

Problem Solving



What is problem?

A problem consists of several basic components

- 1) **An initial state** (*the situation at the beginning of the problem*)
- 2) **A goal state** (*the solution to the problem*)
- 3) **A set of rules** (*constraints that must be followed*)
- 4) **A set of obstacles** (*hindrances that must be overcome*)

Types of Problems

Well and Ill defined problems

1) *Well defined problems* are **clear and structured** i.e., the initial state, goal state and constraints are all understood, and once you reach a solution, it's easily assessed. (e.g., solving an anagram)

2) *Ill defined problems* are **fuzzy and abstract** i.e., the initial state, goal and constraints have gaps in understanding leading to difficulty in accessing the solution (e.g., writing a research paper)

Routine and Non-Routine problem: (familiarity with procedures)

A routine problem is one that can be solved by applying well-practiced procedures (e.g., writing exams)

A non-routine problem is one that cannot be solved by applying well –practiced procedures (e.g., writing research paper)

Challenges with problem solving research

Problem solving is the last hurdle to most cognitive processes and thus takes much longer time to accomplish. Assessing problem solving in terms of accuracy rate provides a rather gross estimate of problem solving proficiency. Measuring solution times provides some useful information but doesn't shed much light on the nature of the processing that occurs during problem solving.

One Saturday night at a local county dance, 40 people, 20 men and 20 women showed up to dance. The dance was a contra dance, in which men and women face each other in lines. From 8-10 pm there were 20 heterosexual couples dancing on the floor. At 10 pm 2 women left leaving 38 people to dance. Could the dance caller make arrangement so that the remaining people could all dance together at the same time in 19 heterosexual couples? The dance caller must remain a caller and cannot take a partner. Answer yes/no giving reasoning behind answer

--- *Gick & McGarry (1992) Learning from mistakes
Journal of Expt Psychol, 18, 623-639*

Verbal protocol: In order to understand the processes of problem solving, researchers have made extensive use of verbal protocols – which are reports generated by problem solvers as they “think aloud” during the solution process. The limitation with such reports are:

1) not everyone has the verbal ability required to reflect accurately on what they are thinking

2) there is no way to assess the accuracy of the verbal report

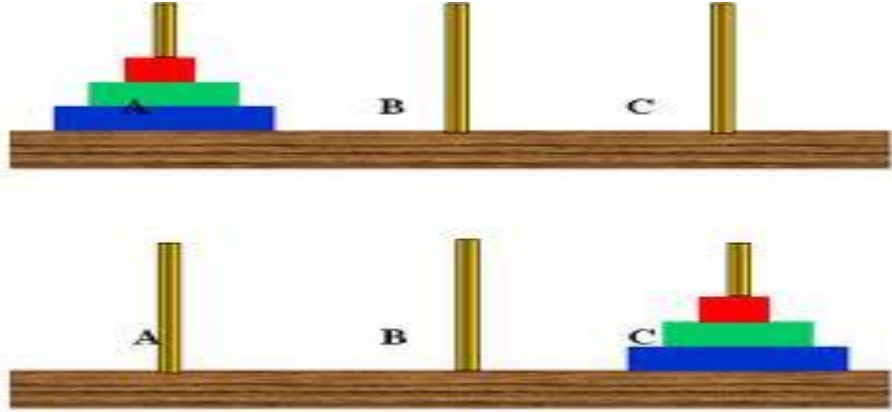
3) the act of thinking out loud may interfere with or change the very nature of the thought process being described.

The varied nature of problems: The complexity of problem solving presents another challenge for researchers. The term *problem* can apply to a diverse set of circumstances from solving math problems to writing a term paper to figuring out an alternative route home during rush hour traffic.

Marr (1992) distinguishes between five types of problems

a) *Transformational*: presents the solver with a goal state, the solver must find the proper strategies that will eventually transform the initial state to the goal state

- 2) *Arrangement problems* – involve presentation of all the necessary elements to solve the problem; the solver must figure out how the elements are to be arranged
- 3) *Induction problems* – involves giving the solver a series of exemplars or instances using which they must figure out the pattern or rule that relates the instances
- 4) *Deduction problems* – presents the solver with premises or conditions and require them to determine whether a conclusion fits the premise
- 5) *Divergent problems* – require the solver to generate as many solutions as possible to a given problem



Transformational

Take a look at the number sequence

8, 5, 4, 1, 7, 6, 10, 0

What is the next number in the sequence?

Induction

“KIGVIN” – rearrange letters to form another word

Arrangement

Think of as many uses of bricks as you can

Divergent

“All professor are caring people” & “All caring people are good”.

Would you accept the statement all professors are good?

