Exceptional precision using non-linear optical sensors at square root singularity

Reference: https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.129.013901

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Agenda

- Introduction
- Setup and preliminary results
- Advantages
- Conclusion

Introduction to exceptional points

- Proposed by Wiersig in 2014
- Degeneracy in non-hermitian Hamiltonian is called an exceptional point (EP)
- Typical systems like 2 coupled non-linear oscillators show EP's
- Hamiltonian (assume $\hbar = 1$)

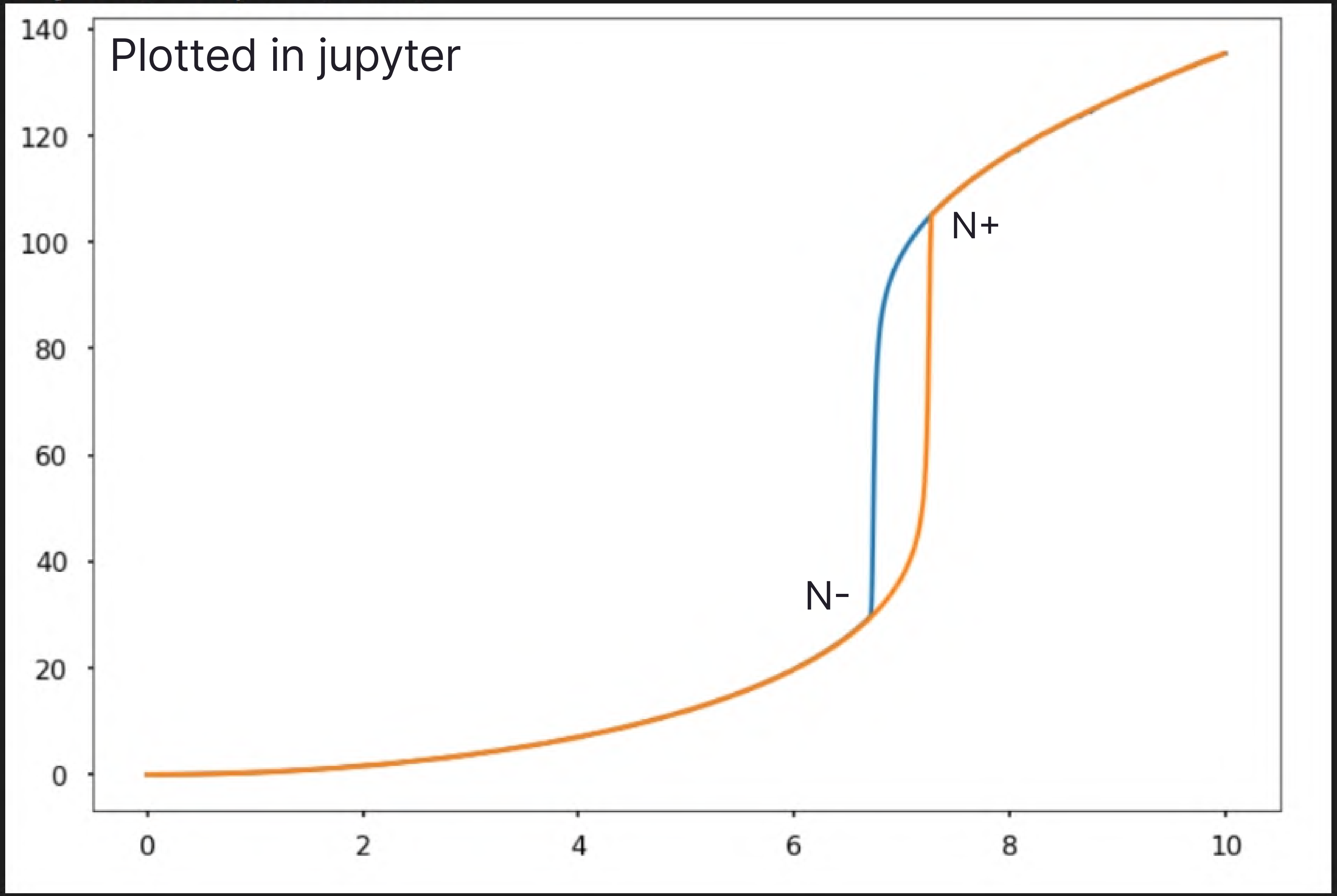
$$\begin{pmatrix} \omega_1 & g \\ g & \omega_2 \end{pmatrix}$$

