Codec 2

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1 Introduction

Codec 2 is an open source speech codec designed for communications quality speech between 700 and 3200 bit/s. The main application is low bandwidth HF/VHF digital radio. It fills a gap in open source voice codecs beneath 5000 bit/s and is released under the GNU Lesser General Public License (LGPL).

Key feature includes:

1. A range of modes supporting different bit rates, currently (Nov 2023): 3200, 2400, 1600, 1400, 1300, 1200, 700C. The number is the bit rate, and the supplementary letter the version (700C replaced the earlier 700, 700A, 700B versions). These are referred to as "Codec 2 3200", "Codec 2 700C" etc.

2. Modest CPU (a few 10s of MIPs) and memory (a few 10s of kbytes of RAM) requirements such that it can run on stm32 class microcontrollers with hardware FPU.

3. Codec 2 has been designed for digital voice over radio applications, and retains intelligible speech at a few percent bit error rate.

4. An open source reference implementation in the C language for C99/gcc compilers, and a *cmake* build and test framework that runs on Linux. Also included is a cross compiled stm32 reference implementation.

5. Ports to non-C99 compilers (e.g. MSVC, some microcontrollers, native builds on Windows) are left to third party developers - we recommend the tests also be ported and pass before considering the port successful.

The Codec 2 project was started in 2009 in response to the problem of closed source, patented, proprietary voice codecs in the sub-5 kbit/s range, in particular for use in the Amateur Radio service.

This document describes Codec 2 at two levels. Section 2 is a high level description aimed at the Radio Amateur, while Section 3 contains a more detailed description with math and signal processing theory. Combined with the C source code, it is intended to give the reader enough information to understand the operation of Codec 2 in detail and embark on source code level projects, such as improvements, ports to other languages, student or academic research projects. Issues with the current algorithms and topics for further work are also included.

This production of this document was kindly supported by an ARDC grant [1]. As an open source project, many people have contributed to Codec 2 over the years - we deeply appreciate all of your support.

2 Codec 2 for the Radio Amateur

2.1 Model Based Speech Coding

A speech codec takes speech samples from an A/D converter (e.g. 16 bit samples at 8 kHz or 128 kbits/s) and compresses them down to a low bit rate that can be more easily sent over a narrow bandwidth channel (e.g. 700 bits/s for HF). Speech coding is the art of "what can we throw away". We need to lower the bit rate of the speech while retaining speech you can understand, and making it sound as natural as possible.

As such low bit rates we use a speech production "model". The input speech is anlaysed, and we extract model parameters, which are then sent over the channel. An example of a model based parameter is the pitch of the person speaking. We estimate the pitch of the speaker, quantise it to a 7 bit number, and send that over the channel every 20ms.

The model based approach used by Codec 2 allows high compression, with some trade offs such as noticeable artefacts in the decoded speech. Higher bit rate codecs (above 5000 bit/s), such as those use for mobile telephony or voice on the Internet, tend to pay more attention to preserving the speech waveform, or use a hybrid approach of waveform and model based techniques. They sound better but require a higher bit rate.

Recently, machine learning has been applied to speech coding. This technology promises high quality, artefact free speech quality at low bit rates, but currently (2023) requires significantly more memory and CPU resources than traditional speech coding technology such as Codec 2. However the field is progressing rapidly, and as the cost of CPU and memory decreases (Moore's law) will soon be a viable technology for many low bit rate speech applications.

2.2 Speech in Time and Frequency

To explain how Codec 2 works, lets look at some speech. Figure 1 shows a short 40ms segment of speech in the time and frequency domain. On the time plot we can see the waveform is changing slowly over time as the word is articulated. On the right hand side it also appears to repeat itself - one cycle looks very similar to the last. This cycle time is the "pitch period", which for this example is around $P = 35$ samples. Given we are sampling at $F_s = 8000$ Hz, the pitch period is $P/F_s = 35/8000 = 0.0044$ seconds, or 4.4ms.

Now if the pitch period is 4.4ms, the pitch frequency or *fundamental* frequency F_0 is about $1/0.0044 \approx 230$ Hz. If we look at the blue frequency domain plot at the bottom of Figure 1, we can see spikes that repeat every 230 Hz. Turns out of the signal is repeating itself in the time domain, it also repeats itself in the frequency domain. Those spikes separated by about 230 Hz are harmonics of the fundamental frequency F_0.

Note that each harmonic has it's own amplitude, that varies across frequency. The red line plots the amplitude of each harmonic. In this example there is a peak around 500 Hz, and another, broader peak around 2300 Hz. The ear perceives speech by the location of these peaks and troughs.

2.3 Sinusoidal Speech Coding

A sinewave will cause a spike or spectral line on a spectrum plot, so we can see each spike as a small sine wave generator. Each sine wave generator has it's own frequency that are all multiples of the fundamental pitch frequency (e.g. $230, 460, 690, ...$ Hz). They will also have their own amplitude and phase. If we add all the sine waves together (Figure 2) we can produce reasonable quality synthesised speech. This is called sinusoidal speech coding and is the speech production "model" at the heart of Codec 2.

The model parameters evolve over time, but can generally be considered constant for a short time window (a few 10s of ms). For example pitch evolves over time, moving up or down as a word is articulated.

As the model parameters change over time, we need to keep updating them. This is known as the *frame rate* of the codec, which can be expressed in terms of frequency (Hz) or time (ms). For sampling model parameters Codec 2 uses a frame rate of 10ms. For transmission over the channel we reduce this to 20-40ms, in order to lower the bit rate. The trade off with a lower frame rate is reduced speech quality.

The parameters of the sinusoidal model are:

1. The frequency of each sine wave. As they are all harmonics of F_0 we can just send F_0 to the decoder, and it can reconstruct the frequency of each harmonic as $F_0, 2F_0, 3F_0, ..., LF_0$. We used 5-7 bits/frame to represent F_0 in Codec 2.

2. The amplitude of each sine wave, $A_1, A_2, ..., A_L$. These "spectral amplitudes" are really important as they convey the information the ear needs to understand speech. Most of the bits are used for spectral amplitude information. Codec 2 uses between 18 and 50 bits/frame for spectral amplitude information.

3. Voicing information. Speech can be approximated into voiced speech (vowels) and unvoiced speech (like consonants), or some mixture of the two.

Figure 1: A 40ms segment from the word "these" from a female speaker, sampled at 8kHz. Top is a plot again time, bottom (blue) is a plot against frequency. The waveform repeats itself every 4.3ms ($F_0 = 230$ Hz), this is the "pitch period" of this segment.