Deduction

Approaches to the study of problem solving

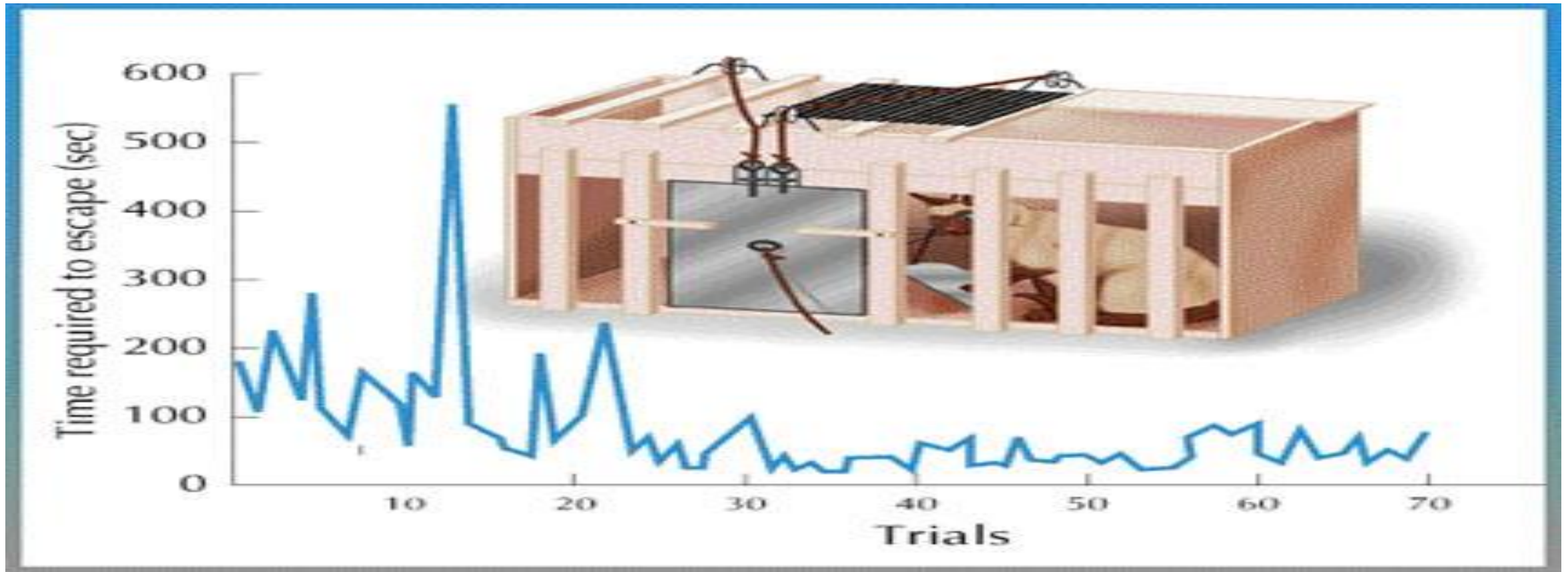
Behaviorism: Problem solving as associative learning

E.L. Thorndike (1800) conducted the first systematic study of problem solving using cats.

Thorndike was interested in knowing whether the ability to solve this confinement problem would appear suddenly as an insight, or gradually through a process of trial and learning.

He found that the cat learned to solve **using trial and error**

Thorndike described this learning process with what he termed as **law of effect - which states if a response leads to a satisfying outcome, the connection between the response and the situation will be strengthened.**



Gestalt Psychology: Problem solving as insight

Gestalt psychology believed that the mind has an inherent tendency to organize incoming information and these organizational processes are the defining features of cognition. Thus rather than defining problem solving as mindless playing out of associations that gradually build up over time, they believe it involves restructuring or reorganizing of problem elements that result in a sudden realization of the solution.

A pioneer study was done by Wolfgang Kohler (1925) with apes to support gestalt approach to problem solving.

For gestalt psychologists problem solving involves a process of restructuring whereby problem elements are suddenly reorganized and seen in a new way. The sudden and successful restructuring of problem elements is termed, *insight* and this is a major focus of the gestalt approach



Cognitive Psychology: Problem solving as information processing

Just as a computer solves problems by executing programs that use information stored in some types of database, humans solve problems by applying mental processes to representations in memory.