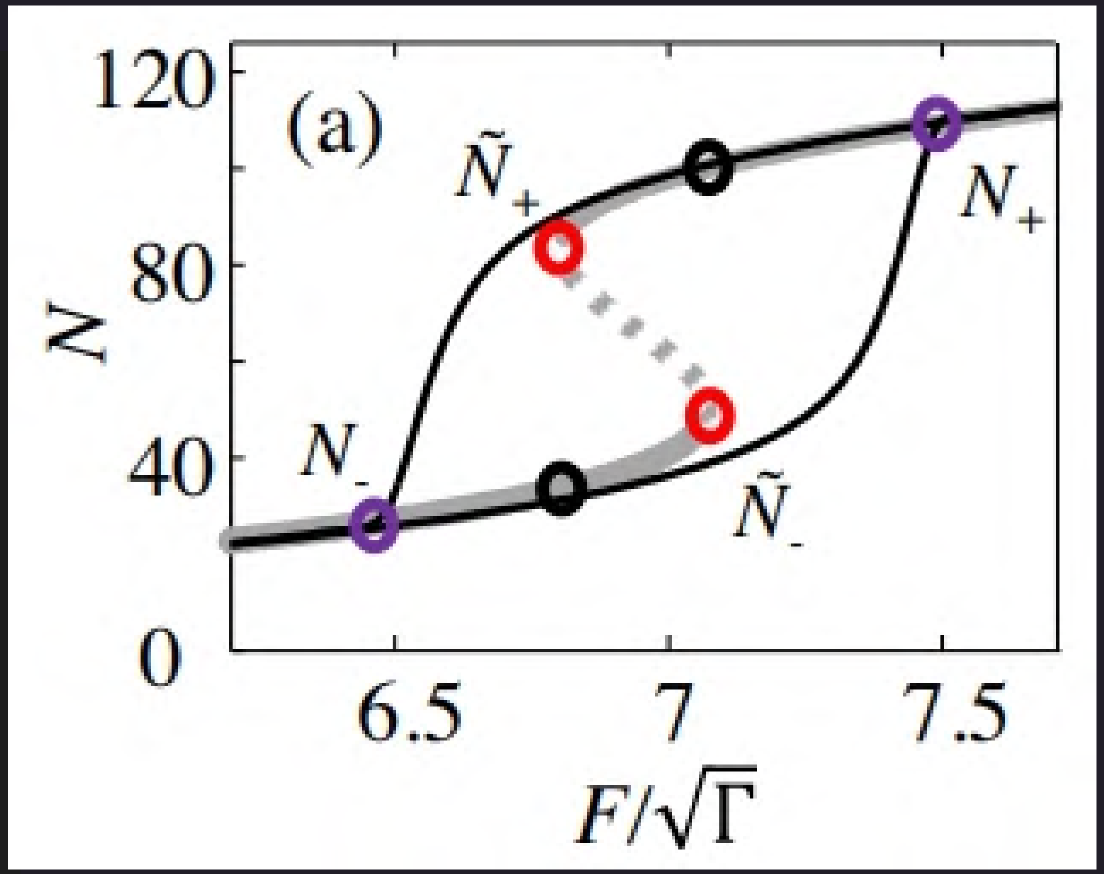
- Eigenvalues: $\omega_\pm = \tilde{\omega}_{av} \pm \tilde{\Delta}/2\sqrt{1 + (\frac{2g}{\tilde{\Delta}})^2}$ where $w_{av} = (\tilde{\omega}_1 + \tilde{\omega}_2)/2$ and $\tilde{\Delta} = \tilde{\omega}_1 - \tilde{\omega}_2$
- Here, exceptional point occurs at $\frac{2g}{\Delta} = i$ (square root singularity)
- Around exceptional point perturbation of the form Re($\tilde{\omega}_i$)+ϵ results in a splitting Re($\omega_+ - \omega_-$) $\propto \sqrt{\epsilon}$ for small ϵ S
- Tradeoff: Highly sensitive to noise due to enhanced sensitivity

Pitch

Alternative: Non-linear optical sensor

- Single mode coherently driven non-linear Kerr oscillator
- Measured signal: Splitting in transmitted intensities at 2 endpoints of hysteresis cycle
- N (y axis) vs $F/\sqrt{\Gamma}$ (x axis)





(reference plot)

For different initial values and parameters

N is the intracavity photon number: will see equation in further slides

Pitch

Non-Linear Optical sensor cont...

- Realised using a Fabry-Perot setup or any other architecture (whispering gallery, ring, photonic crystal etc.) having one mode spectrally distant from other modes and probes an intensity dependent refractive index
- In a frame rotating at driving frequency ω intracavity field α obeys following equation $i\dot{\alpha}(t) = -\Delta - \frac{i\Gamma}{2} + U(|\alpha(t)|^2 - 1)\alpha(t) + i\sqrt{\kappa_L}F + D\xi(t)$
  - α: Intracavity Field
  - Δ: Detuning $(\omega - \omega_0)$ where ω_0 is the resonance frequency
  - Γ: Total Loss=$\gamma + \kappa_L + \kappa_R$ where γ is intrinsic loss, κ_L, κ_R is loss through left and right ports respectively
  - U: Kerr non-linearity strength
  - F: Amplitude of input laser
  - D$\xi(t)$: Gaussian white noise with variance D^2