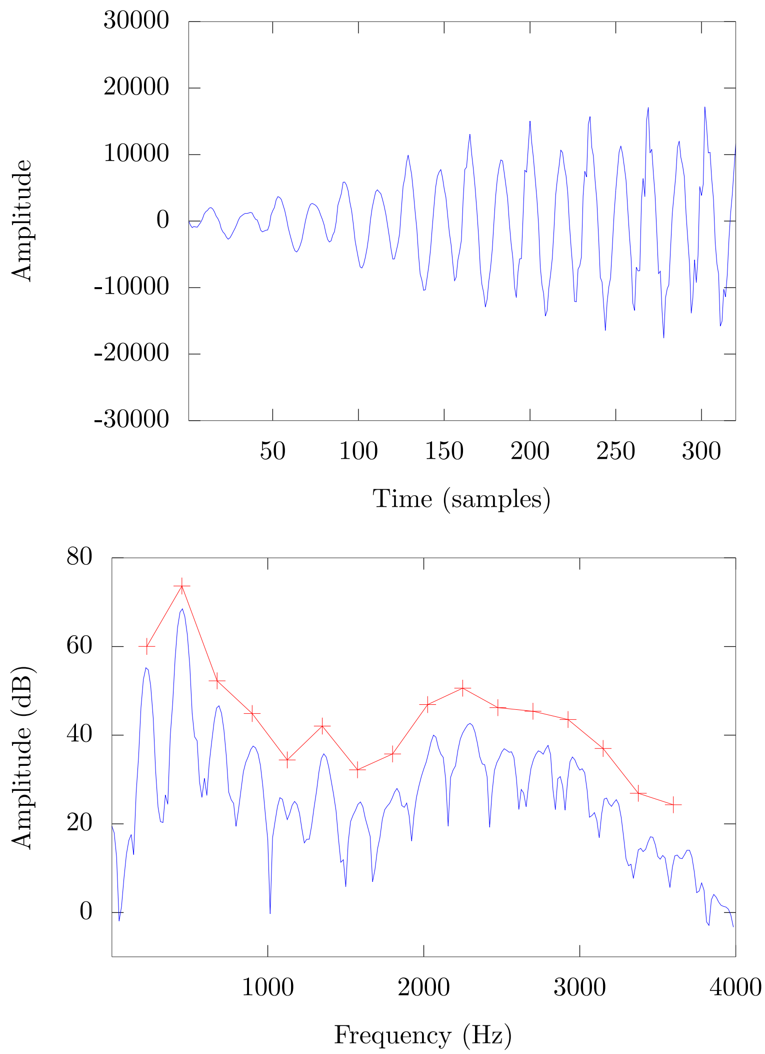
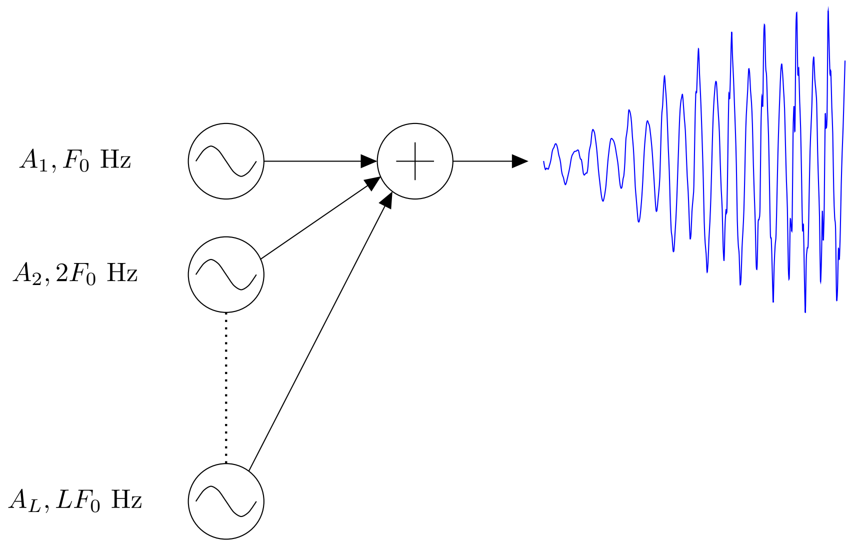


Figure 2: The sinusoidal speech model. If we sum a series of sine waves, we can generate a speech signal. Each sinewave has it's own amplitude ($A_1, A_2, ...A_L$), frequency, and phase (not shown). We assume the frequencies are multiples of the fundamental frequency F_0. L is the total number of sinewaves we can fit in 4 kHz.



The example in Figure 1 above is voiced speech. So we need some way to describe voicing to the decoder. This requires just a few bits/frame.

4. The phase of each sine wave. Codec 2 discards the phases of each harmonic at the encoder and reconstruct them at the decoder using an algorithm, so no bits are required for phases. This results in some drop in speech quality.

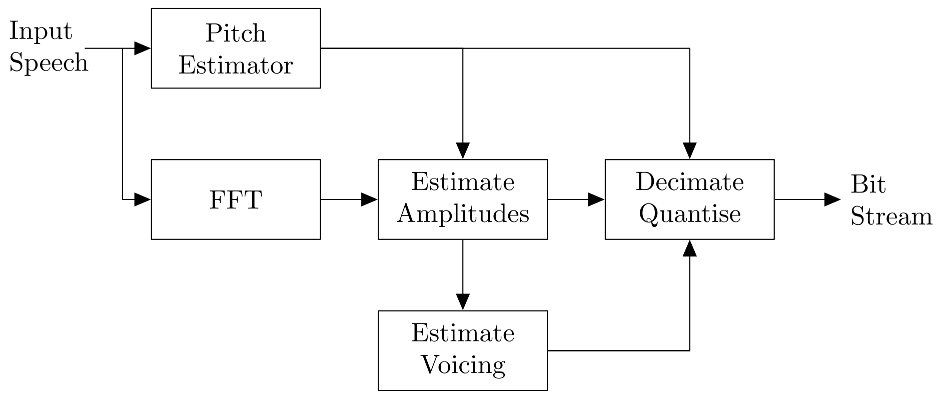
2.4 Codec 2 Encoder and Decoder

This section explains how the Codec 2 encoder and decoder works using block diagrams.

The encoder is presented in Figure 3. Frames of input speech samples are passed to a Fast Fourier Transform (FFT), which converts the time domain samples to the frequency domain. The same frame of input samples is used to estimate the pitch of the current frame. We then use the pitch and frequency domain speech to estimate the amplitude of each sine wave.

Yet another algorithm is used to determine if the frame is voiced or unvoiced. This works by comparing the spectrum to what we would expect for voiced speech (e.g. lots of spectral lines). If the energy is more random and continuous rather than discrete lines, we consider it unvoiced.

Figure 3: Codec 2 Encoder.



Up until this point the processing happens at a 10ms frame rate. However in the next step we “decimate“ the model parameters - this means we discard some of the model parameters to lower the frame rate, which helps us lower the bit rate. Decimating to 20ms (throwing away every 2nd set of model parameters) doesn’t have much effect, but beyond that the speech quality starts to degrade. So there is a trade off between decimation rate and bit rate over the channel.

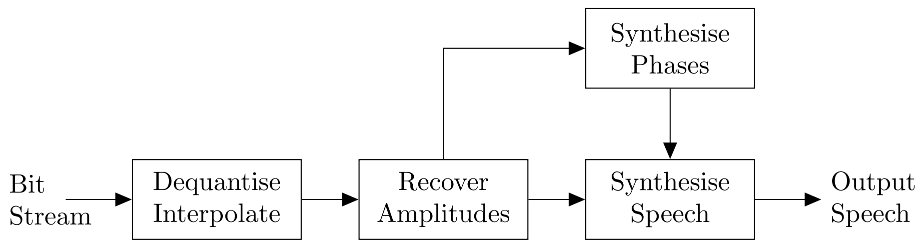
Once we have the desired frame rate, we “quantise” each model parameter. This means we use a fixed number of bits to represent it, so we can send the bits over the channel. Parameters like pitch and voicing are fairly easy, but quite a bit of DSP goes into quantising the spectral amplitudes. For the higher bit rate Codec 2 modes, we design a filter that matches the spectral amplitudes, then send a quantised version of the filter over the channel. Using the example in Figure 1 - the filter would have a band pass peaks at 500 and 2300 Hz. It’s frequency response would follow the red line. The filter is time varying - we redesign it for every frame.

You’ll notice the term “estimate” being used a lot. One of the problems with model based speech coding is the algorithms we use to extract the model parameters are not perfect. Occasionally the algorithms get it wrong. Look at the red crosses on the bottom plot of Figure 1. These mark the amplitude estimate of each harmonic. If you look carefully, you’ll see that above 2000Hz, the crosses fall a little short of the exact centre of each harmonic. This is an example of a “fine” pitch estimator error, a little off the correct value.

