

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

Computer Graphics

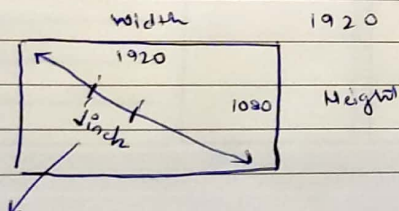
A pixel (Picture element) is the smallest unit of info in an image in 2D space.
Intensity of each pixel is variable.

Pixel $\left\{ \begin{array}{l} \text{Red (8 bits)} \\ \text{Green (8 bits)} \\ \text{Blue (8 bits)} \end{array} \right. \quad 3 \text{ Byte}$

$\rightarrow 2^8 \times 2^8 \times 2^8 = 2^{24}$ shades of color

Let say image size is 1000×750
 $= 750000 \text{ Pixels} = 7.5 \times 10^5 \times 3 \text{ bytes}$

Image Resolution: Width \times Height
 $x \text{ pixel} \times y \text{ pixel}$
 1920×1080



PPI Pixel Per Inch $1920^2 + 1080^2 = x^2$
(Pixel Density) $x = \frac{2205}{\approx 2} = 440 \text{ PPI}$
(≈ 5)

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

Aspect Ratio: Ratio of width to height

Vector / Random Scan display: [Dia]

\rightarrow Vector scan is a technique used for producing images on screen.

\rightarrow Beam is directed to the area where picture is to be drawn.

\rightarrow The display buffer memory stores the 'display list' which contains point & line plotting cmds with (x, y) or (x, y, z) char as well char plotting cmds.

\rightarrow Phosphorus light decays after few milli-second so we have to refresh phosphorus at least 30-60 times per second i.e. 30-60 fps.

Adv:

\rightarrow High resolution than raster scan

\rightarrow Produces smooth line.

\rightarrow Needs less memory to store picture definition.

Dis:

\rightarrow Can't draw realistic image.

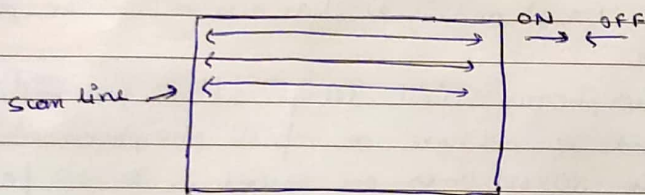
\rightarrow Limitation on clips to be displayed (Max)

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

Raster Scan Display

- Displayed image is stored in form of s/s in the refresh buffer.
- Frame buffer holds Intensity value of Pixel.
- The video controller reads the refresh buffer & produces the Actual image on the screen.
- It does this by scanning one line at a time, from top to bottom & then back to top.



Vector scan v/s Raster scan.

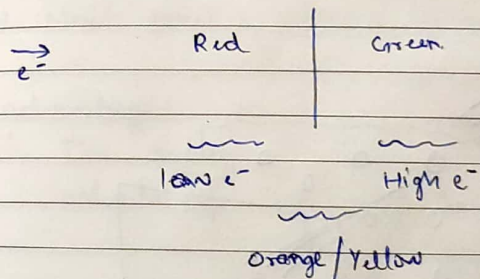
Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

Beam Penetration Technique

- Normal CRT can generate image of single color due to limitation of Phosphor coating.
- It generates a range of colors by combining the emitted light from multilayered Phosphor coated color CRT.
- 2 Techniques for producing colour display.
 - Beam Penetration Tech.
 - Shadow Mask Tech.

