# Class 2

Decision making under uncertainty

# Announcements and Questions

- PSL solutions (videos) will appear after everyone has submitted
- Do you have access to the textbook by now?
- Do you have access to PrecisionTree by now?
- Any other questions or concerns?

# What's on Canvas? All sorts of stuff

- There is a link to the textbook on Canvas. Textbook includes:
  - Excel files: examples & problems
  - Videos explaining concepts/tools
- Exam and PSA guidelines
- Recordings (e.g., PSL and PSA solutions)

# Graded elements & their weights

## 30%: Homework problems (PSL)

- Grading:
  - 0 if not submitted on time
  - 1 if submitted on time and the solution is incomplete, or complete but not your assigned problem or not with the required software (see syllabus for detailed indications)
  - 2 if submitted on time and the solution is complete (**even if not correct!**)
- You can submit multiple files, but only corresponding to the first exercise drawn
- I will post the solution after the due date
- You are welcome to talk to others but <u>make sure to submit your own work</u>

# Graded elements & their weights

## 60%: Exams (no make-up)

- 3 cumulative exams in total; total of 50 MC questions
- I will drop 3 questions with the lowest score
- Focus: core concepts and problem analysis skills

# Graded elements & their weights

## 10%: PSA (no make-up)

- 3 PSA problem to assess your understanding and mastery of the material
- PSA problem sets will be timed via Canvas and available during a specified time window
- <u>Must be prepared individually,</u> with no help from or communication with others
- For each problem set, a student is randomly assigned 2 problems; each problems has 2 questions (12 questions in total)

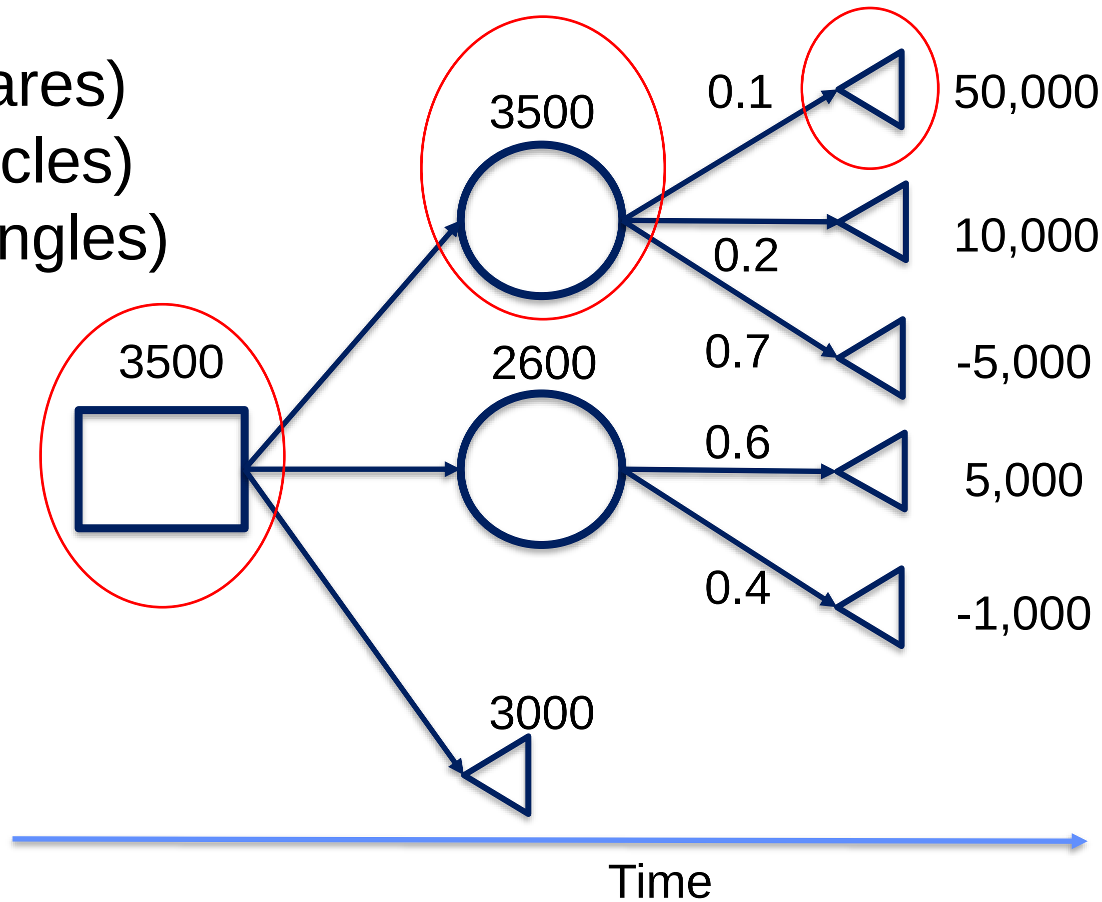
# Graded elements & their weights

## 10%: PSA (no make-up)

- You need to build a model using the tools we learned in class (e.g. decision tree or simulation model) to come up with a solution
- In addition to your answer, you will also need to submit your solution file
- 4 questions with lowest scores are dropped

