**PORTFOLIO OPTIMIZATION USING PARTICLE SWARM OPTIMIZATION**

Report submitted for the

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By

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**Declaration By Student:**

I, **Rhimjhim Daftary (A90555921003)**, a student of **B.Sc. (Hons.) Statistics, (2021-2024),** Department of Statistics, Amity Institute of Applied Sciences, Amity University Kolkata; hereby declare that the NTCC **Major Project**, **STMJ100** entitled “**Portfolio Optimization using Particle Swarm Optimization”** which is submitted by me to Department of Statistics, Amity Institute of Applied Sciences, Amity University, Kolkata, India, done during the tenure between **17/01/2024** to **04/05/2024** in partial fulfillment of requirement for the award of the degree of **B.Sc. (H.) Statistics** has not been previously formed the basis for the award of any degree, diploma or other similar title or recognition.

Place: Kolkata Signature of the Student

Date:

**Certificate by the Faculty Guides**

On the basis of declaration submitted by **Rhimjhim Daftary (A90555921003)**, a student of **B.Sc. Statistics Hons (Batch: 2021-2024 ),** Department of Statistics, Amity Institute of Applied Sciences, Amity University Kolkata; I hereby certify that the NTCC Course, **Major Project**, **STMJ100** entitled “**Portfolio Optimization using Particle Swarm Optimization”** which is submitted to Department of Applied Statistics, Amity Institute of Applied Sciences, Amity University, Kolkata, India, done during the tenure between **17/01/2024** to **04/05/2024** in partial fulfilment of requirement for the award of the degree of **B.Sc. (H.) Statistics**, is an original contribution with existing knowledge and faithful record of work carried out by him/them under my guidance and supervision.

To the best of my knowledge this work has not been submitted in part or full for any Degree Diploma to this University or elsewhere.

Dated:

Signature of the Faculty Guide (Internal) :

Name of Faculty Guide with corresponding affiliation :

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**Abstract**

This research explores the application of a Particle Swarm Optimization (PSO) for portfolio optimization within the context of the NIFTY50, a diversified index representing prominent companies in the Indian stock market. Traditional portfolio optimization methods often face challenges in capturing complex relationships and dynamic market conditions. This study employs monthly data spanning from April 2018 to March 2023 to develop an investment portfolio strategy. Initially, companies with positive average returns and negative skewness are identified. Among these, stocks with low volatility and high sensitivity to market changes are selected using a 4-Quadrant Map. Using Particle Swarm Optimization, an optimum portfolio is developed to minimize risk while maximizing returns. The study aims to assist investors in making prudent investment decisions by providing guidance on stock selection and optimal investment allocation.

**Keywords**

Particle Swarm Optimization, NIFTY50, Portfolio Optimization, Skewness, 4-Quadrant Map.

**Introduction**

Portfolio optimization is a critical concept in finance, offering a systematic framework for investors to strategically allocate assets and manage risk in the pursuit of optimal returns. At its core is the principle of diversification, where investments are spread across different asset classes to mitigate the impact of market volatility. Modern Portfolio Theory, established by Harry Markowitz, lays down the groundwork for portfolio optimization, emphasizing the balance between risk and return. One of its key applications, mean-variance optimization, quantifies this balance by identifying portfolios that either maximize expected returns for a given risk level or minimize risk for a desired return level. The NIFTY50 is a commonly utilized benchmark by investors, analysts, and fund managers to assess the overall performance of the Indian stock market. The NIFTY50 index is computed using a methodology that weighs each constituent stock based on its free-float market capitalization, providing a comprehensive snapshot of the market's health. Companies included in the NIFTY50 are periodically reviewed and can change based on their market capitalization, liquidity, and other criteria. This index serves as a valuable indicator for both domestic and international investors seeking insights into the Indian equity market's trends and stability.