Beam Penetration tech. is used with Vector Scan → 2 layers of phosphor



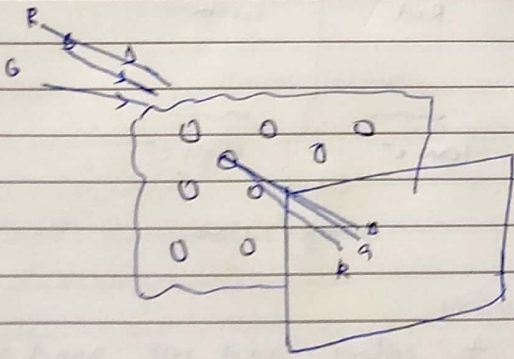
- Inexpensive.
- Only 4 colors and not good quality

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

* Shadow Mask CRT

- Used in Raster Scan & produces wide range of colors.
- 3 phosphor dot of RGB at each Pixel position, emits corresponding color.
- 3 e⁻ beam one for each color & shadow mask grid just behind coated screen.
- 3 e⁻ beam are deflected & focussed as a group on shadow mask & excite a dot Δ by passing hole.
- 1 beam activate only 1 corresponding color.



Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

→ Line Drawing Algorithm

- Line basic concepts.
- Incremental method of line drawing.
- Direct method of line drawing.
- DDA (Digital Differential Analyzer).
- Bresenham's Line Drawing.
- All numerical.

→ Circle Drawing Algo:

[Brute Force Line Algo:]

- (1) Compute m .
- (2) $x_i = x + 1$ $y_i = mx_i + c$.
- (3) Next pixel = $(x_i, \text{round}(y_i))$

Disadvantage:

- ! Too many floating points calculation required in each iteration.

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

→ Digital Differential Analyzer (DDA)

$$y_i = mx_i + b$$

$$y_i = m(x_{i-1} + 1) + b$$

$$y_i = y_{i-1} + m \quad x_i = x_{i-1} + 1$$

Algo:

(x_1, y_1) → (x_2, y_2)
Start End

$$dx = x_2 - x_1 \quad \text{change in } x, y$$

$$dy = y_2 - y_1$$

if $abs(dx) \geq abs(dy)$

$$step = |dx|$$

Use

$$step = |dy|$$

of iteration.

$$x_{inc} = \frac{dx}{step}, \quad y_{inc} = \frac{dy}{step}$$

Plot (x, y)

for $(i=1 \rightarrow steps)$

$$x = x + x_{inc} \quad y = y + y_{inc}$$

Plot (x, y)

dv: No floating point application multiplication

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

Ex Draw line b/w $(5, 4)$ and $(12, 7)$

$$dx = 12 - 5 = 7$$

$$dy = 7 - 4 = 3$$

$$|dx| > |dy| \quad \text{step} = 1$$

$$x_{inc} = \frac{7}{7} = 1 \quad y_{inc} = \frac{3}{7} = 0.4$$

x_i	y_i	x_{i+1}	y_{i+1}
5	4	6	4.4 ~ 4
6	4.4 ~ 4	7	4.8 ~ 5
7	4.8	8	5.2 ~ 5
8	5.2	9	5.6 ~ 6
9	5.6	10	6 ~ 6
10	6	11	6.4 ~ 6
11	6.8	12	6.8 ~ 7

2 $(2, 7)$ $(9, 1)$

$$dx = 9 - 2 = 7$$

$$dy = 1 - 7 = -6$$

$$|dx| > |dy| \quad \text{step} = 7$$

$$x_{inc} = \frac{7}{7} = 1 \quad y_{inc} = \frac{-6}{7} = -0.8$$

x_i	y_i	x_{i+1}	y_{i+1}	x_i	y_i	x_{i+1}	y_{i+1}
2	7	3	6.2	5	4.6	5	3.8
3	6.2	4	5.4	6	3.8	6	3
4	5.4	4	4.6	7	3	7	2.2
				8	2.2	8	1.4
				9	1.4	9	

Ok whatever!