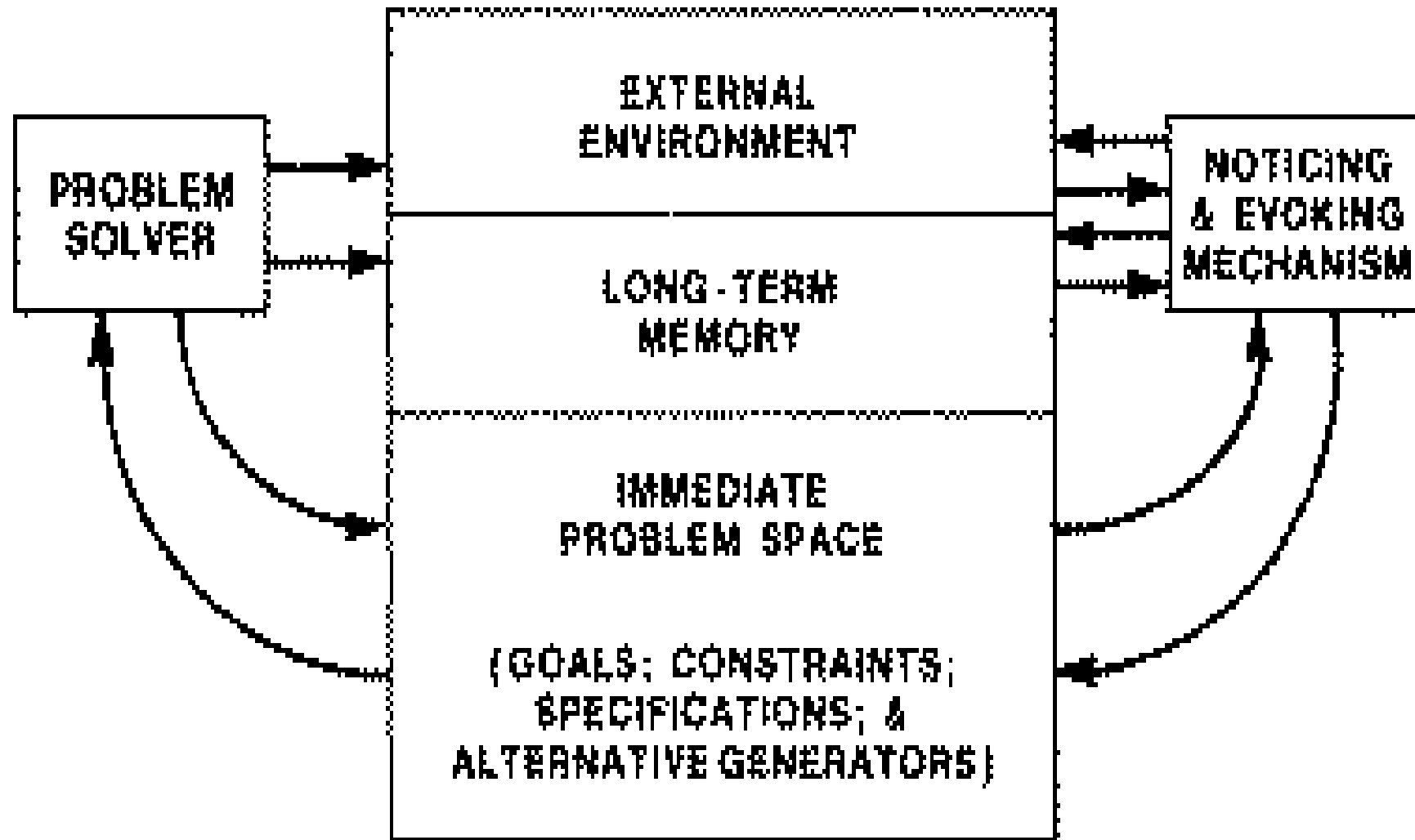
The General Problem Solver:

Newman & Simon (1972) originated the conceptualization of problem solving as a step-by-step progression from an initial state to a goal state. They did so within the framework of a computer program termed as *general problem solver (GPS)*.

The GPS is a general model of human problem solving – one that can be applied to any problem. This approach minimizes the “distance” between an initial state and a goal state by breaking the problem down into a series *sub-goals*

The sub-goal analysis is accomplished through the application of *operators*, which is basically a fancy word for problem-solving techniques. These techniques are applied (at a micro level) to reduce the difference between the current state and the current sub-goal state and (at a macro level) to reduce the difference between the initial state and the final goal state.

GPS is a notion of *problem space* – which basically refers to the problem solvers mental representation of the initial state, the goal state and all possible intermediate (sub-goal) states, and the operators that can be applied to reach these sub-goals



Problem Representation

Problem solving involves the process of converting presented information into some type of internal mental representation. Within the framework of GPS *problem representation*, involves correctly specifying the problem space – i.e. correctly identifying the initial state as well as the operators that may be applied within the constraints of the problem.

Try to solve this problem

Exactly at sunrise one morning, a Buddhist monk set out to climb a tall mountain. The narrow path was not more than a foot or two wide, and it wound around the mountain to a beautiful, glittering temple at the mountain peak. The monk climbed the path at varying rates of speed. He stopped many times along the way to rest and to eat the fruit he carried with him. He reached the temple just before sunset. At the temple, he fasted and meditated for several days. Then he began his journey back along the same path, starting at sunrise and walking, as before, at variable speeds with many stops along the way. However, his average speed going down the hill was greater than his average climbing speed.

Prove that there must be a spot along the path that the monk will pass on both trips at exactly the same time of day (Duncker, 1945)



Try to solve this problem

A man bought a white horse for \$60 and then sold it for \$70. Then he bought it back for \$80 and sold it for \$90. What was his net gain (or net loss) in the horse business? (Maier and Burke, 1967)

Rigidity in Representation

The initial representation of a problem is critical to its eventual solution. Failure in representation might result from a number of factors

- a) the problem elements may not have received sufficient attention
- b) the problem elements may not have been understood
- c) previous experience with similar problems may have led to encoding the problem elements in a rigid manner

Some of the major hindrances to problem solving arise from

a) Mental set: this tendency to rely on habits and procedures used in the past is termed *mental set*. Mental set can interfere with your ability to solve everyday problems (e.g.,)

Solve the problem: there are six eggs in a basket, six people take one of the eggs each. How is it that one egg can still be left in the basket?

Mental set tends to affect the representation phase of problem solving, as past experiences lead to an inappropriate representation of the problem.

b) *Functional fixedness*: refers to people's tendency to view objects in a narrow, fixed sense – i.e. in terms of the typical functions of the object. One development perspective on functional fixedness holds that older children are more likely than younger ones to demonstrate functional fixedness (e.g., writing in space with pen)

Try to solve the problem

two-string problem: Knut is left in a room with a chair and a pair of pliers given the task to bind two strings together that are hanging from the ceiling. The problem he faces is that he can never reach both strings at a time because they are just too far away from each other. What can Knut do?