Portfolio optimization, a central concept in Modern Portfolio Theory (MPT), utilizes the Markowitz formulation to systematically construct investment portfolios. The Markowitz formula, introduced by Harry Markowitz in 1952, is a fundamental component of Modern Portfolio Theory (MPT) and plays a crucial role in identifying the optimal allocation of assets within a portfolio. The formula involves estimating the expected returns and variances of individual assets, as well as the covariances between pairs of assets. Taking into account these variables, the formula computes the allocations of each asset within the portfolio, with the goal of optimizing returns while maintaining a specified level of risk. The Markowitz formulation, often referred to as mean-variance optimization, forms the basis for portfolio construction by identifying the optimal combination of assets that lies on the efficient frontier. This systematic approach to portfolio optimization enhances decision-making for investors by providing a quantitative framework that balances risk and return, aligning with the core principles of MPT.

The Markowitz approach is grounded in statistical analysis, taking into account not only the individual expected returns and variances of assets but also the correlations between them. This comprehensive consideration of risk, return, and diversification distinguishes Markowitz optimization from simpler methods. By accounting for the covariance among assets, Markowitz more accurately reflects the true risk of a portfolio. Additionally, the Markowitz formulation allows for a nuanced exploration of the risk-return trade-off, enabling investors to tailor portfolios to their specific risk preferences. The Markowitz formulation yields a more resilient and effective portfolio construction, optimizing returns for a specified risk level or minimizing risk for a given level of returns. This characteristic makes it the preferred choice among sophisticated and risk-conscious investors when compared to less sophisticated allocation strategies.

Portfolio optimization has been a focal point in financial research for decades, drawing on various methodologies to enhance the risk-return profile of investment portfolios. Early contributions by Harry Markowitz in the 1950s laid the groundwork for Modern Portfolio Theory (MPT), emphasizing the importance of diversification to achieve optimal risk-adjusted returns. Mean-variance optimization, a key application of MPT, quantifies the trade-off between expected returns and portfolio volatility. However, criticisms emerged due to sensitivity to input parameters and assumptions of normality, prompting researchers to explore alternative approaches.

Recent literature explores the integration of advanced optimization techniques, including heuristic algorithms such as Particle Swarm Optimization (PSO) and simulated annealing. These approaches offer solutions to the computational challenges associated with traditional optimization methods and provide flexibility in handling complex constraints. Machine learning, particularly reinforcement learning and neural networks, has gained traction for portfolio optimization, allowing for adaptive strategies and the incorporation of non-linear relationships in financial data.

The NIFTY50, as a stock market index representing 50 large, well-established companies on the National Stock Exchange (NSE) in India, has garnered significant attention in financial literature. Early studies often focused on the construction and methodology of the NIFTY50, providing insights into the criteria used for inclusion and periodic reviews of the index constituents. Researchers have explored the implications of changes in the NIFTY50's composition, studying the impact on market dynamics, investor sentiment, and sectoral trends.

With the rise of algorithmic trading and quantitative strategies, literature related to the NIFTY50 has explored the application of quantitative models for forecasting and trading. This includes studies on technical analysis, machine learning techniques, and statistical arbitrage strategies tailored to the unique characteristics of the NIFTY50.

The impact of macroeconomic variables, such as interest rates and inflation, on the NIFTY50 has been a subject of interest, offering insights into the broader economic implications of the index's movements. Additionally, studies on investor behaviour and sentiment analysis contribute to a comprehensive understanding of the market dynamics surrounding the NIFTY50.

In recent years, the integration of Particle Swarm Optimization (PSO) with machine learning has become a subject of increasing interest. PSO, a population-based stochastic optimization technique inspired by the social behavior of birds and fish, offers unique capabilities in optimizing complex functions and searching high-dimensional spaces. Its application spans various domains, including parameter tuning in machine learning algorithms, optimizing neural network architectures, and fine-tuning hyperparameters in deep learning models.

Ethical considerations concerning the utilization of Particle Swarm Optimization, particularly within the realm of artificial intelligence and autonomous systems, are gaining prominence. Discussions on fairness, transparency, and accountability emerge when deploying PSO in decision-making contexts.

The objective of this research is to employ the Particle Swarm Optimization to achieve an optimal portfolio of the NIFTY50 companies, with the aim of minimizing risk while maximizing returns.