Bresenham's Line Drawing Algo

Disadvantages of DDA

- 1) Floating point Addition
- 2) Round off function

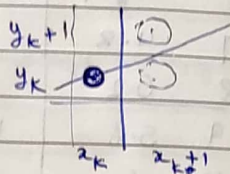
(1) $0 < m < 1$

$$0 < \Delta y / \Delta x < 1 \quad \Delta y < \Delta x$$

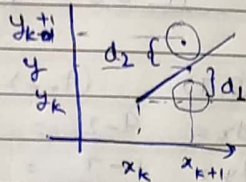
$$(y_2 - y_1) < (x_2 - x_1)$$

$$x_k := x_k + 1$$

$$y_k := y_k + 1 \quad | \quad y_k$$



choose the nearest pixel
 $\min(d_2, d_1)$



$$\text{At } x_k \rightarrow y_k = m x_k + c$$

$$x_{k+1} \quad y_{k+1} = m x_{k+1} + c$$

$$= m(x_k + 1) + c$$

$$y_{k+1} \text{ can be } y_k \text{ or } y_k + 1$$

$$d_1 = y - y_k = (m x_{k+1} + c) - y_k$$

$$d_2 = (m(x_k + 1) + c) - y_k$$

$$d_2 = (y_{k+1} + 1) - y_k$$

$$= y_k + 1 - m(x_k + 1) - c$$

$$d_1 = m(x_k + 1) + c - y_k$$

$$d_2 = y_k + 1 - m(x_k + 1) - c$$

$$d_1 - d_2 = 2m(x_k + 1) - 2y_k + 2c - 1$$

Rearranging using slope equation to remove floating multiplication using $m = \Delta y / \Delta x$

$$\Delta x(d_1 - d_2) = \Delta x [2m(x_k + 1) - 2y_k + 2c - 1]$$

P_k (decision parameter)

$$P_k = 2\Delta y x_k - 2\Delta x y_k + (2\Delta y + \Delta x(2c - 1))$$

constant (d)

$$P_k = 2\Delta y x_k - 2\Delta x y_k + d$$

$$P_k < 0 : d_1 < d_2 : \text{plot lower pix}$$

$$P_k > 0 : d_1 > d_2 : \text{plot upper pix}$$

$$\text{as } \Delta x \text{ is positive } (0 < m < 1)$$

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

For $(k+1)$ iteration

$$P_{k+1} = 2\Delta y x_{k+1} - 2\Delta x y_{k+1} + d$$

$$P_{k+1} - P_k = 2\Delta y (x_{k+1} - x_k) - 2\Delta x (y_{k+1} - y_k)$$

$$P_{k+1} - P_k = 2\Delta y - 2\Delta x (y_{k+1} - y_k)$$

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x (y_{k+1} - y_k)$$

$$\begin{cases} \rightarrow +ve & y_{k+1} = y_k + 1 \\ \rightarrow -ve & y_{k+1} = y_k \end{cases}$$

$$y_{k+1} = y_k$$

$$d_1 - d_2 = 2m(x_{k+1}) - 2y_k + 2c - 1$$

$$P_0 = \Delta x [2mx_0 + 2m - 2y_0 + 2c - 1]$$

$$P_0 = \Delta x [2m - 1]$$

$$P_0 = 2\Delta y - \Delta x$$

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

Algorithm

(1) Find $\Delta x, \Delta y, 2\Delta y, 2\Delta y - 2\Delta x$
 $\Delta x = |x_2 - x_1| \quad \Delta y = |y_2 - y_1|$

(2) Calculate $P_0 = 2\Delta y - \Delta x$

(3) Start at iteration $k=0$,
at each x_k , do the following
if $P_k < 0$

plot (x_{k+1}, y_k)

$$P_{k+1} = P_k + 2\Delta y$$

else

plot (x_{k+1}, y_{k+1})

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x$$

(4) Repeat steps Δx no. of times.

