

Mutually Exclusive:  $A \cap B = \phi$  ;  $P(A \cup B) = P(A) + P(B)$

Collectively Exhaustive events:  $A \cup B = \text{sample space}$

Independent Events =  $P(A \cap B) = P(A) \cdot P(B)$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

Random  $V(X)$

Discrete

Continuous

PMF:  $\sum P(n) \geq 0$

PDF:  $f(n) \geq 0$

$$\sum P(n) = 1$$

$$\int_{-\infty}^{\infty} f(n) = 1$$

→ Binomial

→ Poisson

→ Uniform

→ Normal

→ Exponential

PMF / PDF are same, depends on context.

eg:

x	0	1	2
P(n)	a	b	c

x	Interval $I_1$	$I_2$	$I_3$
P(n)	a	b	c

Cumulative DF

Cumulative density Func.

x	0	1	2
f(x)	a	a+b	a+b+c

Prefix sum.

is same but can't add directly, so we integrate in that range.

Mean =  $\bar{x}$

$$E(n) = \sum x P(n)$$

$$\bar{x} = \int x P(n) = E(n)$$

Variance =  $E(x^2) - (E(x))^2$

Variance

→ calculate

sum of sq. - sq. of sum.

$$E(x^2) = \int x^2 P(n)$$

Gamma function

$$\int_0^{\infty} x^{n-1} e^{-ax} dx = \frac{\Gamma n}{a^n}$$

Mean of general function:

$$E(\phi(n)) = \sum \phi(n) p(n)$$

$$\text{or } \int \phi(n) f(n) dn$$

Binomial Distribution:

$n \rightarrow \text{finite}$

$$P(n) = {}^n C_n p^n q^{n-n} [p+q=1]$$

$$PMF = [p+q]^n = 1^n = 1$$

$$\text{mean} = np$$

$$\text{variance} = npq$$

MGF (Moment generating function) =  $E(e^{xt})$

$$E(e^{xt}) = (pe^t + q)^n$$

← solve

Characteristic Fun:  $E(e^{int})$   $t \rightarrow it$   
 $\rightarrow (pe^{it} + q)^n$

PGF (Probability Generating Function)

$$\rightarrow E(z^n) = (zp + q)^n$$

2<sup>nd</sup> sum syllabus

Poisson Distribution:

$n \rightarrow \infty$

$p \rightarrow 0$

but np is finite.

$$PMF = P(n) = \frac{\lambda^n e^{-\lambda}}{n!}$$

where  $\lambda = \text{mean} = np$

$$\sigma^2 = E(x^2) - (E(x))^2$$

$$= \lambda^2 + \lambda - \lambda^2 = \lambda$$

$$MGF = e^{\lambda(e^t - 1)}$$

$$PGF = e^{\lambda(z - 1)}$$

Mean = Variance

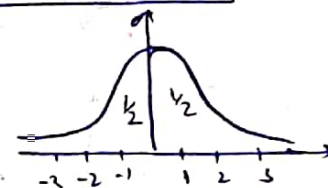
Normal Distribution / Gaussian Distribution

Mean = Median = Mode

$$f(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$\sigma^2 \rightarrow \text{variance}$