“

N which was plotted 2 slides ago is related to intracavity field by the equation

$$N = |\alpha|^2$$

– Shadab Anjum, student @ IITB

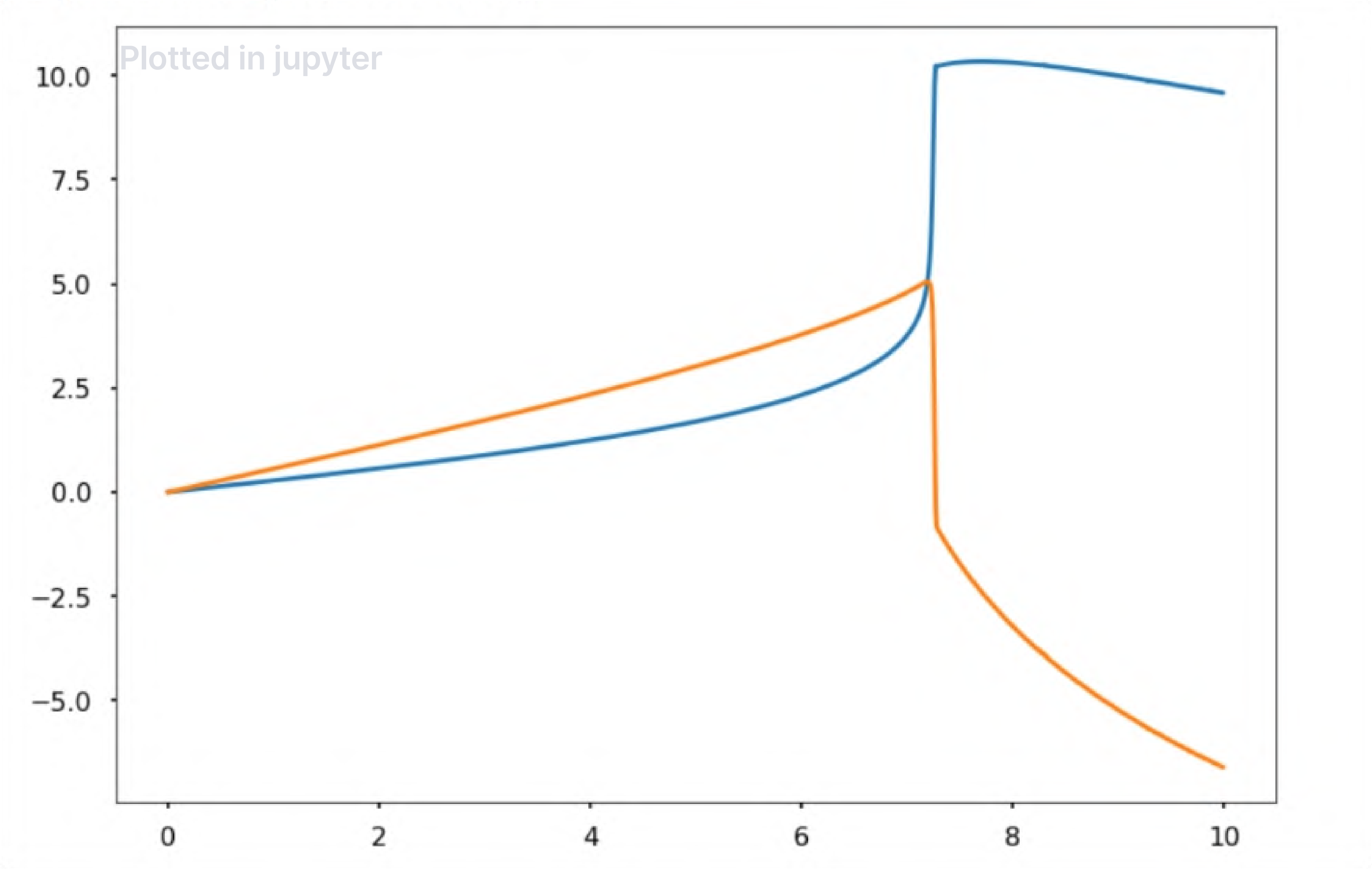
Hence, it is plotted by solving for α using the equation in previous slide

Solving for α

Setting α as a+ib where a and b are real, we transform the equation for $\dot{\alpha}$ to the following coupled non-linear differential equations.

$$\dot{a} = (-\Delta + U(a^2 + b^2 - 1))b - \frac{a\Gamma}{2} + \sqrt{\kappa_L}F$$
$$\dot{b} = \frac{-b\Gamma}{2} - a(-\Delta + U(a^2 + b^2 - 1))$$

Hence we can see U represents the strength of non-linearity in this equation
(blue- variation of a (y axis) with time (x axis))
(orange- variation of b (y axis) with time (x axis))