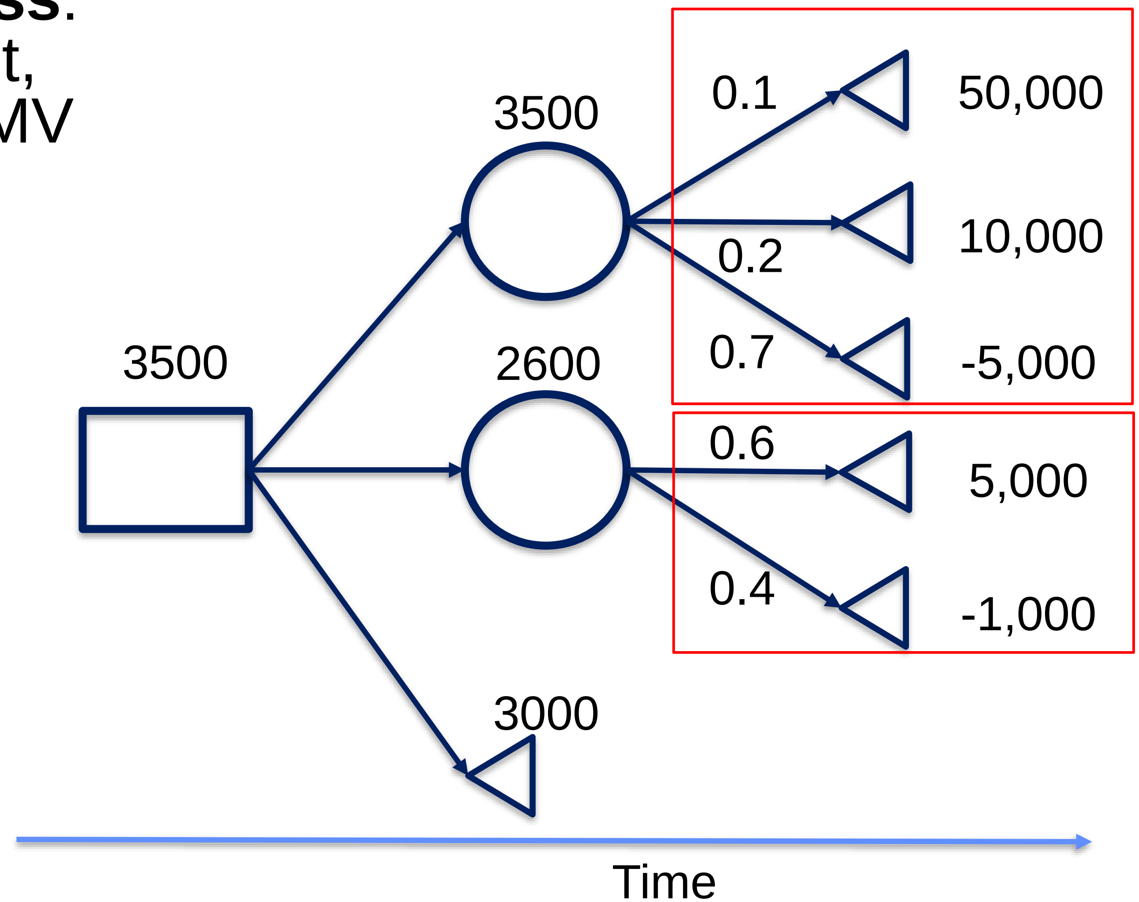
# Recap: Decision Trees

- Key elements of a decision tree:
  - Nodes:
    - Decision (squares)
    - Probability (circles)
    - End node (triangles)
  - Branches



# Recap: Decision Trees

- **Folding back process**: Starting from the right, calculate resulting EMV at each node
- Decision node: max(EMVs)
- Probability node: compute EMV
- Probabilities in a chance node must sum to 1



W. P. Carey School of Business
Arizona State University

# Goal today

- New C9 concepts
  - Multi-stage decision problems
  - Bayes’ rule
  - Value of information (EVI, EVPI)
  - Risk aversion via exponential utility function
  - Risk tolerance parameter
- Greater fluency and comfort in interpreting and using decision trees and PrecisionTree

# Multi-stage Decision Problems

- Many real-world problems involve a set of sequential decisions that might each provide valuable information

- Example 9.2: Product goes through a technology check and could fail. The probability of failing the inspection is 20%. The development cost is $4 million and the marketing cost is $2 million (only incurred if the product passes inspection).

✓ New Product Decisions - Technological Uncertainty.xlsx

✓ How do you feel about the EMV based on the probability of technological success?

