

# Recursion

→ A function that calls itself

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→ PMI → Principle of Mathematical Induction

$$\Rightarrow 1 + 2 + 3 + \dots + n \Rightarrow \frac{n \times (n+1)}{2}$$

Base Condition:

$$n = 1$$

$$1 = 1 \times \frac{(1+1)}{2} \quad \checkmark$$

$$L.H.S = R.H.S$$

② Induction Hypothesis :-  
any  $k \in n$

$$\boxed{1 + 2 + 3 + 4 + \dots + k = \frac{k \times (k+1)}{2}}$$

③ Prove (Induction Step)  
/ / / / /

(K+1) ✓

$$\underbrace{(1+2+3+\dots+k)}_{\text{sum of first } k \text{ natural numbers}} + (k+1) = \frac{(k+1)^2 + (k+1)}{2}$$

$$\Rightarrow \frac{(k+1) * (k+2)}{2}$$

$$\frac{k * (k+1)}{2} + k+1$$

R. H.S

$$\left[ \frac{k^2 + k + 2k + 2}{2} \right]$$

$$\frac{k(k+1) + 2(k+1)}{2}$$

$$\frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2}$$

$$L.H.S = R.H.S$$

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To Solve any Question with  
Recursion

① Base Condition

② Recurrence Relationship

→ equation

$$f_{sum}(n) = \underbrace{1 + 2 + 3 + \dots + (n-1)}_n$$

$$\textcircled{2} f(n) = \boxed{n + f(n-1)}$$

$$\textcircled{1} f(1) = 1$$

def func(n)

if n == 1 :

return 1

else:

return n + func(n-1)

f(4)