**Materials & Methods**

**Data**

The sample for this study consists of all the NIFTY50 stocks. The period of the study is from April 2018 to March 2023. There are altogether 50 stocks and their Monthly Closing Price from April 2018 to March 2023 are collected. We then calculate their monthly return using their Monthly Closing Price (CP) values over time given as

Rt = (CPt – CPt-1)/CPt-1, t = 2, 3….., 60 (1)

where Rt: Return for month t and CPt and CPt-1: Closing Price value for month t and t-1 respectively.

**Selection of Stocks**

The stocks whose average returns are negative are excluded from consideration. The average monthly return for each stock is determined as x̅ = ∑xi / n, where xi represents the return for the ith month, and n is equal to 60. Subsequently, attention is given to stocks that are left-skewed. Skewness is quantified using Bowley’s formula, expressed as

Sk = [(Q3 – Q2)– (Q2 – Q1)] / [(Q3 – Q2) + (Q2 – Q1)],

where Qi denotes the ith quartile of the distribution, ranging from 1 to 3. This statistical measure provides insights into the asymmetry of the return distribution and guides the identification of stocks with a skewed distribution towards negative returns. Negative skewness in return distributions is a critical consideration for investors due to its indication of an asymmetric risk profile. It signifies a higher likelihood of extreme negative returns, emphasizing the importance of downside protection. This asymmetry aligns with behavioral finance principles, reflecting investors' stronger aversion to losses than their preference for gains. Negative skewness influences risk assessment, portfolio construction, and risk management strategies, prompting investors to diversify portfolios and implement risk mitigation measures to address the increased probability of adverse events. Understanding and accounting for negative skewness is crucial for making informed decisions that reflect the true risk dynamics of financial markets.

In the subsequent stage, it is necessary to compute both the SD of the monthly returns and the Beta coefficient for every company included in the portfolio. The standard deviation (SD) serves as a comprehensive gauge of the total risk in an investment, encompassing both unsystematic and systematic risks. It quantifies the extent of deviation of returns from their mean, providing a valuable measure of the investment's overall risk profile. This analysis contributes to a more nuanced understanding of the risk dynamics associated with each company in the portfolio. It is given as

SD = [i - )2 / 59 ]1/2. (2)

Beta (β), which quantifies the systematic risk of an investment relative to the market, is determined using a specific formula. This metric helps investors gauge how an asset's returns correlate with broader market movements, providing insights into the investment's overall risk characteristics. It is found using:

β = Covariance (x, y) / sy2, Covariance (x, y) = i - (yi - ) /59 and Var(y) = i - )2 / 59. (3)

Here xi = return of the ith month, yi = market return of the month i, and x ̅ and y ̅ denote average returns respectively.

Following this, the selection process entails identifying stocks with a standard deviation (SD) lower than the aggregated SD and a beta higher than the average value of beta. This selection is facilitated through the creation of a 4-quadrant Map, where SD and beta are plotted on the respective axis. The map visually represents the place of each stock concerning the average beta and combined SD. Further analysis is conducted on stocks positioned in the second quadrant of the map, indicating lower standard deviation (SD) and higher beta values relative to the averages. This strategic selection process aims to identify stocks with specific risk-return characteristics aligned with the defined criteria.

The combined SD is then calculated by the given formula:

Combined SD = ( i.si2 + i.(i-)2 / i )1/2 (Goon, Gupta and Dasgupta, 2008) (4)

The average beta is calculated as the sum of all beta values divided by the total number of funds remaining at this stage.

The stocks we get after performing the above-mentioned steps will be the ones, we use to create an optimal portfolio using Particle Swarm Optimization.

**Particle Swarm Optimization (PSO)**

PSO was first introduced by Kennedy, Eberhart and Shi. It is based on bird flocking technique. It is inspired by behaviour observed in nature. A PSO uses the positions of the particles in a swarm to find the best position in the swarm and then the best position out of all the swarms formed iteratively. The positions of the swarms are updated with the help of the weight and accelerations provided by the user. The acceleration tells the particles in the swarm with how much acceleration to move from one solution to another and the weight tells us how far to look for a solution. These parameters have to be tweaked to see what works for different problems. This way, multiple solutions are explored till we get the best solution. In case, the maximum number of iterations are reached, it is possible that an optimal solution with the constraints given may not be found. In that, the swarm size, has to increased and the maximum number of iterations also has to be increased. Additionally, it would also be beneficial to check if the problem can converge to an optimal solution given the constraints.