# Multistage Decision Problems

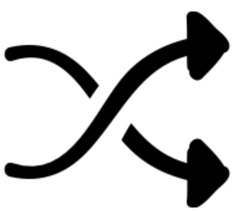
- Often, earlier decisions can provide valuable information that can sharpen our initial intuition

- Probability updating through Bayes' Rule
  - Start with prior probabilities
  - Obtain new information
  - Update prior probabilities into posterior probabilities

# Multistage Decision Problems

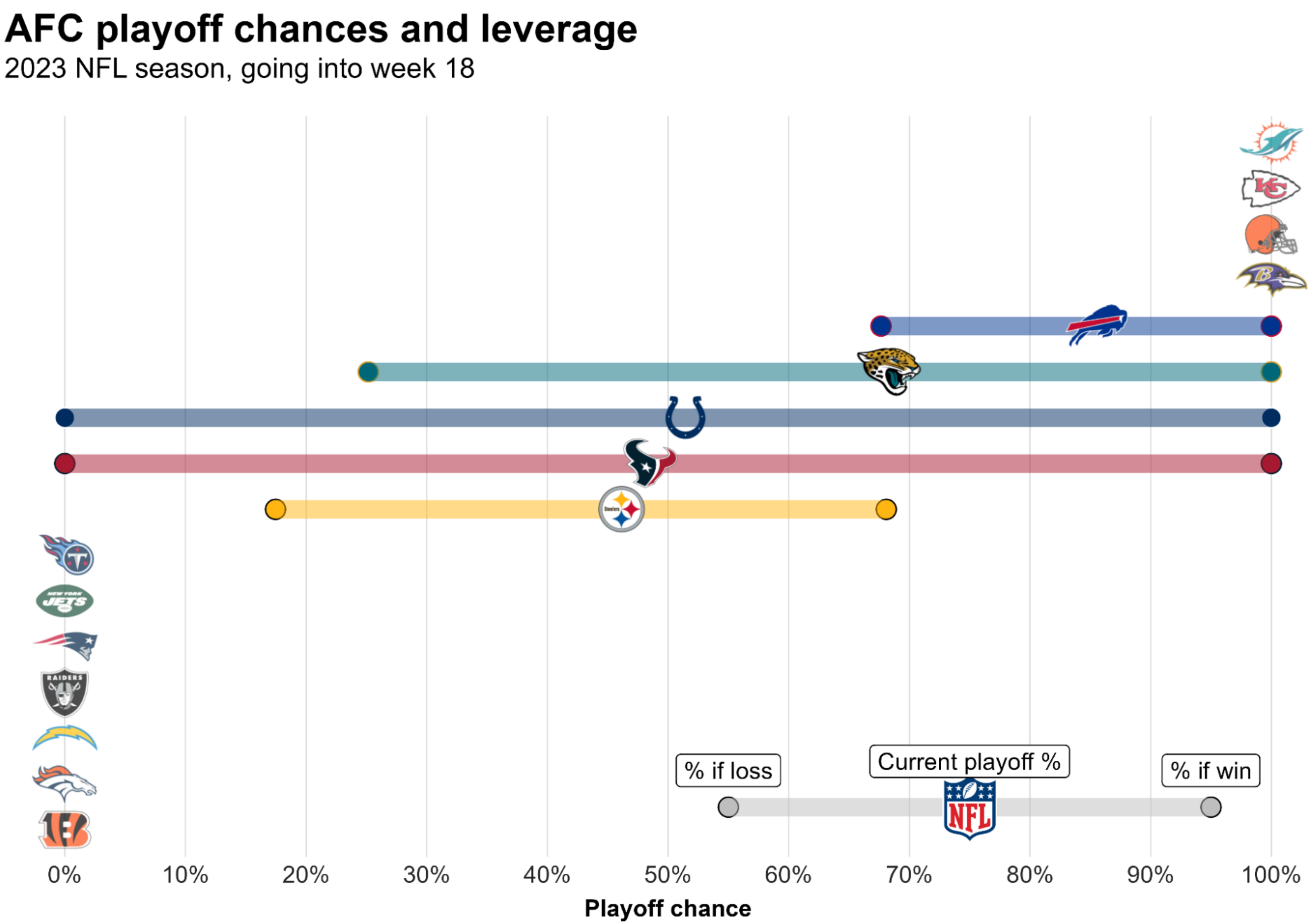
- Your assessment about the likelihood of future outcomes changes as events unfold

- How can companies “update their priors”?



