25

CHAPTER

Shear Force and Bending Moment

25.1 INTRODUCTION

A beam is a structural member whose longitudinal dimension is large compared to its transverse dimension. It is supported along its length and is acted upon by system of loads transverse (at right angles) to its longitudinal axis. It may also be acted upon by some couples.

The effect of loading results in developing shearing force and bending moment at any section of the beam. For designing a beam, information about shear force and bending moment is required. The shearing force and bending moment developed depends upon the combinations of loading and the support conditions of the beam. In this chapter the variation of shear force and bending moment developed along the length of the beam due to different system of loading shall be discussed.

25.2 TYPES OF BEAMS AND LOADING

Types of Beams

1. Cantilever beam
2. Simply supported beam
3. Fixed beam
4. Over-hanging beam
5. Continuous beam

Cantilever Beam. It is a beam which is fixed at one end (*A*) and free at the other end (*B*). There is no deflection or rotation at fixed end. (Fig. 25.1)

Simply Supported Beam. A beam supported freely on supports which may be a knife edge or a roller. There is no deflection or displacement of the beam at the ends. (Fig. 25.2)

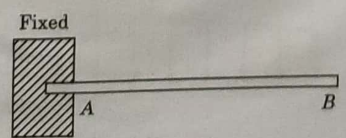

Fig. 25.1

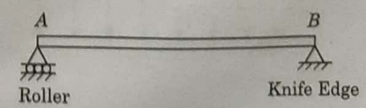

Fig. 25.2

Fixed Beam. A beam whose both ends are fixed. (Fig. 25.3)

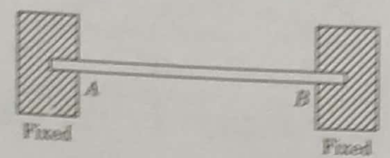

Fig. 25.3

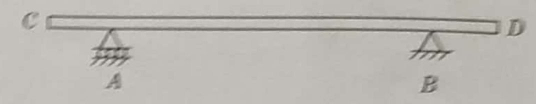

Fig. 25.4

Over Hanging Beam. A beam with one or both ends extended beyond the supports. (Fig. 25.4)

Continuous Beam. A beam with more than two supports is continuous beam. Such a beam may or may not have an overhang (Fig. 24.5).

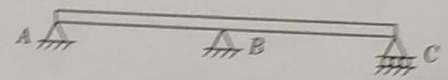

Fig. 25.5

Types of Loads

1. Concentrated load
2. Uniformly distributed load
3. Uniformly varying load
4. A combination of the above loadings

Concentrated Load. A concentrated load is one which is assumed to act at a point. (Fig. 25.6).

Uniformly Distributed Load. A uniformly distributed load is uniformly or evenly distributed over a part or over the entire length of the beam. The rate of loading is expressed in N/m (Fig. 25.7). For solving numerical problems the total uniformly distributed load is converted into equivalent point load acting at the C.G. of uniformly distributed load.

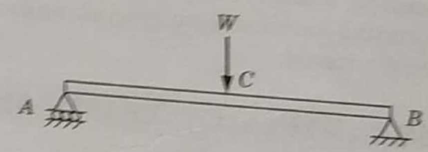

Fig. 25.6

Fig. 25.7

Uniformly Varying Load. A load whose intensity of loading varies linearly or at constant rate along the length. In triangular load (Fig. 25.8) for example, it increases from zero at one end to some value at other end at a constant rate. A trapezoidal loading (Fig. 25.9) is a combination of uniform and triangular loading.

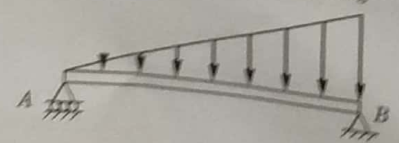

Fig. 25.8

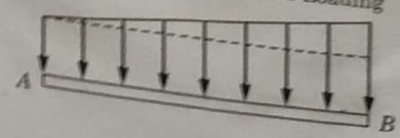

Fig. 25.9

25.3 CONCEPT OF SHEAR FORCE AND BENDING MOMENT

Consider a beam *AB* (Fig. 25.10(*a*)) simply supported and carrying a point load $W = 1000$ N at point *C*.

