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F.E. Semester- I (Revised Course 2016-17)
EXAMINATION SEPTEMBER 2020
Engineering Mathematics-I

[Duration : Two Hours] **[Total Marks : 60]**

Instructions:
1) Answer THREE FULL QUESTIONS with ONE QUESTION FROM EACH PART.
2) Assume suitable data, if necessary.
3) Figure to right indicate full marks.

PART-A

Q.1 a) $\int_0^\infty 3^{-4x^2} dx$ (6)

b) Evaluate $\int_6^8 \sqrt[5]{(x-6)(8-x)}\, dx$ (5)

c) Find all values of $(1+i)^{\frac{2}{3}}$ (5)

d) Prove that $\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2}[\text{erf}(b) - \text{erf}(a)]$ (4)

Q.2 a) Test the convergence of the following series (12)

i) $\sum_{n=1}^{n=\infty} \frac{n!2^n}{n^n}$

ii) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$

iii) $1 - \frac{1}{5} + \frac{1}{9} - \frac{1}{13} + \cdots$

b) Determine 'p' such that the function $f(x,y) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{px}{y}\right)$ is an analytic function. (4)

c) Show that the function $u(x,y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is a harmonic function. (4)

Q.3 a) If $\frac{(1+i)^{x+iy}}{(1-i)^{x-iy}} = a + ib$, then considering the principal value only prove that $\tan^{-1}\left(\frac{b}{a}\right) = \frac{\pi}{2}x + y\log 2$ (6)

b) Considering the principal value only show that the real part of $i^{\log_e(1+i)}$ $is\ e^{-\frac{\pi^2}{8}} \cos\left(\frac{\pi}{4} log\ 2\right)$ (6)

c) Define absolutely convergent and conditionally convergent series. Hence test whether (8)

following series is absolutely convergent or conditionally convergent.
$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$

PART-B

Q.4 a) If $y = e^{\tan^{-1} x}$, prove that, (7)
$(1 + x^2)y_{n+2} + [2(n+1)x - 1]\, y_{n+1} + n(n+1)y_n = 0$

b) By Taylors Theorem expand $2x^3 + 3x^2 - 8x + 7$ in the powers of $(x - 2)$ (6)

c) If $u = \tan^{-1}\left(\frac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x-y}\right)$, (7)
Find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

Q.5 a) Evaluate : (12)
i) $\lim_{x\to 1} \frac{1+\log x - x}{1-2x+x^2}$

ii) $\lim_{x\to 0} (\cos x)^{\frac{1}{x^2}}$

iii) $\lim_{x\to 0} \left[\frac{1}{x} - \frac{1}{e^x - 1}\right]$

b) Form the partial differential equations by eliminating constants 'a' and 'b' (4)
$z = (x^2 + a)(y^2 + b)$

c) Form the partial differential equations by eliminating function (4)
$f(x^2 + y^2 + z^2, x + y + z) = 0$

Q.6 a) If $z = f(u, v)$ where $u = x - y, v = xy$ then show that (8)
$x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial y^2} = (x + y)\left(\frac{\partial^2 z}{\partial u^2} + xy\frac{\partial^2 z}{\partial v^2}\right)$

b) Solve the partial differential equation (6)
$(z - y)p + (x - z)q = (y - x)$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$

c) Use the method of Lagrange's Multipliers to find the maximum and minimum distance of the point (4,3) on the circle $x^2 + y^2 = 1$ (6)

PART-C

Q.7 a) State relation between beta and gamma functions and prove that $\Gamma\frac{1}{2} = \sqrt{\pi}$ (4)

b) If n is positive integer then prove that $\left(\sqrt{3}+i\right)^{n} + \left(\sqrt{3}-i\right)^{n} = 2^{n+1}\cos\frac{n\pi}{6}$ (5)

c) Expand $\log\,(1+\cos x)$ in the powers of x. Find the first four terms. (6)

d) Solve the partial differential equation (5)
$p\cot x + q\cot y = \cot z$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$

Q.8 a) Test the convergence of the following series (5)
$\sum_{n=1}^{n=\infty} \frac{4}{(4n-3)(4n+1)}$

b) Prove that $\cos\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\} = \frac{a^2-b^2}{a^2+b^2}$ (5)

c) Use Taylors series to find the approximate value of $\sqrt{25.15}$ (5)

d) Find the extreme values of the function (5)
$f(x,y) = x^2y - 3x^2 - 2y^2 - 4y + 3$