Steady state solutions

- Set $\dot{\alpha} = 0$ (steady state approximation)
- Also set $\frac{d|F|^2}{dN} = 0$ (turning points)
- Solutions are $\tilde{N}_{\pm} = \frac{2\Delta}{3U} \pm \frac{2\Delta}{6U}\sqrt{1-(\frac{\sqrt{3}\Gamma}{2\Delta})^2}$ (notice similarity with EP sensors)
- https://www.desmos.com/calculator/zhxszbbxrs
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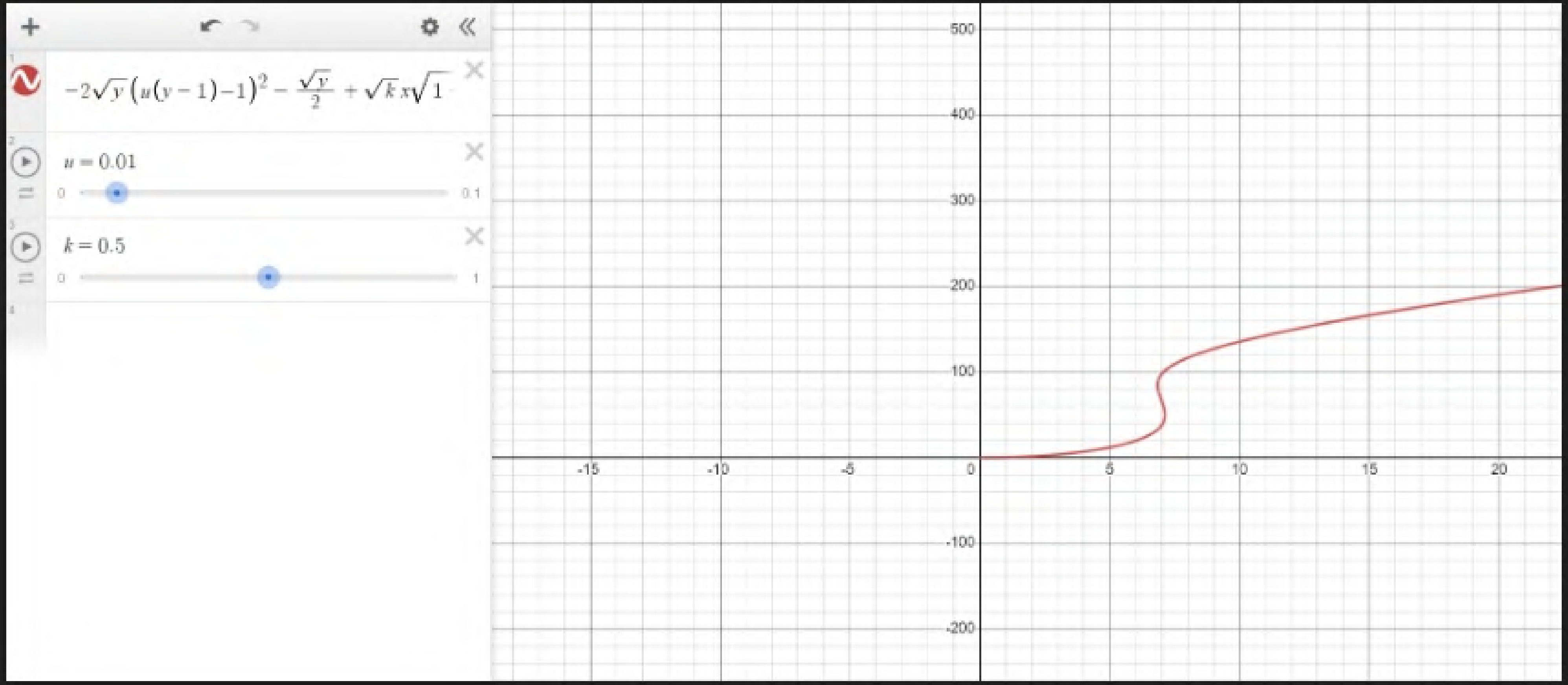
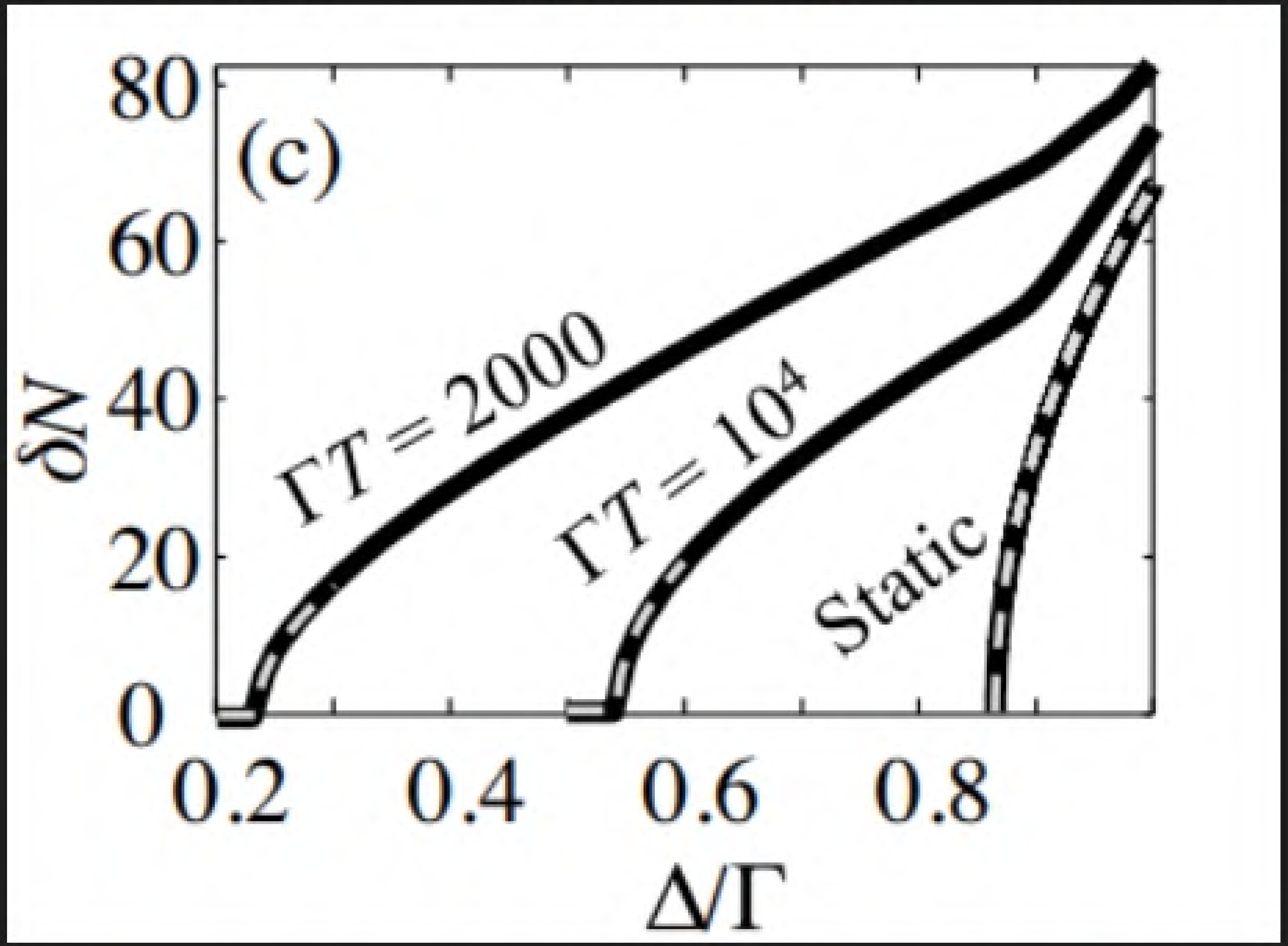


