Cars 4 – Lightnight McQuation

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Abstract

Lightning McQueen, old and rusty, decides to officially retire from the racing scene, leaving Cruz to carry on his legacy.

In search of a new career, he thinks back to all the crazy stunts he'd pulled over the years and realises he has no idea how they were even possible physically. So he has a crazy idea: why not become a physicist and research stunt mechanics?

Who knows, maybe a strong understanding will lay the foundations for a future NASCAR training business . . .

Note to self: learn simulink and experiment further with simulations

1 The Basics of Friction – Walking

2 First Paradigm – Skidding and Drifting

3 Second Paradigm – Banked Tracks

3.1 Deriving a general formula for Friction and Normal forces

The first constraint is that the car travels in a purely horizontal plane i.e. there is no vertical movement and so net vertical force on the car is $\vec{0}$.

For ease of calculation, we deal with both friction down the slope (**Case 1**) and up the slope (**Case 2**) at once using $(\pm)$ and $(\mp)$.

Throughout the calculations, the sign on top (i.e. $(+)$ in $(\pm)$ and $(-)$ in $(\mp)$) is for **Case 1** and the sign on the bottom is for **Case 2**.
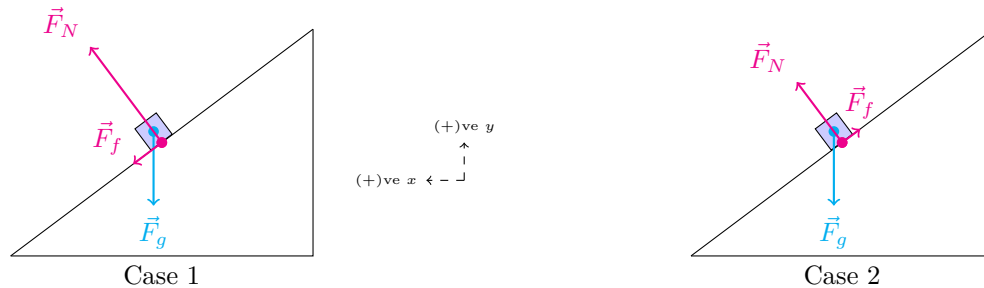

Figure 1: Toy Car on banked track with $m = 0.1$, $r = 2$, $\theta = \arctan\left(\frac{3}{4}\right)$, $\mu = 0.3$, $g = 9.81$ SI units

Setting the upwards direction as positive, we have the following:

$$\sum \vec{F}_y = \vec{0}$$

$$\vec{0} = \vec{F}_g + \vec{F}_{fy} + \vec{F}_{Ny}$$

$$\vec{0} = -mg \mp |F_f|\sin\theta + |F_N|\cos\theta \tag{E1}$$

$$(\text{E1}) \cdot \cos\theta \implies \qquad \vec{0} = -mg\cos\theta \mp |F_f|\sin\theta\cos\theta + |F_N|\cos^2\theta \tag{E2}$$

The second constraint is that the car undergoes uniform circular motion.

Setting the direction in towards the center of the velodrome to be positive, we must have positive centripetal force and hence:

$$\sum \vec{F}_x = +\, m\frac{v^2}{r}$$

$$\vec{F}_{fx} + \vec{F}_{Nx} = m\frac{v^2}{r}$$

$$\vec{0} = m\frac{v^2}{r} - \vec{F}_{fx} - \vec{F}_{Nx}$$

$$\vec{0} = m\frac{v^2}{r} \mp |F_f|\cos\theta - |F_N|\sin\theta \tag{E3}$$

$$(\text{E3}) \cdot \sin\theta \implies \qquad \vec{0} = m\frac{v^2}{r}\sin\theta \mp |F_f|\sin\theta\cos\theta - |F_N|\sin^2\theta \tag{E4}$$