Often the errors interact, for example the fine pitch error shown above will mean the amplitude estimates are a little bit off as well. Fortunately these errors tend to be temporary, and are sometimes not even noticeable to the listener - remember this codec is often used for HF/VHF radio where channel noise is part of the normal experience.

Figure 4 shows the operation of the Codec 2 decoder. We take the sequence

Figure 4: Codec 2 Decoder



of bits received from the channel and recover the quantised model parameters, pitch, spectral amplitudes, and voicing. We then resample the model parameters back up to the 10ms frame rate using a technique called interpolation. For example say we receive a $F_0 = 200$ Hz pitch value then 20ms later $F_0 = 220$ Hz. We can use the average $F_0 = 210$ Hz for the middle 10ms frame.

The phases of each harmonic are generated using the other model parameters and some DSP. It turns out that if you know the amplitude spectrum, you can determine a "reasonable" phase spectrum using some DSP operations, which in practice is implemented with a couple of FFTs. We also use the voicing information - for unvoiced speech we use random phases (a good way to synthesise noise-like signals) - and for voiced speech we make sure the phases are chosen so the synthesised speech transitions smoothly from one frame to the next.

Frames of speech are synthesised using an inverse FFT. We take a blank array of FFT samples, and at intervals of F_0 insert samples with the amplitude and phase for each harmonic. We then inverse FFT to create a frame of time domain samples. These frames of synthesised speech samples are carefully aligned with the previous frame to ensure smooth frame-frame transitions, and output to the listener.

2.5 Bit Allocation

Table 2.5 presents the bit allocation for two popular Codec 2 modes. One additional parameter is the frame energy, this is the average level of the spectral amplitudes, or "AF gain" of the speech frame.

At very low bit rates such as 700 bits/s, we use Vector Quantisation (VQ) to represent the spectral amplitudes. We construct a table such that each row of the table has a set of spectral amplitude samples. In Codec 2 700C the table has 512 rows. During the quantisation process, we choose the table row that best matches the spectral amplitudes for this frame, then send the *index* of the table row. The decoder has a similar table, so can use the index to look up the output values. If the table is 512 rows, we can use a 9 bit number to quantise the spectral amplitudes. In Codec 2 700C, we use two tables of 512 entries each (18 bits total), the second one helps fine tune the quantisation from the first table.

Vector Quantisation can only represent what is present in the tables, so if it sees anything unusual (for example a different microphone frequency response or background noise), the quantisation can become very rough and speech quality poor. We train the tables at design time using a database of speech samples and a training algorithm - an early form of machine learning.

Codec 2 3200 uses the method of fitting a filter to the spectral amplitudes, this approach tends to be more forgiving of small variations in the input speech spectrum, but is not as efficient in terms of bit rate.

Parameter	3200	700C
Pitch F_0	7	5
Spectral Amplitudes $\{A_m\}$	50	18
Energy	5	3
Voicing	2	1
Bits/frame	64	28
Frame Rate	20ms	40ms
Bit rate	3200	700

Table 1: Bit allocation of the 3200 and 700C modes

3 Detailed Design

3.1 Overview

Codec 2 is based on sinusoidal [5] and Multi-Band Excitation (MBE) [2] vocoders that were first developed in the late 1980s. Descendants of the MBE vocoders (IMBE, AMBE etc) have enjoyed widespread use in many applications such as VHF/UHF hand held radios and satellite communications. In the 1990s the author studied sinusoidal speech coding [7], which provided the skill set and a practical, patent free baseline for starting the Codec 2 project:

Some features of the Codec 2 Design:

1. A pitch estimator based on a 2nd order non-linearity developed by the author.

2. A single voiced/unvoiced binary voicing model.

3. A frequency domain IFFT/overlap-add synthesis model for voiced and unvoiced speech.

4. Phases are not transmitted, they are synthesised at the decoder from the magnitude spectrum and voicing decision.

5. For the higher bit rate modes (1200 to 3200 bits/s), spectral magnitudes are represented using LPCs extracted from time domain analysis and scalar LSP quantisation.

6. For Codec 2 700C, vector quantisation of resampled spectral magnitudes in the log domain.

7. Minimal interframe prediction in order to minimise error propagation and maximise robustness to channel errors.

8. A post filter that enhances the speech quality of the baseline codec, especially for low pitched (male) speakers.

3.2 Sinusoidal Analysis

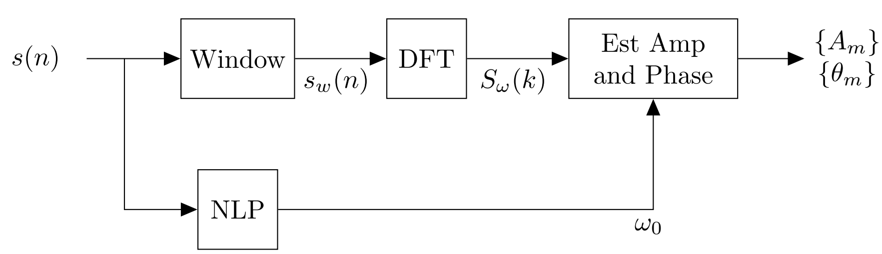
Both voiced and unvoiced speech is represented using a harmonic sinusoidal model:

$$\hat{s}(n) = \sum_{m=1}^{L} A_m cos(\omega_0 mn + \theta_m) \tag{1}$$

where the parameters $A_m, \theta_m, m = 1...L$ represent the magnitude and phases of each sinusoid, ω_0 is the fundamental frequency in radians/sample, and $L = \lfloor \pi/\omega_0 \rfloor$ is the number of harmonics.

Figure 5 illustrates the processing steps in the sinusoidal analysis system at the core of the Codec 2 encoder. This algorithms described in this section is based on the work in [7], with some changes in notation.

Figure 5: Sinusoidal Analysis



For the purposes of speech analysis the time domain speech signal $s(n)$ is divided into overlapping analysis windows (frames) of $N_w = 279$ samples. The centre of each analysis window is separated by $N = 80$ samples, or an internal frame rate or 10ms. To analyse the l-th frame it is convenient to convert the fixed time reference to a sliding time reference centred on the current analysis window:

$$s_w(n) = s(lN + n)w(n), \quad n = -N_{w2}...N_{w2} \tag{2}$$

where $w(n)$ is a tapered even window of N_w (N_w odd) samples with:

$$N_{w2} = \left\lfloor \frac{N_w}{2} \right\rfloor \tag{3}$$

A suitable window function is a shifted Hann window:

$$w(n) = \frac{1}{2} - \frac{1}{2}cos\left(\frac{2\pi(n-N_{w2})}{N_w-1}\right) \tag{4}$$

where the energy in the window is normalised such that:

$$\sum_{n=0}^{N_w-1} w^2(n) = \frac{1}{N_{dft}} \tag{5}$$

To analyse $s(n)$ in the frequency domain the N_{dft} point Discrete Fourier Transform (DFT) can be computed:

$$S_w(k) = \sum_{n=-N_{w2}}^{N_{w2}} s_w(n)e^{-j2\pi kn/N_{dft}} \tag{6}$$

The magnitude and phase of each harmonic is given by:

$$A_m = \sqrt{\sum_{k=a_m}^{b_m-1} |S_w(k)|^2} \tag{7}$$

$$\theta_m = arg\left[S_w(\lfloor mr \rceil)\right] \tag{8}$$

where:

$$\begin{aligned} a_m &= \lfloor (m-0.5)r \rceil \\ b_m &= \lfloor (m+0.5)r \rceil \\ r &= \frac{\omega_0 N_{dft}}{2\pi} \end{aligned} \tag{9}$$

