SEM 1 – 1 (RC 07-08)

F.E. (Semester – I) (Revised in 2007-08) Examination, Nov./Dec. 2015
APPLIED MATHEMATICS – I

Duration : 3 Hours

Total Marks : 100

***Instructions** : 1) Attempt **any five** questions, at least **one** from **each** Module.*
*2) Assume suitable data, **if necessary**.*

MODULE – I

1. a) Evaluate $\int_0^\infty \frac{e^{-x^3}}{\sqrt{x}}\,dx \cdot \int_0^\infty x^4 e^{-x^6}\,dx$. **5**

b) Prove that $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx = B(m, n)$. **5**

c) Evaluate $\int_5^9 \sqrt[4]{(x-5)(9-x)}\,dx$. **5**

d) Prove that error function is an odd function. **5**

2. a) Test the convergence of the following series.

i) $\sum_{n=2}^{\infty} \frac{1}{(\log n)^n}$. **4**

ii) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{2^n}$. **4**

iii) $\frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \ldots$ to ∞. **4**

b) Find the interval of convergence of the following series.

$\frac{x}{1^2} - \frac{x^2}{3^2} + \frac{x^3}{5^2} - \frac{x^4}{7^2} + \ldots$ **8**

P.T.O.

MODULE – II

3. a) Use De Moivre's Theorem and solve $x^7 - 1 = 0$. **6**

b) If $\cos(\theta + i\phi) = r(\cos\alpha + i\sin\alpha)$, prove that $\phi = \frac{1}{2}\log\frac{\sin(\theta-\alpha)}{\sin(\theta+\alpha)}$. **8**

c) If $\frac{(1+i)^{x+iy}}{(1-i)^{x-iy}} = a + ib$ then corresponding the principle values only show that

$\tan^{-1}(b/a) = \frac{\pi x}{2} + y \log 2$. **6**

4. a) Determine the analytic function whose real part is $\log\sqrt{x^2+y^2}$. **6**

b) Show that the function $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. Find the conjugate harmonic function. **6**

c) Show that $i^{i^{i}} = \cos\theta + i\sin\theta$; $\theta = \left(2n + \frac{1}{2}\right)\pi e^{-\left(2n+\frac{1}{2}\right)\pi}$. **8**

MODULE – III

5. a) If $y = e^{p\sin^{-1}x}$, prove that, $(1 - x^2)y_{n+2} - (2n+1)x\, y_{n+1} - (n^2 + p^2)y_n = 0$. **7**

b) Expand $\log(1 + \cos x)$ in powers of x. Find the first 5 terms. **6**

c) Use Taylor's theorem to expand $\tan^{-1} x$ in powers of $\left(x - \frac{\pi}{4}\right)$. **7**

6. a) Evaluate : **12**

i) $\lim_{x\to 0}\frac{\tan x - \sin x}{\sin^3 x}$

ii) $\lim_{x\to 0}\log_{\sin x}\sin 2x$

iii) $\lim_{x\to 0}(a^x + x)^{\frac{1}{x}}$

b) If $z(x, y) = \phi(u, v)$ where $u = l x + m y$, $v = l y - m x$ then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right).$$ **8**

MODULE – IV

7. a) Form the partial differential equations by eliminating constants

i) $2z = (ax + y)^2 + b$

ii) $(x - h)^2 + (y - k)^2 + z^2 = a^2$. **6**

b) Form the partial differential equations by eliminating functions $f(x^2 + y^2 + z^2, x + y + z) = 0$. **6**

c) Solve $z^2(p^2 + q^2 + 1) = 1$. **8**

8. a) If $u = \sec^{-1}\left[\frac{\sqrt{x} - 2\sqrt{y}}{y^3\sqrt{x}}\right]$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. **6**

b) Find the point on the plane $x + 2y + z = 5$ that is closest to the point P(0, 3, 4). **6**

c) Use the method of Lagrange's Multipliers to find the point on the curve $x y^2 = 54$ nearest to the origin. **8**
