INTRODUCTION

In an ideal simple harmonic motion (S.H.M.), the displacement follows a sinusoidal curve as shown in Fig. 8.1(*a*). The amplitude (*a*) of oscillations remain constant for an *infinite time*. This is because there is no loss of energy and total energy remains constant. Such oscillations are called **'Free oscillations'**. However, in actual practice, the situation is different. In our discussion in the last chapter, on harmonic oscillator, we completely ignored the effect of frictional forces on it. Since, however, an oscillator, in actual practice, always experiences *frictional* or *resistive medium*, like air, oil etc., part of its energy is dissipated in overcoming the opposing frictional or viscous forces and its amplitude, therefore goes on decreasing progressively as shown in Fig. 8.1(*b*). Due to the presence of opposing forces, the energy of free oscillator is continuously lost and consequently the amplitude of vibration decreases gradually and ultimately the body comes to rest. Hence, decay of amplitude with time is called **damping**. These opposing forces, being non-conservative nature, produces this **damping effect** are also referred as *damping*, *resistive* or *dissipative forces*. Such oscillations are called **damped harmonic oscillations**.

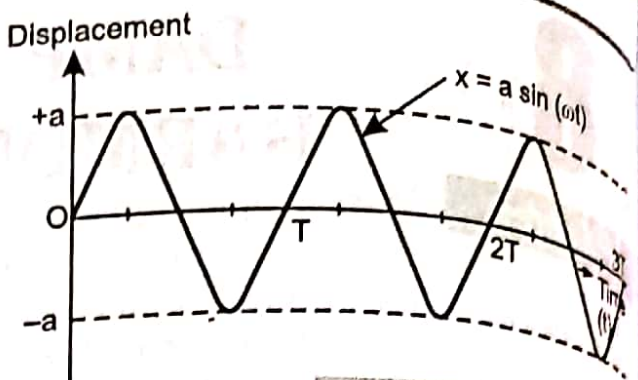

Fig. 8.1(*a*)

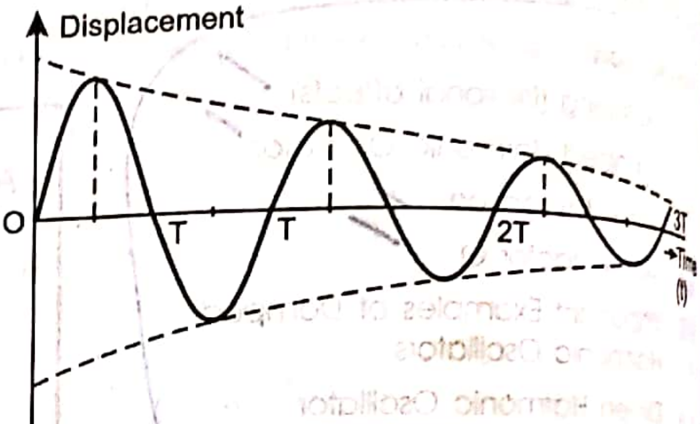

Fig. 8.1(*b*)

In order to maintain the amplitude constant, an *external periodic force* is applied. These forced vibrations initially gains the frequency equal to its natural frequency and then after short time, the oscillator acquires the frequency of the impressed periodic force. The externally applied periodic force is, hence, known as *driven force* and the oscillator is named as *driven harmonic oscillator* or *forced harmonic oscillator*. In this chapter, we will discuss amplitude resonance, quality factor and energy considerations of both damped and driven harmonic oscillators in detail.

We, therefore, have $\dfrac{dv}{dt} + \dfrac{1}{\tau} v = 0$ ...(iii)

The constant $1/\tau = \gamma/m$, or *the resistive force per unit mass per unit velocity*, is often denoted by γ, where k is called the **damping constant** of the medium.

Now, rewriting differential equation (*iii*) in the form $dv/v = -dt/\tau$, we have

$$\int \frac{dv}{v} = -\frac{1}{\tau}\int dt, \text{ which gives } \log_e v = -\frac{t}{\tau} + C,$$

where C is a constant of integration to be determined from the initial conditions.

