DATE

* Cartesian Product:

$A \times B = \{(a,b) \mid a \in A, b \in B\}$

claims $|A| = m$ $|B| = n$ $|A \times B| = mn$

$A \times B \neq B \times A$

It is the maximum relation possible.

One member of one set is realated to a member of other set.

Relⁿ is the subset of cartesian product.

Hence total relation possible $= 2^{mn}$

Complement of Rⁿ

$\bar{R} = \{(a,b) \mid (a,b) \in A \times B \;\&\&\; (a,b) \notin R\}$

$\bar{R} + R = A \times B$

Inverse of Relation:

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$

* Reflexive	symmetric	Transitive
Irreflexive	anti-symmetric	
	asymmetric	

Equivalence r/n

Partial order r/n

Scanned with CamScanner

* A reln R on a set A is said to be reflexive if $a \in A$, $(a,a) \in R$

* sare ke saare diagonal elements hone chaiye.

$A = (a,b,c) \times (a,b,c)$

Reflexive (smallest) $= \{(a,a),(b,b),(c,c)\}$
$= \Delta_A$
= diagonal rnl

	a	b	c
a	✓ aa	ab	
b	-	✓	-
c	-	-	✓

Reflexive (biggest) $= A \times B$.

Reflexive (Total) (possibe) $= 2^{n^2-n}$

* ~~Irreflexive~~ Irreflexive : A relⁿ R on a set A is said to be Irreflexive if $a \forall A$, $(a,a) \in R$.
ek bhi diagnol element nahi hone chaiye.

eg: ✓ $R_1 = \{\phi\}$
✗ $R_2 = A \times A$
$R_4 = \{(a,b),(b,a),(a,a),(b,b)\}$ → Neither a reflexive nor a irreflexive

Irreflexive (Total possible) $= 2^{n^2-n}$.

* Symmetric : A reln R on a set A is said to be symmetric if ~~for~~ every element $\in A$, $(a,b) \in R$ the $(b,a) \in R$.

eg: (1) $\{(a,b),(b,a)\}$
✓(2) $\{(b,c),(c,b),(b,b)\}$
✓(3) $\{(a,a),(b,b),(c,c)\}$
✓(4) ϕ
(5) $A \times B$
✗(6) $\{(a,b),(b,a),(a,c)\}$

DATE

$|A| = n \qquad A \times A = n^2$

$n \to 2^n \qquad \frac{n^2-n}{2} \to 2^{\frac{n^2-n}{2}}$

ab	ba
×	×
✓	✓
×	✓
✓	×

$\Rightarrow 2^n \times 2^{\frac{n^2-n}{2}} \Rightarrow \boxed{2^{\frac{n^2+n}{2}}}$

Q **Antisymmetric:** Symmetry won't exist in non-diagonal element but can exist in diagonal element.

i.e. if (a,b) is present (b,a) should not be and (a,a) are allowed.

Definition: A relⁿ R on a set A is said to be anti-symmetric if $\forall (a,b) \in A$; $(b,a) \in R$ then $(a=$

(1) $\{(a,b)\ (b,c),\ (a,c)\}$

(2) $\{(a,a)\ (b,b)\ (c,c)\}$

(3) $\{(a,b),\ (b,b),\ (c,c)\}$

(4) ϕ

~~(5)~~ $\{(a,b),\ (b,a)\ (a,a)\}$

~~(6)~~ $A \times A$

ab	ba	
✓	✓	×
×	✓	✓
✓	×	✓
×	×	✓

Total $= \boxed{2^n \times 3^{\frac{n^2-n}{2}}}$

DATE

Asymmetric Relⁿ : A relⁿ R is said if $\forall (a,b) \in A$
$(a,b) \in R$ $(b,a) \notin R$

* Diagnol value bhi allowed nahi hai.
(a, a) not allowed.

✓(1) { (a,b), (b,c), (c,a) }
✗(2) { (a,a), (b,b), (c,c) }
✓(3) ϕ

Total = $3^{\frac{n^2-n}{2}}$

Transitive Relⁿ : if $\underbrace{(a,b) \in A, (b,c) \in R}$ then $(a,c) \notin R$

if these pair exist then this won't exist

if these pair don't exist they are transitive

eg: ✓(1) { (a,b), (b,a), (a,a), (b,b) }
✓(2) { (a,a), (b,b), (c,c) }
✓(3) { a, b }
(4) { (a,b), (a,c) }
(5) { (a,b), (c,b) }
✓(6) { (a,b), (b,c), (a,c), (a,a) }
✗(7) { (a,b), (b,c) }

Scanned with CamScanner

* Transistive Closure and Warshall Algo :

$A = \{1, 2, 3\}$, $R\{(1,1), (1,3), (2,2), (3,1), (3,2)\}$

	I	II	III
C	(1,3)	(2,3)	(1)
R	(1,3)	(2)	(1,2)
Cartesian Product C×R	(1,1) (1,3) (3,3) (3,1)	(2,2) (2,3)	(1,1) (1,2)

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 \end{array}$$

3×3

Equivalence Relⁿ → Satisfies all Reflexive, symmetric transistive hai.

eg Smallest eq relⁿ = (1,1) (2,2) (3,3)

eg : {(1,1) (2,2) (3,3) (12) (21) (13) (31)}

• A × A → largest eq relⁿ

Equivalence classes exist is relⁿ is equivalent.