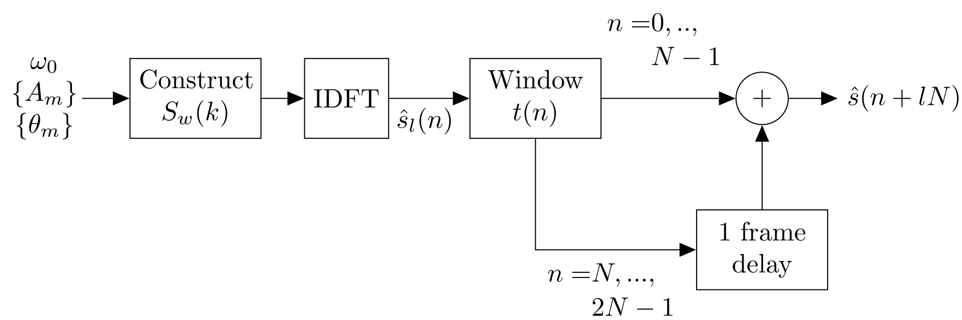
The DFT indexes a_m, b_m select the band of $S_w(k)$ containing the m-th harmonic; r maps the harmonic number m to the nearest DFT index, and $\lfloor x \rceil$ is the rounding operator. This method of estimating A_m is relatively insensitive to small errors in $F0$ estimation and works equally well for voiced and unvoiced speech.

The phase is sampled at the centre of the band. For all practical Codec 2 modes the phase is not transmitted to the decoder so does not need to be computed. However speech synthesised using the phase is useful as a control during development, and is available using the *c2sim* utility.

3.3 Sinusoidal Synthesis

Synthesis is achieved by constructing an estimate of the original speech spectrum using the sinusoidal model parameters for the current frame. This information is then transformed to the time domain using an Inverse DFT (IDFT). To produce a continuous time domain waveform the IDFTs from adjacent frames are smoothly interpolated using a weighted overlap add procedure [5].

Figure 6: Sinusoidal Synthesis. At frame l the windowing function generates $2N$ samples. The first N samples complete the current frame and are the synthesiser output. The second N samples are stored for summing with the next frame.



The synthetic speech spectrum is constructed using the sinusoidal model parameters by populating a DFT array $\hat{S}_w(k)$ with weighted impulses at the harmonic centres:

$$\hat{S}_w(k) = \begin{cases} A_m e^{j\theta_m}, & k = \lfloor mr \rceil, m = 1..L \\ 0, & otherwise \end{cases} \tag{10}$$

As we wish to synthesise a real time domain signal, $S_w(k)$ is defined to be conjugate symmetric:

$$\hat{S}_w(N_{dft} - k) = \hat{S}^*_w(k), \quad k = 1, ..N_{dft}/2 - 1 \tag{11}$$

where $\hat{S}^*_w(k)$ is the complex conjugate of $\hat{S}_w(k)$. This signal is converted to the time domain using the IDFT:

$$\hat{s}_l(n) = \frac{1}{N_{dft}} \sum_{k=0}^{N_{dft}-1} \hat{S}_w(k) e^{j2\pi kn/N_{dft}} \tag{12}$$

Where $N_{dft} > 2N$, to support the overlap add procedure below.

We introduce the notation $\hat{s}_l(n)$ to denote the synthesised speech for the l-th frame. To reconstruct a continuous synthesised speech waveform, we need to smoothly connect adjacent synthesised frames of speech. This is performed by windowing each frame of synthesised speech, then shifting and superimposing adjacent frames using an overlap add algorithm. A triangular window is defined by:

$$t(n) = \begin{cases} n/N, & 0 \le n < N \\ 1 - (n-N)/N, & N \le n < 2N \\ 0, & otherwise \end{cases} \tag{13}$$

The frame size, $N = 80$, is the same as the encoder. The shape and overlap of the synthesis window is not important, as long as sections separated by the frame size (frame to frame shift) sum to 1:

$$t(n) + t(N - n) = 1 \tag{14}$$

The continuous synthesised speech signal $\hat{s}(n)$ for the l-th frame is obtained using:

$$\hat{s}(n+lN) = \begin{cases} \hat{s}(n + (l-1)N) + \hat{s}_l(N_{dft} - N + 1 + n)t(n), & n = 0, 1, ..., N-2 \\ \hat{s}_l(n - N - 1)t(n) & n = N-1, .., 2N-1 \end{cases} \tag{15}$$

From the N_{dft} samples produced by the IDFT (12), after windowing we have $2N$ output samples. The first N output samples $n = 0, ...N - 1$ complete the current frame l and are output from the synthesiser. However we must also compute the contribution to the next frame $n = N, N+1, ..., 2N-1$. These are stored, and added to samples from the next synthesised frame.

3.4 Non-Linear Pitch Estimation

The Non-Linear Pitch (NLP) pitch estimator was developed by the author, and is described in detail in chapter 4 of [7], and portions of this description are reproduced here. The post processing algorithm used for pitch estimation in Codec 2 is different from [7] and is described here. The C code *nlp.c* is a useful reference for the fine details of the implementation, and the Octave script *plnlp.m* can by used to plot the internal states and single step through speech, illustrating the operation of the algorithm.

The core pitch detector is based on a square law non-linearity, that is applied directly to the input speech signal. Given speech is composed of harmonics separated by F_0 the non-linearity generates intermodulation products at F_0, even if the fundamental is absent from the input signal due to high pass filtering.

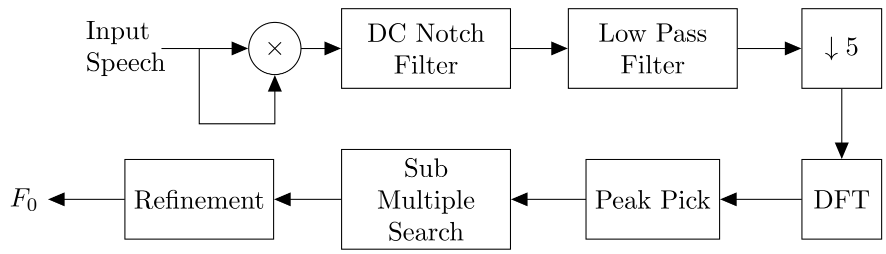
Figure 7 illustrates the algorithm. The fundamental frequency F_0 is estimated in the range of 50-400 Hz. The algorithm is designed to take blocks of $M = 320$ samples at a sample rate of 8 kHz (40 ms time window). This block length ensures at least two pitch periods lie within the analysis window at the lowest fundamental frequency.

The speech signal is first squared then notch filtered to remove the DC component from the squared time domain signal. This prevents the large amplitude DC term from interfering with the somewhat smaller amplitude term at the fundamental. This is particularly important for male speakers, who may have low frequency fundamentals close to DC. The notch filter is applied in the time domain and has the experimentally derived transfer function:

$$H(z) = \frac{1 - z^{-1}}{1 - 0.95z^{-1}} \tag{16}$$

Before transforming the squared signal to the frequency domain, the signal is low pass filtered and decimated by a factor of 5. This operation is performed

Figure 7: The Non-Linear Pitch (NLP) algorithm



to limit the bandwidth of the squared signal to the approximate range of the fundamental frequency. All energy in the squared signal above 400 Hz is superfluous and would lower the resolution of the frequency domain peak picking stage. The low pass filter used for decimation is an FIR type with 48 taps and a cut off frequency of 600 Hz. The decimated signal is then windowed and the $N_{dft} = 512$ point DFT power spectrum $F_w(k)$ is computed by zero padding the decimated signal, where k is the DFT bin.