$-1 < m < 0$ (Symmetry about line)

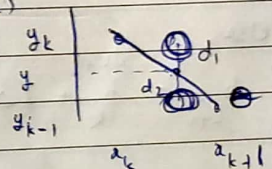
$$d_1 = y_k - y$$

$$d_2 = y - y_{k-1}$$

$$y_{k-1} = y_k / y_k - 1$$

$$d_1 - d_2 < 0 \quad y_k$$

$$> 0 \quad y_k - 1$$



Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

$$\Delta x(d_1 - d_2) = \Delta x [2y_k + 2\Delta y + 2m - 2b + -1]$$

$$P_k = 2\Delta x y_k + 2\Delta y x_k + c$$

$$P_{k+1} = 2\Delta x y_{k+1} + 2\Delta y x_{k+1} + c$$

$$b_k = 2\Delta y - \Delta x$$

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x [y_k - y_{k+1}]$$

$$b_k < 0 \quad y_{k+1} = y_k$$

$$P_{k+1} = P_k + 2\Delta y$$

$$\text{else } y_{k+1} = y_k - 1$$

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x$$

Result same but in, consider if a mirror image about $y=0$.
just d_1 and d_2 changes. u know.

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

Midpoint Circle Algo

→ 8way symmetry : OCTANTS.
($\pm x, \pm y$) ($\pm y, \pm x$)

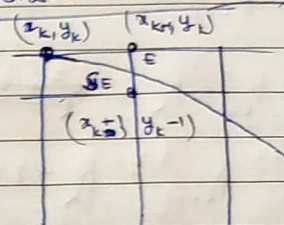
Brute force

$\forall x$, find y i.e. $y = \sqrt{r^2 - x^2}$
where $0 < x < r$

Disadv → sq of calculations.

$E = x_k + 1, y_k$
 $SE = x_k + 1, y_k - 1$
 $M \rightarrow$ Mid Point.

We decided pixel by choosing nearest distance of r' to M



$$\begin{aligned} x^2 + y^2 - r^2 &= 0 && \text{(on circle)} \\ &< 0 && \text{(inside)} \\ &> 0 && \text{(outside)} \end{aligned}$$

We can check for mid point

$$\begin{aligned} x_m^2 + y_m^2 - r^2 &= 0 && \text{choose E / SE} \\ &< 0 && \text{choose E} \\ &> 0 && \text{choose SE} \end{aligned}$$

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

Midpoint = $\left(x_k + 1, y_k - \frac{1}{2} \right)$

decision boundary

$$p_k = x_k^2 + y_k^2 - r^2$$

$$= \left(x_k + 1 \right)^2 + \left(y_k - \frac{1}{2} \right)^2 - r^2$$

$p_{k+1} =$ _____
Solve

$$p_{k+1} - p_k = x_k^2 + 4 + 4x_k$$

$$p_{k+1} = p_k + 2x_k + 3 + y_{k+1}^2 - y_{k+1} - y_k^2 + y_k$$

$p_k < 0 \quad y_{k+1} = y_k$

$$p_{k+1} = p_k + 2x_k + 3$$

$p_k > 0 \quad y_{k+1} = y_k - 1$

$$p_{k+1} = p_k + 2x_k - 2y_k + 5$$

First point $(0, r)$

$$p_0 = (0+1)^2 + (r-\frac{1}{2})^2 - r^2$$

$$= \frac{5}{4} - r$$

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

r is radius, centre is origin

Steps:

(1) Plot $(0, r)$

(2) $p_0 = \frac{5}{4} - r$ (Initial decision Parameter)

(3) If $p_i < 0$, then

$$x_{i+1} = x_i + 1$$

$$y_{i+1} = y_i$$

$$p_{i+1} = p_i + 2x_i + 3$$

(4) $p_i \geq 0$, then

$$x_{i+1} = x_i + 1$$

$$y_{i+1} = y_i - 1$$

$$p_{i+1} = p_i + 2(x_i - y_i) + 5$$

(5) Repeat (3,4) until $x \geq y$

(6) Plot in remaining 7 octants

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

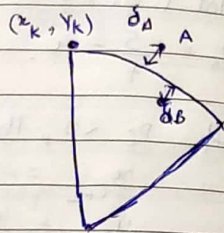
Notes

Bresenham's circle drawing algorithm:

