Pigeonhole in syllabus

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Discrete Mathematics

Q.) Write a program to check if R is equivalence relation or not.

```
int a[10], R[10][10], n, m;
  cout << "Enter Domain";
  // domain input
  // input relation
  for (int i=0; i<n; i++)
      if ( a[i] != R[i][i])
              exit(1);

for (int i=0 ; i<n ; i++)
    for (int j=0; j<m; j++)
         if ( R[i][j] != R[j][i])
              exit(1);

for (int i=0 ; i<n ; i++)
    for (int j=0; j<m; j++)
        for (int k=0; k<n; k++)
           if ( R[i][j] == R[j][k] ) //check both =1
               if (R[i][k] != 1)
                   exit(1);
```

eg $a = [1, 2, 3]$

$R = [(1,1), (2,2), (3,3), (1,2), (2,1)]$

$$R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{array}$$

1, 0

0, 1

* Partial Ordering Set (POSET)

A relation R on set S is said to be partially ordered / partially ordering if it satisfies 3 properties:-

a) Reflexive $\forall a \in S,\ (a,a) \in R$.

b) Anti symmetric if (a,b) & $(b,a) \in R \Rightarrow a = b$.

c) Transitive if (a,b) & $(b,c) \in R \Rightarrow (a,c) \in R$.

[S, R] is called POSET.

representation of POSET = $(P, \leq)$

→ relational operator can be $<, >, /$

Q) is $(Z^+, <)$ a poset?

$1 \in Z^+$ but $1 < 1$ i.e. $(1,1) \notin R$

not poset

Is $(Z^+, \leq)$ a poset? $a \leq b$

$\forall a \in Z^+$

$a \leq a$ ✓ Reflexive ✓

$\forall a, b \in Z^+$

$a \leq b$ & $b \leq a$

only true when $a = b$

Anti symmetric ✓

$\forall a, b, c \in Z^+$

if $a \leq b$ & $b \leq c$

$\Rightarrow a \leq c$ ✓

$\Rightarrow$ Transitive

$(Z^+, \leq)$ poset ✓

Q) $S = \{1, 2, 3\}$ is $(P(S), \subseteq)$ poset?

$\forall$ set $A \in P(S)$

$A \subseteq A$

reflexive ✓

$\{1\} \subseteq \{1, 2\} \subseteq \{1, 2, 3\}$

Transitive ✓

$A \subseteq B$ & $B \subseteq A$

(Q) $(\{1, 2, \ldots\}, /)$

a / a → divisible Reflexive

a / b but (b / a) not possible

a / b & b / c → a / c Transitive

* Comparable Elements

2 elements a & b are said to be comparable if a / b or b / a

aRb or bRa i.e.

But $a \neq b$

* Total Ordering

If every pair of elements from poset is comparable then it is called Total ordering

check on poset

Q) Total ordering or not

$(\{1, 2, 3, 4, 6, 12\}, /)$

(3) & (2, 3)

* Lexicographic Ordering

Let a poset $(A \times A)$, $\leq$

$(a_1, a_2) \leq (b_1, b_2)$

→ either $a_1 < b_1$

or

$a_1 = b_1$ and

discrete < ...

1 / 2

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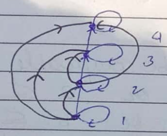
* Hasse Diagram

for eg $[\{1, 2, 3, 4\}, \leq]$

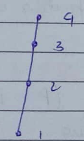
$\Rightarrow \{(1,1), (2,2), (3,3), (4,4), (1,2), (1,3), (1,4), (2,3), (3,4), (2,4)\}$



Step 1: Remove self loops
Step 2: Remove Transitivity
Step 3: If aRb, write

b
$|$ & remove
a arrow

a is lower
RULE



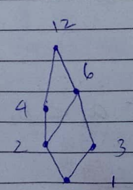
This is poset

Q.) for $[D_{12}, /]$ '/' operator ⇒ a divides b.

$D_{12} \Rightarrow$ divisor of 12.

$\Rightarrow [\{1, 2, 3, 4, 6, 12\} \; /]$

NOTE: for '6'
2 tops present '4' & '3'



→ check for 4, if No, go to its lower '2'
→ Also check for 3.