Thus, if at $t = 0$, $v = v_0$, we have $\log_e v_0 = C$. And, therefore,

$$\log_e v - \log_e v_0 = -t/\tau, \quad \text{Or,} \quad v = v_0 e^{-t/\tau}, \qquad ...(iv)$$

clearly showing that *the velocity decreases exponentially with time*, as shown by the curve in Fig. 8.2 between the function $e^{-t/\tau}$ and t. We express this by saying that *the velocity is damped, with time constant* τ.

As will be readily seen, at $t = \tau$, $v = v_0 e^{-1} = v_0/e = v_0/2.718 = 0.368\ v_0$.

This enables us to define the *time constant* (or the *relaxation time*) τ *as the time in which the velocity of the oscillating particle falls to* $1/e$th (*i.e.*, 0.368 *or, roughly, one-third*) *of its initial value.*

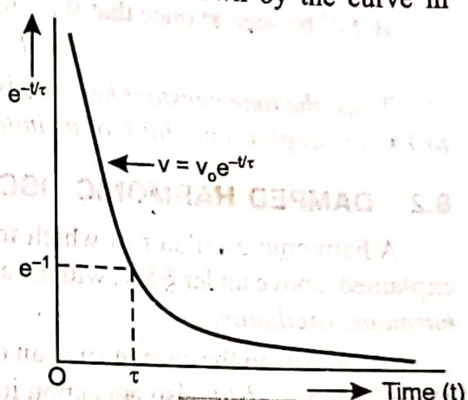

Fig. 8.2

And, since the kinetic energy of the oscillating particle is given by $T = \frac{1}{2}mv^2$, we have, on substituting the value of v from relation (iv) above, $T = \frac{1}{2}mv_0^2 e^{-2t/\tau}$. Or, representing the initial kinetic energy $\frac{1}{2}mv_0^2$ by T_0, we have $T = T_0 e^{-2t/\tau}$ indicating that *the kinetic energy of the oscillating particle too falls exponentially with time, with a relaxation time half that for velocity, i.e.,* $\tau/2$, which is only to be expected since $K.E. \propto (\text{velocity})^2$.

Putting dx/dt for v in relation (*iv*) above, we have $dx/dt = v_0 e^{-t}/\tau$, which, on integration, gives $x = -v_0 \tau e^{-t/v} + C$, where C is a constant of integration.

At $t = 0$, $x = 0$, so that $C = v_0 t$. And, therefore, $x = v_0 \tau(1 - e^{-t}/\tau)$.

Now, as $t \to \infty$, $e^{-t/v} \to 0$ and, therefore, $x \to v_0 \tau$.

Thus, *the maximum value of x is the distance that would be covered by the particle in time* τ *if its velocity remained constant at its initial value* v_0.

Examples. (*i*) As an example of the resistive or damping force of the type represented by relation (*i*) above may be cited the force experienced by a flat disc moving normally to its plane through a gas (or air), at very low pressure, at a speed very much smaller than that of its molecules.

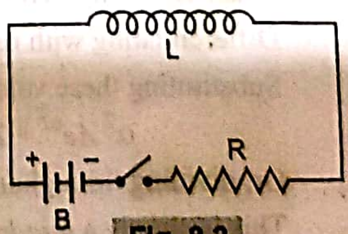

Fig. 8.3

(*ii*) Or, perhaps a more familiar example is that furnished by the ohmic resistance in an electrical circuit containing an inductance, such as the one shown in Fig. 8.3. On suddenly breaking the circuit, an induced *emf*, $-LdI/dt$, is set up across the inductance L, where I is the value of the current flowing through the circuit at the instant it is broken. Since there is now no external *emf* operative, we have

$$RI = -LdI/dt. \quad \text{Or,} \quad LdI/dt + RI = 0 \qquad ....(v)$$

where RI is the potential drop across the resistance R.