$\delta_A =$

$$A = (x_k + 1, y_k)$$

$$B = (x_k + 1, y_k - 1)$$



$$\delta_A = dA = \sqrt{(x_k + 1)^2 + y_k^2}$$

$$\delta_B = \sqrt{(x_k + 1)^2 + (y_k - 1)^2}$$

$$\delta A = dA - r$$

$$\delta B = dB - r \quad (-ve)$$

Only Integer calculations in Bresenham's.

$$\delta'_A = \delta A^2 - r^2$$

$$\delta'_B = \delta B^2 - r^2$$

Decision Parameter: $\delta'_A + \delta'_B$

+ve if $\delta'_A > \delta'_B$ (Choose B)

-ve if $\delta'_A < \delta'_B$ (Choose A)

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

$$P_k = \delta A' + \delta B'$$

$$P_k = (dA^2 - r^2) + (dB^2 - r^2)$$

$$= (x_k + 1)^2 + y_k^2 + (x_k + 1)^2 + (y_k - 1)^2 - 2r^2$$

$$P_k = 2(x_k + 1)^2 + y_k^2 + (y_k - 1)^2 - 2r^2$$

$$P_{k+1} = 2(x_{k+1} + 1)^2 + y_{k+1}^2 + (y_{k+1} - 1)^2 - 2r^2$$

$P_{k+1} - P_k =$ simplify $x_{k+1} \rightarrow x_k + 1$

$$P_{k+1} = P_k + 4x_k + 2(y_{k+1}^2 - y_k^2) - 2(y_{k+1} - y_k) + 6$$

if $P_k < 0$ choose A

$$P_{k+1} = P_k + 4x_k + 6$$

else choose B

$$P_{k+1} = P_k + 4x_k - 4y_k + 10$$

(0, r)

$$P_0 = 3 - 2r$$

Steps:

→ Determine r , inscribe circle at origin

→ First pixel at $(0, r)$

→ $p_0 = 3 - 2r$

→ Repeat $(x \leq y)$

If $p_k < 0$

$$p_{k+1} = p_k + 4x_k + 6$$

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k$$

Else

$$p_{k+1} = p_k + 4(x_k - y_k) + 10$$

$$x_{k+1} = x_k$$

$$y_{k+1} = y_k - 1$$

Plot (x_k, y_k)

→ Other octants.

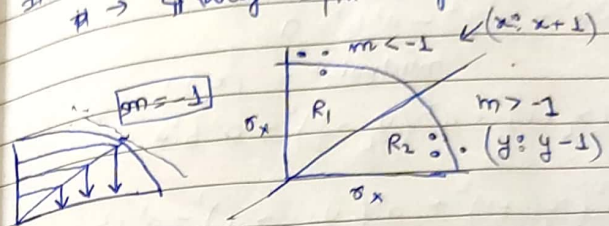
For centres x_c, y_c

$$x'_{k+1} = x_{k+1} + x_c$$

$$y'_{k+1} = y_{k+1} + y_c$$

Mid point Ellipse

→ 4 way symmetry. (Quadrants)



→ on boundary slope of curve = -1

$$x^2 + y^2 = r^2$$

$$b^2 x^2 + a^2 y^2 - a^2 b^2 = 0$$

$$\frac{dy}{dx} (b^2 x^2 + a^2 y^2 - a^2 b^2) = 0$$

$$\Rightarrow \frac{dy}{dx} (y^2) = \frac{dy}{dx} \left(\frac{a^2 b^2 - b^2 x^2}{a^2} \right)$$