Good Write

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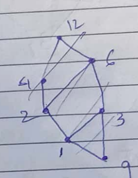
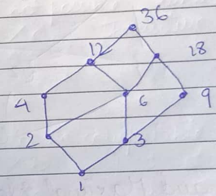
Q) $[D_{36}, /]$ Hasse Diagram.

$D_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$.

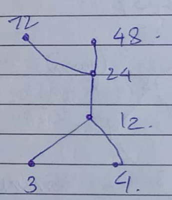




NOTE

for 9 because $9 \to 3 \to 1$ connects 1
we need not explicitly mention 1

Q.) $(\{3, 4, 12, 24, 48, 72\}, /)$

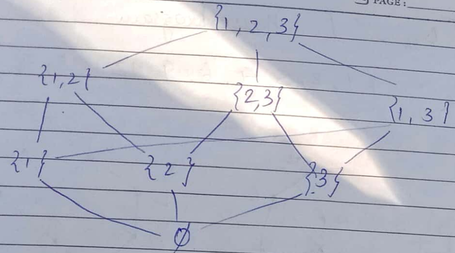


Imp

Q) $\{ \cancel{3}, \cancel{4}, \cancel{12}, \cancel{24}, \cancel{48}, \cancel{72} \}$ $S = \{1, 2, 3\}$ don't forget $\varnothing$!
$[P(S), \subseteq]$

$[\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}]$

Good Write



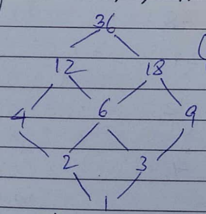
* Minimal and Maximal Element.

for $[P, \leq]$ & $S \subseteq P$.

$a \in S$ is a minimal element only if $b \leq a$. relation of

i.e. lowest element of Hasse Diagram.

Q.) minimal from $S = \{1, 2, 3\}$.



(given)

minimal element $\Rightarrow (1)$.

b) $S = \{4, 6, 3\}$

Ans = 3 & 4

as $3 \to 6$ & 4 does not have relation

Hence lowest & highest both.

c) $S = \{3, 4, 6, 12, 18, 9\}$.

since no element before 4.

$\Rightarrow$ 3 & 4.

$a \in S$, a is maximal if there is no b such that $a < b$.

If a set is non-empty poset, there will be atleast 1 minimal & maximal element.

(least) (greatest) other names.

* Minimum and Maximum element.

A element $a \in S$ is minimum if $a \leq b \ \forall b \in S$. → relation of

$(S \subseteq P)$

Q.) a) $S = \{1, 2, 3\}$

$1 \to 1$, $1 \to 2$, $1 \to 3$

$\to$ Minimum = 1

b) $S = \{4, 3, 6\}$

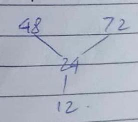
$3 \leq 6$ but $3 \nleq 4$

$4 \nleq 6$

$\Rightarrow$ No Minimum

A element $a \in S$ is maximum element if $a \geq b \ \forall b \in S$.

Q.) (a) $S = \{12, 24, 48, 72\}$



minimum = 12.

maximum = ✗ $48 \nleq 72$.

minimal = 12

maximal = 48 & 72.

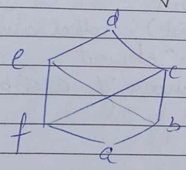
* Upper Bound & Lower Bound.

for $S \subseteq [P, \leq]$

An element(s) $y \in P$ is called the upper bound if $x \leq y \quad \forall x \in S$

An element(s) $y \in P$ is called the lower bound element if $x \geq y \quad \forall x \in S$.

eg



minimal (f, b, c, e, a) = a

maximal (") e, c

minimum (") a

maximum (") $\emptyset$.

UB (f, b) $\{d, c, e\}$

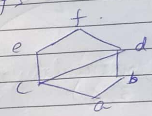
LB (f, b) $\{a\}$

UB (e, c, b) = $\{d\}$

LB (e, c, b) = $\{a, b\}$

Q.) Upper bound for $\{c, b\} = \{d, f\}$ * & lower bound for $\{c, b\} = \{a\}$

$\{a, e\} = \{e, f\}$ $\{e, a\} = \{a\}$.



* NOTE not 'e' as e is not connected to b.