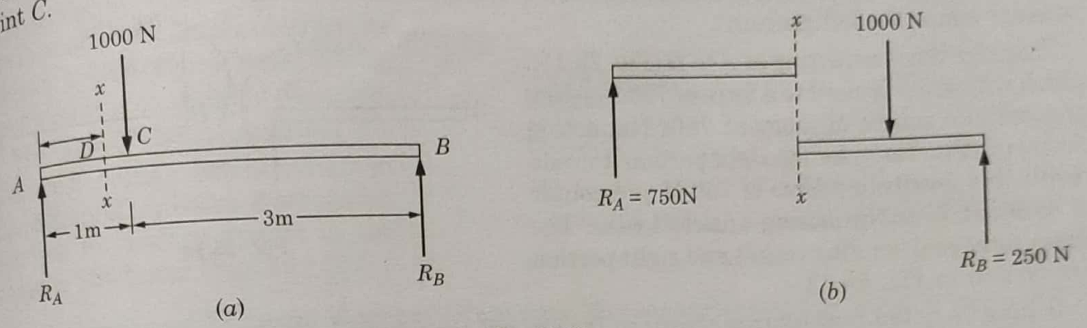

Fig. 25.10

Reactions R_A and R_B at the supports can be determined as,

$\Sigma M_A = 1000 \times 1 - R_B \times 4$

$R_B = 250$ N

$\Sigma F_y = 0 \qquad R_A + R_B - 1000 = 0$

$$R_A = 750 \text{ N}$$

Assume a cutting plane *x–x*, a distance *x* from *A* dividing the beam into two segments (Fig. 25.10(*b*)). On the left portion an unbalanced vertical force (resultant) equal to $R_A = 750$ N is acting upwards.

Also, on the right portion an equal unbalanced vertical force (resultant), $W - R_B = 750$ N is acting, but, in opposite direction to that acting on left portion i.e. downward.

This resultant force acting on the section to the left or right of the section is called *shear force*.

Now taking moment of all the external forces about *x–x* on the left and the right portions.

$$M_{\text{left}} = R_A \times x = 750x \text{ Nm (clockwise)}$$

$$M_{\text{right}} = -250\ (4 - x) + 1000\ (1 - x)$$

$$= -1000 + 250x + 1000 - 1000x$$

$$M_{\text{right}} = -750x \text{ Nm (anticlockwise)}$$

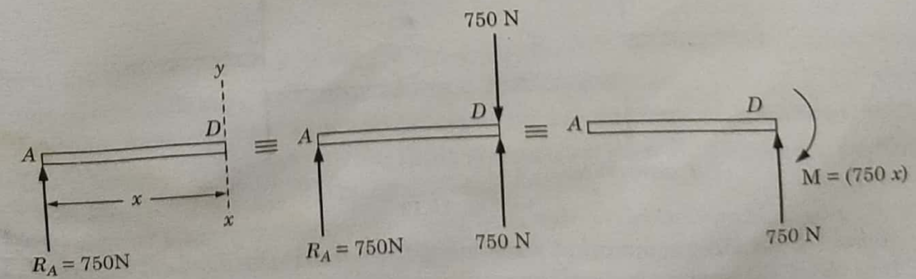

Fig. 25.11

The moment of the *resultant force* about the section acting either to the left or right is called the *bending moment*. Again it may noted that bending moment acting on the left portion and right portion are equal in magnitude but opposite in sign.

For a better understanding of the above concepts consider again the left portion.

Transfer the force acting at A to D (Fig. 25.11), which will be equivalent to a force of 750 N acting upward and couple of moment $750x$ Nm acting clockwise. Similarly, for the right portion it would result in a downward force of 750 N and couple of moment $750x$ Nm acting anticlockwise. The shear force and bending on left and right portion are shown in Fig. 25.12.

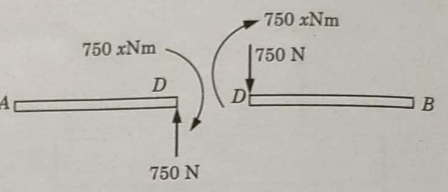

Fig. 25.12

It may be noted that figures shown so far are *not the free body diagrams* of beams or their sections. Infact, in free body diagrams *resisting* forces and *resisting* couples developed in the beam are to be included so as to satisfy the conditions of equilibrium.

25.4 DEFINITION OF SHEAR FORCE AND BENDING MOMENT AND SIGN CONVENTION

Shear Force. Shear force at a section of the beam is the force that is trying to shear-off the section of the beam.

Shear force is obtained by the algebraic summation of all the external forces (loads and reactions) acting normal to the axis of the beam, acting either on left side or right side of the section.

Sign Convention. When the left portion of the section is considered, if the resultant of vertical external forces is acting *upwards*, the shear force on the section is considered *positive*. If the resultant is acting downwards then is considered negative.

When right portion of the section is considered, if the resultant of vertical external forces is acting *downwards* it is considered *positive*. If acting upwards it is considered negative.

For positive shear force the above rule of *sign* produces the effect that tends to move the left hand portion of the beam upwards with respect to right portion. When the shear force is negative, left portion tends to move downward with respect to right portion (Fig. 25.13).