This is a relation, identical in form with relations (*ii*) and (*iii*) above, with the time constant $\tau = L/R$, clearly indicating that the ohmic resistance plays the same part here as the damping force there, so that *the current in the circuit falls exponentially with time*. For, putting relation (*v*) in the form $dI/I = -(R/L)\,dt$ and integrating, we have

$$\log_e I = -\frac{R}{L}t + C, \text{ where } C \text{ is a constant of integration.}$$

Since at $t = 0$, $I = I_0$, its initial (or maximum) value, we have $C = \log_e I_0$. And, therefore,

$$\log_e I = -\frac{R}{L}t + \log_e I_0.$$

Or,
$$\log_e \frac{I}{I_0} = -\frac{R}{L}t, \text{ whence, } I = I_0\, e^{-(R/L)t}.$$

It will be seen at once that if t = *time constant* L/R, we have

$$I = I_0 e^{-1} = I_0/e.$$

Thus, *the time constant here too is the time in which the value of the current in the circuit falls to 1/e, or roughly, one-third, of its initial or maximum value.*

8.2 DAMPED HARMONIC OSCILLATOR

A harmonic oscillator in which the oscillations are damped on account of resistive forces, as explained, above under § 8.1, with its amplitude progressively decreasing to zero, is called a *damped harmonic oscillator.*

Obviously, in the case of such an oscillator, in addition to the restoring force $-Cx$, a resistive or damping force $\gamma dx/dt$ also acts upon it, where dx/dt is its velocity at displacement x. We, therefore, have

$$m\frac{d^2x}{dt^2} = -\gamma\frac{dx}{dt} - Cx. \quad \text{Or,} \quad \frac{d^2x}{dt^2} + \frac{\gamma dx}{mdt} + \frac{C}{m}x = 0.$$

Or, since $\gamma/m = 2k$ (where k is the *damping constant* of the resistive medium) and $\sqrt{C/m} = \omega_0$, the *natural angular frequency of the oscillating particle, i.e.*, its frequency in the absence of damping, we have

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega_0^2 x = 0, \quad ...(vi)$$

which is called the **differential equation of a damped harmonic oscillator.**

Solution of the equation. The differential equation, which is a *homogeneous linear type, of the second order*, has at least one solution of the form $x = Ae^{\alpha t}$, where A and α are both *arbitrary constants*. We shall, therefore, use this as a trial solution.

Differentiating with respect to t, we have $dx/dt = \alpha Ae^{\alpha t}$ and $d^2x/dt^2 = \alpha^2 Ae^{\alpha t}$.

Substituting these values in the differential equation (*vi*) above, we have

$$\alpha^2 Ae^{\alpha t} + 2k\alpha Ae^{\alpha t} + \omega_0^2 Ae^{\alpha t} = 0.$$

Or, dividing throughout by $Ae^{\alpha t}$, we have $\alpha^2 + 2k\alpha + \omega_0^2 = 0.$ ...(vii)

This is clearly a *quadratic equation* in α and, therefore,

$$\alpha = -k + \sqrt{k^2 - \omega_0^2}$$

The differential equation (*vi*) is thus satisfied by two values of x, *viz.*,

$$x = Ae^{(-k+\sqrt{k^2-\omega_0^2})t} \quad \text{and} \quad x = Ae^{(-k-\sqrt{k^2-\omega_0^2})t}.$$

The equation being a linear one, the linear sum of the two linearly independent solutions of the equation is also a — and, indeed, the most — general solution. Thus, the general solution is

$$x = A_1 e^{(-k+\sqrt{k^2-\omega_0^2})t} + A_2 e^{(-k-\sqrt{k^2-\omega_0^2})t} \qquad \text{...(viii)}$$

Now, $k = 1/2\tau$ and if we put $\sqrt{k^2-\omega_0^2} = \beta$, we may also put the solution in the form

$$x = A_1 e^{-\frac{t}{2\tau}+\beta t} + A_2 e^{-\frac{t}{2\tau}-\beta t}$$

where the values of the *arbitrary constants* A_1 and A_2 may be determined as follows:

Differentiating expression (*viii*) with respect to t, we have

$$\frac{dx}{dt} = (-k+\sqrt{k^2-\omega_0^2})A_1 e^{(-k+\sqrt{k^2-\omega_0^2})t}$$
$$+(-k-\sqrt{k^2-\omega_0^2})A_2 e^{(-k-\sqrt{k^2-\omega_0^2})t} \qquad \text{...(ix)}$$

So that, if the displacement x be the *maximum*, equal to $x_{max} = a_0$, say, at $t = 0$ and, therefore, the velocity $dx/dt = 0$, we have from relation (*viii*) above,

$$x_{max} = a_0 = (A_1 + A_2) \qquad \text{...(A)}$$

and from relation (*ix*).

