Mutually Exclusive : $A \cap B = \phi$; $P(A \cup B) = P(A) + P(B)$

Collectively Exhaustive events : $A \cup B$ = sample space.

Independent Events = $P(A \cap B) = P(A) \cdot P(B)$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

Random V(X)

Discrete	continuous.
PMF : $1 \geq P(x) \geq 0$	PDF : $f(x) \geq 0$
$\sum P(x) = 1$	$\int_{-\infty}^{\infty} f(x) = 1$
→ Binomial → Poisson	→ Uniform → Normal → Exponential

PMF / PDF are same, depends on context.

eg:

x	0	1	2
P(x)	a	b	c

x	Interval	I_2	I_3
P(x)	a	b	c

Cumulative DF

x	0	1	2
f(x)	a	a+b	a+b+c

Prefix sums.

Cumulative density Func. is same but can't add directly, so we integrate in that range.

Mean = $\bar{x}$

$E(x) = \sum x P(x)$

$\bar{x} = \int x P(x) = E(x)$

Variance = $E(x^2) - (E(x))^2$

sum of sq. − sq. of sum.

Variance → calculate $E(x^2) = \int x^2 P(x)$

$$\boxed{\text{Gamma function } \int_0^\infty x^{n-1} e^{-ax} dx = \frac{\Gamma n}{a^n}}$$

Mean of general function :

$E(\phi(x)) = \sum \phi(x) p(x)$

or $\int \phi(x) f(x) dx$

Binomial Distribution : $n \to$ finite.

$P(x) = {}^nC_x \, p^x q^{n-x}$ $[p+q=1]$ ~~p+q~~

PMF = $[p+q]^n = 1^n = 1$

$$\boxed{\begin{aligned} \text{mean} &= np \\ \text{variance} &= npq \end{aligned}}$$

MGF (Moment generating function) = $E(e^{xt})$

$E(e^{xt}) = (pe^t + q)^n$ ← solve

Characteristic Fun : $E(e^{ixt})$ $t \to it$

$\Rightarrow (pe^{it} + q)^n$

PGF (Probability Generating Function)

$\to E(z^x) = (zp+q)^n$

2nd sem syllabus

Poisson Distribution : $n \to \infty$ $p \to 0$ but np is finite.

PMF = $P(x) = \frac{\lambda^x e^{-\lambda}}{x!}$

where λ = mean = np

$\boxed{\text{Mean = Variance}}$

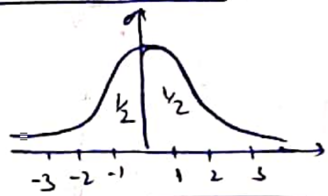
$\sigma^2 = E(x^2) - (E(x))^2$

$= \lambda^2 + \lambda - \lambda^2 = \lambda$

MGF = $e^{\lambda(e^t - 1)}$ PGF = $e^{\lambda(z-1)}$

Normal Distribution / Gaussian Distribution

Mean = Median = Mode



$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$\sigma^2 \to$ variance

$\mu \to$ mean.

$\boxed{\frac{x-\mu}{\sigma} = Z}$

Area : $(-1 < Z < 1)$: 0.68

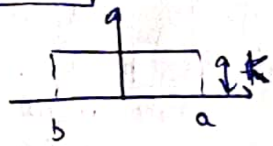
$(-2 < Z < 2)$: 0.95

$(-3 < Z < 3)$: 0.99

$(-\infty < Z < \infty)$: 1

Rectangular / Uniform distribution

$$f(x) = \begin{cases} K & b < x < a \\ 0 & \text{else} \end{cases}$$



We know $\int_{-\infty}^{\infty} f(x) = 1$ = Area

→ so $\boxed{K = \frac{1}{a-b}}$

$$\boxed{\begin{aligned} \text{mean} &= \frac{a+b}{2} = E(x) \\ \text{Variance} &= \frac{(b-a)^2}{12} \end{aligned}}$$

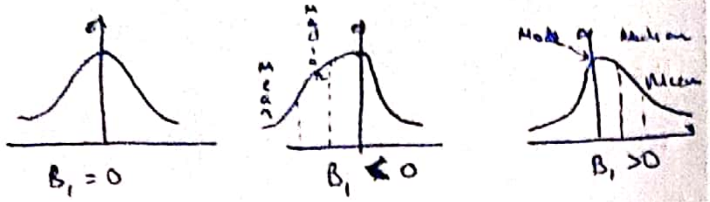
Moments | Representation → $\mu \to \mu^0$ $\mu' \to \mu^1$ $\mu'' \to \mu^2$

about Origin	Mean	Any value
$\mu^1_r = E(x-0)^r$	$\mu^0_r = E(x-\bar{x})^r$	$\mu^2_r = E(x-A)^r$
$\mu'_0 = 1$	$\mu^0_0 = 1$	
$\boxed{\mu^1_1 = \bar{x} = \text{Mean}}$	$\boxed{\mu^0_1 = 0}$	
$\mu'_2 = E(x^2)$	$\boxed{\mu^0_2 = \mu'_2 - (\mu'_1)^2 \text{ Variance}}$	
$\mu'_3 = E(x^3)$	$\mu^0_3 = \mu'_3 - 3\mu'_2\mu'_1 + (2\mu'_1)^3$	
⋮	$\mu^0_4 = \mu'_4 - 4\mu'_2\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1$	
	BORING	