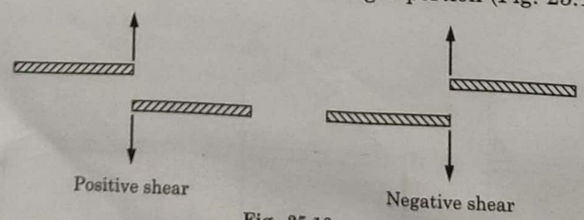

Fig. 25.13

Bending Moment. Bending moment at a section of the beam is the moment that tends to bend the beam. It is obtained by the algebraic summation of moments of all the external force (loads and reactions) about the section acting either to the left or right portion of the section.

Sign Convention. If the left portion of the section is considered the bending moment is considered positive when the sum of all moments of external forces (loads and reactions) is clockwise. When acting anticlockwise it is considered negative.

If the right portion of the section is considered the bending moment is considered positive when the sum of all moments of external forces is anticlockwise. When acting clockwise it is considered negative.

The above *sign* convention for positive bending moment results in the sagging of the beam. For negative bending moment it results in the hogging the beam (Fig. 25.14).

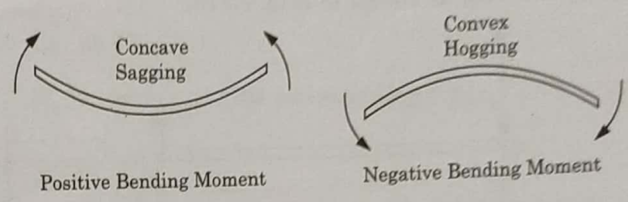

Fig. 25.14

Shear Force and Bending Moment Diagrams. The shear force and bending moment acting on the beam generally vary along the length of the beam. Their variations are normally represented as shear force and bending moment diagrams. The abscissa or x-axis indicates the position of the section.

The ordinate or y-axis indicates the values of shear force or bending moment (positive or negative).

Notes:

1. The curve for bending moment in a portion of the beam is one degree higher than the curve for shear force.
2. The portion in which S.F. is constant. B.M. curve is a straight line.
3. The portion in which S.F. is varying linearly B.M. curve is parabolic.
4. Rate of change of shear force is equal to the rate of loading.

$$\frac{d\,(\text{Shear Force})}{dx} = w$$

5. Shear force is basically the rate of change of bending moment with respect to x.

$$\text{S.F.} = \frac{d\,(\text{B.M.})}{dx}$$

So, at maximum or minimum value of B.M. i.e. when,

$$\frac{d\,(\text{B.M.})}{dx} = 0, \quad \text{S.F.} = 0$$

So, at the point where S.F. is zero in the diagram, B.M. is either maximum or minimum.

6. Point of contraflexure is a point where B.M. changes sign.
7. The shear force changes suddenly at a section where there is a vertical point load.
8. When a beam is subjected to a couple at the section, then B.M. changes suddenly at the section but S.F. remains unchanged at the section.
9. If an inclined load is acting on the beam, it is resolved into two components, horizontal and vertical. Vertical component will contribute to the S.F. and B.M.. Whereas, horizontal component is resisted by hinge as reaction and results is an axial force in the beam.

25.5 SHEAR FORCE AND BENDING MOMENT DIAGRAMS FOR STANDARD CASES

Example 25.1. Draw the shear force and bending moment diagrams for a cantilever beam acted by a point load as shown in (Fig. 25.15).

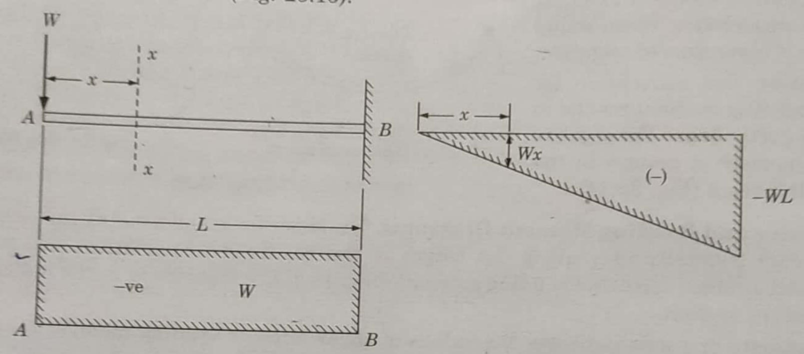

Fig. 25.15

Solution: At the fixed end B, reaction will consist of a force and a couple but it need not be determined.

Consider a section x–x at a distance x from the end A. Consider the left portion, shear force at the section x–x, F_x

$$F_x = -W$$

resultant force W is acting downwards and hence taken as negative.

The shear force is constant at all section as it is not varying with x. So S.F. diagrams is as shown in Fig. 25.15.

