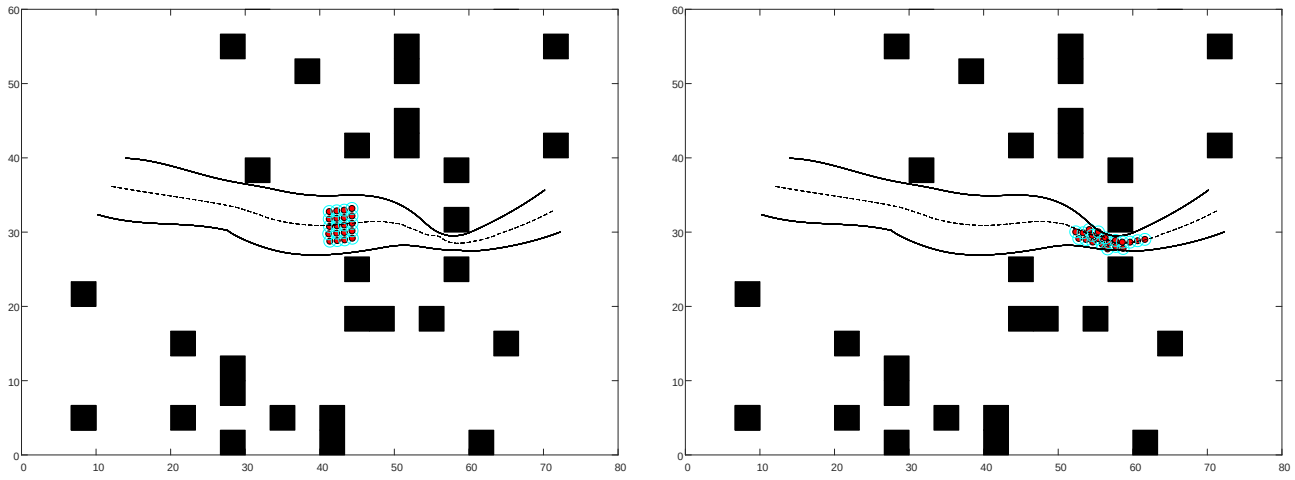
Supplementary Fig.14. The passing-through process of the swarm in scene 19 according to the method proposed. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