* Equivalence classes : of x is denoted by $[x]$.

$[x] = \{y \mid y \in A \text{ and } (x,y) \in R\}$

eg $A = \{1, 2, 3, 4, 5\}$

$R = \{(11), (22), (33), (44), (55), (12), (21), (45), (54)\}$

$[1] = \{1, 2\}$

$[2] = \{2, 1\} = \{1, 2\}$

$[3] = \{3\}$

$[4] = \{4, 5\}$

$[5] = \{4, 5\}$

Scanned with CamScanner

= (1, 2) , (3) , (4, 5)
↑ ↑ ↑
Partision P_1 P_2 P_3

Also $P_1 \cup P_2 \cup P_3 = A$

$P_1 \cap P_2 \cap P_3 = \phi$

Also max partition = 5 ;

min —— = 1 ;

2 partitions are not related to each other.

If P_1 and P_2 are there, then find relation that existed will be $(P_1 \times P_1)$ and $(P_2 \times P_2)$

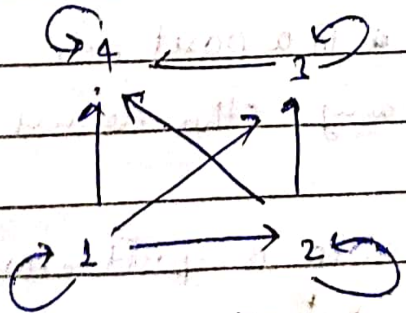
Partial ordering relⁿ:

(i) Reflexive. (ii) anti-symmetric (c) Transitive.

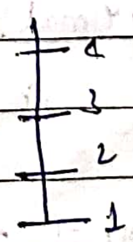
* Hass diagram / POSET diagram:

① $A = \{1, 2, 3, 4\}$

$R = \{ (1,1) (1,2) (1,3) (1,4) (2,2) (2,3) (2,4) (3,3) (3,4) (4,4) \}$



⇒

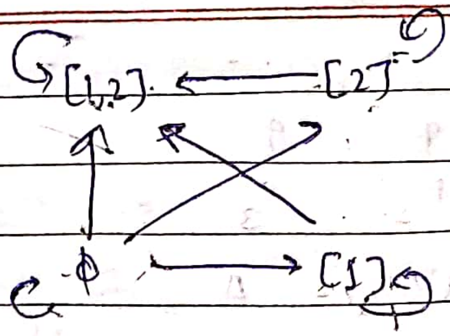


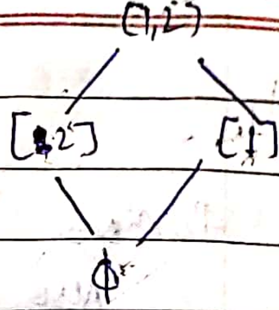
② ~~$A = \{1, 2\}$~~

$P(A) = (\phi, \{1\}, \{2\}, \{1,2\})$

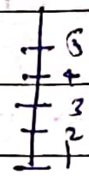
~~$R = \{ (\phi, \{1\}) (\phi, \{2\}) (\phi, \{1,2\}) (\{1\}\{1\}) (\{1\}, \{1,2\})$~~

~~$(\{2\}\{2\}) (\{2\}, \{1,2\}) (\{1,2\}, \{1,2\}) \}$~~

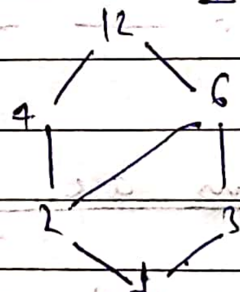




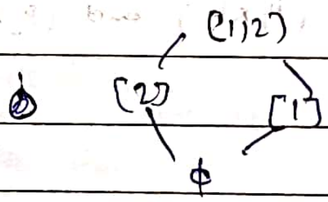
[[1,2,3, 4,5] ≤]] →



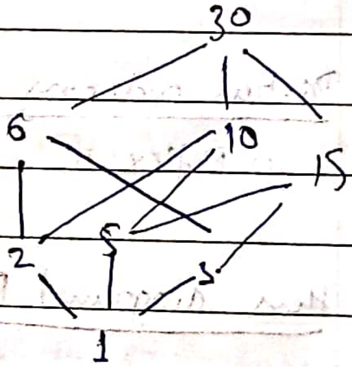
[(1,2,3, 4,6,12)/]



(ϕ [1] [2] [1,2) ≤)



(1, 2,3,5,6, 10, 15, 30)



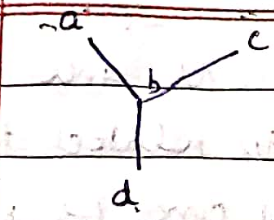
A valid hasse diagram don't have horizontal line.