- Then, after these events, your assessment of future market demand may change; how it changes depends on the unknown outcome of these events

# Bayes' rule



AFC playoff chances and leverage
2023 NFL season, going into week 18
% if loss
Current playoff %
% if win
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Playoff chance

# Bayes' rule

- A and B are two possible outcomes, or "events"
  - $P$[A | B]: The chance of observing A given that B happens
  - $P$[A & B]: The chance of observing A and B
- $P[A \& B] = P[A \mid B]\, P[B] = P[B \mid A]\, P[A]$
- (If A and B are independent: $P[A \mid B] = P[A]$, $P[B \mid A] = P[B]$)
- Rearranging $P[A \mid B]\, P[B] = P[B \mid A]\, P[A]$
- **Bayes' rule:** $P[A \mid B] = P[B \mid A]\, P[A] / P[B]$
- **Law of Total Probability:** $P(B) = P[A \& B] + P[A^c \& B] = P[B \mid A]\, P[A] + P[B \mid A^c]\, P[A^c]$

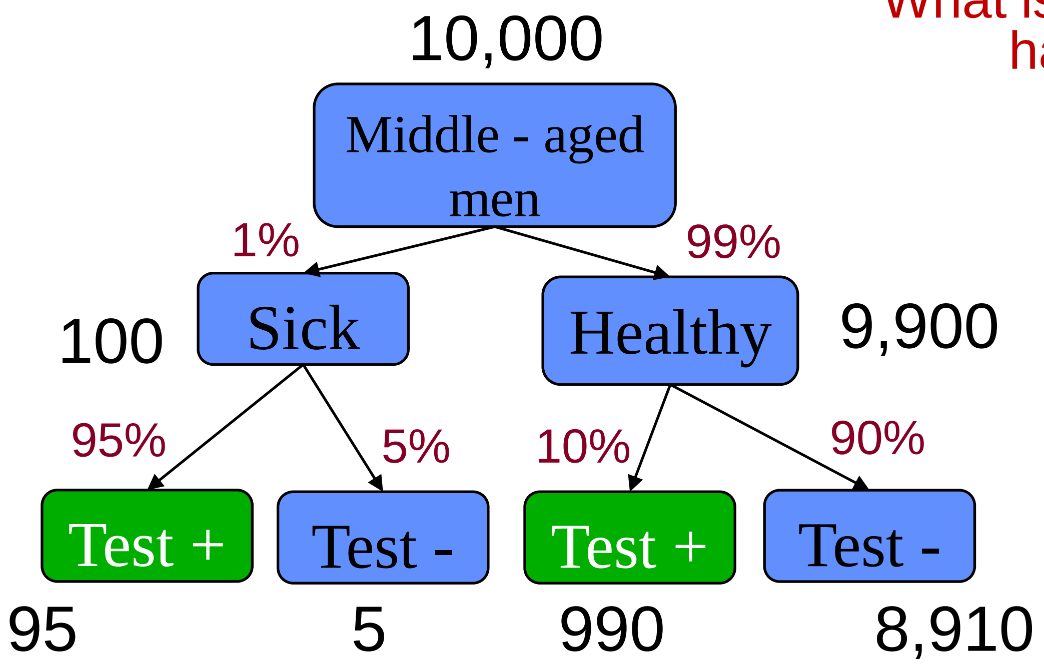
# The Disease example

A middle-aged man, Joe, has tested positive for a rare disease

Likelihoods (prior: 1% of the population is sick )

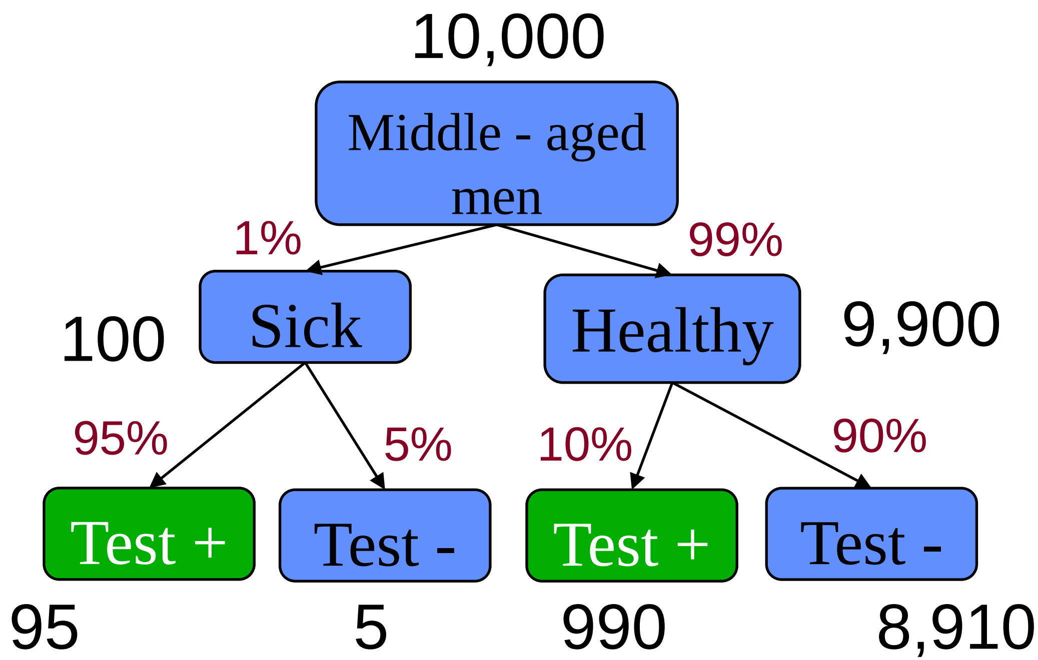
| | Positive | Negative |
|---|---|---|
| Sick | 95% | 5% |
| Healthy | 10% | 90% |

What is the chance that Joe has the disease?