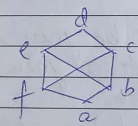
* Least upper bound & Greatest lower bound.

(join (v)) superior (meet (^)) inferior

Ex The least (minimum) of set y is called least upper bound.

The greatest (maximum) of set y is called ~~the~~ greatest lower bound.

eg



Now UB $(f, b) = \{d, c, e\}$

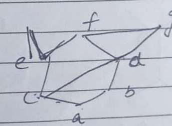
LB $(f, b) = \{a\}$.

for join minimum of $\{d, c, e\} = \emptyset$ - doesnot exist.

for meet maximum of $\{a\} = a$.

Q.) LUB $\{c, d, e\}$ = ?

GLB (c, d, e) = ?



UB = $\{g, f\}$

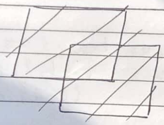
LUB = $\{f\}$

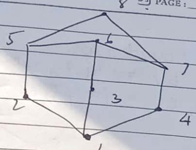
LB = $\{a, c\}$

GLB = $\{c\}$

Good Write

Q.)





minimal, minimum, maximal, maximum, UB, LB
LUB, GLB of (4, 3, 6)

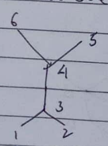
minimal = {3, 4}.
minimum = ∅
maximal = {6}
maximum = {6}
LUB = UB = {6}.
GLB = LB = {~~2~~}

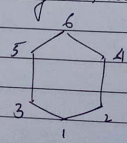
* Join Semi Lattice

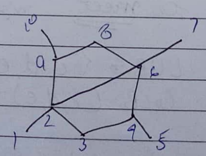
A poset [P, ≤] is said to be join semi lattice if for all pair (a,b) there exist a join

$\forall \ (a, b) \ \exists \ (a \vee b) \ \forall \ a, b \in [P, \leq]$

Q.) Which one is semi join lattice







(3,4) (5,6)
LUB = ∅.

✓

(8, 7)
X.

Good Write

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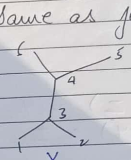
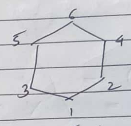
* Meet Semi lattice

Same as join semi lattice but check for meet.

Q.)



X

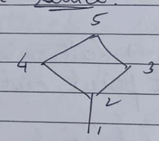


✓.

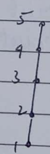
* Lattice

If a Hasse diagram satisfies the condition for both join semi lattice & meet semi lattice, it is a lattice.

Q.)



Yes.



Yes.

Properties of Lattice

1) Idempotent law.

$X \wedge X = X.$
$X \vee X = X.$

Good Write

2) Commutative Property

$X \vee Y = Y \vee X$.

$X \wedge Y = Y \wedge X$.

) Associative Property

$X \vee (Z \vee Y) = (X \vee Z) \vee Y$.

$X \wedge (Y \wedge Z) = (X \wedge Y) \wedge Z$.

) Absorption law.

$X \wedge (X \vee Y) = X$

$X \vee (X \wedge Y) = X$.

Compliment element of lattice.

If the compliment element exists, $\Rightarrow$ Lattice is bounded

i.e. highest join point & lowest meet point exist.

i.e. LUB = highest element / point (rep. by 1)

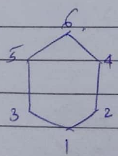
GLB = lowest element / point (rep by 0)

If a has compliment X then,

$a \vee X = 1$

and $a \wedge X = 0$.

eg



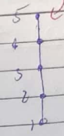
compliment of

$1 \longrightarrow 6$

$\emptyset$ $\quad$ U

for $3 \longrightarrow 2, 4$.

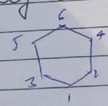
TOSET



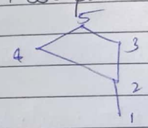
compliment of $2 \longrightarrow \emptyset$

* Complimented Lattice

A lattice L is said to be complimented lattice if every element has atleast 1 compliment.



✓



X (2)

* Distributive Lattice

A Lattice L is said to be distributive if it satisfy following 2 properties $\forall$ x, y $\in$ L.

$X \wedge (Y \vee Z) = (X \wedge Y) \vee (X \wedge Z)$

$X \vee (Y \wedge Z) = (X \vee Y) \wedge (X \wedge Z)$