SEM 1 – 1 (RC 07 – 08)

F.E. (Semester – I) (Revised Course 2007-08) Examination, Nov./Dec. 2014
APPLIED MATHEMATICS – I

Duration : 3 Hours Total Marks : 100

Instructions : *1) Attempt **any five** questions, at least **one** from **each** Module.*
*2) **Assume** suitable data, **if** necessary.*

MODULE – I

1. a) Show that $\int_0^{\infty} x^n e^{-a^2x^2} dx = \frac{1}{2a^{n+1}} \Gamma\left(\frac{n+1}{2}\right)$ and hence deduce that $\int_0^{\infty} e^{-a^2x^2} dx = \frac{\sqrt{\pi}}{2a}$. 6

b) Evaluate $\int_0^{\infty} e^{-\sqrt{x}} \sqrt[4]{x} dx$. 4

c) Prove that $\int_0^1 \left(1 - x^{1/n}\right)^m dx = \frac{m!n!}{(m+n)!}$. 5

d) Show that $erf_c(x) + erf_c(-x) = 2$. 5

2. a) State Cauchy's Root test for the convergence of a series. 2

b) When is a series said to be absolutely convergent ? Give an example of absolutely convergent series. Also show that every absolutely convergent series is convergent. 6

c) Test the convergence of the following series : 12

i) $\sum_{n=0}^{\infty} \frac{2^n n!}{n^n}$

ii) $\sum_{n=1}^{\infty} \left(2 + \frac{1}{\sqrt{n}}\right)^{-n}$

iii) $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \ldots$

P.T.O.

MODULE – II

3. a) If $\tan(\theta+\phi) = e^{i\alpha}$. Prove that $\theta = \frac{n\pi}{2}+\frac{\pi}{4}$ and $\phi\ \frac{1}{2}\log_e \tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$. **6**

b) If $\tan\left(\frac{x}{2}\right) = \tanh\left(\frac{u}{2}\right)$. Prove that $u = \log_e \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$. **4**

c) Express $\sin 5\theta$ in terms of multiples of $\sin\theta$. **5**

d) Prove that $\cos^{-1} x = \log_e\left(x+\sqrt{x^2-1}\right)$. **5**

4. a) Prove that $\log_e(-\log_e i) = \log_e \frac{\pi}{2} - i\frac{\pi}{2}$. **5**

b) Considering principal value only, separate $\left(\sqrt{i}\right)^{\sqrt{i}}$ into real and imaginary parts. **5**

c) Solve the equation $x^5 + 1 = 0$. **5**

d) Show that $f(z) = \log_e z$ is analytic everywhere in the complex plane except at origin and find its derivative. **5**

MODULE – III

5. a) If $y = a\cos(\log_e x) + b\sin(\log_e x)$ then show that $x^2 y_{n+2} + 2(n+1)xy_{n+1} + (n^2+1)y_n = 0$. **7**

b) Show that $\log_e(1-\log_e(1-x)) = x + \frac{x^3}{6} + \ldots$ **7**

c) Use Taylors theorem to express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x-2)$. **6**

6. Evaluate :

a) i) $\lim_{x\to 1}\left(1-x^2\right)^{1/\log_e(1-x)}$

ii) $\lim_{x\to a}\log_e\left(2-\frac{x}{a}\right)\cot(x-a)$

iii) $\lim_{x\to 0}\left[\frac{1}{x^2}-\frac{1}{x\tan x}\right]$. **12**

b) If z= f(u, v), where $u = x^2 - y^2$ and $v = 2xy$, prove that

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=4(x^2+y^2)\left(\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}\right).$$ 8

MODULE – IV

7. a) Form the partial differential equation eliminating the arbitrary functions

   i) $z = f(x^2 + y^2) + x + y$

   ii) $f(x + y + z, x^2 + y^2 + z^2) = 0.$ 8

   b) Solve the partial differential equation

   i) $x(z^2 - y^2)\frac{\partial z}{\partial x} + y(x^2 - z^2)\frac{\partial z}{\partial y} = z(y^2 - x^2)$

   ii) $\frac{y^2 z}{x}\frac{\partial z}{\partial x} + xz\frac{\partial z}{\partial y} = y^2.$ 12

8. a) If $u = \sin^{-1}\left(\sqrt{x^2+y^2}\right)$, find the value of $x^2\frac{\partial^2 u}{\partial^2 x} + 2xy\frac{\partial^2 u}{\partial y \partial x} + y^2\frac{\partial^2 u}{\partial^2 y}$. 8

   b) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$. 6

   c) Examine the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ for extreme values. 6