**Algorithm:**

A diagram of a diagram

Description automatically generated with medium confidence

The expected return of the individual fund is given by

(2)

where denotes the weight of the ith fund and denotes the expected return of fund i.

The expected return of portfolio P is then given by: , where n is the number of funds, and the objective function of portfolio return to be maximized is written as:

Max (3)

The Portfolio risk is given as

: Variance of ith fund

: Covariance between fund *i* and fund j

The objective function of portfolio risk to be minimized is given as:

Min (4)

Thus, the multi-objective function to be minimized is given as:

(5)

Subject to the constraints:

,  (6)

The fitness function is given as:

Fitness = K (7)

**Results & Discussions**

The monthly returns of 50 stocks were calculated using equation (1). After discarding the stocks with negative average returns and positive skewness, we have 29 stocks.

We got:

combined SD = 0.10897

average beta = 0.00389

The investor’s perception map is given below. It is prepared by taking (combined SD, average beta) as its origin.

From the Investor’s Perception Map above, we can clearly see that there are 5 companies in the second quadrant. The companies are Larsen & Toubro Limited (LT.NS), Titan Company Limited (TITAN.NS), Adani Ports and Special Economic Zone Limited (ADANIPORTS.NS), Grasim Industries Limited (GRASIM.NS) and ICICI Bank Limited (ICICIBANK.NS).

After applying Particle Swarm Optimization on the 5 stocks with the following parameters:

* Maximum number of iterations = 1000
* Swarm size = 100
* w = 0.7
* c1 = 1.7
* c2 = 1.9

We get the weights for the stocks as follows:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Companies | Larsen & Toubro Limited (LT.NS) | Titan Company Limited (TITAN.NS) | Adani Ports and Special Economic Zone Limited (ADANIPORTS.NS) | Grasim Industries Limited (GRASIM.NS) | ICICI Bank Limited (ICICIBANK.NS) |
| Weights | 12% | 39% | 4% | 7% | 38% |

Also,

Best fitness value = -0.00241

Expected Return = 0.01065

Risk = 0.00825

Here, we can see that the annual return is approximately coming to be 14% which is greater than the current annual market return (11%-12%) and the risk is 0.008 which is fairly low. We can conclude that the optimal portfolio’s proportions or stock weight obtained using Particle Swarm Optimization accounts for the average returns on the shares as well as stock risk.

**Conclusion**

Investing in the stock market also has risks associated with it but we can always try to minimize it while maximizing our return so that, we may profit in the long run as a result of our investments. The aim of this study was to create an optimal portfolio using Particle Swarm Optimization to minimize risk and maximize return. We were successful at doing it but there are as always present, certain drawbacks to this study. They are:

* There is no certainty if this will be the optimal portfolio for a long period of time, indicating that this may or may not be viable for long term.
* The Modern Portfolio Theory doesn’t model the current market fluctuations.
* There may be some investors who like to take bigger risks for a higher possibility of returns, this study, doesn’t take them into consideration.

Despite the drawbacks, the study still provides an optimal portfolio that can be used by multiple investors currently, additionally, we also have plans of improving the study.

**Future Scope**

In the future, we would like to expand the scope of this study by comparing the results given by different optimization techniques and getting an idea about which technique gives us the best result. Specifically with a very commonly used algorithm for portfolio optimization called Genetic Algorithm. Also, we would like to compare the results with another up-and-coming portfolio optimization technique called Dynamic Programming. We could also try to create a new algorithm combining the best parts of all three algorithms but that is a far-fetched future goal as it would require considerable time and effort.

Also, forecasting the prices of the stock would make this a comprehensive study and would also decrease the drawbacks of this study by making it viable for a longer period.

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