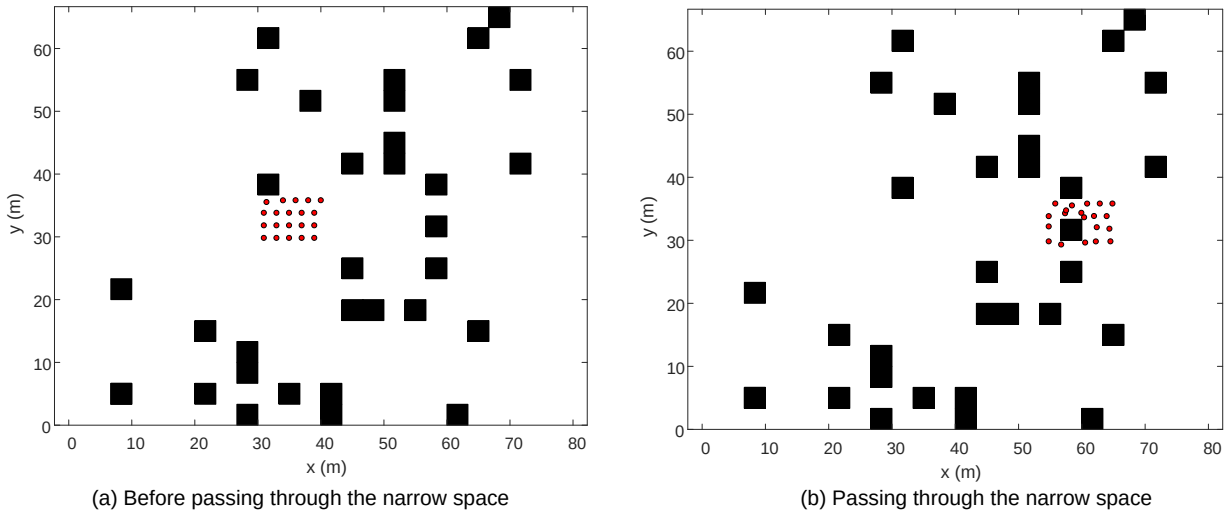
(b) Passing through the narrow space

Supplementary Fig.15. The passing-through process of the swarm in scene 19 according to the basic virtual tube control method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



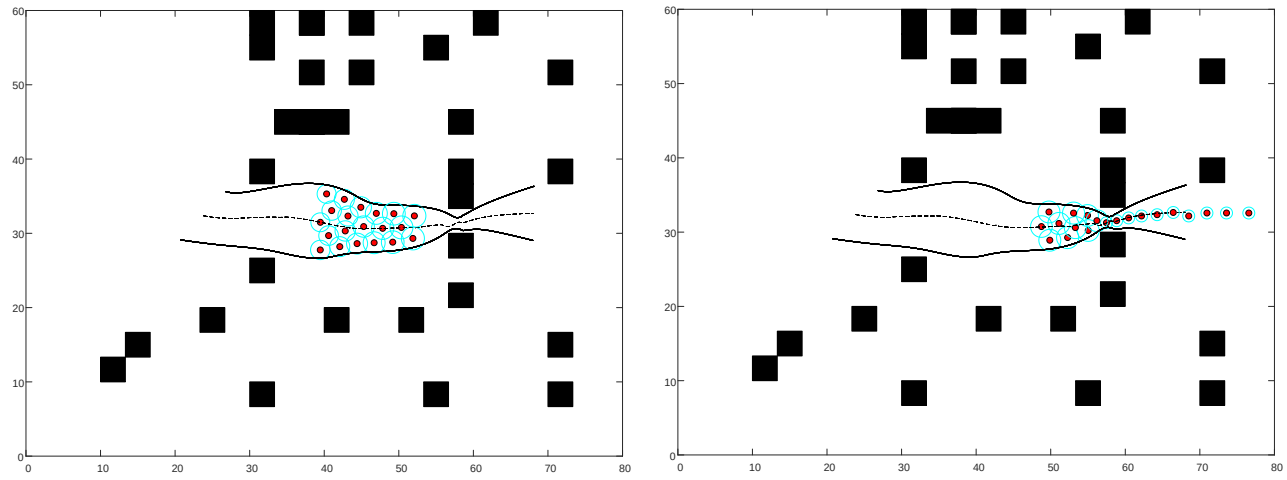
(a) Before passing through the narrow space (b) Passing through the narrow space

Supplementary Fig.16. The passing-through process of the swarm in scene 19 according to the flocking method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