https://www.desmos.com/calculator/zhxszbbxrs

Properties

- Notice that the transmitted signal ($\delta N = N_+ - N-$) depends on Δ and measurement time.
- The dependence (as plotted by the authors) has been shown below

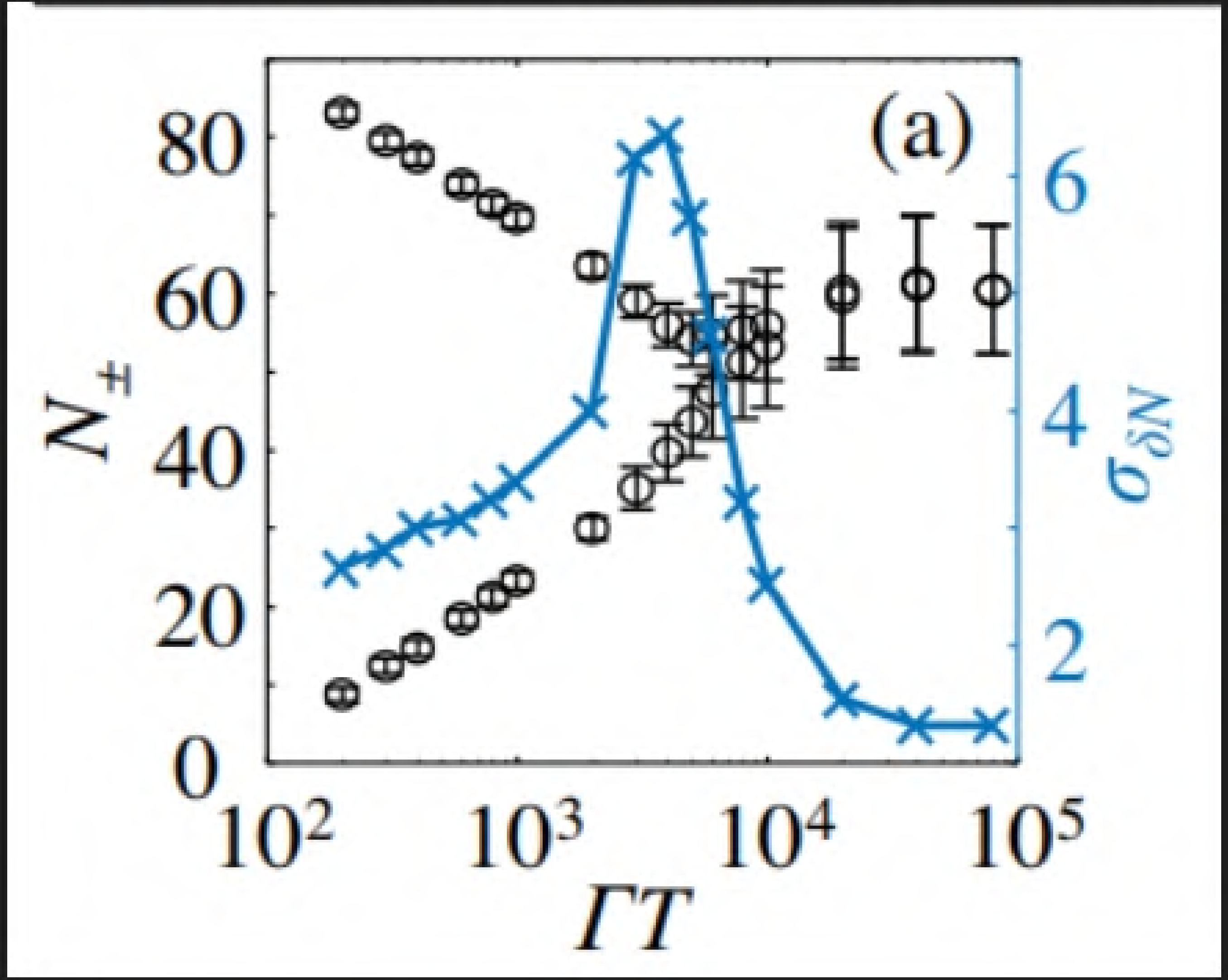


- Observe: For faster measurement time, less detuning needed for signal $\Rightarrow$ better sensitivity for faster measurement times.
- Dashed Lines: Square root fit near each singularity

Pitch

Effects of Noise

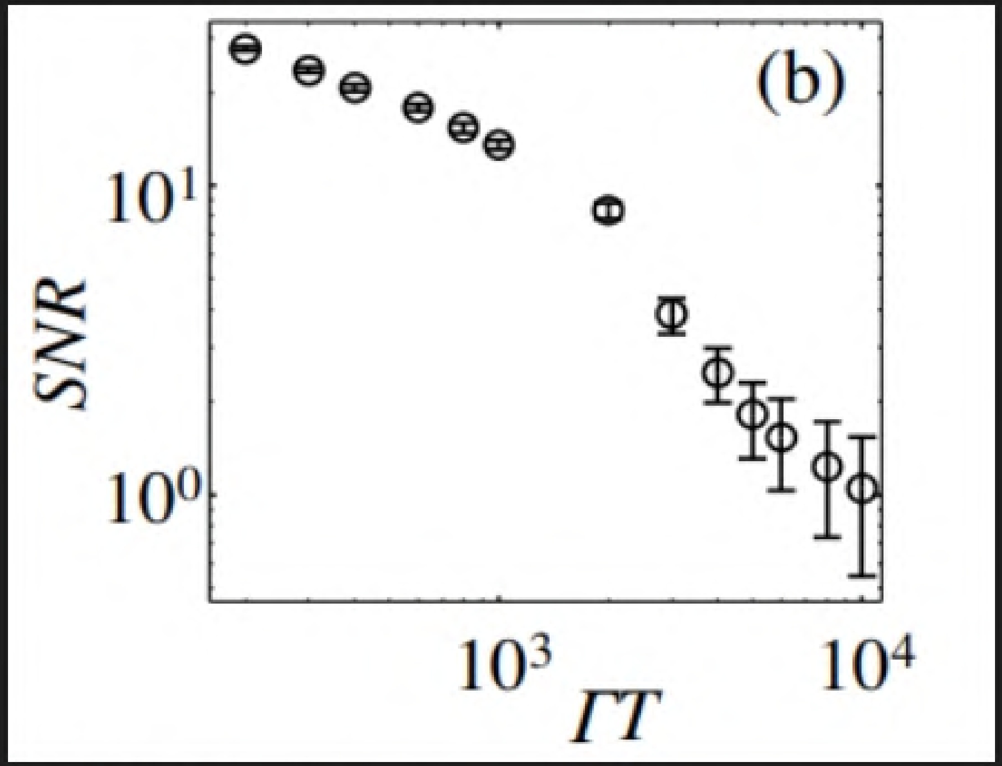
- TIll now analysis was based on assumption D=0
- WIth noise, solve numerically with different random seeds