$\mu \rightarrow \text{mean}$



$$\frac{x - \mu}{\sigma} = Z$$

Area:  $(-1 < Z < 1) : 0.68$

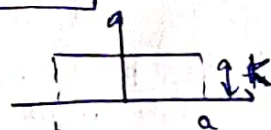
$(-2 < Z < 2) : 0.95$

$(-3 < Z < 3) : 0.99$

$(-\infty < Z < \infty) : 1$

Rectangular / Uniform distribution

$$f(n) = \begin{cases} K & b < x < a \\ 0 & \text{else} \end{cases}$$



We know  $\int_{-\infty}^{\infty} f(n) = 1 = \text{Area}$

$$\rightarrow \text{so } K = \frac{1}{a-b}$$

$$\text{mean} = \frac{a+b}{2} = E(n)$$

$$E(n^2) \text{ Variance} = \frac{(b-a)^2}{12}$$

Moments

Representation

about Origin

Mean

Any value

$$\mu_0^1 = E(n-0)^1$$

$$\mu_0^0 = E(n-\bar{n})^0$$

$$\mu_r^2 = E(n-A)^r$$

$$\mu_0^1 = 0$$

$$\mu_0^1 = 1$$

$$\mu_1^1 = \bar{x} = \text{Mean}$$

$$\mu_1^0 = 0$$

$$\mu_2^1 = E(x^2)$$

$$\mu_2^0 = \mu_2^1 - (\mu_1^1)^2 \text{ Variance}$$

$$\mu_3^1 = E(x^3)$$

$$\mu_3^0 = \mu_3^1 - 3\mu_2^1\mu_1^1 + 2(\mu_1^1)^3$$

$$\mu_4^0 = \mu_4^1 - 4\mu_3^1\mu_1^1 + 6\mu_2^1\mu_1^2 - 3(\mu_1^1)^4$$

$$\mu_5^0 = \mu_5^1 - 5\mu_4^1\mu_1^1 + 10\mu_3^1\mu_1^2 - 10\mu_2^1\mu_1^3 + 6\mu_1^1\mu_1^5$$

$$\mu_6^0 = \mu_6^1 - 6\mu_5^1\mu_1^1 + 15\mu_4^1\mu_1^2 - 20\mu_3^1\mu_1^3 + 15\mu_2^1\mu_1^4 - 6\mu_1^1\mu_1^6$$

$$\mu_7^0 = \mu_7^1 - 7\mu_6^1\mu_1^1 + 21\mu_5^1\mu_1^2 - 35\mu_4^1\mu_1^3 + 35\mu_3^1\mu_1^4 - 21\mu_2^1\mu_1^5 + 7\mu_1^1\mu_1^7$$

$$\mu_8^0 = \mu_8^1 - 8\mu_7^1\mu_1^1 + 28\mu_6^1\mu_1^2 - 56\mu_5^1\mu_1^3 + 70\mu_4^1\mu_1^4 - 56\mu_3^1\mu_1^5 + 28\mu_2^1\mu_1^6 - 8\mu_1^1\mu_1^8$$

$$\mu_9^0 = \mu_9^1 - 9\mu_8^1\mu_1^1 + 36\mu_7^1\mu_1^2 - 84\mu_6^1\mu_1^3 + 126\mu_5^1\mu_1^4 - 126\mu_4^1\mu_1^5 + 84\mu_3^1\mu_1^6 - 36\mu_2^1\mu_1^7 + 9\mu_1^1\mu_1^9$$

$$\mu_{10}^0 = \mu_{10}^1 - 10\mu_9^1\mu_1^1 + 45\mu_8^1\mu_1^2 - 120\mu_7^1\mu_1^3 + 210\mu_6^1\mu_1^4 - 252\mu_5^1\mu_1^5 + 210\mu_4^1\mu_1^6 - 120\mu_3^1\mu_1^7 + 45\mu_2^1\mu_1^8 - 10\mu_1^1\mu_1^{10}$$

$$\mu_{11}^0 = \mu_{11}^1 - 11\mu_{10}^1\mu_1^1 + 55\mu_9^1\mu_1^2 - 165\mu_8^1\mu_1^3 + 330\mu_7^1\mu_1^4 - 462\mu_6^1\mu_1^5 + 462\mu_5^1\mu_1^6 - 330\mu_4^1\mu_1^7 + 165\mu_3^1\mu_1^8 - 55\mu_2^1\mu_1^9 + 11\mu_1^1\mu_1^{11}$$

$$\mu_{12}^0 = \mu_{12}^1 - 12\mu_{11}^1\mu_1^1 + 66\mu_{10}^1\mu_1^2 - 220\mu_9^1\mu_1^3 + 495\mu_8^1\mu_1^4 - 792\mu_7^1\mu_1^5 + 792\mu_6^1\mu_1^6 - 495\mu_5^1\mu_1^7 + 220\mu_4^1\mu_1^8 - 66\mu_3^1\mu_1^9 + 12\mu_2^1\mu_1^{10} - 12\mu_1^1\mu_1^{12}$$

$$\mu_{13}^0 = \mu_{13}^1 - 13\mu_{12}^1\mu_1^1 + 78\mu_{11}^1\mu_1^2 - 286\mu_{10}^1\mu_1^3 + 672\mu_9^1\mu_1^4 - 1287\mu_8^1\mu_1^5 + 1287\mu_7^1\mu_1^6 - 672\mu_6^1\mu_1^7 + 286\mu_5^1\mu_1^8 - 78\mu_4^1\mu_1^9 + 13\mu_3^1\mu_1^{10} - 13\mu_2^1\mu_1^{11} + 13\mu_1^1\mu_1^{13}$$