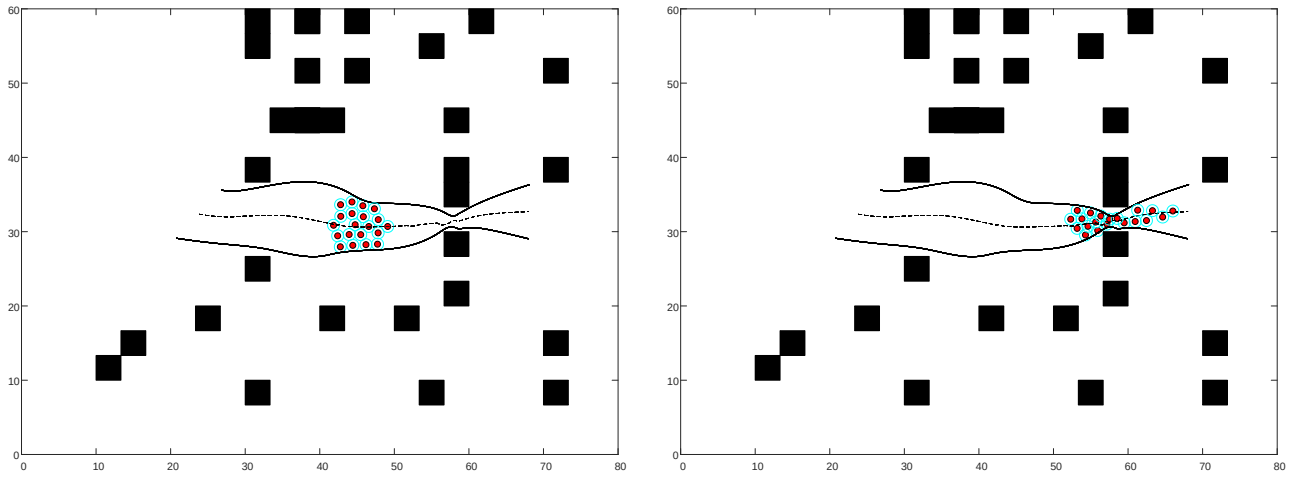
(a) Before passing through the narrow space (b) Passing through the narrow space

Supplementary Fig.17. The passing-through process of the swarm in scene 19 according to the NMPC method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



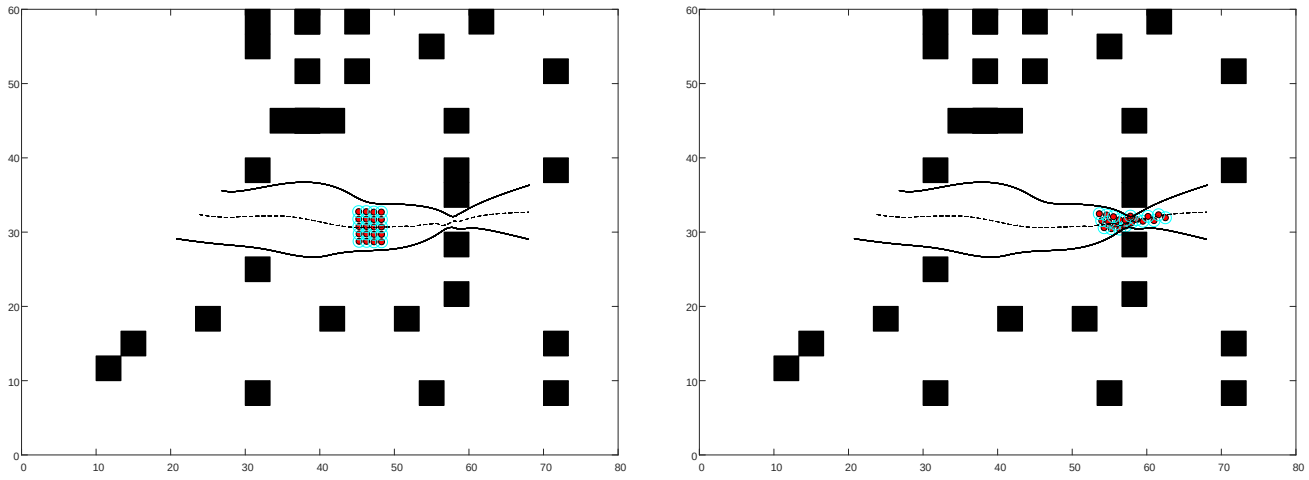
(a) Before passing through the narrow space (b) Passing through the narrow space

Supplementary Fig.18. The passing-through process of the swarm in scene 20 according to the method proposed. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space (b) Passing through the narrow space