$$(-k+\sqrt{k^2-\omega_0^2})A_1 + (-k-\sqrt{k^2-\omega_0^2})A_2 = 0.$$

Or, $$-k(A_1+A_2)+\sqrt{k^2-\omega_0^2}\,(A_1-A_2) = 0.$$

Or, $$\sqrt{k^2-\omega_0^2}\,(A_1-A_2) = k(A_1+A_2) = k\,a_0,$$

whence, $$(A_1-A_2) = ka_0\Big/\sqrt{k^2-\omega_0^2} \qquad \text{...(B)}$$

Adding relations A and B, therefore, we have

$$2A_1 = a_0 + ka_0\Big/\sqrt{k^2-\omega_0^2}$$

And $\therefore$ $$A_1 = \frac{1}{2}\left(a_0 + \frac{ka_0}{\sqrt{k^2-\omega_0^2}}\right) = \frac{1}{2}a_0\left(1+\frac{k}{\sqrt{k^2-\omega_0^2}}\right)$$

$$= \frac{1}{2}a_0\left(1+\frac{1}{2\beta\tau}\right)$$

$$A_2 = (A_1+A_2) - A_1 = a_0 - \frac{1}{2}a_0\left(1+\frac{k}{\sqrt{k^2-\omega_0^2}}\right)$$

$$= \frac{1}{2}a_0\left(1-\frac{k}{\sqrt{k^2-\omega_0^2}}\right) = \frac{1}{2}a_0\left(1-\frac{1}{2\beta\tau}\right)$$

Substituting these values in expression (*viii*) above, we have

$$x = \frac{1}{2}a_0 e^{-kt}\left[\left(1+\frac{k}{\sqrt{k^2-\omega_0^2}}\right)e^{\sqrt{(k^2-\omega_0^2)}t} + \left(1+\frac{k}{\sqrt{k^2-\omega_0^2}}\right)e^{-\sqrt{(k^2-\omega_0^2)}t}\right] \qquad \text{...(x)}$$

Or, since $k = 1/2\tau$ and $\sqrt{k^2 - \omega_0^2} = \beta$, we have

$$x = \frac{1}{2}a_0 e^{-t/2\tau}\left[\left(1 + \frac{1}{2\beta\tau}\right)e^{\beta t} + \left(1 - \frac{1}{2\beta t}\right)e^{-\beta t}\right]$$

Now, *three important cases arise:*

1. When k (or 1/2τ) > ω_0 — (Case of overdamping). In case of high damping such as this, clearly, $\sqrt{k^2 - \omega_0^2}$ (or $\sqrt{1/4\tau^2 - \omega_0^2}$) is a *real quantity* with a *positive value, less than k*. So that, each of the two terms on the right hand side of equation (*x*) or (*xi*) has an exponential term with a negative power. The displacement, after attaining its maximum value, therefore, dies off exponentially with time, without changing direction. There is thus no oscillation and the motion is, therefore, called **overdamped, aperiodic** or **dead beat**, as we have in the case of a *dead beat galvanometer* (§ 8.7, I) or that of a pendulum oscillating in a viscous fluid like oil.

2. When k (or 1/2τ) = ω_0 — (Case of critical damping). In this case, obviously, $\sqrt{k^2 - \omega_0^2}$ (or $\sqrt{1/4\tau^2 - \omega_0^2}$) = 0, so that each of the two terms on the right hand side of equation (*x*) or (*xi*) above, becomes infinite and the solution breaks down.