Now we combine to obtain an equation involving F_f but not F_N

$$(\text{E4}) - (\text{E2}) \implies \qquad \vec{0} - \vec{0} = m\frac{v^2}{r}\sin\theta + mg\cos\theta + (\vec{0}) + |F_N| \cdot (-\sin^2\theta - \cos^2\theta)$$

$$\vec{0} = m\frac{v^2}{r}\sin\theta + mg\cos\theta + |F_N| \cdot (-1)$$

$$|F_N| = m\left(\frac{v^2}{r}\sin\theta + g\cos\theta\right) \tag{E5}$$

This gives us the normal force! Now, plugging into (E1), we find the friction force

$$(\text{E5}) \to (\text{E1}) \implies \qquad \vec{0} = -mg \mp |F_f|\sin\theta + m\left(g\cos\theta + \frac{v^2}{r}\sin\theta\right)\cos\theta$$

$$\pm|F_f|\sin\theta = m\left(\frac{v^2}{r}\sin\theta\cos\theta + g(-1 + \cos^2\theta)\right)$$

$$= m\left(\frac{v^2}{r}\sin\theta\cos\theta + g(-\sin^2\theta)\right)$$

$$|F_f| = \pm m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right)$$

Summarising our results thus far, we have,

$$\boxed{\vec{F}_N = m\left(\frac{v^2}{r}\sin\theta + g\cos\theta\right) \ \text{ up perpendicular to slope}}$$

and splitting the answer for friction into the two cases:

$$\vec{F}_{f(1)} = m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right) \text{ down the slope} \qquad \text{or} \qquad \vec{F}_{f(2)} = m\left(g\sin\theta - \frac{v^2}{r}\cos\theta\right) \text{ up the slope}$$

3.2   Deriving Range of Speeds

We now wish to find the maximum value of $|v|$ that meets our constraints for horizontal circular motion. This occurs in **Case 1** with maximal $|F_f|$, as F_f has a maximal centre-seeking component, and so does F_N as it increases to balance out both friction and gravity. Thus, they both contribute as much as possible to the centripetal force. Hence, we have:

$$|F_{f(1)}| \le \mu|F_N|$$

$$m\left(\frac{v^2}{r}\cos\theta - g\sin\theta\right) \le \mu m\left(\frac{v^2}{r}\sin\theta + g\cos\theta\right)$$

$$\frac{v^2}{r}\cos\theta - g\sin\theta \le \mu\left(\frac{v^2}{r}\sin\theta + g\cos\theta\right)$$

$$\frac{v^2}{r}(\cos\theta - \mu\sin\theta) \le g(\sin\theta + \mu\cos\theta)$$

$$\frac{v^2}{r} \le g\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}$$

$$|v| \le \sqrt{rg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)}$$

Similarly, to minimise $|v|$, we want F_f to be maximally directed up the slope (i.e. replace $(\pm)$ with $(-)$). The primary reason is that this leads to a maximal component away from the center of the velodrome. The vertical component also "lessens the burden" of F_N in opposing the constant F_g downwards, and thereby weakens F_N alongside its center-seeking component to a min. Both consequences minimise centripetal force and thus speed as required.

Thus,

$$|F_{f(2)}| \le \mu|F_N|$$

$$m\left(g\sin\theta - \frac{v^2}{r}\cos\theta\right) \le \mu m\left(\frac{v^2}{r}\sin\theta + g\cos\theta\right)$$

$$g(\sin\theta - \mu\cos\theta) \le \frac{v^2}{r}(\cos\theta + \mu\sin\theta)$$

$$|v| \ge \sqrt{rg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)}$$

Thus we have the full range of speeds for which static friction and the normal force can work to ensure there is horizontal circular motion:

$$\sqrt{rg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)} \le |v| \le \sqrt{rg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)}$$

or, dividing out $\cos\theta$ to involve $\tan\theta$ only:

$$\sqrt{rg\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)} \leq |v| \leq \sqrt{rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$$

4   New Business – Lightnight McQuation