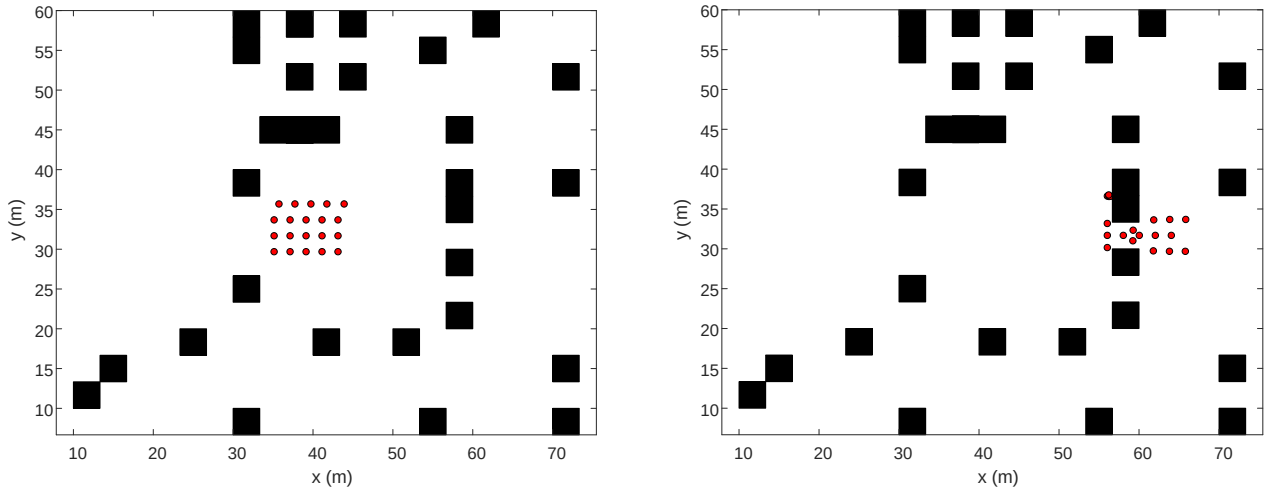
Supplementary Fig.19. The passing-through process of the swarm in scene 20 according to the basic virtual tube control method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



(a) Before passing through the narrow space

(b) Passing through the narrow space

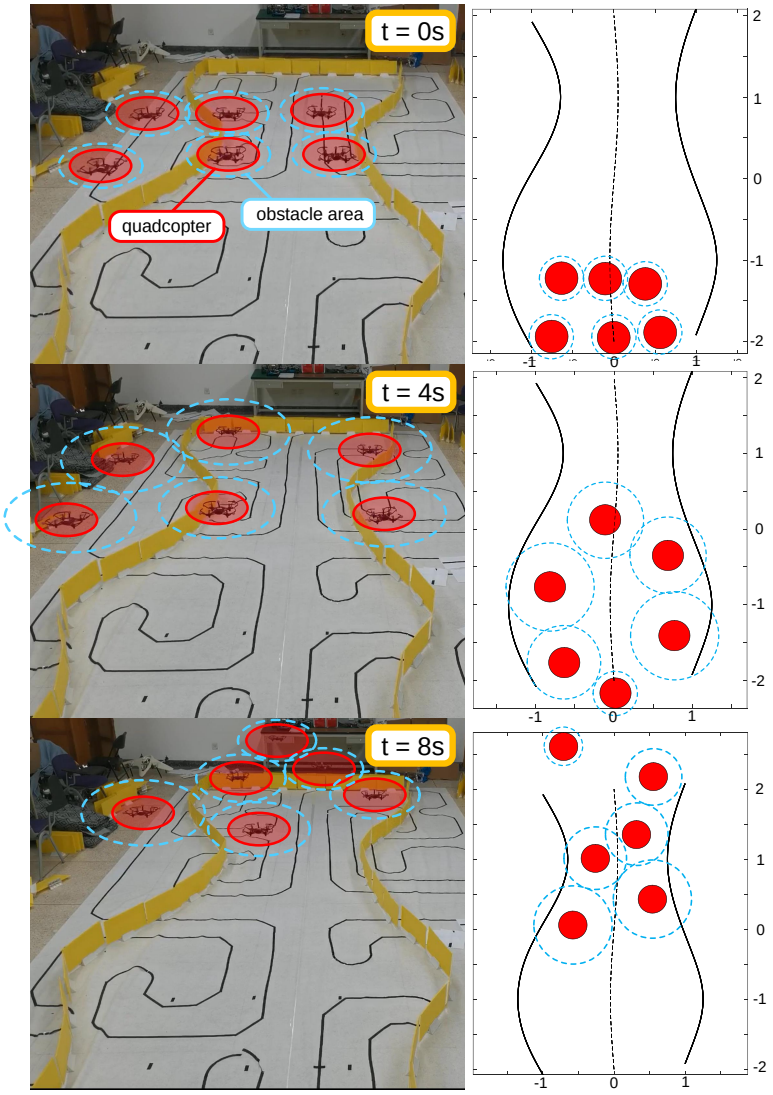
Supplementary Fig.20. The passing-through process of the swarm in scene 20 according to the flocking method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