Let us, however, consider the case when $\sqrt{k^2 - \omega_0^2} = h$, *a very small quantity but not zero*. We shall then have, from relation (*viii*) above,

$$x = A_1 e^{(-k+h)t} + A_2 e^{(-k-h)t} = e^{-kt}(A_1 e^{ht} + A_2 e^{-ht}).$$

Or,

$$x = e^{-kt}\left[A_1\left(1 + ht + \frac{h^2t^2}{2!} + \frac{h^3t^3}{3!} + \ldots\right) + A_2\left(1 - ht + \frac{h^2t^2}{2!} - \frac{h^3t^3}{3!}\right)\right]$$

Or, neglecting terms containing the second and higher powers of *h*, we have

$$x = e^{-kt}[A_1(1 + ht) + A_2(1 - ht)] = e^{-kt}[(A_1 + A_2) + (A_1 - A_2)ht]$$

Or, putting $(A_1 + A_2) = M$ and $(A_1 - A_2)h = N$, we have

$$x = e^{-kt}(M + Nt).$$

Now, taking $x = x_{max} = a_0$ and $dy/dt = 0$ at $t = 0$, we have

$$M = x_{max} = a_0 \quad \text{and} \quad N = ka_0.$$

So that,

$$x = e^{-kt}(a_0 + ka_0 t) = a_0 e^{-kt}(1 + kt) = a_0 e^{-t/2\tau}\left(1 + \frac{t}{2\tau}\right).$$

Here, the second term $a_0 + kte^{-kt}$ [or $(a_0 t/2\pi)e^{-t/2\tau}$] decays less rapidly than the first term $a_0 e^{-kt}$ (or $a_0 e^{-t/2\tau}$) and the displacement of the oscillator first increases but it then returns back quickly to its equilibrium position. The motion of the oscillator thus becomes *just aperiodic or non-oscillatory i.e., it just ceases to oscillate*. This is called the case of **critical damping**, the necessary condition for which, as we have just seen, is the $k \rightarrow \omega_0$. It finds an application in many pointer-type instruments like galvanometers where the pointer moves at once to, and stays at, the correct position, without any annoying oscillations.

3. When *k* (or 1/2τ) < ω_0 — (Case of underdamping). Here, clearly, the quantity $\sqrt{k^2 - \omega_0^2}$ will be an *imaginary* one, say, equal to *i*ω, where $i = \sqrt{-1}$ and $\omega = \sqrt{\omega_0^2 - k^2}$, a *real quantity*. Expression (*viii*) above thus becomes $x = A_1 e^{(-k+i\omega)t} + A_2 e^{(-k-i\omega)t}$

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$$x = e^{-kt}[A_1(\cos\omega t + i\sin\omega t) + A_2(\cos\omega t - i\sin\omega t)]$$
$$= e^{-kt}[(A_1 + A_2)\cos\omega t + i(A_1 - A_2)\sin\omega t]$$

putting $(A_1 + A_2) = A$ and $i(A_1 - A_2) = B$, we have

$$x = e^{-kt}(A\cos\omega t + B\sin\omega t)$$

If A, B and a_0 be related as shown in Fig. 8.4, we have

$$x = e^{-kt}\left(a_0\cos\omega t\frac{A}{a_0} + a\sin\omega t\frac{B}{a_0}\right)$$
$$= e^{-kt}(a_0\cos\omega t\sin\phi + a_0\sin\omega t\cos\phi)$$
$$x = a_0e^{-kt}\sin(\omega t + \phi)$$
$$= a_0e^{-t/2\tau}\sin(\omega t + \phi)$$

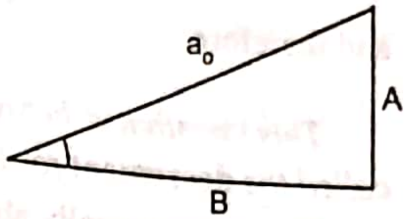

Fig. 8.4

This is the equation of a damped harmonic oscillator, with amplitude a_0e^{-kt} (or $a_0e^{-t/2\tau}$) and ...(xii)
frequency $\omega/2\pi = \sqrt{\omega_0^2 - k^2}/2\pi$. It is so called because the sine term in the equation suggests the oscillatory character of the motion and the exponential term, the gradual damping out of the oscillations.

Damping thus clearly produces two effects:

(i) *The frequency of the damped harmonic oscillator, $\omega/2\pi$ is smaller than its undamped or natural frequency $\omega_0/2\pi$, i.e.*, damping somewhat decreases the frequency or increases the time-period of the oscillator. In actual practice, in a majority of cases, particularly in the case of musical instruments, the damping is small and its effect on the frequency or the time-period of the oscillator, therefore, quite negligible.