$$\mu_{14}^0 = \mu_{14}^1 - 14\mu_{13}^1\mu_1^1 + 91\mu_{12}^1\mu_1^2 - 364\mu_{11}^1\mu_1^3 + 900\mu_{10}^1\mu_1^4 - 1701\mu_9^1\mu_1^5 + 1701\mu_8^1\mu_1^6 - 900\mu_7^1\mu_1^7 + 364\mu_6^1\mu_1^8 - 91\mu_5^1\mu_1^9 + 14\mu_4^1\mu_1^{10} - 14\mu_3^1\mu_1^{11} + 14\mu_2^1\mu_1^{12} - 14\mu_1^1\mu_1^{14}$$

$$\mu_{15}^0 = \mu_{15}^1 - 15\mu_{14}^1\mu_1^1 + 105\mu_{13}^1\mu_1^2 - 429\mu_{12}^1\mu_1^3 + 1001\mu_{11}^1\mu_1^4 - 2002\mu_{10}^1\mu_1^5 + 2002\mu_9^1\mu_1^6 - 1001\mu_8^1\mu_1^7 + 429\mu_7^1\mu_1^8 - 105\mu_6^1\mu_1^9 + 15\mu_5^1\mu_1^{10} - 15\mu_4^1\mu_1^{11} + 15\mu_3^1\mu_1^{12} - 15\mu_2^1\mu_1^{13} + 15\mu_1^1\mu_1^{15}$$

$$\mu_{16}^0 = \mu_{16}^1 - 16\mu_{15}^1\mu_1^1 + 112\mu_{14}^1\mu_1^2 - 504\mu_{13}^1\mu_1^3 + 1201\mu_{12}^1\mu_1^4 - 2520\mu_{11}^1\mu_1^5 + 2520\mu_{10}^1\mu_1^6 - 1201\mu_9^1\mu_1^7 + 504\mu_8^1\mu_1^8 - 112\mu_7^1\mu_1^9 + 16\mu_6^1\mu_1^{10} - 16\mu_5^1\mu_1^{11} + 16\mu_4^1\mu_1^{12} - 16\mu_3^1\mu_1^{13} + 16\mu_2^1\mu_1^{14} - 16\mu_1^1\mu_1^{16}$$

$$\mu_{17}^0 = \mu_{17}^1 - 17\mu_{16}^1\mu_1^1 + 119\mu_{15}^1\mu_1^2 - 595\mu_{14}^1\mu_1^3 + 1360\mu_{13}^1\mu_1^4 - 2807\mu_{12}^1\mu_1^5 + 2807\mu_{11}^1\mu_1^6 - 1360\mu_{10}^1\mu_1^7 + 595\mu_9^1\mu_1^8 - 119\mu_8^1\mu_1^9 + 17\mu_7^1\mu_1^{10} - 17\mu_6^1\mu_1^{11} + 17\mu_5^1\mu_1^{12} - 17\mu_4^1\mu_1^{13} + 17\mu_3^1\mu_1^{14} - 17\mu_2^1\mu_1^{15} + 17\mu_1^1\mu_1^{17}$$

$$\mu_{18}^0 = \mu_{18}^1 - 18\mu_{17}^1\mu_1^1 + 126\mu_{16}^1\mu_1^2 - 672\mu_{15}^1\mu_1^3 + 1512\mu_{14}^1\mu_1^4 - 3150\mu_{13}^1\mu_1^5 + 3150\mu_{12}^1\mu_1^6 - 1512\mu_{11}^1\mu_1^7 + 672\mu_{10}^1\mu_1^8 - 126\mu_9^1\mu_1^9 + 18\mu_8^1\mu_1^{10} - 18\mu_7^1\mu_1^{11} + 18\mu_6^1\mu_1^{12} - 18\mu_5^1\mu_1^{13} + 18\mu_4^1\mu_1^{14} - 18\mu_3^1\mu_1^{15} + 18\mu_2^1\mu_1^{16} - 18\mu_1^1\mu_1^{18}$$

