

SEM 1 – 1 (RC 16-17)

**F.E. (Semester – I) (Revised in 2016 – 2017)**  
**Examination, November/December 2017**  
**ENGINEERING MATHEMATICS – I**

Duration : 3 Hours

Max. Marks : 100

- Instructions :** 1) Answer **five** questions, atleast **two** from Part – A, **two** from Part – B and **one** question from Part – C.  
 2) Assume suitable data, if **necessary**.  
 3) Figures to **right** indicate **full** marks.

**PART – A**

1. a) Prove that  $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ . 6

b) Prove that  $\int_0^1 x^3(1-\sqrt{x})^5 dx = \frac{1}{5148}$ . 6

c) If  $(1+i)^{x+iy} = a+ib$  then considering the principal value only prove that

$2 \tan^{-1}\left(\frac{b}{a}\right) = \frac{\pi}{2}x + y \log 2$  8

2. a) Test the nature of the following series.

i)  $\sum_{n=1}^{\infty} \left(2 + \frac{1}{n}\right)^n$

ii)  $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

iii)  $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{4n-3}$ . 12

b) Find the value of P such that  $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1}\left(\frac{Px}{y}\right)$  is an analytic function. 4

c) Find the analytic function whose real part is  $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ . 4

P.T.O.





3. a) Evaluate  $\int_0^{\infty} \frac{dx}{2^{3x^2}}$ . 5
- b) Prove that  $n \beta (m+1, n) = m \beta (m, n+1)$ . 5
- c) If  $\tan\left(\frac{\pi}{6} + i\alpha\right) = x + iy$  prove that  $x^2 + y^2 + \frac{2x}{\sqrt{3}} = 1$ . 5
- d) Prove that  $\sinh^{-1} x = \frac{1}{2} \operatorname{cosech}^{-1} \left( \frac{1}{2x(1+x^2)^{1/2}} \right)$ . 5

## PART – B

4. a) If  $y = e^{\tan^{-1}x}$  show that  $(1+x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$ . 7
- b) Use Taylor's theorem to expand  $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$  in the powers of  $(x-1)$ . 6

- c) If  $u = x^3 \sin^{-1}\left(\frac{y}{x}\right) + x^4 \tan^{-1}\left(\frac{y}{x}\right)$  find the value of

$$\frac{x^2 \partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}.$$

5. a) Evaluate :

i)  $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$

ii)  $\lim_{x \rightarrow a} \log\left(2 - \frac{x}{a}\right) \cot(x-a)$

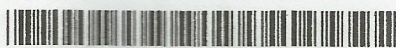
iii)  $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$ .

- b) Solve the partial differential equations.

i)  $y^2 z p + x^2 z q = xy^2$

ii)  $(z-y)p + (x-z)q = (y-x)$  where  $p = \frac{\partial z}{\partial x}$  and  $q = \frac{\partial z}{\partial y}$ .





6. a) If  $z = f(x, y)$

$$x = u + v, y = u.v$$

$$\text{Prove that } \frac{\partial^2 z}{\partial x^2} - y \frac{\partial^2 z}{\partial y^2} = \frac{1}{u-v} \left[ \frac{u \partial^2 z}{\partial u^2} - \frac{v \partial^2 z}{\partial v^2} \right].$$

8

b) Form the partial differential equation by eliminating function 'f'

$$z = e^{ay} f(x + by).$$

6

c) Use method of lagrange multipliers to find the largest product of the numbers  $x, y$  and  $z$  when  $x^2 + y^2 + z^2 = 9$ .

6

### PART – C

7. a) Prove that  $\operatorname{erf}_c(x) + \operatorname{erf}_c(-x) = 2$ .

5

$$\text{b) Prove that } \sin^{-1}(\operatorname{cosec} \theta) = \frac{\pi}{2} + i \log \left( \cot \frac{\theta}{2} \right).$$

5

$$\text{c) Prove that } \sqrt{1 + \sin x} = 1 + \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{48} + \frac{x^4}{384} + \dots \infty.$$

5

d) Determine and classify extreme points of  $f(x, y) = y^2 + 4xy + 3x^2 + x^3$ .

5

$$\text{8. a) Prove that } \int_0^1 \frac{x^2}{\sqrt{1-x^2}} dx \int_0^1 \frac{dx}{(1+x^4)^{1/2}} = \frac{\pi}{4\sqrt{2}}.$$

6

b) Use De Moivre's theorem to solve  $x^4 + 1 = 0$ .

4

c) Use Taylor's series expansion to find the approximate value of  $\sqrt{25.15}$ .

5

d) Solve the partial differential equation

$$(z^2 - 2yz - y^2) p + (xy + zx) q = xy - zx \text{ where } p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}.$$

5