The DFT power spectrum of the squared signal $F_w(k)$ generally contains several local maxima. In most cases, the global maxima will correspond to F_0, however occasionally the global maxima $|F_w(k_{max})|$ corresponds to a spurious peak or multiple of F_0. Thus it is not appropriate to simply choose the global maxima as the fundamental estimate for this frame. Instead, we look at submultiples of the global maxima frequency $k_{max}/2, k_{max}/3, ...k_{min}$ for local maxima. If local maxima exists and is above an experimentally derived threshold we choose the submultiple as the F_0 estimate. The threshold is biased down for F_0 candidates near the previous frames F_0 estimate, a form of backwards pitch tracking.

The accuracy of the pitch estimate in then refined by maximising the function:

$$E(\omega_0) = \sum_{m=1}^{L} |S_w(\lfloor rm \rceil)|^2 \tag{17}$$

where $r = \omega_0 N_{dft}/2\pi$ maps the harmonic number m to a DFT bin. This function will be maximised when $m\omega_0$ aligns with the peak of each harmonic, corresponding with an accurate pitch estimate. It is evaluated in a small range about the coarse F_0 estimate.

There is nothing particularly unique about this pitch estimator or it's performance. There are occasional artefacts in the synthesised speech that can be traced to "gross" and "fine" pitch estimator errors. In the real world no pitch estimator is perfect, partially because the model assumptions around pitch break down (e.g. in transition regions or unvoiced speech). The NLP algorithm could benefit from additional review, tuning and better pitch tracking. However it appears sufficient for the use case of a communications quality speech codec, and is a minor source of artefacts in the synthesised speech. Other pitch estimators could also be used, provided they have practical, real world implementations that offer comparable performance and CPU/memory requirements.

3.5 Voicing Estimation

Voicing is determined using a variation of the MBE voicing algorithm [2]. Voiced speech consists of a harmonic series of frequency domain impulses, separated by ω_0. When we multiply a segment of the input speech samples by the window function $w(n)$, we convolve the frequency domain impulses with $W(k)$, the DFT of the (w). Thus for the m-th voiced harmonic, we expect to see a cop yof the window function $W(k)$ in the band $Sw(k), k = a_m, ..., b_m$. The MBE voicing algorithm starts with the assumption that the band is voiced, and measures the error between $S_w(k)$ and the ideal voiced harmonic $\hat{S}_w(k)$.

For each band we first estimate the complex harmonic amplitude (magnitude and phase) using [2]:

$$B_m = \frac{\sum_{k=a_m}^{b_m} S_w(k)W^*(k - \lfloor mr \rceil)}{|\sum_{k=a_m}^{b_m} W(k - \lfloor mr \rceil)|^2} \tag{18}$$

where $r = \omega_0 N_{dft}/2\pi$ is a constant that maps the m-th harmonic to a DFT bin, and $\lfloor x \rceil$ is the rounding operator. As $w(n)$ is a real and even, $W(k)$ is real and even so we can write:

$$B_m = \frac{\sum_{k=a_m}^{b_m} S_w(k)W(k + \lfloor mr \rceil)}{\sum_{k=a_m}^{b_m} |W(k + \lfloor mr \rceil)|^2} \tag{19}$$

Note this procedure is different to the A_m magnitude estimation procedure in (7), and is only used locally for the MBE voicing estimation procedure. Unlike (7), the MBE amplitude estimation (19) assumes the energy in the band of $S_w(k)$ is from the DFT of a sine wave, and B_m is complex valued.

The synthesised frequency domain speech for this band is defined as:

$$\hat{S}_w(k) = B_m W(k + \lfloor mr \rceil), \quad k = a_m, ..., b_m - 1 \tag{20}$$

The error between the input and synthesised speech in this band is then:

$$\begin{aligned} E_m &= \sum_{k=a_m}^{b_m-1} |S_w(k) - \hat{S}_w(k)|^2 \\ &= \sum_{k=a_m}^{b_m-1} |S_w(k) - B_m W(k + \lfloor mr \rceil)|^2 \end{aligned} \tag{21}$$

A Signal to Noise Ratio (SNR) ratio is defined as:

$$SNR = \sum_{m=1}^{m_{1000}} \frac{A_m^2}{E_m} \tag{22}$$

where $m_{1000} = \lfloor L/4 \rceil$ is the band closest to 1000 Hz, and $\{A_m\}$ are computed from (7). If the energy in the bands up to 1000 Hz is a good match to a harmonic series of sinusoids then $\hat{S}_w(k) \approx S_w(k)$ and E_m will be small compared to the energy in the band resulting in a high SNR. Voicing is declared using the following rule:

$$v = \begin{cases} 1, & SNR > 6\text{dB} \\ 0, & otherwise \end{cases} \tag{23}$$

The voicing decision is post processed by several experimentally derived rules to prevent common voicing errors, see the C source code in *sine.c* for details.

3.6 Phase Synthesis

In Codec 2 the harmonic phases $\{\theta_m\}$ are not transmitted, instead they are synthesised at the decoder from the remaining model parameters, $\{A_m\}$, ω_0, and v. The phase model described in this section is referred to as "zero order" or *phase0* in the source code, as it requires zero model parameters to be transmitted over the channel.

Consider the source-filter model of speech production:

$$\hat{S}(z) = E(z)H(z) \tag{24}$$

where $E(z)$ is an excitation signal with a relatively flat spectrum, and $H(z)$ is a synthesis filter that shapes the magnitude spectrum. The phase of each harmonic is the sum of the excitation and synthesis filter phase:

$$\begin{split} arg\left[\hat{S}(e^{j\omega_0 m})\right] &= arg\left[E(e^{j\omega_0 m})H(e^{j\omega_0 m})\right] \\ \hat{\theta}_m &= arg\left[E(e^{j\omega_0 m})\right] + arg\left[H(e^{j\omega_0 m})\right] \\ &= \phi_m + arg\left[H(e^{j\omega_0 m})\right] \end{split} \tag{25}$$

For voiced speech $E(z)$ is an impulse train (in both the time and frequency domain). We can construct a time domain excitation pulse train using a sum of sinusoids:

$$e(n) = \sum_{m-1}^{L} cos(m\omega_0(n - n_0)) \tag{26}$$

Where n_0 is a time shift that represents the pulse position relative to the centre of the synthesis frame $n = 0$. By finding the DTCF transform of $e(n)$ we can determine the phase of each excitation harmonic:

$$\phi_m = -m\omega_0 n_0 \tag{27}$$

As we don't transmit any phase information the pulse position n_0 is unknown at the decoder. Fortunately the ear is insensitive to the absolute position of pitch pulses in voiced speech, as long as they evolve smoothly over time (discontinuities in phase are a characteristic of unvoiced speech).

The excitation pulses occur at a rate of ω_0 (one for each pitch period). The phase of the first harmonic advances by $N\phi_1$ radians over a synthesis frame of N samples. For example if $\omega_1 = \pi/20$ (200 Hz), then over a (10ms $N = 80$) sample frame, the phase of the first harmonic would advance $(\pi/20)80 = 4\pi$ radians or two complete cycles. We therefore derive n_0 from the excitation phase of the fundamental, which we treat as a timing reference. Each frame we advance the phase of the fundamental:

$$\phi_1^l = \phi_1^{l-1} + N\omega_0 \tag{28}$$

Given ϕ_1 we can compute n_0 and the excitation phase of the other harmonics:

$$\begin{aligned} n_0 &= -\phi_1/\omega_0 \\ \phi_m &= -m\omega_0 n_0 \\ &= m\phi_1 \qquad m = 2, ..., L \end{aligned} \tag{29}$$

For unvoiced speech $E(z)$ is a white noise signal. At each frame we sample a random number generator on the interval $-\pi...\pi$ to obtain the excitation phase of each harmonic. We set $F_0 = 50$ Hz to use a large number of harmonics $L = 4000/50 = 80$ for synthesis to best approximate a noise signal.