Bending moment at section $x - x$,

$$M_x = (-)\, W \cdot x$$

Anticlockwise B.M. on left portion is taken as –ve as it is producing hogging.

Bending moment is varying linearly with x, at $x = L$,

$$M_{x=L} = -WL$$

and B.M. diagram is as shown in Fig. 25.15.

Example 25.2. Draw the shear force and bending moment diagrams for a cantilever beam loaded as shown in Fig. 25.16.

Solution: Intensity of loading is w per unit length (w N/m). Consider a section x–x at a distance x from the end A. Consider the left portion.

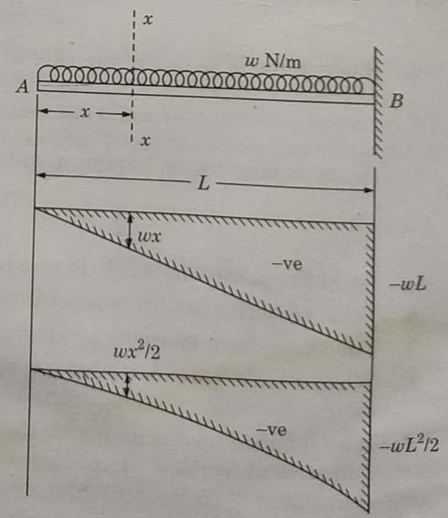

Fig. 25.16

Uniformly distributed load on length x is equivalent to a point load of magnitude wx acting at the C.G. of the section i.e. at a distance of $x/2$ from A.

Shear force $F_x = -wx$ (Acting downward, hence negative)

At A, $x = 0$ $\quad F_A = 0$

At B, $x = L$ $\quad F_B = -wL$

The variation of S.F. is shown as in Fig. 25.16.

Bending moment $\quad M_x = -wx\left(\frac{x}{2}\right)$ Anticlockwise Moment

$$M_x = -\frac{wx^2}{2}$$

At A, $x = 0$ $\quad M_A = 0$

At B, $x = L$ $\quad M_B = -\frac{wL^2}{2}$

B.M. diagram is quadratic or parabolic curve as shown in Fig. 25.16.

Example 25.3. A simply supported beam AB is loaded by a point load W at C. Draw the shear force and bending moment diagrams.

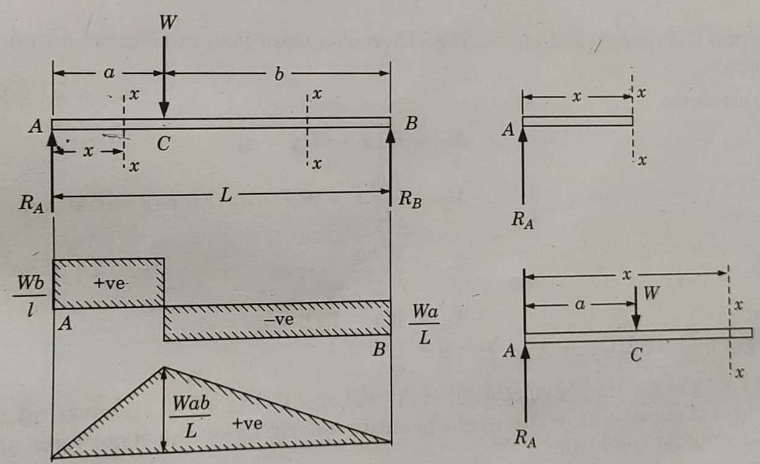

Fig. 25.17

Solution: Let the reactions at supports be R_A and R_B

$\Sigma F_y = 0$

$$-W + R_A + R_B = 0,$$
$$R_A + R_B = W$$

$\Sigma M_A = 0$

$$Wa - R_B L = 0$$
$$R_B = \frac{Wa}{L},$$
$$R_A = W - \frac{Wa}{L} = \frac{Wb}{L}$$

Scanned by CamScanner

Consider a section x–x at a distance x from A lying between AC($0 < x < a$).

Shear force $F_x = R_A = + \dfrac{Wb}{L}$ (Acting upward hence positive)

Shear force is constant between A and C.

Bending moment,

$$M_x = R_A x = \frac{Wb}{L} x \quad \text{(Positive clockwise)}$$

at $x = 0$, $\quad M_A = 0$

$x = a$, $\quad M_C = \dfrac{Wab}{L}$

Consider a section x–x lying between CB ($a < x < L$)

Shear force $F_x = R_A - W = - W + \dfrac{Wb}{L} = - \dfrac{Wa}{L}$ (downward is negative)

$$F_c = - \frac{Wa}{L} \quad \text{(Constant)}$$

S.F. diagram is as shown in Fig. 25.17. As there is a point load at C, there is sudden change in shear force.

