SEM 2 – 1 (RC 07-08)

F.E. (Semester – II) (RC 2007-08) Examination, May/June 2018
APPLIED MATHEMATICS – II

Duration : 3 Hours   Total Marks : 100

Instructions : 1) *Attempt **any five** questions, at least **one** from **each** Module.*
2) *Assume suitable data, **if** necessary.*

MODULE – I

1. a) Assuming the validity of differentiation under integral sign rule evaluate the following integrals : $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$. 7

   b) Find the length of astroid $x = a \cos^3 t$, $y = a \sin^3 t$. 6

   c) Find the area of lemniscate $r^2 = a^2 \cos 2\theta$. 7

2. a) Show that $\overline{r}(t) = A(e^{2t})i + B\,e^{-3t}j$ satisfies $\frac{d^2r}{dt^2} + \frac{dr}{dt} - 6r = 0$. 6

   b) For the following curve $\overline{r} = \cos ti + \sin tj + tk$, find curvature and torsion. 6

   c) The acceleration of a particle at any time is given by $a = e^{-t} i - 6(t+1)j + 3 \sin tk$. Find velocity V and displacement r at any time 't' given that V = 0 when t = 0 and r = 0 when t = 0. 8

MODULE – II

3. a) Evaluate the integral $\int_0^1 \int_0^x xx^2 + y^2 dxdy$. 6

   b) Change the order of integration and evaluate $\int_3^5 \int_0^{\frac{4}{x}} (xy) dydx$. 6

   c) Using double integration find the area bounded by the parabolas $x = y^2$, $x = 2y - y^2$. 8

4. a) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\,dz\,dx\,dy$. 6

   b) Find the volume of the solid under the plane $3x + 2y + z = 12$ and above the rectangle $R = (x, y)/0 \leq x \leq 1, -2 \leq y \leq 3$ by the planes $x = 0, y = 0, z = 0, x + y + z = a$. 6

   c) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a\cos\theta} \int_0^{\sqrt{a^2 - r^2}} r dz dr d\theta$. 8

MODULE – III

5. a) Find the maximum directional derivative of $xy(x - y + 2z)$ at $(1, 1, 0)$. 7

   b) If $\bar{F} = (4x + 3y + az)i + (bx - y + z)j + (2x + cy + z)k$ is irrotational, find constants a, b, c. 6

   c) Find the work done in moving a particle from $A(1, 0, 1)$ to $B(2, 1, 2)$ along a straight line AB in the force field $\bar{F} = x^2 i + (x - y)j + (y + z)k$. 7

6. a) Verify Greens theorem in plane for $\oint \left[\left(\frac{1}{y}\right)dx + \left(\frac{1}{x}\right)dy\right]$ where C is the region bounded by $x = 1$, $x = 4$, $y = \sqrt{x}$. 8

   b) Verify Stoke's theorem for $\bar{F} = (x^2 - y^2)i + 2xyj$ in the xy plane and region bounded by $y = 0$, $x = 2$, $y = x$. 12

MODULE – IV

7. Solve the following differential equations.

   a) $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$. 5

   b) $x\sin\left(\frac{y}{x}\right)dy = \left[y\sin\left(\frac{y}{x}\right) - x\right]dx$. 5

   c) $\frac{dy}{dx} = \frac{2y - x - 4}{y - 3x + 3}$. 5

   d) $\frac{dy}{dx} + \tan x = \cos y \cos^3 x$. 5

8. Solve the following differential equations.

   a) $\frac{d^2y}{dx^2} + 9y = x\cos 2x$. 5

   b) $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = x^2$. 5

   c) $(D^2 - 3D + 2)y = e^{4x} + \sin 3x + x^2$. 5

   d) $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$. 5