F.E. (Semester – I) (RC 2007-08) Examination, Nov./Dec. 2016
APPLIED MATHEMATICS – I

Duration : 3 Hours   Total Marks : 100

Instructions : *1) Attempt **any five** questions, atleast **one** from **each** Module.*
*2) **Assume** suitable data, if **necessary**.*

MODULE – I

1. a) Evaluate $\int_0^1 x^6(1-x^2)^{\frac{1}{2}}dx$. **5**

b) Evaluate $\int_0^1 (x \log x)^3 .dx$. **5**

c) Show that $B(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}$. **5**

d) Prove that $\frac{d}{dx}[\text{erf}(ax)] = \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$ **5**

2. a) Test the convergence of the following series.

i) $\sum_{n=1}^{\infty}\left(\frac{n!}{n^n}\right)^n$. **4**

ii) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt{n+1}}$. **4**

iii) $2-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\ldots$ to ∞. **4**

b) Define interval of convergence of power series and hence find it for the series.

$x-\frac{x^2}{\sqrt{2}}+\frac{x^3}{\sqrt{3}}-\frac{x^4}{\sqrt{4}}+\ldots$ **8**

MODULE – II

3. a) Use De Moivre's theorem and solve $x^9 - x^5 + x^4 - 1 = 0$. 6

b) If $\sin(\alpha + i\beta) = x + iy$ prove that :

i) $\dfrac{x^2}{\cosh^2\beta} + \dfrac{y^2}{\sinh^2\beta} = 1$

ii) $\dfrac{x^2}{\sin^2\alpha} - \dfrac{y^2}{\cos^2\alpha} = 1$. 8

c) Separate into real and imaginary part $\sqrt{i}^{\sqrt{i}}$. 6

4. a) Determine the analytic function whose real part is $\cos x \cosh y$. 6

b) Show that the function $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. Find the conjugate harmonic function. 6

c) Show that $i^{i} = \cos\theta + i\sin\theta$; $\theta = \left(2n + \dfrac{1}{2}\right)\pi e^{-\left(2n+\frac{1}{2}\right)\pi}$. 8

MODULE – III

5. a) If $y = \log\left(x + \sqrt{x^2 + p^2}\right)^2$, prove that,

$(x^2 + p^2)y_{n+2} + (2n + 1)\,x\,y_{n+1} + n^2 y_n = 0$. 7

b) Expand $\log(1 + e^x)$ in powers of x. Find the first 5 terms. 6

c) Use Taylor's theorem to show that $\sqrt{1 + x + 2x^2} = 1 + \dfrac{x}{2} + \dfrac{7}{8}x^2 - \dfrac{7}{16}x^3 + \ldots$. 7

6. a) Evaluate : 12

i) $\lim_{x\to a} \dfrac{\log(x-a)}{\log(e^x - e^a)}$ ii) $\lim_{x\to a}\left(2 - \dfrac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ iii) $\lim_{x\to 0} \dfrac{e^x - e^{\sin x}}{x - \sin x}$

b) If $x = p\cos\theta - q\sin\theta$, $y = p\sin\theta + q\cos\theta$ and u is function of x and y, then show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial p^2} + \frac{\partial^2 u}{\partial q^2}.$$ 8

MODULE – IV

7. a) Form the partial differential equations by eliminating constants

   i) $z = (x - a)(y - b)$.

   ii) $(x - h)^2 + (y - k)^2 + z^2 = a^2$. **6**

   b) Form the partial differential equations by eliminating functions $z = f(z^2 - xy, x/z)$. **6**

   c) Solve $x^2p^2 + y^2q^2 = z^2$. **8**

8. a) If $u = \sec^{-1}\left[\frac{\sqrt{x} - 2\sqrt{y}}{y^3\sqrt{x}}\right]$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. **6**

   b) Obtain the maxima and minima of the function $x^3 + y^3 - 63(x + y)$. **6**

   c) Use the method of Lagrange's multipliers to find the greatest and smallest values that the function $f(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$. **8**
