Assignment-1 Date....................

① Given- algo of order $O(n^2)$
Give output in 5 sec when input size $n=10$

$n=10$, therefore $10^2 = 5$ sec
$n=50$, therefore $(5\times10)^2 = 25$ sec.

∴ for $n=10$ the output time will be 25 sec.

② Given- $T_{A(n)} = n^3$ --- ①

$T_{B(n)} = 2n^2$ --- (ii)
or
for cut-off of break ① = (ii)

$n^3 = 2n^2 \Rightarrow n^2(n-2)=0$
$\cancel{n=0}, n=2$

∴ cutoff or break point is $n=2$

③ average case- It is the function which performs an average number of steps on input data of n elements.

worst case- It is the funⁿ which performs the maximum no. of steps on input data of size n

Note- Average performance and worst-case performance are the most used in algo analysis.

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b) Big O Notation - The Big O notation defines an upper bound of an algo, it bounds a funⁿ only from above.

Ω Notation - Just as Big O notation provides an asymptotic upper bound on a funⁿ, Ω notation provides an asymptotic lower bound. Ω notation can be used when we have lower bound on time complexity of an algo.

③ We have to prove $n2^n = O(4^n)$

we know that if $f(n) = O(g(n))$

$$\therefore \lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$$

here, $f(n) = n2^n$
$g(n) = 4^n$

$$\therefore \lim_{n\to\infty} \frac{n2^n}{4^n} = \lim_{n\to\infty} \frac{n}{2^n}$$

Using LH Rule

$$\lim_{n\to\infty} \frac{1}{2^n \ln 2}$$

As $n \to \infty \Rightarrow 2^n \to \infty$

$$\therefore \lim_{n\to\infty} \frac{1}{2^n \ln 2} = 0$$

So, $n2^n$ is $O(4^n)$.

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(6) $n^4 + \log n + 17$

Sol: As we increase the value of n.

n^4 increases much more time than $\log n$

∴ we can neglect the other values.

∴ $n^4 + \log n + 17$ is a $O(n^4)$.

(7) (a)

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K = 1
while K <= n
K = K+1
END While
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Time complexity is $O(n)$.

(b) for i — $O(n-1)$

for j — $O(n-1)$

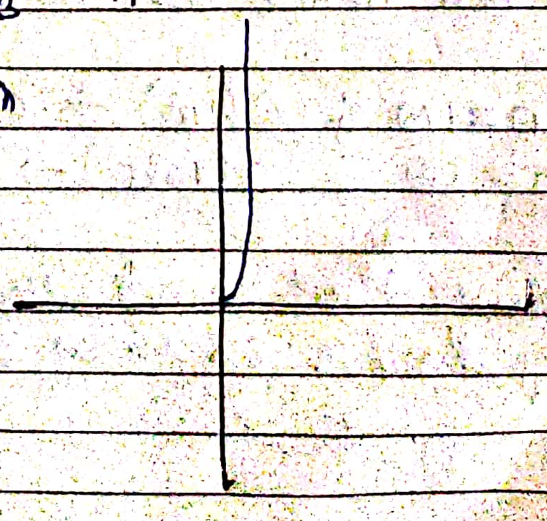
swap — $O(1)$

∴ $O(n-1) * O(n-1) * O(1) \approx O(n^2)$

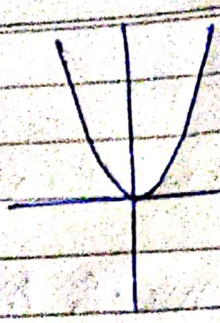
(9) Given - $T_A = 100^n$

$T_B = n^4$

Graph of 100^n



Graph of n^4



As clearly T_A grows faster than T_B So T_A is better than T_B when $n \to \infty$.

(10) $\log(n!) = \log[n(n-1)(n-2) \cdots 1)$

$= \log(1) + \log(2) + \log(3) \cdots \log(n)$

$\leq \log(n) + \log(n) + \log(n) \cdots \log(n)$

$\leq n\log(n)$

So $n\log(n) \in O(\log(n!))$.

(8) Let an algo of order $O(n^2)$ takes 10 sec to complete the answers for an input instance $n = 10$.

So we increase the input instance to $n = 50$ then we will see it will take time 50. [Prove]

$$\begin{pmatrix} n=10 \; ; \; O(10^2) \to 10 \text{ sec.} \\ n=50 \; ; \; O(10 \times 5^2) \to 50 \text{ sec.} \end{pmatrix}$$

$\therefore$ we observe that quadratic algo runtime is nearly double as input size increases.

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(11) a) $2^{n-1} + 4^{n+1}$

$2^{n-1} + 2^{2(n+1)} \approx 2^{n-1} + 2^{2n}$

$\therefore \Theta(2^{n})$ Ans

b) $(n^2+6)^8$

$(n^2)^8 + - - -$

$\therefore \Theta(n^{16})$ Ans