Skewness : lack of symmetry.

$$\boxed{B_1 = \frac{(\mu_3^\circ)^2}{(\mu_2^\circ)^3}}$$

$\mu_r^\circ \to$ about mean



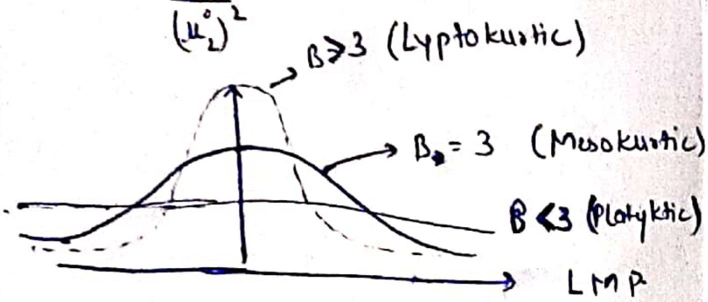
$B_1 = 0$ — Mean = Median = Mode

$B_1 < 0$ — -ve skewed.

$B_1 > 0$ — +ve skewed

Kurtosis | Flatness of curve | Bell shape

$$B_2 = \frac{\mu_4^\circ}{(\mu_2^\circ)^2}$$



Mean $= \dfrac{\sum x_i f_i}{\sum f_i}$ ← used while calculating moments etc.

Preferably find about a point and on back page replace $\mu' \to \mu^2$ to calculate about mean (lengthy)

Bivariate RV | 2 dimension |

$P(X = x_i, Y = y_i) = P_{ij}$

Joint PMF = (i) $P_{ij} \geq 0$

(ii) $\sum\sum P_{ij} = 1$

~~pair~~ triplet $= (x_i, y_i, P)$

Marginal PMF :

for $P(X = x_i) = \sum_j P_{ij} = P_{i1} + P_{i2} + P_{i3} \cdots P_{im}$

collection of pair $(x_i, P_i) \to$ MGF of x

for $P(Y = y_j) = \sum_i P_{ij} = P_{1j} + P_{2j} + P_{3j} \cdots P_{nj}$

If X and Y are independent:

$$P_{XY}(x,y) = P_X(x)\, P_Y(y)$$

i.e solve in 1d and *

$x = 0, 1 \cdots n$

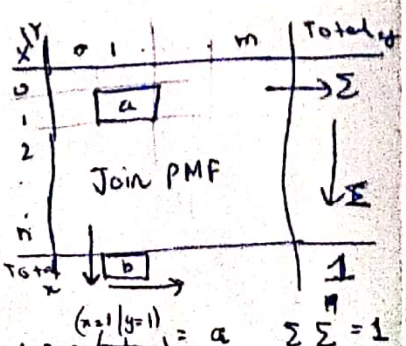
$y = 0, 1 \cdots m$

Marginal X

X	Total y
0	
1	
2	
n	

Marg Y

Y	Total x
0	
1	
2	
m	



conditional PMF $(x=1 \mid y=1) = \dfrac{a}{b}$

Distributions with replacement → independent trial; without → not independent

(i) Binomial : → $P(x) = {}^nC_x\, p^x (1-p)^x$ — independent trials

$\boxed{\text{mean} = np \qquad \sigma^2 = npq}$

(ii) Hypergeometric : dependent trials.

$n \to$ objects $\quad a \to$ success $\quad (N-a)$ failure

$$P(X = x) = \frac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}}$$

↳ solve normally without natural formula.

$\boxed{\text{Mean} = n\dfrac{a}{N} \qquad \sigma^2 = ?}$

Sometime Ans of Binomic ~ Hyper geo ($\sim 5\%$ error)

(iii) Geometric: First success on x^{th} trial.

i.e. (i) $(x-1) \to$ failure i.e $(1-p)^{x-1}$

(ii) x^{th} must be success p

PMF $= P(X = x) = (1-p)^{x-1} p$

$\boxed{\text{mean} = \dfrac{1}{p} \qquad \sigma^2 = \dfrac{1-p}{p^2}}$ $\quad P(X \leq x) = 1-(1-p)^x$

Negative Binomial :

(i) First $(x-1)$ trials → $(r-1)$ success (solve via Binomial)

(ii) x^{th} → success

$\boxed{\text{mean} = \dfrac{r}{p} \qquad \sigma^2 = \dfrac{r(1-p)}{p^2}}$

Multinomial

$$P(X = x_1, Y = y_1) = {}^nC_x\, p^x q^y = \frac{n!}{x!(n-x)!} p^x q^y = \frac{n!}{x!\,y!} p^x q^y$$

General

$$P(X = x_1 \ldots Y = x_n) = \frac{n!}{\lfloor x_1 \lfloor x_2 \cdots \lfloor x_n} p_1^{x_1} p_2^{x_2} \cdots (p_n)^{x_n}$$

$x_i = 0, 1, 2 \ldots n$

$\sum x_i = 1$