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B - 732 (Pages : 04) **SEM 1-1 (RC 07-08)**

[Total No. of Questions : 8]

F.E. (Semester - I) (Revised 2007-08 Course) Examination, Nov./Dec. - 2011

APPLIED MATHEMATICS - I

Duration : 3 Hours **Total Marks : 100**

Instructions : 1) Attempt any five questions, at least one from each Module.

2) Assume suitable data, if necessary.

MODULE - I

Q1) a) $\int_0^1 x^{n-1}\left[\log_e\left(\frac{1}{x}\right)\right]^{m-1} dx = \frac{1}{n^m}\sqrt{m}$ [5]

b) Evaluate $\int_0^1 \frac{\mathrm{P}^{a-1}+\mathrm{P}^{b-1}}{(1+\mathrm{P})^{a+b}}\,dp$. [5]

c) Prove that $\int_0^1 \frac{x^3-2x^4+x^5}{(1+x)^7}\,dx = \frac{7}{60}$. [6]

d) S.T. $erf\ (x) = \frac{2}{\sqrt{\pi}}\left[x - \frac{x^3}{3} + \frac{1}{2!}\frac{x^5}{5} - \frac{1}{3!}\frac{x^7}{7} + - - - - - -\right]$. [4]

Q2) a) Test the convergence of the following series [12]

i) $\Sigma \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$.

ii) $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + - - - -$

iii) $2 - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + - - - -$

P.T.O.

b) Define absolutely convergent series and conditionally convergent series. Find out the type of the following series. **[4]**

$$5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+----$$

c) State and prove Leibnitz's Rule for convergence of on alternating series. **[4]**

MODULE - II

Q3) a) Use De Moivre's theorem to solve the equation $x^4 - x^3 + x^2 - x + 1 = 0$. **[4]**

b) If $u = \log_e \tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$. **[4]**

Prove that $\tan h\frac{u}{2} = \tan\frac{6}{2}$.

c) If $\sin(\theta + i\,\phi) = \tan\alpha + i\sec\alpha$ show that $\cos 2\theta \cos h\, 2\,\phi = 3$. **[6]**

d) Considering the principal value only prove that the real part of $\left(1+i\sqrt{3}\right)^{1+i\sqrt{3}}$ is

$2\,e^{-\pi/\sqrt{3}} \cos\left(\frac{\pi}{3}+\sqrt{3}\log 2\right)$. **[6]**

Q4) a) Prove that $\cos h^{-1}\sqrt{1+x^2} = \tan h^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$. **[6]**

b) Find p and q if the following function is analytic **[6]**

$f(z) = \cos x\,(\cos h\,y + p\sin h\,y) + i\sin x\,(\cos h\,y + q\sin h\,y)$

c) If $f(z) = u + iv$ is analytic function then find $f(z)$ in terms of z if **[8]**

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - 2\cosh y}$$

MODULE - III

Q5) a) If $y = \left[\log\left(x+\sqrt{x^2+a^2}\right)\right]^2$ show that $(x^2 + a^2)\,y_{n+2} + (2n+1)\,x\,y_{n+1} + n^2 y_n = 0$

Hence deduce $y_n(0)$ **[9]**

b) Prove that **[6]**

$$\log(1+\sin x) = x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 - - - - -$$

c) Expand the polynomial **[5]**

$f(x) = x^5 + 2x^4 - x^2 + x + 1$

in powers of $(x + 1)$.

Q6) a) If $u = f(x, y)$, where $x = e^r \cos\theta$ $y = e^r \sin\theta$, then show that **[8]**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2r}\left[\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial \theta^2}\right].$$

b) Evaluate **[12]**

i) $\lim_{x\to 0} \frac{\cos hx - \cos x}{x \sin x}$

ii) $\lim_{x\to 0} (\sin x)^{\tan x}$

iii) $\lim_{x\to 0} \left(\frac{1}{x} - \operatorname{cosec}^2 x\right)$

MODULE - IV

Q7) a) Examine the function **[7]**

$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ for extreme values.

b) Find the area of a greatest rectangle that can be inscribed in an ellipse **[7]**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

c) If $u = \sin^{-1}\left(x^3 + y^3\right)^{2/5}$ **[6]**

Find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$.

Q8) a) Form a partial differential equation by eliminating arbitrary constant **[8]**

i) $ax^2 + by^2 + z^2 = 1$

ii) $(x-h)^2 + (y-k)^2 + z^2 = a^2$.

b) Solve the following partial differential equation **[6]**

$(y + zx)\, p - (x + yz)\, q = x^2 - y^2$.

c) Solve the following partial differential equation **[6]**

$p\,(1 + q^2) = q\,(z - a)$.

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