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F.E. Semester-II (Revised Course 2007-08)
EXAMINATION MARCH 2021
Applied Mathematics-II

[Duration : Two Hours] **[Total Marks : 60]**

Instruction:
1) Answer THREE FULL QUESTIONS with ONE QUESTION from ANY THREE MODULES.
2) Assume suitable data if necessary

Module -I

Q.1

a) Assuming the validity of differentiation under integral prove that $\int_0^{\infty} \frac{\log(1+\propto x^2)}{x^2} dx = \pi\sqrt{\propto}$ **7**

b) Find the length of the cardiode $r = a(1 + \cos\theta)$ lying outside $r = a$ **7**

c) Find the surface area of the surface generated by the revolution of $x = \frac{y^3}{3}; 0 \le y \le 1$ about the y-axis **6**

Q.2

a) Show that $\bar{r}(t) = Ae^{2t}\bar{\imath} + Be^{-3t}\bar{\jmath}$ satisfies $\frac{d\bar{r}^2}{dt^2} + \frac{d\bar{r}}{dt} - 6\bar{r} = 0$ **6**

b) State and prove serret – Fernet formula **7**

c) Find the principal normal N and binormal B of **7**

$$\bar{r}(t) = sint\hat{\imath} + (t+1)\hat{\jmath} + cost\hat{k}\ at\ t = \frac{\pi}{2}$$

Module –II

Q.3

a) Evaluate $\iint_R (x^2 - y^2)dxdy$ where R is the triangle whose vertices are (0,0) (1,0) and (0,1) **6**

b) Write the following as one double integral and evaluate **8**

$$\int_{-1}^{0}\int_{0}^{x+1} 2y + 3\,dxdy + \int_{0}^{1}\int_{0}^{1} 2y + 3dxdy$$

c) Evaluate $\iint_R rsin\theta + 3drdo$ over the region $\{1 \le r \le 2\cos\theta;\ 0 \le \theta \le \frac{\pi}{3}\}$ **6**

Q.4

a) Find the area bounded by $y = x^2 and\ y = 2x + 3$ **6**

b) Evaluate $\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^1 3r^3\cos^2\theta drd\theta d\emptyset$ **7**

c) Use triple integration to find the volume of the 3-D region bounded by $x^2 + y^2 = g\ Z =$ **7**

$0 \; and \; Z = 2$

Module –III

Q.5 a) Find the directional derivative of $f(x,y,Z) = xy^2 + xZ^3 \; at \; (2,-1,1)$ in the direction of $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ **5**

b) Find the unit normal to the surface $x^2 + y^2 + Z^2 = 25$ at the point (4,3,0) **5**

c) Define divergence of a vector field show that $\nabla.(\nabla f) = \nabla^2 f$ **3**

d) Show that $\bar{F} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$ is irrotational and hence find its scalar potential. Also find the line integral of $\bar{F}$ from (1,2) to (2,1) **7**

Q.6 a) Verify greens theorem in the plane for $\oint_c (xy + 1)dx + (4x^2)dy$ where C is the boundary of the region bounded by y=0 , x=1 and y=x . **10**

b) Verify stokes theorem for $\bar{F} = xy\,\hat{\imath} - 2yZ\hat{\jmath} - Z\hat{k}$ where S is the open surface bounded by x=0 , x=1, y=0 , y=2 and $Z = 3$ excluding the XY plane **10**

Module –IV

Q.7 a) Solve the following differential equations **5X4=20**

a) $e^y(1 + x^2)\frac{dy}{dx} - 2x(1 + e^y) = 0$

b) $\frac{dy}{dx} + \frac{y}{x} = x^3y^4$

c) $\frac{dy}{dx} = \frac{x+2y+1}{2x+4y+3}$

d) $(1 + xy)ydx + (1 - xy)xdy = 0$

Q.8 Solve the following differential equations **5X4=20**

a) $(D^2 + 2D + 1)y = e^{3x} + 5$

b) $(D^2 + 3D - 4)y = e^{-2x}Sin\,3x$

c) $(D^2 - D - 6)y = 3x^2 - 5x + 2$

d) $(D^2 + 1)y = tanx$