* <u>Maximal element</u> : if a in a poset, an element is not related to any other element.

<u>Minimal element</u> : if a in a poset, no element is related to ~~that~~ element.

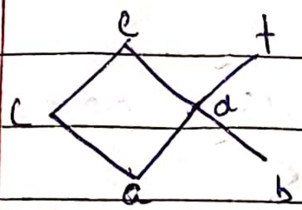
DATE

example



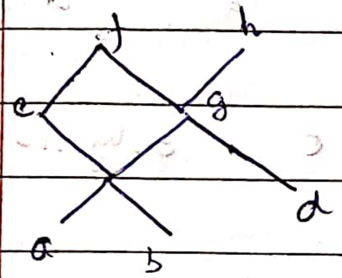
Minimal = d

Maximal = a, c



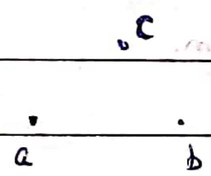
Minimal - a, b.

Maximal = e, f.



Minimal = a, b, d

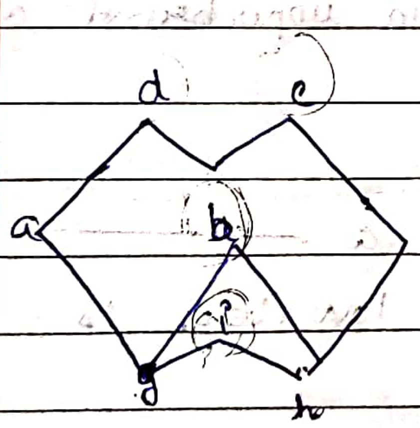
Maximal = f, h.



Maximal - a, c, b

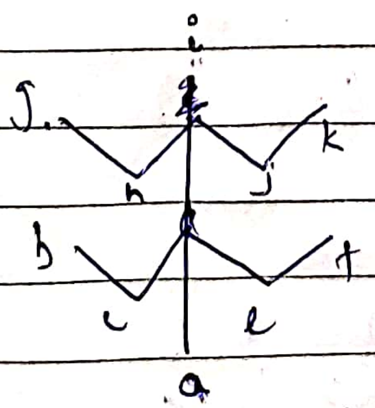
Minimal - a, b, c

• a is maximal minimal.



Max → d, e, b, i

Min → g, h.



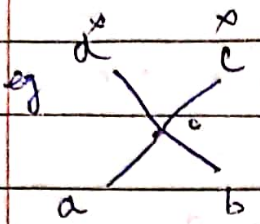
Max = g, i, k, b, f

Min - a, c, e, h, j

Maximum element: if it is maximal and every element is related to it.

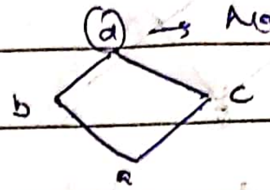
Minimum element: If it is minimal and its related to every element in poset.

eg



only maximal

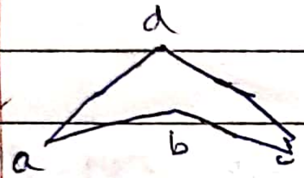
a, b are not minimum



(a) → Maximum + maximal

a is minimum

• a . a . b . c only maximal.



no maximal.

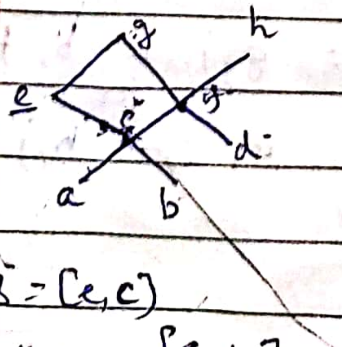
a, c are not minimum.

Upper bound: let B be the a subset of a set A. An element $x \in A$ is in upperbound of B if $(y, x) \in$ Poset $\forall y \in B$.

lower bound: let B be the a ______

______ lowerbound is -

(x, y) $\forall y \in B$.



B = [e, c]	B = [c, f, d]
L.B = [a, b, c]	L.B. = [ϕ]
U.B = [g, e]	U.B = [f, g, h]