An additional phase component is provided by sampling $H(z)$ at the harmonic centres. The phase spectra of $H(z)$ is derived from the filter magnitude response using minimum phase techniques. The method for deriving the phase spectra of $H(z)$ differs between Codec 2 modes and is described below in Sections 3.7 and 3.8. This component of the phase tends to disperse the pitch pulse energy in time, especially around spectral peaks (formants).

The zero phase model tends to make speech with background noise sound "clicky". With high levels of background noise the low level inter-formant parts of the spectrum will contain noise rather than speech harmonics, so modelling them as voiced (i.e. a continuous, non-random phase track) is inaccurate. Some codecs (like MBE) have a mixed voicing model that breaks the spectrum into voiced and unvoiced regions. However (5-12) bits/frame (5-12) are required to transmit the frequency selective voicing information. Mixed excitation also requires accurate voicing estimation (parameter estimators always break occasionally under exceptional conditions).

In our case we use a post processing approach which requires no additional bits to be transmitted. The decoder measures the average level of the background noise during unvoiced frames. If a harmonic is less than this level it is made unvoiced by randomising it's phases. See the C source code for implementation details.

Comparing to speech synthesised using original phases $\{\theta_m\}$ the following observations have been made:

1. Through headphones speech synthesised with this model drops in quality. Through a small loudspeaker it is very close to original phases.

2. If there are voicing errors, the speech can sound clicky or staticy. If voiced speech is mistakenly declared unvoiced, this model tends to synthesise

annoying impulses or clicks, as for voiced speech $H(z)$ is relatively flat (broad, high frequency formants), so there is very little dispersion of the excitation impulses through $H(z)$.

3. When combined with amplitude modelling or quantisation, such that $H(z)$ is derived from $\{\hat{A}_m\}$ there is an additional drop in quality.

4. This synthesis model (e.g. a pulse train exciting a LPC filter) is effectively the same as a simple LPC-10 vocoders, and yet (especially when $H(z)$ is derived from unquantised $\{A_m\}$) sounds much better. Conventional wisdom (AMBE, MELP) says mixed voicing is required for high quality speech.

5. If $H(z)$ is changing rapidly between frames, it's phase contribution may also change rapidly. This approach could cause some discontinuities in the phase at the edge of synthesis frames, as no attempt is made to make sure that the phase tracks are continuous (the excitation phases are continuous, but not the final phases after filtering by $H(z)$).

6. The recent crop of neural vocoders produce high quality speech using a similar parameters set, and notably without transmitting phase information. Although many of these vocoders operate in the time domain, this approach can be interpreted as implementing a function $\{\hat{\theta}_m\} = F(\omega_0, \{Am\}, v)$. This validates the general approach used here, and as future work Codec 2 may benefit from being augmented by machine learning.

3.7 LPC/LSP based modes

In this and the next section we explain how the codec building blocks above are assembled to create a fully quantised Codec 2 mode. This section discusses the higher bit rate (3200 - 1200) modes that use a Linear Predictive Coding (LPC) and Line Spectrum Pairs (LSPs) to quantise and transmit the spectral magnitude information. There is a great deal of information available on these topics so they are only briefly described here.

The source-filter model of speech production was introduced above in Equation (24). A relatively flat excitation source $E(z)$ excites a filter $H(z)$ which models the magnitude spectrum of the speech. Linear Predictive Coding (LPC) defines $H(z)$ as an all pole filter:

$$H(z) = \frac{G}{1 - \sum_{k=1}^{p} a_k z^{-k}} = \frac{G}{A(z)} \tag{30}$$

where $\{a_k\}, k = 1..10$ is a set of p linear prediction coefficients that characterise the filter's frequency response and G is a scalar gain factor. An excellent reference for LPC is [4].

To be useful in low bit rate speech coding it is necessary to quantise and transmit the LPC coefficients using a small number of bits. Direct quantisation of these LPC coefficients is inappropriate due to their large dynamic range

(8-10 bits/coefficient). Thus for transmission purposes, especially at low bit rates, other forms such as the Line Spectral Pair (LSP) [3] frequencies are used to represent the LPC parameters. The LSP frequencies can be derived by decomposing the p-th order polynomial $A(z)$, into symmetric and anti-symmetric polynomials $P(z)$ and $Q(z)$, shown here in factored form:

$$\begin{aligned} P(z) &= (1+z^{-1})\prod_{i=1}^{p/2}(1-2cos(\omega_{2i-1}z^{-1}+z^{-2}) \\ Q(z) &= (1-z^{-1})\prod_{i=1}^{p/2}(1-2cos(\omega_{2i}z^{-1}+z^{-2}) \end{aligned} \tag{31}$$

where ω_{2i-1} and ω_{2i} are the LSP frequencies, found by evaluating the polynomials on the unit circle. The LSP frequencies are interlaced with each other, where $0 < \omega_1 < \omega_2 <, ..., < \omega_p < \pi$. The separation of adjacent LSP frequencies is related to the bandwidth of spectral peaks in $H(z) = G/A(z)$. A small separation indicates a narrow bandwidth. $A(z)$ may be reconstructed from $P(z)$ and $Q(z)$ using:

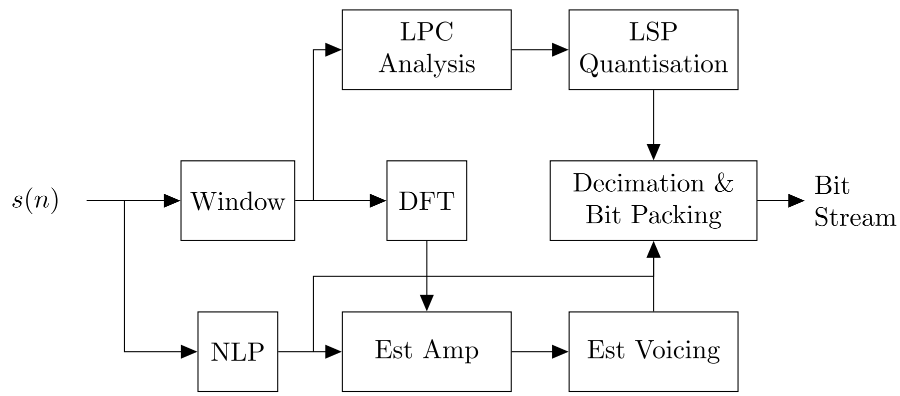
$$A(z) = \frac{P(z)+Q(z)}{2} \tag{32}$$

Thus to transmit the LPC coefficients using LSPs, we first transform the LPC model $A(z)$ to $P(z)$ and $Q(z)$ polynomial form. We then solve $P(z)$ and $Q(z)$ for $z = e^{j\omega}$to obtain p LSP frequencies $\{\omega_i\}$. The LSP frequencies are then quantised and transmitted over the channel. At the receiver the quantised LSPs are then used to reconstruct an approximation of $A(z)$. More details on LSP analysis can be found in [7] and many other sources.

