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F.E. (Sem-I) (Revised Course 2016-17)
EXAMINATION Nov/Dec 2019
Engineering Mathematics - I

[Duration : Three Hours]

[Total Marks : 100]

Instructions:

- 1) Attempt any two questions from part A, any two from part B and any one from part C.
- 2) Assume missing data, if any.

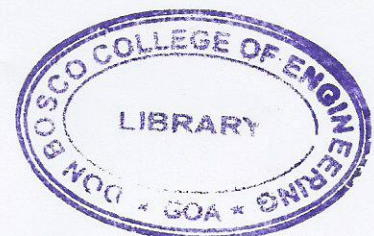
PART-A

(Answer Any Two Questions)

- Q.1**
- a) Prove that $\int_0^{\infty} \frac{e^{-x^2}}{\sqrt{x}} dx \int_0^{\infty} x^2 e^{-x^4} dx = \frac{\pi}{4\sqrt{2}}$ (6)
 - b) Prove that $\operatorname{erf}(x) + \operatorname{erfc}(x) = 1$. (4)
 - c) Use DeMoivre's theorem to evaluate all the values of $i^{\frac{1}{4}}$ (5)
 - d) Evaluate $\int_0^{\infty} \frac{x^3(1-x^4)}{(1+x)^{11}} dx$ using a suitable form of the Beta function. (5)
- Q.2**
- a) Test the convergence of the following series (12)
 - (i) $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$
 - (ii) $\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \frac{4^2 \cdot 5^2}{4!} + \dots$
 - (iii) $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots$
 - b) If 'n' is a positive integer then prove that (4)

$$(\sqrt{3} + i)^n + (\sqrt{3} - i)^n = 2^{n+1} \cos\left(\frac{n\pi}{6}\right)$$
 - c) Determine the values of a, b, c, d so that the function. (4)

$$f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$$
 is analytic



Q.3

a) Prove that $i \log \left(\frac{x-i}{x+i} \right) = \pi - 2 \tan^{-1} x$ (6)

b) Prove $\sinh^{-1} x = \frac{1}{2} \operatorname{cosech}^{-1} \frac{1}{2x(1+x^2)^{1/2}}$ (6)

c) Find the radius and interval of convergence for the power series (8)

$$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \dots \infty$$

PART-B

(Answer Any Two Questions)

Q.4

a) If $u = \cos^{-1} \left(\frac{2x^4 - y^4}{x - y} \right)$ then evaluate $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy}$ (6)

b) If $y = (\sin^{-1} x)^2$ then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$ (7)

c) Apply Taylor's series expansion formula to find the approximate value of $\sqrt[3]{8.01}$ (7)

Q.5

a) Evaluate (12)

(i) $\lim_{x \rightarrow 0} \log_{\sin x} (\sin 2x)$

(ii) $\lim_{x \rightarrow 0} \frac{2 \cos x - 2 + x^2}{x^4}$

(iii) $\lim_{x \rightarrow 0} (\cos x)^{1/x^2}$

b) Form a partial differential equation by eliminating a and b from (4)

$$z = (x - a)^2 + (y - b)^2$$

c) Form a partial differential equation by eliminating the arbitrary function from (4)

$$z = e^{ay} f(x + by)$$

Q.6

a) If $z = f(u, v)$ where $u = lx + my$, $v = ly - mx$; l and m being constants, then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right)$ (8)

b) Solve the partial differential equation $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$ (6)

c) Find and classify the critical points of $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ (6)

PART-C
(Answer Any One Question)

- Q.7**
- a) Evaluate $\int_0^1 x(\log x)^6 dx$ (6)
- b) Prove (8)
- (i) $\sin\{i \log(\frac{a-ib}{a+ib})\} = \frac{2ab}{a^2+b^2}$
- (ii) $\cos\{i \log(\frac{a-ib}{a+ib})\} = \frac{a^2-b^2}{a^2+b^2}$
- c) Prove that $\log(1 - \log(1 - x)) = x + \frac{x^3}{6} + \dots$ (6)
- Q.8**
- a) Evaluate $\int_0^1 x^3(1 - \sqrt{x})^5 dx$ using Beta function (6)
- b) Use Lagrange's method to find the maximum distance of the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$ (7)
- c) If $u = \frac{x^3 y^3}{e^{x^2+y^2}}$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 4u \log(u)$ (7)