Knut has to recognize he can use the pliers in a novel function – as weight for a pendulum. He can bind them to one of the :strings, push it away, hold the other string and just wait for the first one moving towards him. If necessary, Knut can even climb :on the chair, but he is not that small, we suppose.....



Stereotypes as a threat to problem representation

stereotype threat occurs when a member of a negatively stereotyped group feels that the stereotype might be used to judge their behavior thus resulting in a negative judgment that will propagate the problem. Quinn & Spencer (2001) has men and women solve either math word problem or numeric/algebraic equivalents of the word problem

Solve the problem

A sporting goods store sold 64 Frisbees in one week, some for \$3 and rest for \$4 each. If recipients from Frisbee sales for the week totaled \$204, what is the fewest number of \$4 Frisbees that could have been sold?

Or

Solve: $3(64-x) + 4(x) = 204$

Results form Quinin and Spencer

<i>Correct Sol %</i>	<i>word problem</i>	<i>Numeric</i>
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<i>Gender</i>		
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<i>male</i>	<i>20</i>	<i>40</i>
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<i>female</i>	<i>8</i>	<i>38</i>
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Problem Solution

once the problem has been successfully transformed from externally presented information into an internal representation, the next phase of the problem-solving process involves searching for; testing and evaluating solutions. Within the context of Newell & Simon's (1972) IP approach, problem solution amounts to travelling through the problem space.

Algorithms – are basically a set of rules that can be applied systematically to solve certain types of problems. A mathematical formula is a good example of algorithm.

e.g., two short sides of a right angled triangle are 3cm and 4 cm. find the length of the hypotenuse

Algorithms are very powerful problem solving techniques; applied correctly, an algorithm will always lead to the correct solution, if one exists.

e.g., solve the anagram “kigvin” using algorithm

there are several shortcomings of using algorithm

a) the exhaustive nature make them overly tedious and quite impractical

b) there simply aren't any for most of the problems in our daily life

Heuristics – are general strategies or rules of thumb, than can be applied to various problems. Heuristics serve as “shortcuts” through problem space. Given the strengths and limitations of human problem solver, along with the fact that most problems are ill-defined and have relatively large problem spaces, heuristic problem solving is much more effective.

Solving the anagram “kigvin”

Heuristics say no word in english start with gk, vg or ikn so these are eliminated and so on.....

Solve

You are purchasing three items at the store, at these prices:

\$19.95,

\$39.98,

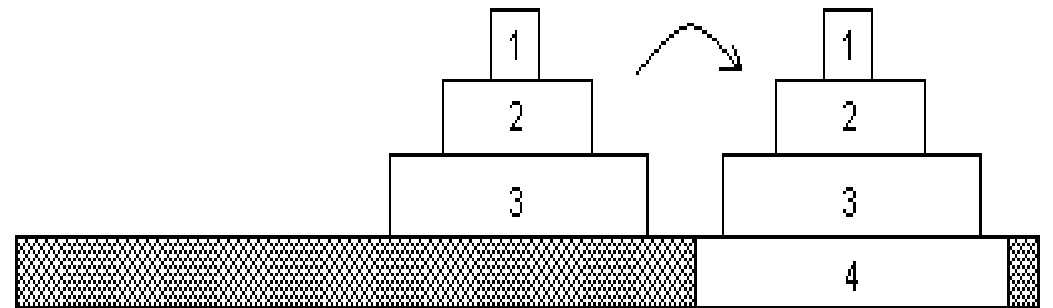
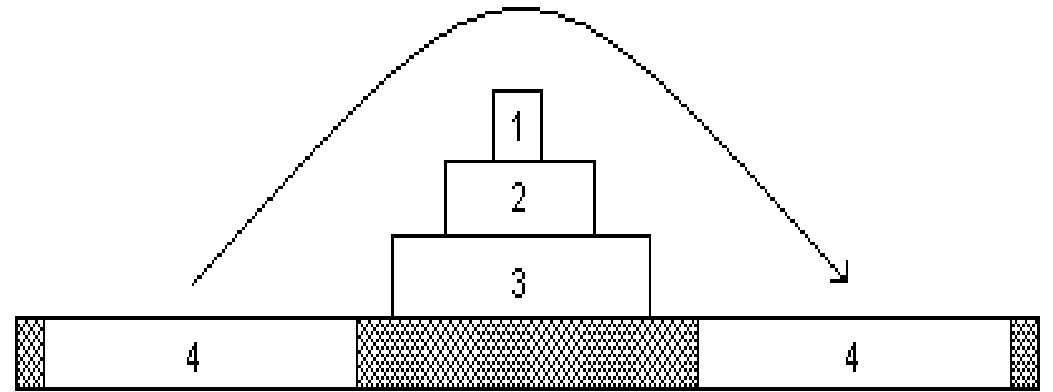
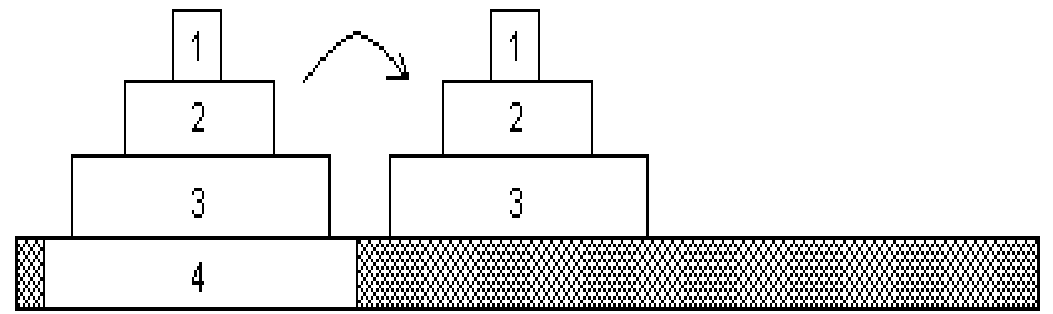
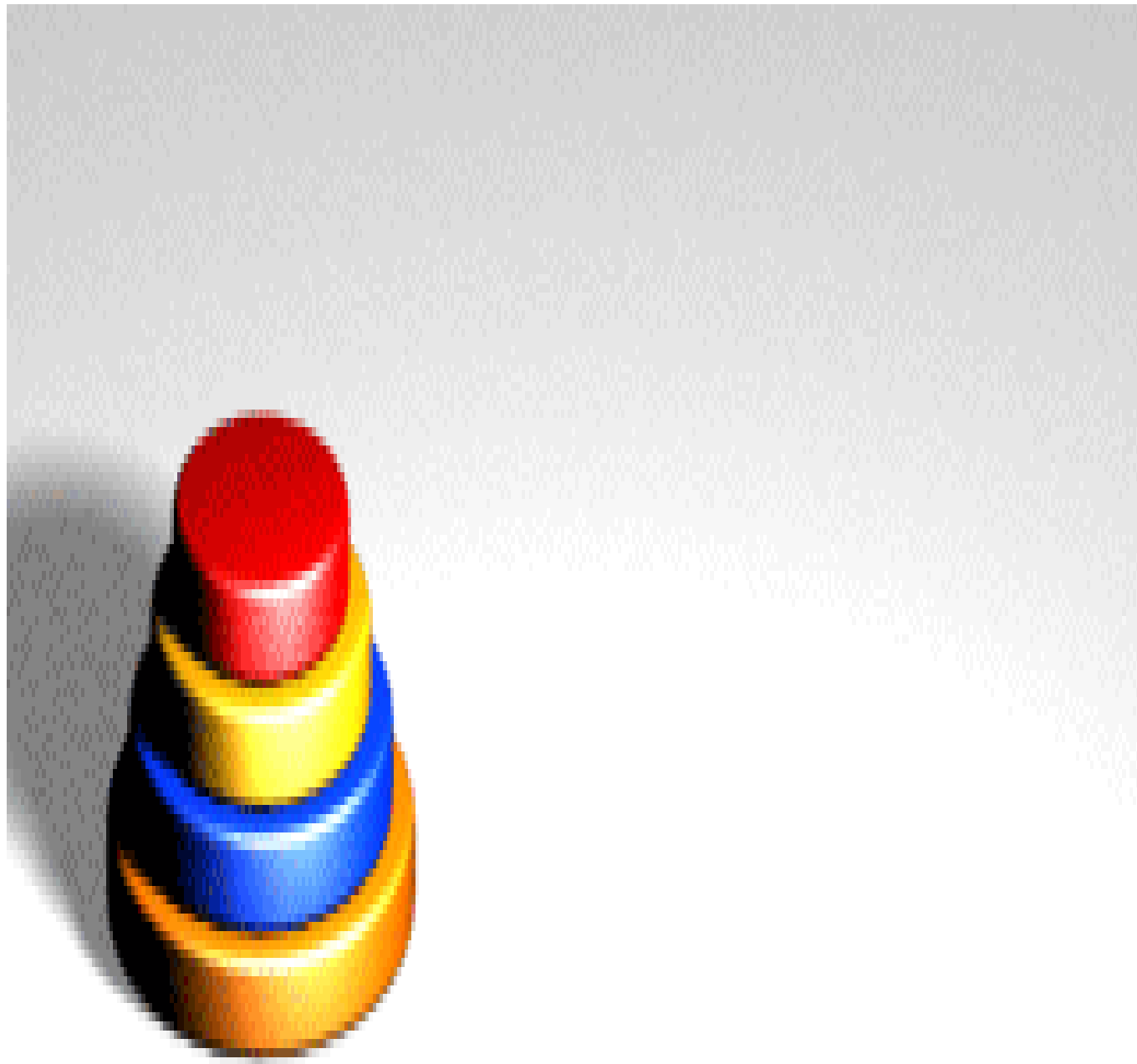
\$29.97

About how much money are you spending?

Solution: The fastest way to solve this problem is to round off and approximate. The first item costs about \$20, the second about \$40, and the third about \$30; therefore, you are spending about \$90 on your shopping spree.

Means-End-Analysis

The general problem solver developed by Newman & Simon utilizes the heuristic known as *mean-end-analysis*. MEA involves breaking a problem into smaller sub-goals in which accomplishing each sub-goal moves the solver closer to the final goal – the problem solution. As the term implies in MEA the solver systematically attempts to devise means to get to the each sub-goal's ends. MEA can be an effective way to solve transformational problems



Surface vs. Structural Features

One of the primary steps in solving problems by analogies involve the use mapping between original and new problem. Gick & Holyoak term this mapping process as *schema induction*. Our ability to notice, map and develop schemata depends on the particular *type of similarity*. These can of two types (Gentner, 1989).

a) *surface features* – are the specific elements of the problem. If two problems share surface similarities, this means that the parts of the problem look pretty similar. (e.g.,)

the way paper jams trays on printers

b) *structural feature* – are the underlying relationships among the surface features of the two problems. If two problems are structurally similar, they may look quite different on the surface but have underlying similarities in terms of their relationships. (e.g.,)

the dog chased the cat

the cat was chased by the dog

How do experts solve problems?