Figure 8 presents the LPC/LSP mode encoder. Overlapping input speech frames are processed every 10ms ($N = 80$ samples). LPC analysis determines a set of $p = 10$ LPC coefficients $\{a_k\}$ that describe a filter the spectral envelope of the current frame and the LPC energy $E = G^2$. The LPC coefficients are transformed to $p = 10$ LSP frequencies $\{\omega_i\}$. The source code for these algorithms is in *lpc.c* and *lsp.c*. The LSP frequencies are then quantised to a fixed number of bits/frame. Other parameters include the pitch ω_0, LPC energy E, and voicing v. The quantisation and bit packing source code for each Codec 2 mode can be found in *codec2.c*. Note the spectral magnitudes $\{A_m\}$ are not transmitted, but are still required for voicing estimation (22).

One of the problems with quantising spectral magnitudes in sinusoidal codecs is the time varying number of harmonic magnitudes, as $L = \pi/\omega_0$, and ω_0 varies from frame to frame. As we require a fixed bit rate for our uses cases, it is desirable to have a fixed number of parameters. Using a fixed order LPC model is a neat solution to this problem. Some disadvantages [4] are that the energy minimisation property means the LPC residual spectrum is rarely flat, i.e. it doesn't follow the spectral magnitudes A_m exactly. The slope of the LPC spectrum near 0 and π must be 0, which means it does not track perceptually important low frequency information well. For high pitched speakers, LPC tends

Figure 8: LPC/LSP Modes Encoder



to place poles around single pitch harmonics, rather than tracking the spectral envelope.

In CELP codecs these problems can be accommodated by the (high bit rate) excitation, and some low rate codecs such as MELP supply supplementary low frequency information to "correct" the LPC model.

Before bit packing, the Codec 2 parameters are decimated in time. An update rate of 20ms is used for the highest rate modes, which drops to 40ms for Codec 2 1300, with a corresponding drop in speech quality. The number of bits used to quantise the LPC model via LSPs is also reduced in the lower bit rate modes. This has the effect of making the speech less intelligible, and can introduce annoying buzzy or clicky artefacts into the synthesised speech. Lower fidelity spectral magnitude quantisation also results in more noticeable artefacts from phase synthesis. Neverthless at 1300 bits/s the speech quality is quite usable for HF digital voice, and at 3200 bits/s comparable to closed source codecs at the same bit rate.

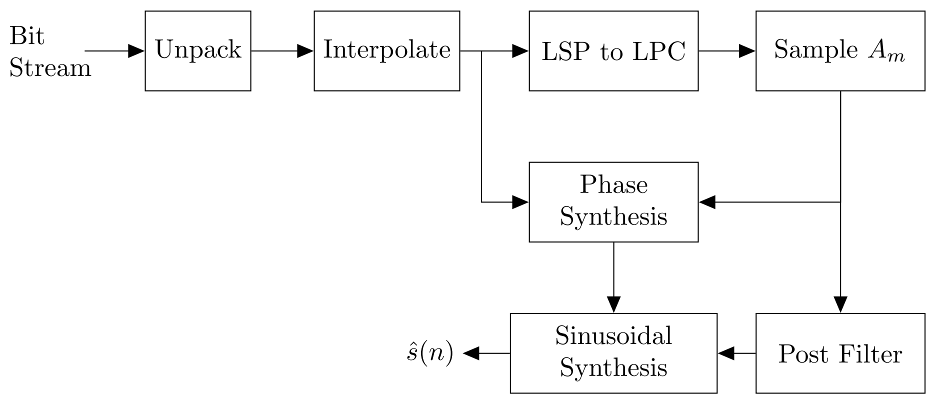
Figure 9 shows the LPC/LSP mode decoder. Frames of bits received at the frame rate are unpacked and resampled to the 10ms internal frame rate using linear interpolation. The spectral magnitude information is resampled by linear interpolation of the LSP frequencies, and converted back to a quantised LPC model $\hat{H}(z)$. The harmonic magnitudes are recovered by averaging the energy of the LPC spectrum over the region of each harmonic:

$$\hat{A}_m = \sqrt{\sum_{k=a_m}^{b_m-1} |\hat{H}(k)|^2} \tag{33}$$

where $H(k)$ is the N_{dft} point DFT of the received LPC model for this frame. For phase synthesis, the phase of $H(z)$ is determined by sampling $\hat{H}(k)$ in the centre of each harmonic:

$$arg\left[H(e^{j\omega_0 m})\right] = arg\left[\hat{H}(\lfloor mr \rceil)\right] \tag{34}$$

Figure 9: LPC/LSP Modes Decoder



3.8 Codec 2 700C

To efficiently transmit spectral amplitude information Codec 2 700C uses a set of algorithms collectively denoted *newamp1*. One of these algorithms is the Rate K resampler which transforms the variable length vectors of spectral magnitude samples to fixed length K vectors suitable for vector quantisation.

Consider a vector $\mathbf{a}$ of L harmonic spectral magnitudes in dB:

$$\mathbf{a} = \left[20log_{10}A_1, 20log_{10}A_2, \ldots 20log_{10}A_L\right] \tag{35}$$

$$L = \left\lfloor \frac{F_s}{2F_0} \right\rfloor = \left\lfloor \frac{\pi}{\omega_0} \right\rfloor \tag{36}$$

F_0 and L are time varying as the pitch track evolves over time. For speech sampled at $F_s = 8$ kHz F_0 is typically in the range of 50 to 400 Hz, giving L in the range of $10 \ldots 80$.

To quantise and transmit $\mathbf{a}$, it is convenient to resample $\mathbf{a}$ to a fixed length K element vector $\mathbf{b}$ using a resampling function:

$$\begin{aligned} \mathbf{y} &= \left[Y_1, Y_2, \ldots Y_L\right] = H(\mathbf{a}) \\ \mathbf{b} &= \left[B_1, B_2, \ldots B_K\right] = R(\mathbf{y}) \end{aligned} \tag{37}$$

Where H is a filter function chosen to smooth the spectral amplitude samples A_m while not significantly altering the perceptual quality of the speech; and

R is a resampling function. To model the response of the human ear B_k are sampled on K non-linearly spaced points on the frequency axis:

$$\begin{aligned} f_k &= warp(k,K) \text{ Hz} \quad k = 1 \dots K \\ warp(1,K) &= 200 \text{ Hz} \\ warp(K,K) &= 3700 \text{ Hz} \end{aligned} \tag{38}$$

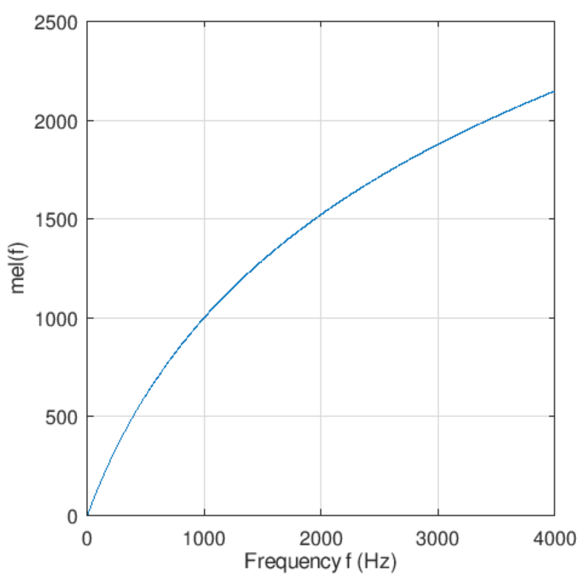
where $warp()$ is a frequency warping function. Codec 2 700C uses $K = 20$, $H = 1$, and $warp()$ is defined using the Mel function [6, p 150] (Figure 10) which samples the spectrum more densely at low frequencies, and less densely at high frequencies:

$$mel(f) = 2595 log_{10}(1 + f/700) \tag{39}$$

The inverse mapping of f in Hz from $mel(f)$ is given by:

$$f = mel^{-1}(x) = 700(10^{x/2595} - 1); \tag{40}$$

Figure 10: Mel function



We wish to use $mel(f)$ to construct $warp(k,K)$, such that there are K evenly spaced points on the $mel(f)$ axis (Figure 11). Solving for the equation of a straight line we can obtain $mel(f)$ as a function of k, and hence $warp(k,K)$

