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F. E. Semester-II (Revised Course 2007-2008) EXAMINATION MAY/JUNE 2019
Applied Mathematics-II

[Duration : Three Hours]

[Max. Marks : 100]

Please check whether you have got the right question paper.

Instructions :

1. Attempt **any five** questions, at least one from each module.
2. Assume suitable data if necessary

MODULE-I

- Q.1
- a) Evaluate $\int_0^\infty \frac{\log_e(1+at^2)dt}{t^2}$ by applying differentiation under the integral sign. 08
 - b) Find the length of $y = 2x^{3/2} + 1$ $1 \leq x \leq 3$ 06
 - c) Find the perimeter of the cardioid $r = a(1 + \cos \theta)$ 06
- Q.2
- a) Find the unit tangent vector, and principal normal of the space curve $\vec{r}(t) = 2t^3\vec{i} + 2t\vec{j} + (t+2)\vec{k}$ at $t=1$. 06
 - b) Define Curvature and Torsion of a space curve and prove Serret – Frenet formula. 08
 - c) Solve $\frac{d\vec{r}^2}{dt^2} = 2\vec{i} - 3\vec{k}$, $\vec{r}(0) = 2\vec{i} - \vec{j}$ and $\frac{d\vec{r}}{dt}|_{t=0} = \vec{k}$ 06

MODULE-II

- Q.3
- a) Evaluate $\int_0^1 \int_y^1 2x + 1 dx dy$. 05
 - b) Change the order of integration and evaluate $\int_0^1 \int_{x^2}^x y + 3 dx dy$ 08
 - c) Evaluate $\int \int r^2 + 3 \sin \theta dr d\theta$ over the region $r \leq 1$ above the initial line. 07
- Q.4
- a) Find by double integration the volume of the solid generated by the revolution of the region $y^2 \leq x$ and $x \leq 1$ about the x-axis. 06
 - b) Evaluate $\int_{-1}^1 \int_0^x \int_0^{(x+y)} 2x dz dy dx$ 06
 - c) Find the volume of the region $\{(x, y, z)/x^2 + y^2 \leq 4, 0 \leq z \leq 1, y \geq 0\}$. 08

MODULE-III

- Q.5 a) If $\vec{f}(t)$ is a vector field having constant magnitude show that $\vec{f}(t) \cdot \frac{d}{dt}(\vec{f}(t)) = 0$ 05
- b) Define Curl of a vector field. Show that if \vec{a} is any constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then show that $\text{curl}(\vec{a} \times \vec{r}) = 2\vec{a}$. 05
- c) In what direction is the directional derivative of $f(x, y, z) = x^2z + 2xy$ at the point $(-1, 2, 1)$ maximum and what is its magnitude? 05
- d) Show that the vector field $\vec{F} = (6xy + 3z)\vec{i} + 3x^2\vec{j} + (6z + 3x)\vec{k}$ is irrotational find its scalar potential. 05
- Q.6 a) Find the work done in moving a particle in a force field $\vec{F} = 3y^2\vec{i} + 2y\vec{j} + xz\vec{k}$ along the curve $x = 3t, y = 2t^2, z = 2t$ from $t = 0$ to $t = 1$. 06
- b) Verify Green theorem in the plane for $\oint_C (x + 3y^2)dx + (x^2 + 2y)dy$ where C is the boundary of the region bounded by $y=x, x=1$ and $y=0$ 08
- c) Use Stoke's theorem to evaluate $\int_S \nabla \times \vec{F} \cdot \vec{n} ds$ where $\vec{F} = x^2z\vec{i} + x^2\vec{j} + y^2\vec{k}$ \vec{n} is the unit normal vector to S , the surface of the region bounded by $x = 0, y = 0, z = 0$ and $x + y + z = 1$ excluding the surface in the xy plane 06

MODULE-IV

- Q.7 Solve the following differential equations. 20
- a) $\frac{dy}{dx} = e^{x-2y} + xe^{-2y}$
- b) $(x + 2y \cos x)dx + (y + 2 \sin x)dy = 0$
- c) $\frac{dy}{dx} = \frac{2y-x-4}{y-3x+3}$
- d) $(1+x)\frac{dy}{dx} + y = x^2$
- Q.8 Solve the following differential equations 20
- a) $(D^2 + 3D + 2)y = 4e^{-3x}$
- b) $(D^2 + 4)y = 2 \cos^2 x$

c) $(D^2 + 4D + 4)y = 2\sinh 2x$

d) $(x + 3)^2 \frac{d^2 y}{dx^2} - 4(x + 3) \frac{dy}{dx} + 6y = x$