(ii) *The amplitude of the oscillator does not remain constant at a_0, which represents the amplitude in the absence of damping, but decays exponentially with time, to zero, in accordance with the term e^{-kt}, called the damping factor.*

This is illustrated by the time-displacement curve of the damped harmonic oscillator, shown in Fig. 8.5.

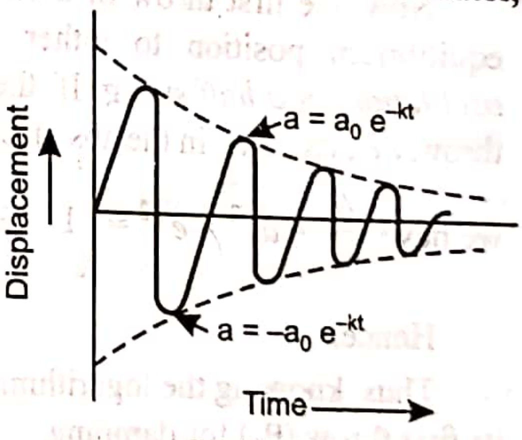

Fig. 8.5

Since the maximum values of $\sin(\omega t + \phi)$ are $+1$ and -1 alternately, the time-displacement graph of the oscillating particle is bounded by the dotted curves

$a = a_0e^{-kt}$ (or $a = a_0e^{-t/2\tau}$) and $a = -a_0e^{-kt}$ (or $a = -a_0e^{-t/2\tau}$).

Thus, although its amplitude decreases exponentially with time, the underdamped harmonic oscillator does perform a sort of oscillatory motion. *The motion does not, of course, repeat itself and is thus not periodic in the usual sense of the term.* However, it has still a time-period $= 2\pi/\omega =$
$2\pi/\sqrt{\omega_0^2 - k^2}$

Or,
$$x = e^{-kt}\,[A_1(\cos\omega t + i\sin\omega t) + A_2(\cos\omega t - i\sin\omega t)]$$
$$= e^{-kt}\,[(A_1 + A_2)\cos\omega t + i\,(A_1 - A_2)\sin\omega t]$$

Or, putting $(A_1 + A_2) = A$ and $i(A_1 - A_2) = B$, we have
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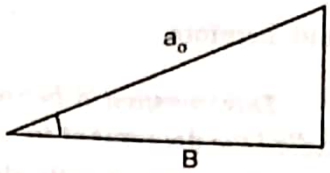

Fig. 8.4

$$x = e^{-kt}\left(a_0\cos\omega t\,\frac{A}{a_0} + a\sin\omega t\,\frac{R}{a_0}\right)$$
$$= e^{-kt}\,(a_0\cos\omega t\sin\phi + a_0\sin\omega t\cos\phi)$$

Or,
$$x = a_0 e^{-kt}\sin(\omega t + \phi)$$
$$= a_0 e^{-t/2\tau}\sin(\omega t + \phi) \qquad ...(xii)$$

This is the equation of a damped harmonic oscillator, with amplitude $a_0 e^{-kt}$ (or $a_0 e^{-t/2\tau}$) and *frequency* $\omega/2\pi = \sqrt{\omega_0^2 - k^2}/2\pi$. It is so called because the sine term in the equation suggests the oscillatory character of the motion and the exponential term, the gradual damping out of the oscillations.

Damping thus clearly produces two effects:

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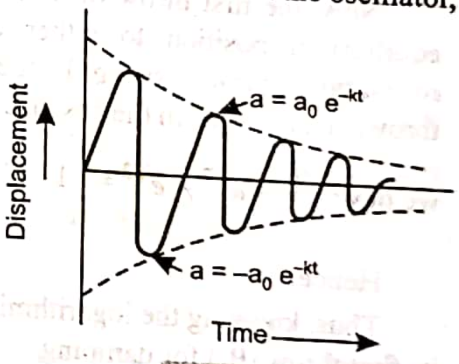

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Thus, although its amplitude decreases exponentially with time, the underdamped harmonic oscillator does perform a sort of oscillatory motion. *The motion does not, of course, repeat itself and is thus not periodic in the usual sense of the term.* However, it has still a time-period $= 2\pi/\omega = 2\pi/\sqrt{\omega_0^2 - k^2}$, which is the time-interval between its successive passages in the same direction past the equilibrium point. It is obviously also the time-interval between successive maximum displacements on the same side of the equilibrium point.