$$\Rightarrow 2y \frac{dy}{dx} = - \frac{2b^2 x}{a^2}$$

$$y' = - \frac{b^2 x}{a^2 y} \quad \text{] slope}$$

$$m = 1 \quad b^2 x = a^2 y$$

Date: Mon. Tue. Wed. Thu. Fri. Sat. Sun $x_{k+1} = x_k + 1$ Notes

Region 1 $y_{k+1} = y_k, y_k - 1$

$$Eg = b^2 x^2 + a^2 y^2 - a^2 b^2 = 0$$

$$\text{midpoint} = (x_k + 1, y_k - \frac{1}{2})$$

$$p_k = b^2 (x_k + 1)^2 + a^2 (y_k - \frac{1}{2})^2 - a^2 b^2$$

$$p_{k+1} = b^2 (x_{k+1} + 1)^2 + a^2 (y_{k+1} - \frac{1}{2})^2 - a^2 b^2$$

$$p_{k+1} - p_k = b^2 \{ 2(x_k + 1) + 1 \} + a^2 \{ y_{k+1}^2 - y_k^2 - y_{k+1} + y_k \}$$

$$y_{k+1}$$

$$p_k < 0 : y_k$$

$$p_k \geq 0 : y_k - 1$$

$$p_k < 0 : p_{k+1} = p_k + b^2 2x_{k+1} + b^2$$

$$p_k \geq 0 : p_{k+1} = p_k + 2x_{k+1} b^2 + b^2 - 2y_{k+1} a^2$$

Initial point for $R_1 = (0, b)$

$$p_0 = b^2 + \frac{a^2}{4} - b a^2$$

Date: Mon. Tue. Wed. Thu. Fri. Sat. Sun Notes

Region 2 $x_{k+1} = x_k, x_k + 1$
 $y_{k+1} = y_k - 1$

$$\text{mid point} = (x_k + \frac{1}{2}, y_k - 1)$$

$$p_k = b^2 (x_k + \frac{1}{2})^2 + a^2 (y_k - 1)^2 - b^2 a^2$$

$$p_{k+1} - p_k = b^2 \{ x_{k+1}^2 + x_{k+1} - x_k^2 - x_k \} + a^2 \{ 1 - 2y_{k+1} \}$$

$$\text{if } p_k > 0 : x_{k+1} = x_k$$

$$p_{k+1} = p_k - 2y_{k+1} a^2 + b^2$$

$$p_k \leq 0 : x_{k+1} = x_k + 1$$

$$p_{k+1} = p_k + b^2 (2x_{k+1}) - 2y_{k+1} a^2 + a^2$$

last point of Region 1

$$p_0 = b^2 (x + \frac{1}{2})^2 + a^2 (y - 1)^2 - a^2 b^2$$

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

Algo

Region 1

$a, b,$

$$dx = 2b^2x \quad dy = 2a^2y$$

Region 1

$$p_0 = b^2 + a^2/4 - ba^2$$

while ($dx < dy$)

plot (x_k, y_k)

($p_k \leq 0$) {

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k$$

$$p_{k+1} = p_k + b^2 2x_{k+1} + b^2$$

($p_k > 0$) {

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k - 1$$

$$p_{k+1} = p_k + 2x_{k+1}b^2 + b^2 - 2y_{k+1}a^2$$

}

$$p_k = p_{k+1}$$

$$x_k = x_{k+1}$$

$$y_k = y_{k+1}$$

$$dx = 2b^2x_{k+1}$$

$$dy = 2a^2y_{k+1}$$

Date: _____
Mon. Tue. Wed. Thu. Fri. Sat. Sun

Notes

Region 2

while ($y > 0$) {

plot (x_k, y_k)

($p_k > 0$)

$$x_{k+1} = x_k$$

$$y_{k+1} = y_k - 1$$

$$p_{k+1} = p_k - 2y_{k+1}a^2 + b^2$$

else

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k - 1$$

$$p_{k+1} = p_k + b^2(2x_{k+1}) - a^2 2y_{k+1} + a^2$$

$p_k, x_k, y_k, dx, dy.$

$$dx = 2x_k b^2$$

$$dy = 2a^2y_k$$