# The Disease example

A middle-aged man, Joe, has tested positive for a rare disease




What is the chance that Joe has the disease?

P(sick | Positive) =

95/(95+990) = 8.75%

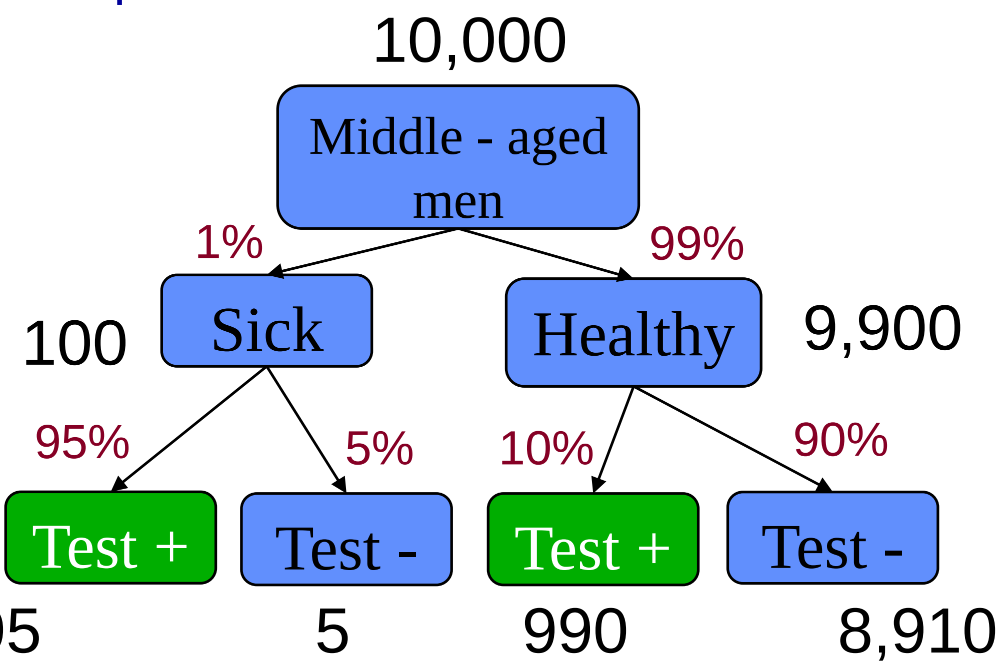
What is the chance of testing positive for an average individual?

P(positive) =

(990 + 95)/10,000 = 10.9%

# The Disease example

A middle-aged man, Joe, has tested positive for a rare disease




Likelihoods (prior: 1% of the population is sick )

| | Positive | Negative |
|---|---|---|
| Sick | 95% | 5% |
| Healthy | 10% | 90% |

**Bayes' rule computation**

P(positive)

= P(positive | sick) * P(sick) + P(positive | healthy) * P(healthy)

= (0.95)(0.01) + (0.1)(0.99)

= 0.109

P(sick | positive)

= P(positive | sick) * P(sick) / P(positive) = 0.95 * 0.01 / 0.109 = 8.75%

# Example 9.3

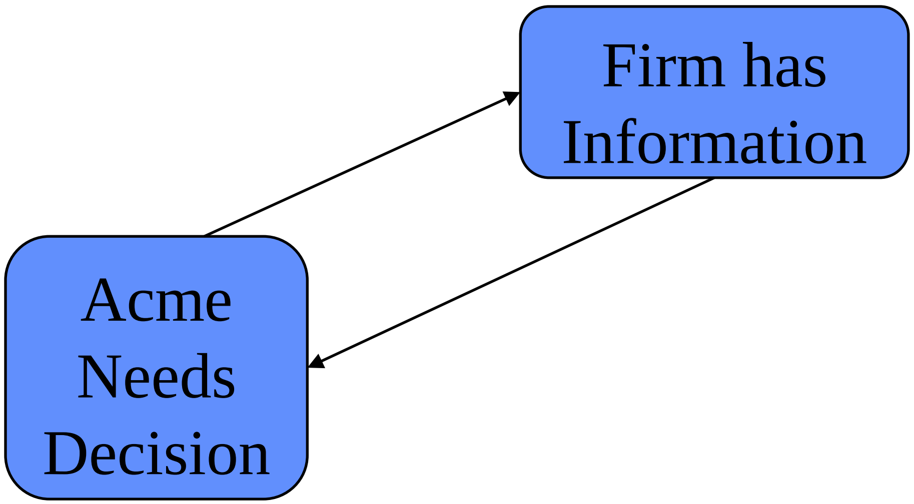
- **Product development continued**
  - Suppose that fixed development costs are no longer an issue (ignore the $4M), and technological failure is no longer a possibility
  - If we decide to market the product, there will be a fixed cost of $2M. There are two possible market outcomes, good or bad (demand 600,000 and 100,000 units, respectively), with probabilities 0.4 and 0.6, respectively.
  - The firm has the option to hire a well-respected marketing research firm for $0.15M, and act upon the outcome of market research conducted by this firm.