Bending moment,

$$M_x = R_A x - W(x - a)$$

$$M_x = \frac{Wb}{L} x - W(x - a) \quad \text{(Linear variation)}$$

at $x = a$, $\quad M_A = \dfrac{W ba}{L}$ (+ve)

at $x = L$ $\quad M_B = Wb - W(L - a) = 0$

B.M. diagram is a shown in Fig. 25.17.

Example 25.4. A beam simply supported at ends A and B (Fig. 25.18) is carrying a uniformly distributed load of w per unit length over the entire length. Draw the shear force and bending moment diagrams for the beam.

Solution: Reactions at the supported can be determined easily as,

$$R_A = \frac{wL}{2}$$

and $\quad R_B = \dfrac{wL}{2}$

Consider a left section x–x at a distance x from A.

Shear force $F_x = R_A - wx = + \dfrac{wL}{2} - wx$ (Linear variation with x)

at $x = 0$ i.e. at A, $F_A = +\dfrac{wL}{2}$ (Upward, +ve)

at $x = L$ i.e. B, $F_B = \dfrac{wL}{2} - wL = -\dfrac{wL}{2}$ (downward, –ve)

at $x = \dfrac{L}{2}$ at the centre C, $F_C = \dfrac{wL}{2} - \dfrac{wL}{2} = 0$

So shear is upwards (+ve) till $x = L/2$ and then acts downwards (–ve) becoming zero at the centre C of the beam as shown in Fig. 25.18.

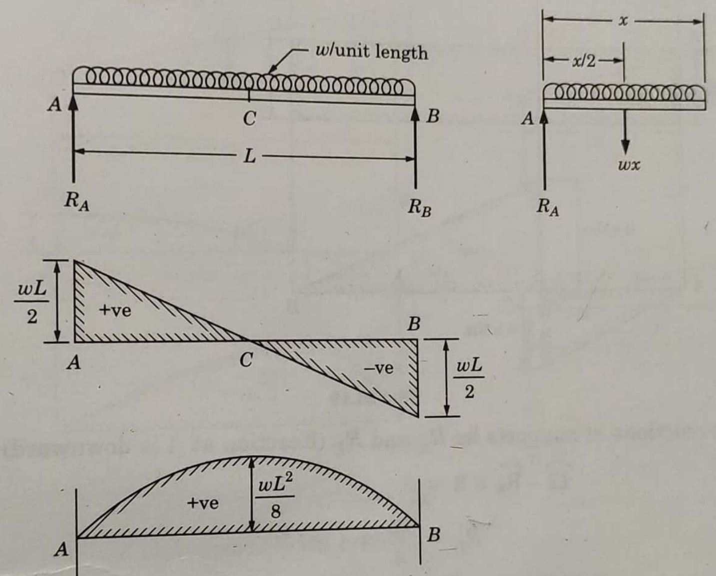

Fig. 25.18

Bending moment. At section $x–x$ bending moment is,

$$M_x = R_A x - wx\frac{x}{2}$$

(+ve) (–ve)

$$= \frac{wL}{2}x - \frac{wx^2}{2} \quad \text{(Parabolic variation)}$$

At, $x = 0$, i.e. at A $\quad M_A = 0$

$x = L$ i.e. at B $\quad M_B = 0$

$x = \dfrac{L}{2}$ i.e. at C $\quad M_C = \dfrac{wL}{2}\cdot\dfrac{L}{2} - \dfrac{w\left(\dfrac{L}{2}\right)^2}{2} = \dfrac{wL^2}{8}$

Bending moment varies parabolically and is +ve as it tends to sag the beam (concave upwards).

Example 25.5 A simply supported beam AB of length 3m is hinged at A and roller supported at B. It is subjected to clockwise couple of 12 kNm at a distance of 1 m from the left end A (Fig. 25.19). Draw of S.F. and B.M. diagram.