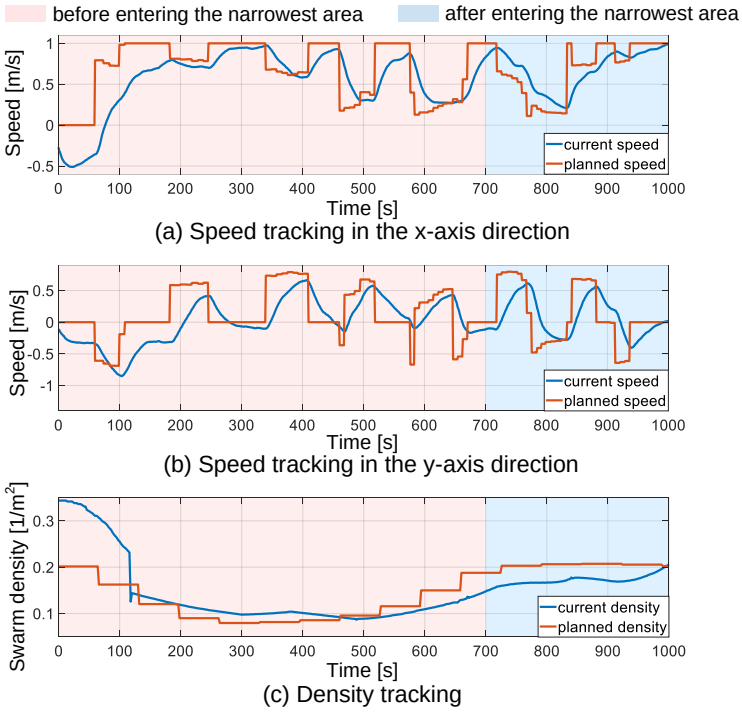
(a) Before passing through the narrow space

(b) Passing through the narrow space

Supplementary Fig.21. The passing-through process of the swarm in scene 20 according to the NMPC method. The black squares are the obstacles, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue circle denotes the avoidance area of the robot.



Supplementary Fig.22. Flight experiment on quadcopters in a narrowing virtual tube. Six speed-constrained quadcopters are used to do experiments within a narrowing curved virtual tube and simulate in real time. On the left, the yellow boards denote the tube boundaries. On the right, the black solid curves are the tube boundaries, and the black dotted curve is the tube center curve. The red circle denotes the robot, and the blue dotted circle denotes the avoidance area of the robot. There is an obvious expansion of the quadcopter swarm before entering the narrowest part of the virtual tube at the 4 second due to the increase of r_{a}. Finally, the quadcopter swarm passes through the narrowest part of the virtual tube without conflict or surpassing the tube boundary at the 8 second.



Supplementary Fig.23. Speed and density tracking of one quadcopter in the flight experiment with real obstacles in the paper.