Experts have something called *expertise* – *exceptional knowledge and/or performance in some specific problem domain*. Since 1950 expertise is no more viewed as the product of innate capacities but rather an outgrowth of learning and repetition over the course of years that produces an extensive body of knowledge and an extremely well learned set of skills. Experts are thus *skilled memorizers*

According to Ericsson & Polson's 1988 *skilled memory theory* – there are several differences between experts and novices, all to the advantage of experts.

- a) the semantic network are richly elaborated in experts

b) experts have quicker and more direct access to long term memory

c) information is more easily encoded onto long term memory by experts and the speed of this encoding improves with practice

Advantages for expert problem solvers

The core of problem solving is memory – the long term memory that allows for the storage of domain related general knowledge and specific episodes and the working memory that allows for quick and efficient online processing of problem information.

- a) Studies by Chase & Simon (1973), de Groot (1978) point that experts can instantly recognize problem configurations based on extensive knowledge & experience base
- b) Ericsson & Kintsch (1995) propose the idea of long term working memory to explain expert advantage in online processing of problems. This theory states that experts can bypass the limits of working memory by using the information of working memory to directly access LTM.
- c) Experts problem solvers use general strategies that differ from those of novice. They tend to search the problem space in a forward fashion, reasoning from givens towards goals.

d) Experts are much better at picking up on structural features of problems, and recognize analogous problems when faced with novel problem

e) Lemaire & Siegler (1995) propose the *adaptive strategy model* which distinguish experts from novice on four layers

1) *strategy existence* – experts have more strategies at disposal

2) *strategy base rate* – experts in a given domain know which strategies tend to work and select those strategies

3) *strategy choice* – refers to experts advantage in discerning which strategies should be chosen for a specific circumstance

4) *strategy execution* – refers to the expert advantage in actually carrying out the strategy in terms of speed accuracy

Expert disadvantages

- a) Novices are actually better at understanding randomly arranged problems than experts.
- b) Novice remember more information about specific problems than experts and this is termed as *intermediate effect*
- c) Experts may actually solve problems using mental set as compared to novice users

Insight and Creativity

What is creativity? What process lead to creative products?

Wallas (1926) proposed that the process leading up to a creative breakthrough can be described in terms of four stages

- 1) Preparation – in which the solver gathers information and makes initial attempts at problem solution*
- 2) Incubation – which might described as a period of productivity inactivity*
- 3) Illumination – a stage where the solver arrives at critical insight*
- 4) Verification – in which the solver verifies if the solution will actually work*

Wallas theory has not be accepted in entirety however it has pointed out two major question in problem solving

1) what is the nature of insight?

2) if sudden breakthrough in problem solving is a reality, can these be encouraged by a period of incubation

Insight

Insight involves the sudden realization of a problem solution (or of any key idea necessary to the solution). The problem with insight is that theorist believe problem solving to be an incremental process and not sudden realization, another problem lies with the clear definition and non experimental proof of the existence of insight by the Gestalt psychologists

Gilhooly & Murphy, 2005 made an observation that even if insight was a reality it could not be applied to all types of problems. They distinguished between problems as

- 1) Non-insightful problems – are those that are likely solved through incremental process. They require analytical step-by-step processing.*
- 2) Insightful problems – are those in which solutions appear suddenly*

Two key assumptions about insightful problems solving are

- 1) it involves a mistaken assumption that once removed will clear the way to successful solution of the problem*
- 2) that the solver is hit with the solution suddenly and has what might be termed as an “Aha!” experience*

Insightful problems

Water lilies double in area every 24 hours. At the beginning of summer there is one water lily on the lake. It takes 60 days for the lake to become completely covered with water lilies. On which day is the lake half covered?

A prisoner was attempting to escape from a tower. He found in his cell a rope which was half long enough to permit him to reach the ground safely. He divided the rope in half, and tied the two parts together and escaped? How could he have done this?

Non Insightful problems

three people play a game in which one person loses and two people win each round. The one who loses must double the amount of money that each player has at that time. The three player agree to play three games. At the end of the three games, each player has lost one game, and each person has \$8. what was the original stake of each player?