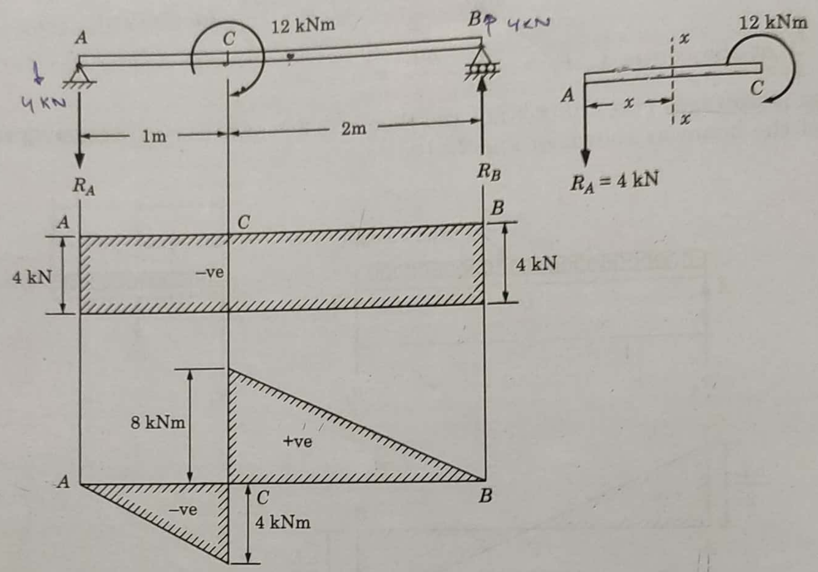

Fig. 25.19

Solution: Let reactions at supports be R_A and R_B (Reaction at A is downward)

$\Sigma M_A = 0$

$$12 - R_B \times 3 = 0$$

$$R_B = \frac{12}{3} = 4 \text{ kN} \uparrow$$

$\Sigma M_B = 0$

$$R_A \times 3 + 12 = 0$$

$$R_A = -\frac{12}{3} = -4 \text{ kN} \downarrow$$

S.F.

Shear force at A, $F_A = -4$ kN (downward)

It remains constant between A and B. S.F. diagram is shown in Fig. 25.19.

B.M.

Take section x–x at distance x from A

$$M_x = R_A \times x \text{ (Anticlockwise)}$$

B.M. at A, $\quad M_A = 0$

B.M. at a section just before point C,

$$M_C = -4 \times 1 = -4 \text{ kNm (Anticlockwise –ve)}$$

B.M. at a section after point C

at $x = 1$ $\quad M_x = -R_A \cdot x + 12$ kN

at point B, $x = 3$, $\quad M_C = -4 \times 1 + 12 = +8$ kN

$M_B = 0$

There is a sudden change in B.M. at C due to couple acting at C (Fig. 25.19).

Example 25.6. Draw the shear force and bending moment diagrams for a simply supported beam carrying a uniformly varying load from zero at one end to w per unit length at the other end (triangular load) as shown in Fig. 25.20.

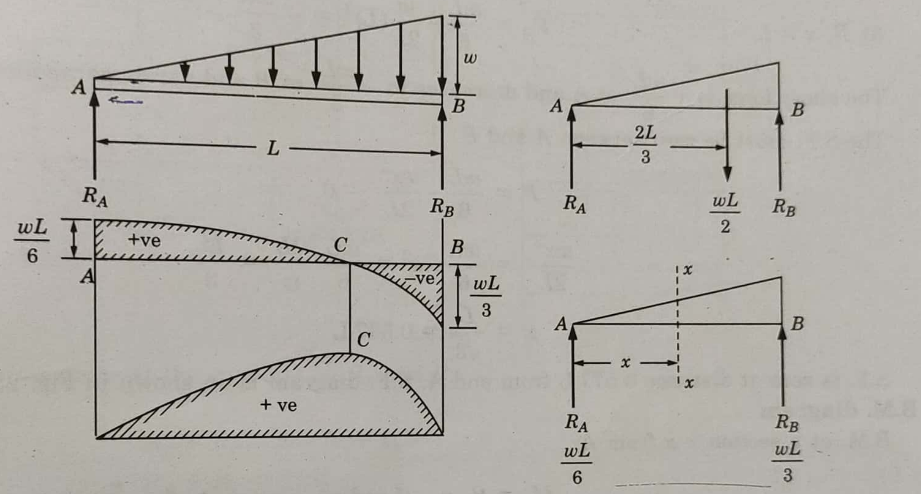

Fig. 25.20

Solution: Let reaction at supports be R_A and R_B

Total load on beam $= \frac{wL}{2}$ (area of load diagram), and is acting at the centroid of load diagram,

that is, at distance of $\frac{2}{3}L$ from A.

$\Sigma M_A = 0 \qquad -R_B L + \left(\frac{wL}{2}\right)\frac{2}{3}L = 0$

$$R_B = \frac{wL}{3}$$

$\Sigma F_y = 0 \qquad \frac{wL}{2} - R_A + R_B = 0$

$$R_A = \frac{wL}{6}$$

Load over length $Ax = \left(\frac{w}{L}x\right)\frac{x}{2} = \frac{wx^2}{2L}$

S.F. Consider any section at distance x from A

$$\text{Shear force } F = R_A - \frac{wx}{L} \cdot \frac{x}{2}$$

$$F = \frac{wL}{6} - \frac{wx^2}{2L}$$

It is seen to be varying parabolically (second degree in x).