**Prediction Accuracy of Marketing Research Firm**

| Actual/Predicted | Good | Bad |
|---|---|---|
| Good | 0.8 | 0.2 |
| Bad | 0.3 | 0.7 |

# Example 9.3

- **Questions:**
  - Should Acme hire the firm?
  - What is the EMV for the decision tree?




- Relying on marketing research, the firm can obtain advanced predictions about the sales volume
- However, this information comes with a cost of 0.15M

✓ Practice: New Product Decisions – Information Option.xlsx

# Example 9.3

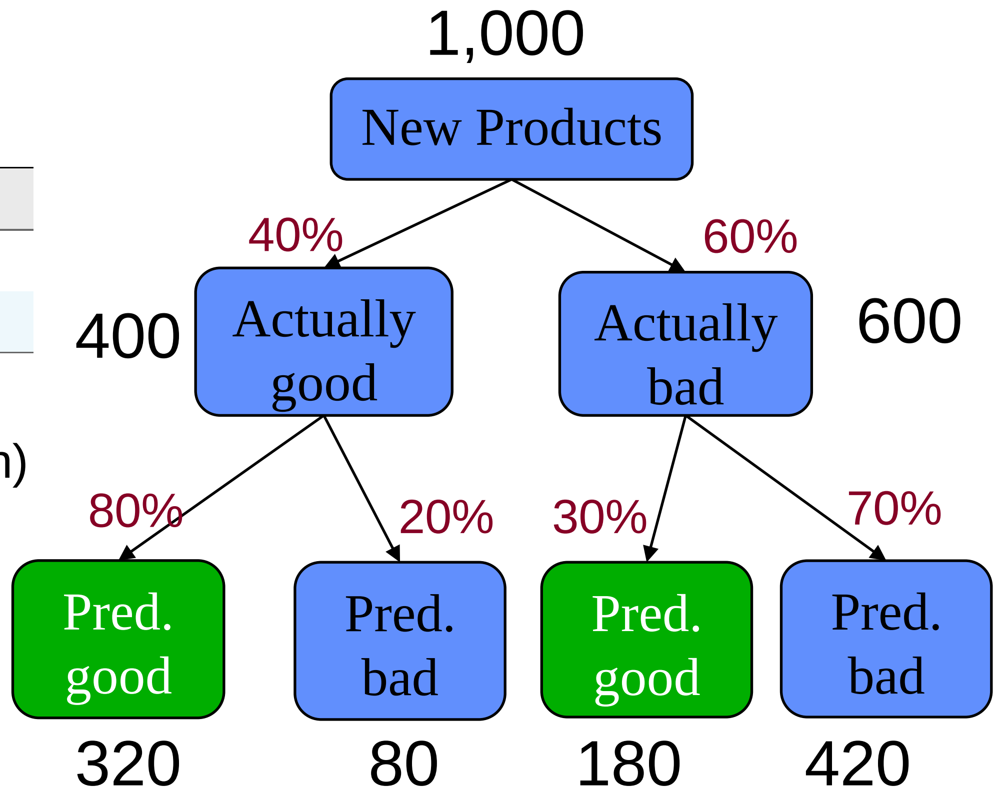
- Let us first get the tree structure in place and input all the numbers we have …
- Then, we need the probability of the market outcome given the prediction of the firm (for decision tree analysis)
- We get this by using the Bayes Rule to turn the probabilities of a prediction given an outcome (given)

# Example 9.3 Bayes' rule

*"Formal mechanism of updating probabilities as new information becomes available"*

Likelihoods (Prior: Good = 0.4)

| Actual/Predicted | Good | Bad |
|---|---|---|
| Good | 0.8 | 0.2 |
| Bad | 0.3 | 0.7 |




- P(Good market | good prediction)
  = 320/(320+180) = 64%
- P(Good market | bad prediction)
  = 80/(420+80) = 16%
- P(Bad market | good prediction)
  = 180/(320+180) = 36%
- P(Bad market | bad prediction)
  = 420/(420+80) = 84%

- P(Good prediction) = (320+180)/1,000 = 50%
- P(Bad prediction) = (80 +420)/1,000 = 50%

# Example 9.3 Bayes' rule

*"Formal mechanism of updating probabilities as new information becomes available"*

Likelihoods (Prior: Good = 0.4)

| Actual/Predicted | Good | Bad |
|---|---|---|
| Good | 0.8 | 0.2 |
| Bad | 0.3 | 0.7 |