(reference plot)

Parameters: $\Delta = 0.7\Gamma$

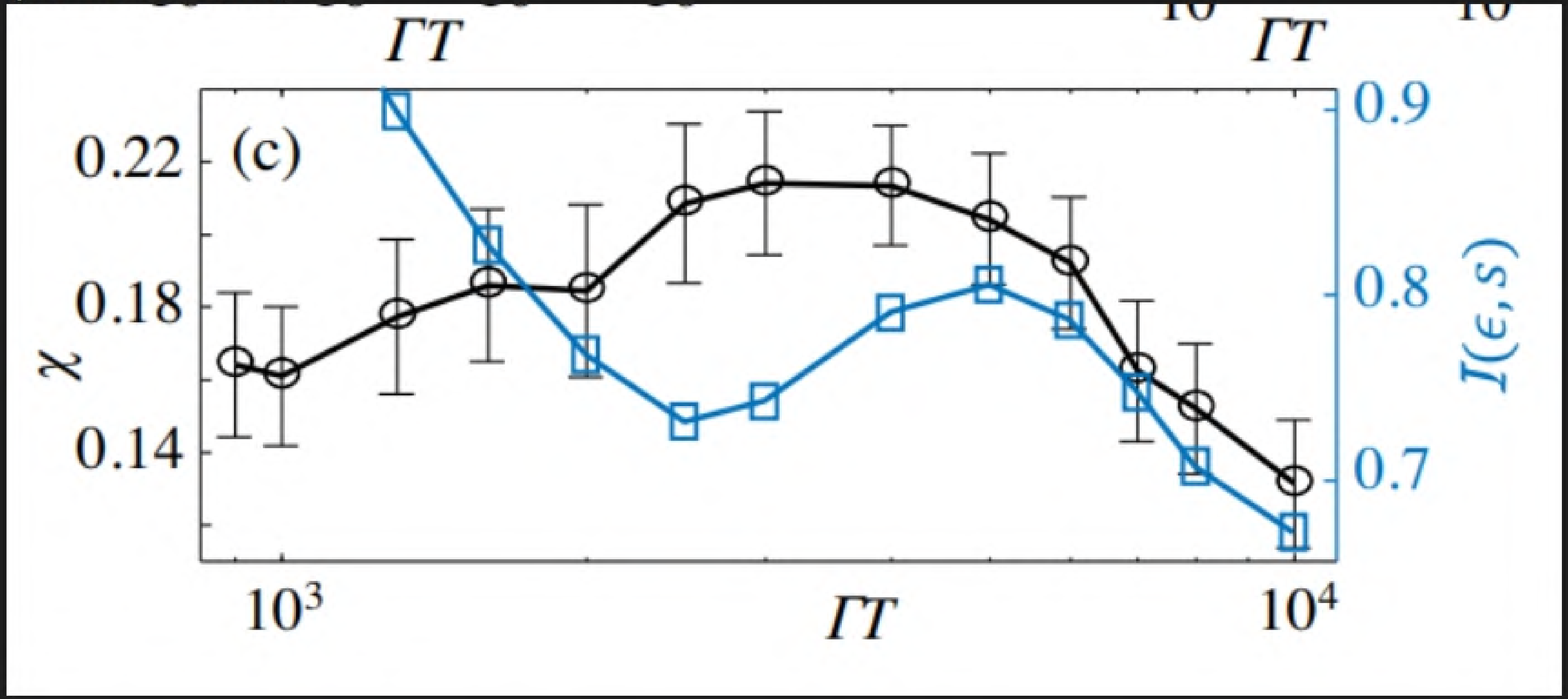
Advantages cont...



SNR increases with decrease in measurement time

(plots from reference)

Peak occurs near Tss: demonstrates precision enhancement by square root singularity



Tradeoff between measurement time and precision

SNR: Signal to noise ratio defined as $\frac{\delta N}{\sigma_{\delta N}}$

χ : Precision figure of merit: $\frac{\delta \bar{N}_\epsilon - \delta \bar{N}_0}{\sigma_0 + \sigma_\epsilon}$; Quantifies the mean change in signal relative to measurement uncertainty