~~f(4) = 3~~

	1	
A	2 + f(1)	3
	3 + f(2)	6
	4 + f(3)	10
	f(4)	10

$\boxed{Cx - 2}$

0

2

2

2

2

$$\Rightarrow 1^2 + 2 + 3 + 4 + \dots + n$$

$$\Rightarrow \text{Base Cond} \Rightarrow f(1) = 1$$

$\Rightarrow$  Recurrence Relation

$$n^2 + f(n-1)$$

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def func(n):
    if n == 1:
        return 1
    else:
        return n2 + f(n-1)
```

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

① Base Condition

$$f(1) = 1$$

② Recurrence R

$$\left(\frac{1}{n}\right) + f(n-1) \leftarrow \dots \left(\frac{1}{n-1}\right)$$

$\Rightarrow$

Ex-3

$$f(n, p) \Rightarrow 4, 3$$

$4^r$

Base Condition

$$f(n, 1) \Rightarrow n \checkmark$$

$$f(n, 0) \Rightarrow 1 \checkmark$$

Recursion Relationship

$$n \times (f(n, p-1))$$

def func\_pow(n, p):

if p == 1:

return n

else:

return n \* func\_pow(n, p-1)

$f(2,1)$	②	$2 \times (f(2,1)) \Rightarrow 2 \times 2 = 4$
$f(2,2)$		
$f(2,3)$		$\Rightarrow 2 \times f(2,2) \Rightarrow 2 \times 4$
		$\Rightarrow 8$

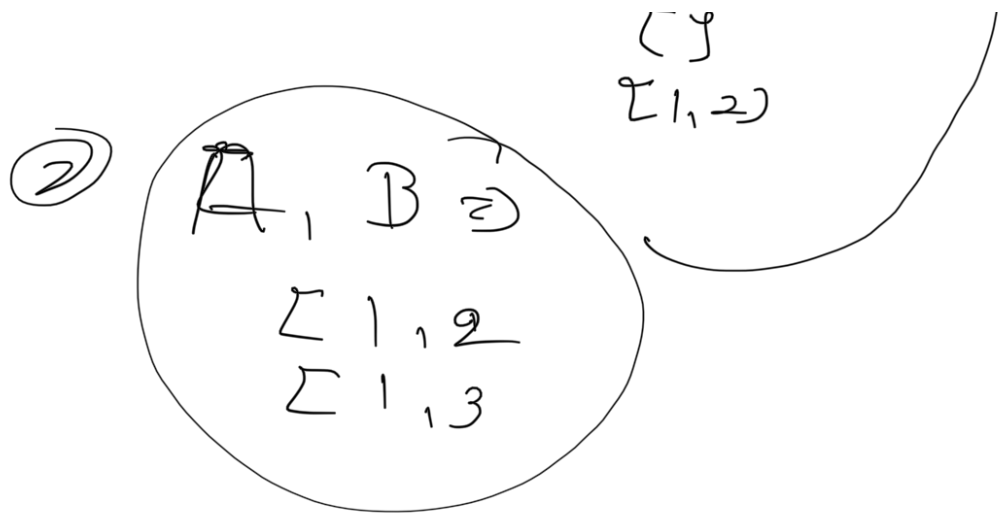
$\Rightarrow$  H.W.  $1, 2, 3 \checkmark$   
 $f(4) \begin{cases} 1^2 + 2^2 + 3^2 + \dots + n^2 \times \\ 1 + \frac{1}{2} + \frac{1}{3} \dots \frac{1}{n} \times \end{cases}$

$f(2,3) \checkmark$  Pouch

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① Base-Condition  
Recursion R

② all subsets  $\checkmark$   
 $\Rightarrow \{1, 2, 3\}$



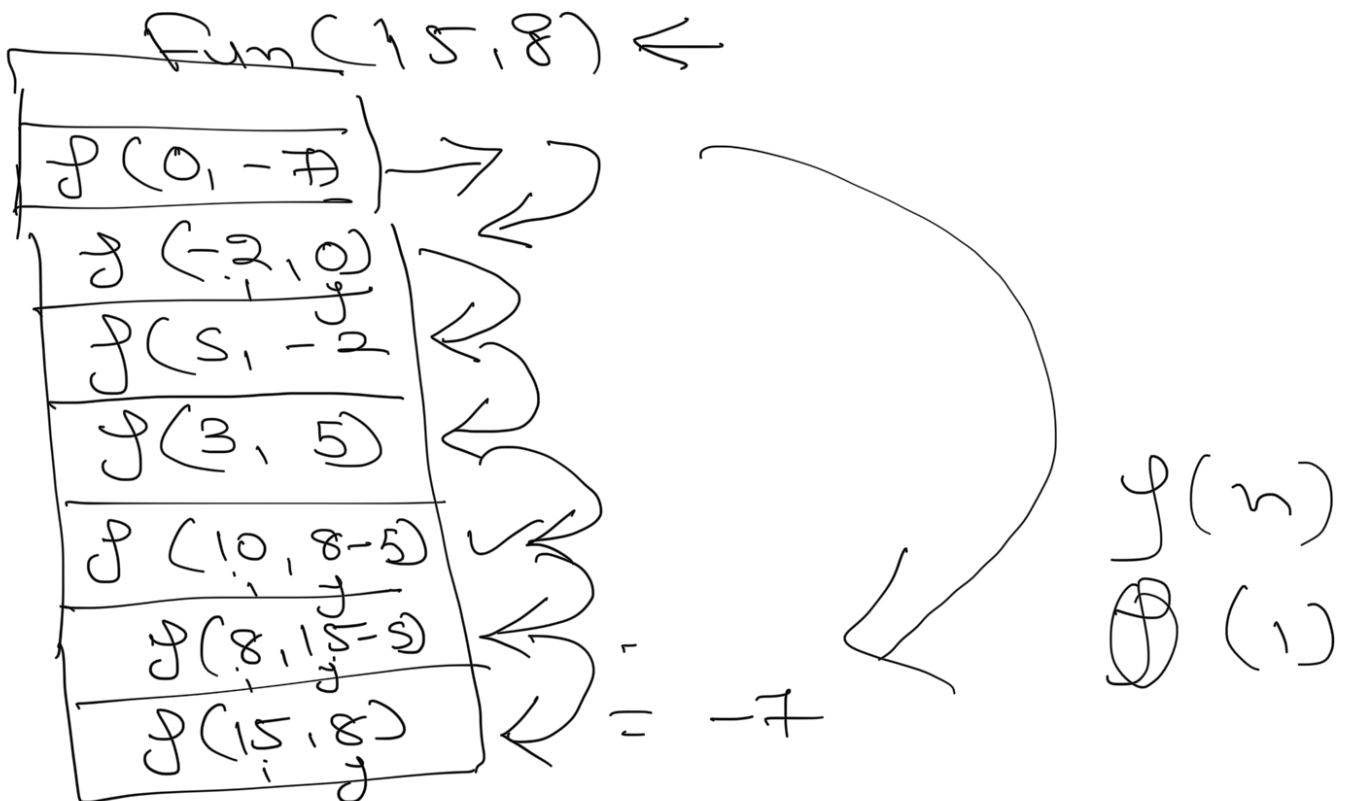
Def  $fun(i, j)$ :

if  $i == 0$ :

return  $j$

else:

return  $fun(j, i-5)$



Count = 'hi'  
String = abbhicc hicc d

[2]

Def CountHi (String, count=0)

if len(String) < 2:  
return count

else:

R "R"

if string == 'hi':  
return count + 1

hi (

Count on R: 1 → 1 + 1



$S \subseteq x \dots j = x$      $\text{if } x \in S \Rightarrow j(x) = x$   
 $j = 1$      $\text{if } a \in S \Rightarrow j(a) = a$   
 $j = 1$      $\text{if } a \in S \Rightarrow j(a) = a$

(2)

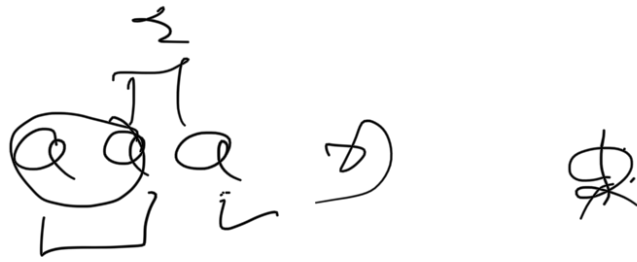
first 2 letters are Hi  
 $\Rightarrow$  reduce string by 2  
 $\Rightarrow$  increment Count


$\times j(2)$  not 'Hi'  
 $\Rightarrow$  reduce string by 1  
 $\Rightarrow$  no  $\Rightarrow c$


Hi Shi  
 $\Rightarrow$  Count +1


$j(\text{ishi})$      $S[1]: X$

$j(\text{shii})$      $S[2]:$




 $\Rightarrow 1, 'aa'$  jumpSize = 2


 $\Rightarrow 2, 'aa'$  jumpSize = 1



$\uparrow$   
 $f(hisc)$  can't jump  
 $f(hisc)$

$\Rightarrow 10, (10-2, 10-4)$

$\Rightarrow 10, \boxed{8, 6, \dots}$

$\Rightarrow 10 + (f(8))$

$$\Rightarrow 10 + (f(10-2) + \dots)$$

$\uparrow$                        $\uparrow$   
 $n$                        $(n-2)$

$$f(8) = 8 + f(6)$$

$$10 + f(8) \Rightarrow 8 + f(8-2)$$

$$f(9), f(7), f(5), f(3), f(1), \dots$$

if  $n < 1 =$  return 0 ✓

if  $n == 0$  return 0