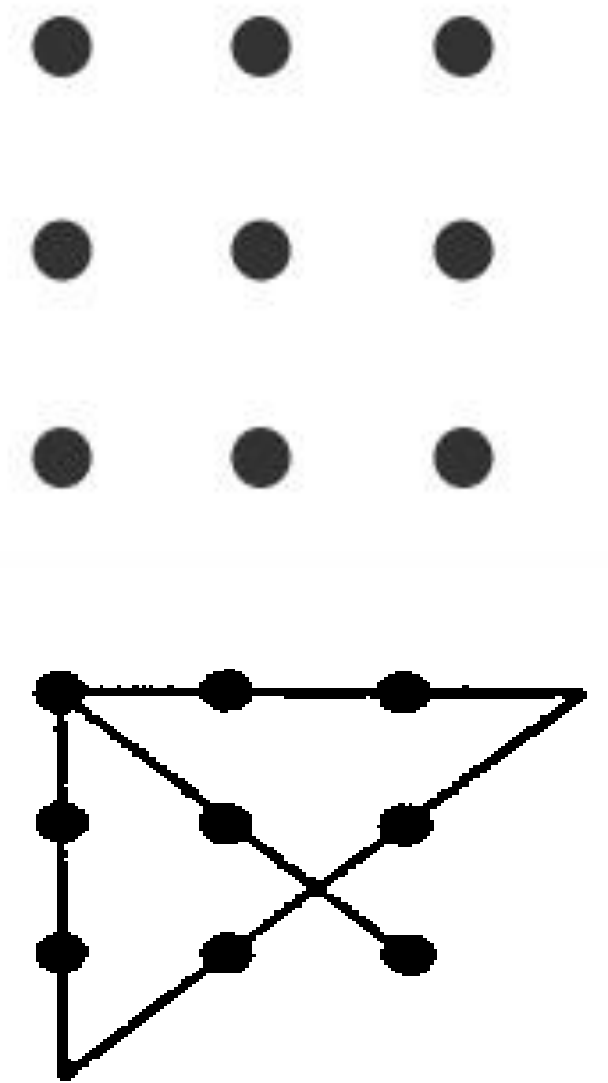
Next week I am going to lunch with my friend, visit the new gallery, go to the social security office and have my teeth checked at the dentist. My friend cannot meet me on Wednesday, the social security office is closed on weekends, the dentist has office hours on tue, fri and sat and the gallery is closed on tue, thurs and weekends. What day can I do everything I planned?

Removal of a mistaken assumption?

The difficulty of insightful problems is that they involve one key –but errant-perception or assumption that if removed, would lead to an easy solution. One of the most studied problem in this regard is the nine-dot problem.

Nine dot problem - connect all the dots with one continuous line.

Due to the mental set of staying within the boundary the problem appears unsolvable



According to Kershaw and Ohlsson (2004) three sources of difficulty are evident in the nine-dot problem

1) perceptual factors – relate to the Gestalt organizational principles

2) process factors – relate to the information processing demand of the problem

3) knowledge factors – involves prior knowledge that a solver brings to bear on the problem

Any/all of these factors might come into play and prevent people from solving problems

The “Aha!” Experience

another defining feature of the insight experience is a sudden and tangible feeling of discovery, usually described as an “Aah!” experience. The issue of what someone is thinking as they think related to metacognition. Metacognition refers to a person’s knowledge of their own thought process. Using metacognition and warmth ratings while solving insightful and non-insightful problems, Metcalfe & Weibe (1987) reported that, accuracy was higher for the later than for the former.

Intuition as Insight

The issue of whether insight problem solving involves special, unconscious processes-like sudden restructuring of problem elements or removal of some mistaken information-still stirs controversy in the field

Does incubation lead to insight?

One of the most controversial notions within the study of problem solving is incubation, or the idea that taking a break in problem solving leads to a quicker solution than does continuing effort. The idea is that break allows for the elements of the problems to be reorganized or for unconscious processes to continue to work on the problem and that this unconscious is limited by conscious work on the problem.

Can you come up with a saying represented by each?

YOU JUST ME

r|e|a|d|i|n|g|

0

M.Sc PhD M.D

Fly night

Smith (1995) suggests that incubation effects do occur, but only under specific circumstances – namely, when a problem is doable and when the solver is blocked in some way from the solution. Smith offers a contextual view of incubation, which is basically the encoding specificity principle in reverse

the *contextual view* of incubation states that when problem solving is stymied, a solution will come more easily if there is a contextual change from the previous situation

Creativity

creative individuals are able to think “outside the box” – to come up with new ideas, view old problems from a fresh perspective, and connect seemingly disparate problem situations.

What is creativity?

experts on creativity generally agree that creative solutions have two components – novelty and appropriateness (Lubart, 1994 & Sternberg, 2004). Creative solutions are novel, different from previous solutions and usually unexpected. The solution of creative problems must also satisfy the constraints of the problem at hand; it must fulfill a need and be sensible and useful.

one of the most – cited framework for describing and investigating creativity was originally proposed by Rhodes, who suggested that creativity can be informed by a focus on several dimensions, which he labeled *person, process, press and product*.

1) person – creativity to some extent is related to aspects of person / personality. Creative persons are thought to exhibit a number of personality like – broad interests, appreciation of complexity, tolerance of ambiguity, self-confidence, independence and sensible risk taking

Simonton believes that creativity is not always the product of a particular comfortable environment. In fact creative depends on diverse set of life experiences

2) *Process – creativity also refers to specific set of processes. Two contradictory ideas about cognitive processing in creativity has been proffered –*

one view asserts that creativity involves special processes and abilities like the ability to quickly restructure problem information and to connect seemingly remote possibilities.

another view contends that creative thinking is the product of the garden variety cognitive processing as attention and memory.

The creative cognitive approach (smith 2003) argues that the answer is probably that creative thinking can be the result of either type of processes or both.

Sternberg & Davidson (1995) cite three processes as important in reaching creative insights.