N.B. Actually, as we shall see later, a damped oscillation may, in terms of *Fourier's theorem*, be imagined to result from the superposition of a very large number (*i.e.*, an infinite series) of undamped oscillations, with their frequencies varying continuously on either side of the main or principal frequency $\omega/2\pi$ and with their amplitudes diminishing in proportion to the departure of their frequencies from the principal one.

Logarithmic decrement. As we have just seen above, the amplitude in the case of a damped harmonic motion goes on decreasing progressively. So that, if a_n and $a_{(n+1)}$ be the successive amplitudes of the oscillating particle on the two sides of the equilibrium position respectively, the time-interval between them is clearly half the time-period (T) of oscillation and is thus $T/2$. We thus have

$$a_n = a_0 e^{-kt} \quad \text{and} \quad a_{(n+1)} = a_0 e^{-k(t+T/2)}$$

and therefore,
$$\frac{a_n}{a_{(n+1)}} = \frac{e^{-kt}}{e^{-k(t+T/2)}} = e^{kT/2} = d, \text{ a constant.}$$

This constant d between two successive amplitudes of a given damped harmonic motion is called the **decrement** *for that motion.*

The same naturally also applies to angular amplitudes, where we have

$$\frac{\theta_n}{\theta_{(n+1)}} = d.$$

The logarithm of the decrement, *i.e.*, $\log_e d = kT/2 = \lambda$. Or, $d = e^{\lambda}$.

This constant, λ, which is obviously the *natural logarithm of the decrement or the ratio between two successive amplitudes of the oscillation* is referred to as the **logarithmic decrement** for that motion or oscillation.

Use is made of this in applying the necessary correction for damping to the *first deflection* or the *first 'throw'* θ_1 of a ballistic galvanometer as follows:

Each half oscillation, as we know, comprises one swing from θ_1 to θ_2 or from θ_2 to θ_3, *i.e.*, from the extreme position on one-side to the extreme position on the other, as shown in Fig. 8.6.

So that, $\theta_1/\theta_2 = \theta_2/\theta_3 = d = e^{\lambda}$.

Now, the first throw of a ballistic galvanometer, from the mean or equilibrium position to either extreme, constitutes only a *quarter oscillation on a half swing*. If, therefore, θ be the *true* value of this first throw, *i.e.*, its value in the absence of damping, and θ_1, its observed value,

Fig. 8.6

we have $$\frac{\theta}{\theta_1} = d^{1/2} = e^{\lambda/2} = \left(1 + \frac{\lambda}{2} + \frac{(\lambda/2)^2}{2!} + \ldots\right) = 1 + \frac{\lambda}{2} \text{ very nearly.}$$

Hence, $$\theta = \theta_1 (1 + \lambda/2).$$

Thus, knowing the logarithmic decrement (λ) for the given galvanometer, we can easily correct its first throw (θ_1) for damping.

8.3 POWER DISSIPATION

Since the amplitude of a damped harmonic oscillator goes on falling exponentially with time on account of the resistive or damping forces it has to overcome, it is clear that its energy gets continuously dissipated during its oscillation. Let us calculate its *rate of dissipation of energy* or its *power dissipation*, as it is more commonly called (because *energy or work/time = power*).

Since the displacement of the oscillator is given by

$$x = a_0 e^{-kt} \sin(\omega t + \phi),$$

we have, *velocity of the particle at a given instant t, i.e.,*

$$dx/dt = a_0 e^{-kt} [-k \sin(\omega t + \phi) + \omega \cos(\omega t + \phi)]$$

$\therefore$ *K.E. of oscillation of the particle at the given instant t*

$$= \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 = \frac{1}{2} m a_0^2 e^{-2kt} [-k \sin(\omega t + \phi) + \omega \cos(\omega t + \phi)]^2$$

$$= \frac{1}{2} m a_0^2 e^{-2kt} [k^2 \sin^2(\omega t + \phi) + \omega^2 \cos^2(\omega t + \phi)$$
$$- 2k\omega \sin(\omega t + \phi) \cos(\omega t + \phi)]$$