At A, $x = 0$ $$F_A = \frac{wL}{6} - 0 = \frac{wL}{6}$$

At B, $x = L$ $$F_B = \frac{wL}{6} - \frac{w}{2L}(L)^2 = -\frac{wL}{3}$$

The shear force is $+\frac{wL}{6}$ at A and decreases to $-\frac{wL}{3}$ at B and varies parabolically.

The S.F. must be zero between A and B

$$F = \frac{wL}{6} - \frac{wx^2}{2L} = 0$$

$$\frac{wx^2}{2L} = \frac{wL}{6},\ x^2 = \frac{wL}{6} \cdot \frac{2L}{w} = \frac{L^2}{3}$$

$$x = \frac{L}{\sqrt{3}} = 0.577\,\text{L}$$

S.F. is zero at distance 0.577 L from end A. S.F. diagram is an shown in Fig. 25.20.

B.M. diagram

B.M. at a section x–x from A,

$$M_x = R_A x - (\text{Load on portion } A\text{–}x) \times \frac{x}{3}$$

$$M_x = \frac{wL}{6}x - \frac{wx^2}{2L}\,\frac{x}{3} = \frac{wL}{6}x - \frac{wx^3}{6L}$$

At $x = 0$, $$M_A = 0$$

At $x = L$, $$M_B = \frac{wL}{6}L - \frac{wL^3}{6L} = 0$$

B.M. varies between A and B as three degree curve (cubically)

B.M. is maximum where, S.F. is zero.

Max B.M. occur its $x = \frac{L}{\sqrt{3}}$

$$\text{Maximum B.M.} = \frac{wL}{6}\left(\frac{L}{\sqrt{3}}\right) - \frac{w}{6L}\left(\frac{L}{\sqrt{3}}\right)^3$$

$$\text{Maximum B.M.} = \frac{wL^2}{9\sqrt{3}}$$

B.M. diagram is shown in Fig. 25.20.

Example 25.7. A overhanging beam is loaded and supported as shown in Fig. 25.21. Draw the shear force and bending moment diagrams.

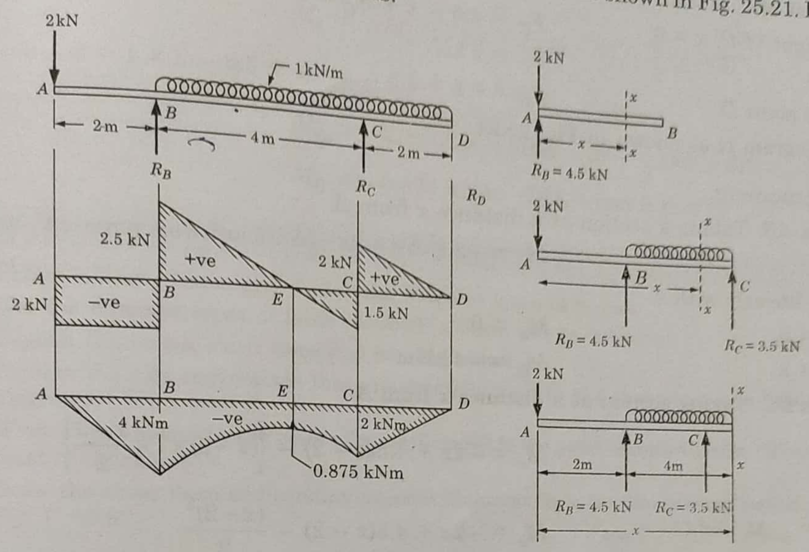

Fig. 25.21

Solution: Reactions at the supports be R_B and R_C

Applying,

$\Sigma R_y = 0$ $\quad R_B + R_C - 2 - (4 + 2) \times 1 = 0$

$R_B + R_C = 8$ kN ...(*i*)

$\Sigma M_A = 0$ $\quad -R_B \times 2 - R_C \times 6 + 4 \times 4 + 2 \times 7 = 0$

$2R_B + 6R_C = 30$

$R_B + 3R_C = 15$ ...(*ii*)

Solving (*i*) and (*ii*) $R_B = 4.5$ kN $\quad R_C = 3.5$ kN

Shear Force. Taking a section *x–x* from *A* (Between *A* and *B*)

$F_x = -2$ kN (down force, –ve)

Shear force is constant from *A* to just left of *B*

$F_A = -2$ kN

S.F. just left of $B = -2$ kN

S.F. just right of $B = -2 + R_B = -2 + 4.5 = +2.5$ kN

(It changes because of reaction at *B*)

Taking section *x–x* from *A* (Between *A* and *C*)

Shear force $F_x = -2 + 4.5 - 1 \times (x - 2)$

$= 2.5 - x + 2 = 4.5 - x$

Just left of C, $x = 6$, $\quad F_C = 4.5 - 6 = -1.5$ kN

Just right of C, $x = 6$, $\quad F_C = 4.5 - x + R_C = 4.5 - x + 3.5$

$$F_C = 2 \text{ kN}$$

S.F. at point D $\quad F_D = -2 + 4.5 - 4 \times 1 + 3.5 - 2 \times 1 = 0$

S.F. diagram is as shown in Fig. 25.21.