(Figure 12):

$$
\begin{aligned}
g &= \frac{mel(3700) - mel(200)}{K-1} \\
mel(f) &= g(k-1) + mel(200)
\end{aligned} \tag{41}
$$

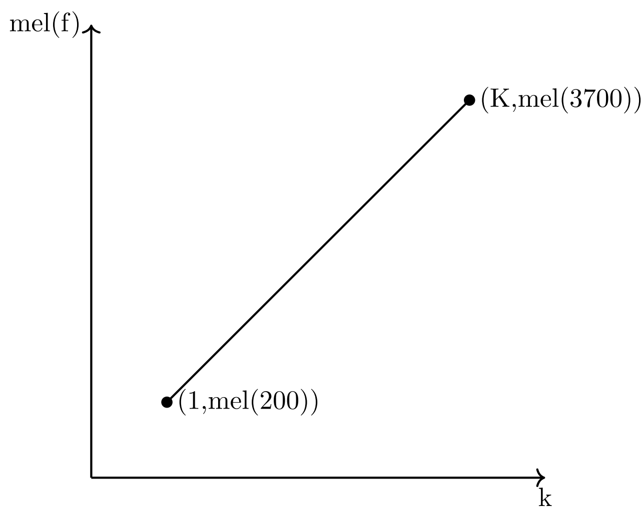
Substituting (40) into the LHS:

$$
\begin{aligned}
2595log_{10}(1+f/700) &= g(k-1) + mel(200) \\
f = warp(k,K) &= mel^{-1}(g(k-1) + mel(200))
\end{aligned} \tag{42}
$$

and the inverse warp function:

$$
k = warp^{-1}(f,K) = \frac{mel(f) - mel(200)}{g} + 1 \tag{43}
$$

Figure 11: Linear mapping of $mel(f)$ to Rate K sample index k



The rate K vector $\mathbf{b}$ is vector quantised for transmission over the channel:

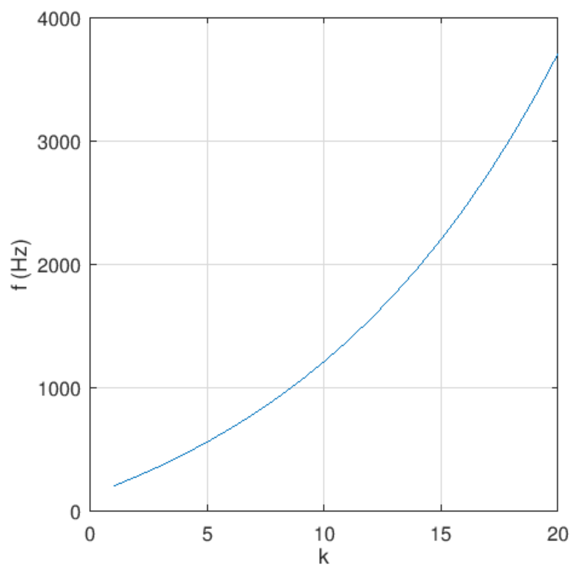
$$
\hat{\mathbf{b}} = Q(\mathbf{b}) \tag{44}
$$

Codec 2 700C uses a two stage VQ with 9 bits (512 entries) per stage. The rate filtered rate L vector can then be recovered by resampling $\hat{\mathbf{b}}$ using another resampling function:

$$
\hat{\mathbf{y}} = S(\hat{\mathbf{b}}) \tag{45}
$$

TODO: Microphone equaliser. ratek study

Figure 12: $warp(k, K)$ function for $K = 20$

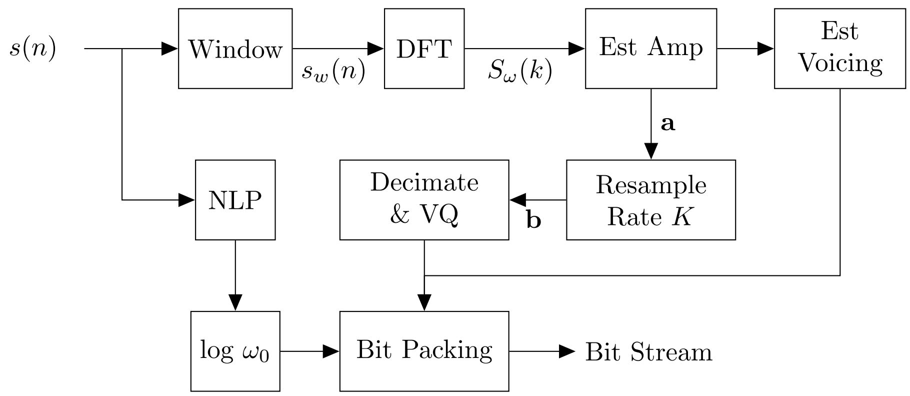


4 Further Work

Summary of mysteries/interesting points drawn out above.

1. Some worked examples aimed at the experimenter - e.g. using c2sim to extract and plot model parameters. Listen to various phases of quantisation.
2. How to use Octave tools to single step through codec operation
3. Table summarising source files with one line description
4. Add doc license (Creative Commons?)
5. Energy distribution theory. Need for V model, neural vocoders, non-linear function. Figures and simulation plots would be useful. Figure of phase synthesis.

Figure 13: Codec 2 700C (newamp1) encoder



5 Codec 2 Modes

Mode	Frm (ms)	Bits	A_m	E	ω_0	v	Comment
3200	20	64	50	5	7	2	LSP differences
2400	20	50	36	8	-	2	Joint ω_0/E VQ, 2 spare bits
1600	40	64	36	10	14	4	
1400	40	56	36	16	-	4	
1300	40	52	36	5	7	4	Joint ω_0/E VQ
1200	48	40	27	16	-	4	LSP VQ, Joint ω_0/E VQ, 1 spare
700C	40	28	18	4	6	-	VQ of log magnitudes

Table 2: Codec 2 Modes

6 Glossary

Acronym	Description
DFT	Discrete Fourier Transform
DTCF	Discrete Time Continuous Frequency Fourier Transform
IDFT	Inverse Discrete Fourier Transform
LPC	Linear Predictive Coding
LSP	Line Spectrum Pair
MBE	Multi-Band Excitation
NLP	Non Linear Pitch (algorithm)

Table 3: Glossary of Acronyms

Symbol	Description	Units
$A(z)$	LPC (analysis) filter	
a_m	lower DFT index of current band	
b_m	upper DFT index of current band	
$\{A_m\}$	Set of harmonic magnitudes $m = 1, ...L$	dB
F_0	Fundamental frequency (pitch)	Hz
F_s	Sample rate (usually 8 kHz)	Hz
$F_w(k)$	DFT of squared speech signal in NLP pitch estimator	
L	Number of harmonics	
P	Pitch period	ms or samples
$\{\theta_m\}$	Set of harmonic phases $m = 1, ...L$	dB
r	Maps a harmonic number m to a DFT index	
$s(n)$	Input speech	
$s_w(n)$	Time domain windowed input speech	
$S_w(k)$	Frequency domain windowed input speech	
ϕ_m	Phase of excitation harmonic	
ω_0	Fundamental frequency (pitch)	radians/sample
$\{\omega_i\}$	set of LSP frequencies	
v	Voicing decision for the current frame	

Table 4: Glossary of Symbols

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