$$\mu_{19}^0 = \mu_{19}^1 - 19\mu_{18}^1\mu_1^1 + 133\mu_{17}^1\mu_1^2 - 729\mu_{16}^1\mu_1^3 + 1639\mu_{15}^1\mu_1^4 - 3363\mu_{14}^1\mu_1^5 + 3363\mu_{13}^1\mu_1^6 - 1639\mu_{12}^1\mu_1^7 + 729\mu_{11}^1\mu_1^8 - 133\mu_{10}^1\mu_1^9 + 19\mu_9^1\mu_1^{10} - 19\mu_8^1\mu_1^{11} + 19\mu_7^1\mu_1^{12} - 19\mu_6^1\mu_1^{13} + 19\mu_5^1\mu_1^{14} - 19\mu_4^1\mu_1^{15} + 19\mu_3^1\mu_1^{16} - 19\mu_2^1\mu_1^{17} + 19\mu_1^1\mu_1^{19}$$

$$\mu_{20}^0 = \mu_{20}^1 - 20\mu_{19}^1\mu_1^1 + 140\mu_{18}^1\mu_1^2 - 780\mu_{17}^1\mu_1^3 + 1770\mu_{16}^1\mu_1^4 - 3640\mu_{15}^1\mu_1^5 + 3640\mu_{14}^1\mu_1^6 - 1770\mu_{13}^1\mu_1^7 + 780\mu_{12}^1\mu_1^8 - 140\mu_{11}^1\mu_1^9 + 20\mu_{10}^1\mu_1^{10} - 20\mu_9^1\mu_1^{11} + 20\mu_8^1\mu_1^{12} - 20\mu_7^1\mu_1^{13} + 20\mu_6^1\mu_1^{14} - 20\mu_5^1\mu_1^{15} + 20\mu_4^1\mu_1^{16} - 20\mu_3^1\mu_1^{17} + 20\mu_2^1\mu_1^{18} - 20\mu_1^1\mu_1^{20}$$

$$\mu_{21}^0 = \mu_{21}^1 - 21\mu_{20}^1\mu_1^1 + 147\mu_{19}^1\mu_1^2 - 819\mu_{18}^1\mu_1^3 + 1848\mu_{17}^1\mu_1^4 - 3773\mu_{16}^1\mu_1^5 + 3773\mu_{15}^1\mu_1^6 - 1848\mu_{14}^1\mu_1^7 + 819\mu_{13}^1\mu_1^8 - 147\mu_{12}^1\mu_1^9 + 21\mu_{11}^1\mu_1^{10} - 21\mu_{10}^1\mu_1^{11} + 21\mu_9^1\mu_1^{12} - 21\mu_8^1\mu_1^{13} + 21\mu_7^1\mu_1^{14} - 21\mu_6^1\mu_1^{15} + 21\mu_5^1\mu_1^{16} - 21\mu_4^1\mu_1^{17} + 21\mu_3^1\mu_1^{18} - 21\mu_2^1\mu_1^{19} + 21\mu_1^1\mu_1^{21}$$

$$\mu_{22}^0 = \mu_{22}^1 - 22\mu_{21}^1\mu_1^1 + 154\mu_{20}^1\mu_1^2 - 858\mu_{19}^1\mu_1^3 + 1938\mu_{18}^1\mu_1^4 - 3960\mu_{17}^1\mu_1^5 + 3960\mu_{16}^1\mu_1^6 - 1938\mu_{15}^1\mu_1^7 + 858\mu_{14}^1\mu_1^8 - 154\mu_{13}^1\mu_1^9 + 22\mu_{12}^1\mu_1^{10} - 22\mu_{11}^1\mu_1^{11} + 22\mu_{10}^1\mu_1^{12} - 22\mu_9^1\mu_1^{13} + 22\mu_8^1\mu_1^{14} - 22\mu_7^1\mu_1^{15} + 22\mu_6^1\mu_1^{16} - 22\mu_5^1\mu_1^{17} + 22\mu_4^1\mu_1^{18} - 22\mu_3^1\mu_1^{19} + 22\mu_2^1\mu_1^{20} - 22\mu_1^1\mu_1^{22}$$