Bending moment

Portion AB: Taking a section at a distance x from A

$$M_x = -2 \times x = -2x \quad \text{(Anticlockwise moment –ve)}$$

Varies linearly with x

at $x = 0$, $\quad M_A = 0$

at $x = 2$, $\quad M_B = -4$ kNm

Portion BC: Taking section at a distance x from A

$$M_x = -2x + 4.5(x - 2) - \left\{(x-2)\times 1 \times \frac{x-2}{2}\right\}$$

$$M_x = -2x + 4.5(x - 2) - \frac{(x-2)^2}{2}$$

At $x = 2$ $\quad M_B = -4$ kNm

At $x = 6$ $\quad M_C = -12 + 18 - 8 = -2$ kNm

Variation between BC is parabolic.

Portion CD. Taking section at distance x from A

$$M_x = -2x + 4.5(x - 2) - \frac{1}{2}(x - 2)^2 + 3.5(x - 6)$$

at $x = 0$ $\quad M_C = -2$ kN

at $x = 8$ $\quad M_D = -2 \times 8 + 4.5(6) - \frac{1}{2} \times (6)^2 + 3.5(2)$

$$M_D = 0$$

Variation in this portion is again parabolic.

B.M. diagram is as shown in Fig. 25.21.

Note that shear force changes sign from +ve to –ve at E.

Location of E w.r.t. B is

$$\frac{2.5}{BE} = \frac{1.5}{EC},$$

$$\frac{2.5}{BE} = \frac{1.5}{4 - BE}$$

$$BE = 2.5 \text{ m}$$

B.M. at E (Portion BC)

$$M_E = -M_x = -2x + 4.5(x-2) - \frac{1}{2}(x-2)^2$$

Putting, $x = 2 + 2.5 = 4.5$ m

$$M_E = -2 \times 4.5 + 4.5(4.5-2) - \frac{1}{2}(4.5-2)^2$$

$$M_E = -0.875 \text{ kNm} \quad \text{(BM, when S.F. = 0)}$$

PROBLEMS

25.1. Define a beam. Name and sketch the different types of beams.

25.2. List the different types of loads to which a beam can be subjected.

25.3. Explain the terms, shear force and bending moment.

25.4. Explain the sign conventions that are generally used to plot shear force and bending moment diagrams.

25.5. What do you meant by the point of contraflexure? Is the point of contraflexure different from the point of inflexion?

25.6. Draw the shear force and bending moment diagrams for a cantilever beam loaded as shown in Fig. P. 25.6. [F_{max} = –12 kN M_{max} = – 35 kNm]

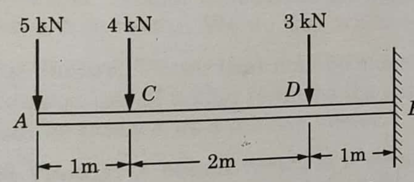

Fig. P. 25.6

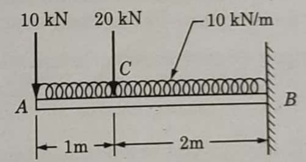

Fig. P. 25.7

25.7. Draw the shear force and bending moment diagram for a cantilever loaded by point loads and uniformly distributed load as shown in Fig. P. 25.7. [F_{max} = 60 kN, M_{max} = – 115 kNm]

25.8. A simply supported beam of length 6 m, carries two point loads as shown in Fig. P. 25.8. Draw shear force and bending moment diagrams for the beam. [F_{max} = – 5 kN, M_{max} = 10 kNm]

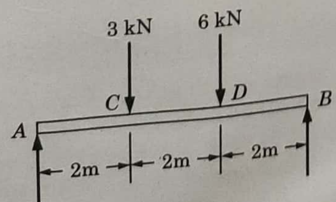

Fig. P. 25.8

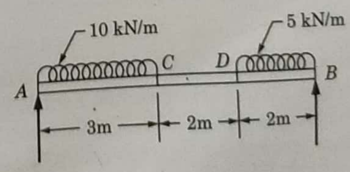

Fig. P. 25.9

25.9. A simply supported beam of length 7 m is carrying uniformly distributed load as shown in Fig. P. 25.9. Draw the shear force and bending moment diagrams for the beam. [F_{max} = + 25 kN, M_{max} = 31.25]

(Shear force is zero at distance 2.5 m from A)