1) selective encoding – involves distinguishing between relevant and irrelevant information in the domain of expertise. Creative individuals are better at distinguishing useful information from red herrings

2) selective combination – involves going beyond discovering and encoding the information to the combination of the information in new and productive way

3) selective comparison – involves relating new information to old information in novel ways.

3) press – refers to the notion that creative behavior does not occur in a vacuum, that it's subjects to various external pressures and contextual factors. Creative acts are also products of interpersonal, disciplinary and socio-cultural environments. (brainstorming)

4) products – which refers to the outcome yielded by the creative process be it a painting, poem, design or new technology. The analysis of creativity from this perspective is a challenge as its evaluation requires some type of objective standard

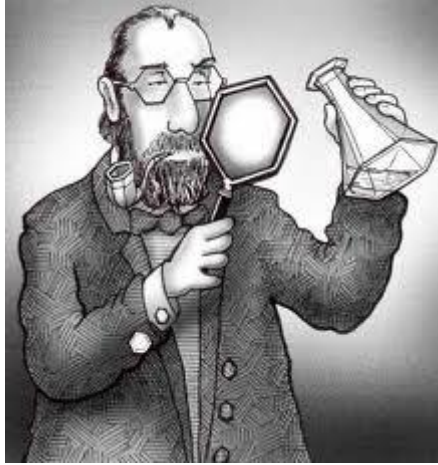
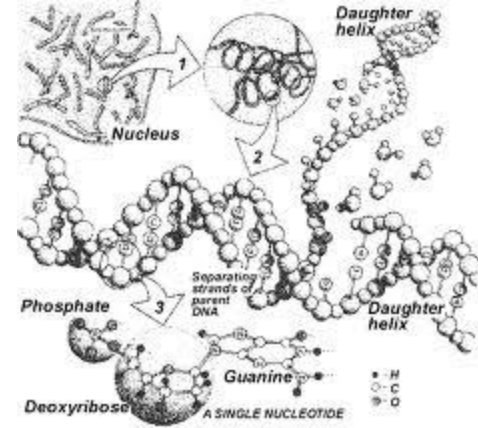
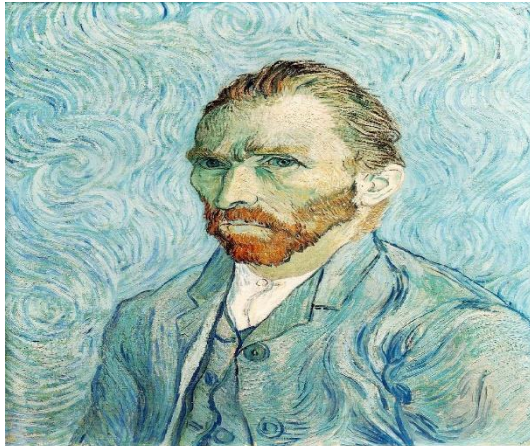

A taxonomy of creative processes and products

Dietrich (2004) proposes a useful scheme that imposes some order on the disparate research into creativity. Dietrich's characterizes creative insights as a product of two distinct dimensions: processing mode and knowledge domain

1) processing mode – refers to whether creative insight emerges as result of a deliberate and effortful search, or whether it emerges spontaneously as an unexpected flash of insight.

2) knowledge domain – refers to the nature of the creative insight, and whether it is characterized more by a cognitive or by a emotional breakthrough

Processing Mode

	Spontaneous	Deliberate
Cognitive	<p>Kekule's Dream (Benzene ring)</p>  A black and white illustration of a man with a beard and glasses, wearing a suit. He is holding a hexagonal benzene ring in his right hand and a flask in his left hand.	<p>Mapping of human Genome</p>  A diagram illustrating the process of DNA replication. It shows a double helix being unwound, with labels for 'Nucleus', 'Phosphate', 'Deoxyribose', 'Guanine', 'A SINGLE NUCLEOTIDE', 'Separating strands of parent DNA', and 'Daughter helix'. The diagram is numbered 1, 2, and 3, showing the progression of the replication fork.
Emotional	<p>Van Gogh's self portrait</p>  A reproduction of Van Gogh's self-portrait, showing him with a beard and a blue jacket, looking directly at the viewer. The background is a swirling, textured blue.	<p>Insight during psychotherapy</p>  A cartoon illustration of a psychotherapy session. A therapist in a blue suit is sitting on a chair, holding a notepad. A client is lying on a purple couch, looking up at the therapist. The background is a bright orange circle.

Try to solve the following problems

- 1) How could a baby fall out of a twenty-story building onto the ground and live?
- 2) A man and his son are in a car crash. The father is killed and the child is taken to hospital gravely injured. When he gets there, the surgeon says, 'I can't operate on this boy - for he is my son!!!' How can this possibly be?
- 3) Three cannibals and three anthropologists have to cross a river. The boat they have is only big enough for two people. The cannibals will do as requested, even if they are on the other side of the river, with one exception. If at any point in time there are more cannibals on one side of the river than anthropologists, the cannibals will eat them. What plan can the anthropologists use for crossing the river so they don't get eaten? *Note: One anthropologist can not control two cannibals on land, nor can one anthropologist on land control two cannibals on the boat if they are all on the same side of the river. This means an anthropologist will not survive being rowed across the river by a cannibal if there is one cannibal on the other side.*
- 4) *A police officer saw a truck driver clearly going the wrong way down a one-way street, but did not try to stop him. Why not?*