- P(Good prediction) = (0.8*0.4 + 0.3*0.6) = 50%
- P(Bad prediction) = (0.2*0.4 + 0.7*0.6) = 50%

- P(Good market | good prediction) =

  (0.8) (0.4) / (0.5) = 64%

- P(Good market | bad prediction) =

  (0.2) (0.4) / (0.5) = 16%

- P(Bad market | good prediction) =

  (0.3) (0.6) / (0.5) = 36%

- P(Bad market | bad prediction) =

  (0.7) (0.6) / (0.5) = 84%

# Example 9.3

- **Questions:**
  - Should Acme hire the firm?
  - What is the EMV for the decision tree?
  - If Acme was forced to hire the firm, what would the decision be?
  - How does Acme influence the firm’s decision?

  - For information to be valuable, it needs to be actionable!

# Value of Information

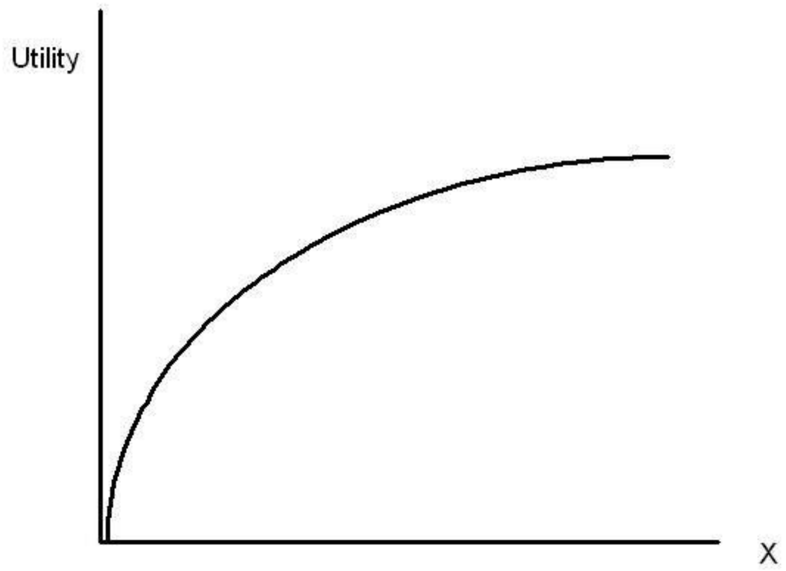
- **Expected value of information (EVI)** = EMV with (free) information – EMV without information
  - The maximum amount we are willing to pay for **this** information
  - It measures the worth of information
- **Expected value of perfect information (EVPI)** = EMV with (free) perfect information – EMV without information
  - The maximum amount we are willing to pay for **any** information
  - What is perfect information?
- Let us revisit Example 9.3 and assume the fixed marketing cost is $4M

# Utility Functions

- Does EMV maximization really make sense?
  - Gamble 1: win $20 (50%) and lose $10 (50%)
  - Gamble 2: win $200,000 (50%) and lose $100,000 (50%)
- Rational decision makers are willing to violate the EMV maximization criterion when large amounts of money are at stake.
  - In other words, most people are risk averse and are willing to sacrifice some EMV to avoid a risky gamble
- Most researchers believe that in general people are expected utility maximizers
  - Every extra dollar of payoff is worth less than the previous dollar and every dollar of cost is considered more costly
- Utility function is therefore increasing and concave