$$\mu_{23}^0 = \mu_{23}^1 - 23\mu_{22}^1\mu_1^1 + 161\mu_{21}^1\mu_1^2 - 897\mu_{20}^1\mu_1^3 + 2017\mu_{19}^1\mu_1^4 - 4034\mu_{18}^1\mu_1^5 + 4034\mu_{17}^1\mu_1^6 - 2017\mu_{16}^1\mu_1^7 + 897\mu_{15}^1\mu_1^8 - 161\mu_{14}^1\mu_1^9 + 23\mu_{13}^1\mu_1^{10} - 23\mu_{12}^1\mu_1^{11} + 23\mu_{11}^1\mu_1^{12} - 23\mu_{10}^1\mu_1^{13} + 23\mu_9^1\mu_1^{14} - 23\mu_8^1\mu_1^{15} + 23\mu_7^1\mu_1^{16} - 23\mu_6^1\mu_1^{17} + 23\mu_5^1\mu_1^{18} - 23\mu_4^1\mu_1^{19} + 23\mu_3^1\mu_1^{20} - 23\mu_2^1\mu_1^{21} + 23\mu_1^1\mu_1^{23}$$

$$\mu_{24}^0 = \mu_{24}^1 - 24\mu_{23}^1\mu_1^1 + 168\mu_{22}^1\mu_1^2 - 936\mu_{21}^1\mu_1^3 + 2112\mu_{20}^1\mu_1^4 - 4224\mu_{19}^1\mu_1^5 + 4224\mu_{18}^1\mu_1^6 - 2112\mu_{17}^1\mu_1^7 + 936\mu_{16}^1\mu_1^8 - 168\mu_{15}^1\mu_1^9 + 24\mu_{14}^1\mu_1^{10} - 24\mu_{13}^1\mu_1^{11} + 24\mu_{12}^1\mu_1^{12} - 24\mu_{11}^1\mu_1^{13} + 24\mu_{10}^1\mu_1^{14} - 24\mu_9^1\mu_1^{15} + 24\mu_8^1\mu_1^{16} - 24\mu_7^1\mu_1^{17} + 24\mu_6^1\mu_1^{18} - 24\mu_5^1\mu_1^{19} + 24\mu_4^1\mu_1^{20} - 24\mu_3^1\mu_1^{21} + 24\mu_2^1\mu_1^{22} - 24\mu_1^1\mu_1^{24}$$

$$\mu_{25}^0 = \mu_{25}^1 - 25\mu_{24}^1\mu_1^1 + 175\mu_{23}^1\mu_1^2 - 975\mu_{22}^1\mu_1^3 + 2205\mu_{21}^1\mu_1^4 - 4410\mu_{20}^1\mu_1^5 + 4410\mu_{19}^1\mu_1^6 - 2205\mu_{18}^1\mu_1^7 + 975\mu_{17}^1\mu_1^8 - 175\mu_{16}^1\mu_1^9 + 25\mu_{15}^1\mu_1^{10} - 25\mu_{14}^1\mu_1^{11} + 25\mu_{13}^1\mu_1^{12} - 25\mu_{12}^1\mu_1^{13} + 25\mu_{11}^1\mu_1^{14} - 25\mu_{10}^1\mu_1^{15} + 25\mu_9^1\mu_1^{16} - 25\mu_8^1\mu_1^{17} + 25\mu_7^1\mu_1^{18} - 25\mu_6^1\mu_1^{19} + 25\mu_5^1\mu_1^{20} - 25\mu_4^1\mu_1^{21} + 25\mu_3^1\mu_1^{22} - 25\mu_2^1\mu_1^{23} + 25\mu_1^1\mu_1^{25}$$

$$\mu_{26}^0 = \mu_{26}^1 - 26\mu_{25}^1\mu_1^1 + 182\mu_{24}^1\mu_1^2 - 1014\mu_{23}^1\mu_1^3 + 2292\mu_{22}^1\mu_1^4 - 4584\mu_{21}^1\mu_1^5 + 4584\mu_{20}^1\mu_1^6 - 2292\mu_{19}^1\mu_1^7 + 1014\mu_{18}^1\mu_1^8 - 182\mu_{17}^1\mu_1^9 + 26\mu_{16}^1\mu_1^{10} - 26\mu_{15}^1\mu_1^{11} + 26\mu_{14}^1\mu_1^{12} - 26\mu_{13}^1\mu_1^{13} + 26\mu_{12}^1\mu_1^{14} - 26\mu_{11}^1\mu_1^{15} + 26\mu_{10}^1\mu_1^{16} - 26\mu_9^1\mu_1^{17} + 26\mu_8^1\mu_1^{18} - 26\mu_7^1\mu_1^{19} + 26\mu_6^1\mu_1^{20} - 26\mu_5^1\mu_1^{21} + 26\mu_4^1\mu_1^{22} - 26\mu_3^1\mu_1^{23} + 26\mu_2^1\mu_1^{24} - 26\mu_1^1\mu_1^{26}$$