Alternative viewpoint: Information theory

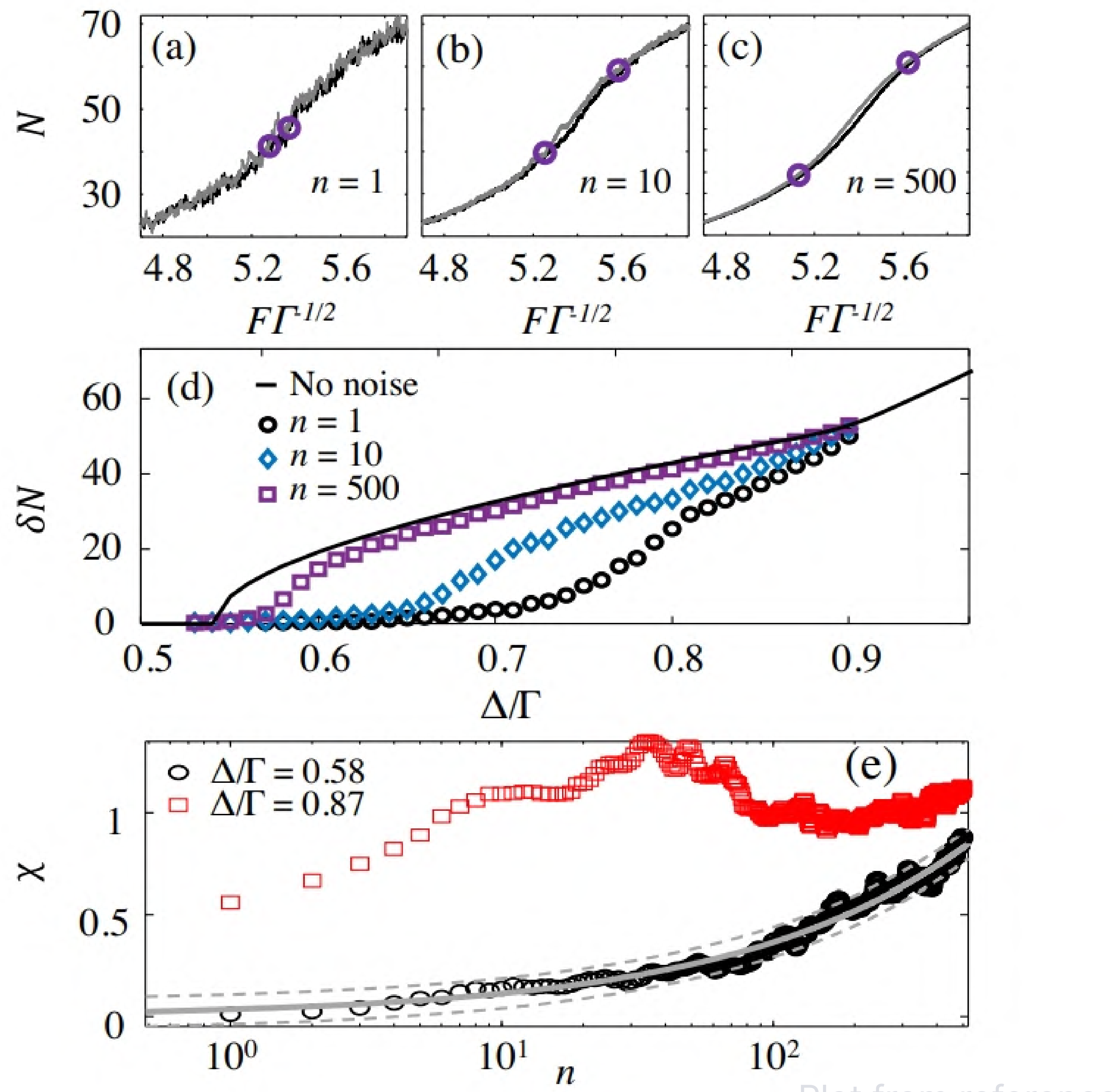
- Mutual information between 2 random variables: Measure of certainty of a random variable given knowledge of the other variable
- $I(\epsilon;S) = \sum_{s\in S}\sum_{\epsilon\in E} p_{\epsilon,S}(\epsilon,s) log\frac{p_{(\epsilon,S)}(\epsilon,s)}{p_\epsilon(\epsilon)p_S(s)}$ (from KL divergence between $p_{(\epsilon,S)}(\epsilon,s)$ and $p_\epsilon p_S(s)$)
- KL divergence: Measure of similarity between to probability distributions P and Q
- Set $p_\epsilon(\epsilon)$ gaussian with mean $\Gamma/100$ and $\sigma = \Gamma/1000$ and calculate $p_s(s)$ numerically using detuning $\Delta_0 = 0.7\Gamma$. Then based on value of s we get the joint probability distribution
- Observe the enhancement of information content near T_{ss}
- Correlation between square root singularity, mutual information and precision.
- Results are general for $\epsilon << \Gamma$

Pitch

Effects of averaging

- Smoothens effects of noise thus improving precision
- For same protocol F, it enhances sensitivity
- Tradeoff: Measurement time vs precision: averaging detrimental to fast sensing
- Non-trivial dependence of χ on number of cycles n.
- Peak followed by fall (unexpected) followed by gradual rise (expected)

$\Rightarrow$ Possibility of model parameters having best of both worlds (speed and precision)



Plot from reference

Summary

Square root singularity improved precision, information and sensitivity

Reduced Measurement time resulted in better signal to noise ratio

Simple monochromatic intensity measurements as sensing strategy, no error prone spectral fittings as in EPs

Square root singularity achieved easily using commercially available amplitude modulators

Thank You !!