# Utility Functions

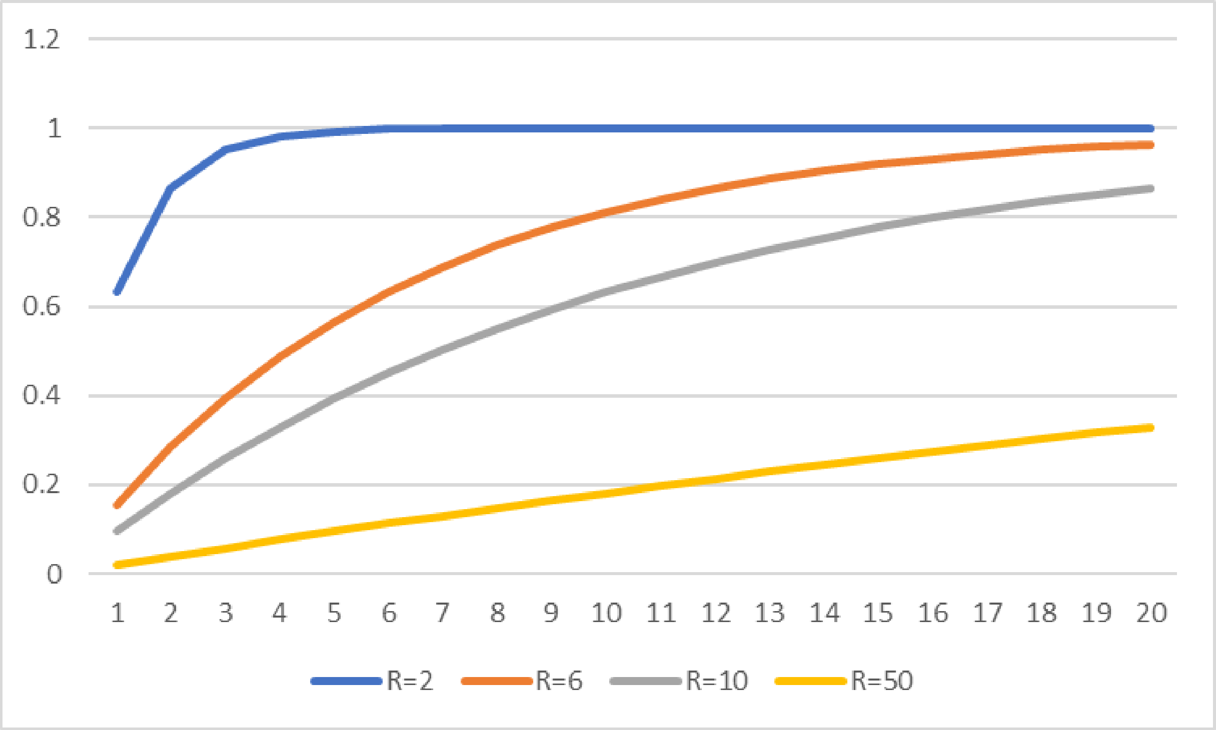
- Increasing and concave utility function




- Exponential utility function are typically used: $U(x) = 1 - e^{-x/R}$
  - R: risk tolerance parameter
  - The higher the risk tolerance, less risk averse
  - $U(0) = 0$

# What is the risk tolerance parameter?

Larger values of R tend to approximate EMV-maximizing behavior (i.e., less risk aversion)




It is the most risky bet that you are willing to take where you win \$R and lose \$R/2 with probability 0.5

# What is the certainty equivalent of a bet?

Ok, I'm out!

- The sure dollar amount that would give you the same utility as the expected utility from a bet

- Mathematically, $E[U(X)] = U(y)$, where $U(X)$ is the utility distribution of the payoffs and $y$ is the certainty equivalent to the gamble

W. P. Carey School of Business
Arizona State University

# Upcoming assignment

- Homework
  - 9.12: Sensitivity analysis (extension of example 9.2)
  - 9.22: Analysis using Bayes' rule – extending (modifying example 9.3)
  - 9.29: Utility maximization instead of EMV maximization

- Goal is to understand all of the assigned exercises, but
  - make sure to **upload your assigned problem before class**

# Certainty equivalent

- What is the certainty equivalent of a bet where I stand to make $300 with 50% probability and lose $200 with 50% probability? Assume my risk tolerance parameter is $1,000

# Bayes' rule example*

- "Money can't buy you happiness because a Harvard study found that only 10% of happy people are rich."

- Let's use Bayes' rule to consider whether this is a convincing argument. First, some other statistics:
  - 40% of people are happy
  - 5% of people are rich

- What % of rich people are happy, or in other words, what is the probability one is happy given rich?
  - P(happy | rich) = P(rich | happy) * P(happy)/P(rich) = 80%

*Example from Quora

# Utility Functions



## Leicester City fan cashes out £72k on Premier League bet

7 March 2016

BBC

The winner's bet would have been upped to £91,000 if he had held on a few hours longer

- “A Leicester City fan, who stood to win £250,000 from a £50 bet on his team winning the Premier League, has cashed out for £72,000.”
- “Odds on them winning the Premier League title are now 5-4, said Ladbrokes.”
- “The winner said: ‘It will mean so much if we win, so there's no point in being greedy.’”
- Jessica Bridge of Ladbrokes said: "It's a life-changing amount of money."

- **How much money did this fan leave on the table in terms of EMV?**

# Candidate exercises

- I am considering investing $100,000 to the stock market. If I do not do this, I also have the option of investing it into a bank to obtain $2,000 as the interest.
- If I invest my money on stock. With probability 0.5, I may earn $10,000 dollars. With probability 0.5, I may lose $10,000 dollars.
- What is the EVPI?

# Candidate exercises

- Problem 9.24
  - In the original OJ Simpson trial, it was accepted that OJ had battered his wife. OJ's lawyer tried to negate the impact of this information by stating that in a one-year period, only 1 out of 2500 battered women are murdered, so the fact that OJ battered his wife does not give much evidence that he was the murderer. The prosecution (foolishly!) let this go unchallenged. Here are the relevant statistics: In a typical year 6.25 million women are battered, 2500 are battered and murdered, and 2250 of the women who were battered and murdered were killed by the batterer. How should the prosecution have refuted the defense's argument?
  - Challenge is to find a **weakness in a point related** to probabilities made by OJ Simpson's defense in his murder trial