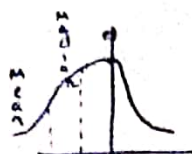
Skewness : lack of symmetry.

$$B_1 = \frac{(\mu_3')^2}{(\mu_2')^3}$$

$\mu_r' \rightarrow$  about mean



$$B_1 = 0$$



$$B_1 < 0$$



$$B_1 > 0$$

Mean = Median = Mode -ve skewed. +ve skewed

Kurtosis | Flatness of curve | Bell shape

$$B_2 = \frac{\mu_4'}{(\mu_2')^2}$$

$B_2 \geq 3$  (Leptokurtic)

$B_2 = 3$  (Mesokurtic)

$B_2 < 3$  (Platykurtic)



LMP

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i} \leftarrow \text{used while calculating moments etc.}$$

Preferably find about a point and on back page replace  $\mu^1 \rightarrow \mu^2$  to calculate about mean (lengthy)

Bivariate RV

2 dimension

$$P(X=x_i, Y=y_j) = P_{ij}$$

Joint PMF = (i)  $P_{ij} \geq 0$

(ii)  $\sum \sum P_{ij} = 1$  triplet =  $(x_i, y_j, p)$

Marginal PMF:

for  $P(X=x_i) = \sum_j P_{ij} = P_{i1} + P_{i2} + P_{i3} \dots P_{in}$

collection of pair  $(x_i, p_i) \rightarrow$  MGF of  $x$

for  $P(Y=y_j) = \sum_i P_{ij} = P_{1j} + P_{2j} + P_{3j} \dots P_{nj}$

If  $X$  and  $Y$  are independent:

$$P_{XY}(x, y) = P_X(x) P_Y(y)$$

i.e. solve in 1d and +

$x = 0, 1, \dots, n$		$y = 0, 1, \dots, m$			
Marginal X	Total	Y	Total	X \ Y	Total
0		0		a	$\rightarrow \Sigma$
1		1			
2		2			
					$\downarrow \Sigma$

Conditional PMF  $(x=1, y=1) = \frac{a}{b}$

Distributions

with replacement  $\rightarrow$  independent trial  
without  $\rightarrow$  not independent

(i) Binomial

$$P(x) = {}^n C_x p^x (1-p)^{n-x}$$

independent trials

$$\text{mean} = np, \sigma^2 = npq$$

(ii) Hypergeometric : dependent trials.

$n \rightarrow$  objects  $a \rightarrow$  success  $(N-a)$  failure

$$P(X=x) = \frac{{}^a C_x {}^{N-a} C_{n-x}}{{}^N C_n}$$

$$\text{mean} = n \frac{a}{N}$$

$\hookrightarrow$  solve normally  $\binom{N}{n}$  without natural formula.

$$\sigma^2 = ?$$

Sometime Ans of Binomial  $\sim$  Hypergeometric (v.s. error)

(iii) Geometric : First success on  $x^{\text{th}}$  trial.

i.e. (i)  $(x-1) \rightarrow$  failure i.e.  $(1-p)^{x-1}$

(ii)  $x^{\text{th}}$  must be success  $p$

$$\text{PMF} = P(X=x) = (1-p)^{x-1} p$$

$$\text{mean} = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

$$P(X \leq x) = 1 - (1-p)^x$$

Negative binomial:

(i) First  $(r-1)$  trials  $\rightarrow$   $(r-1)$  success

(solve via Binomial)

(ii)  $x^{\text{th}}$   $\rightarrow$  success

$$\text{mean} = \frac{r}{p}, \sigma^2 = \frac{r(1-p)}{p^2}$$

Multinomial

$$P(X=x_1, Y=y_1) = {}^n C_{x_1} p^{x_1} q^{y_1} = \frac{n!}{x_1! y_1!} p^{x_1} q^{y_1}$$

General

$$P(X=x_1, \dots, Y=y_n) = \frac{n!}{x_1! x_2! \dots x_n!} p_1^{x_1} p_2^{x_2} \dots p_n^{x_n}$$

$x_i = 0, 1, 2, \dots, n$

$$\sum x_i = 1$$