SEM 1 – 1 (RC 07 – 08)

F.E. (Semester – I) (RC 2007 – 08) Examination, November/December 2018
APPLIED MATHEMATICS – I

Duration : 3 Hours Total Marks : 100

Instructions : *1) Attempt **any five** questions at least **one** from **each** Module.*
*2) Assume **suitable** data **if necessary**.*

MODULE – I

1. a) Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$. 5

b) Show that $\int_0^1 (1 - x^{1/n})dx = \frac{n!m!}{(n+m)!}$. 5

c) Prove that $\int_0^\infty \frac{x^5}{5^x} dx = \frac{120}{(\log 5)^6}$. 5

d) Prove that er f $(\infty) = 1$. 5

2. a) Test the convergence of the following series.

i) $\sum_{n=1}^{\infty}\left(2 + \frac{1}{n}\right)^n$ 4

ii) $\frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \frac{5}{26} - ...$ 4

iii) $\sum_{n=1}^{\infty}\frac{n^3}{e^n}$ 4

b) Define absolutely convergent and conditionally convergent series. Test whether the following series is absolutely convergent or conditionally convergent series.

$\sum_{n=0}^{\infty}(-1)^n \frac{1}{(3n+1)^2}$ 8

P.T.O.

MODULE – II

3. a) Use De Moivre's Theorem and solve $x^6 + 1 = 0$. 6

b) Express $\sin 6\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$. 6

c) Separate into real and imaginary parts.

$\cos^{-1}\left(\dfrac{3i}{4}\right)$. 8

4. a) Determine 'p' such that the function

$\dfrac{\log(x^2+y^2)}{2} + i\tan^{-1}\left(\dfrac{px}{y}\right)$ is analytic function. 6

b) Show that the function

$u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic. Find the analytic function. 6

c) If $\tan\left(\dfrac{\pi}{6} + i\alpha\right) = x + iy$ prove that $x^2 + y^2 + \dfrac{2x}{\sqrt{3}} = 1$. 4

d) If 'n' is a positive integer then prove that

$\left(\sqrt{3}+i\right)^n + \left(\sqrt{3}-i\right)^n = 2^{n+1}\cos\left(\dfrac{n\pi}{6}\right)$. 4

MODULE – III

5. a) If $y = \cos(m\log x)$, prove that, $x^2 y_{n+2} + (2n+1)xy_{n+1} + (m^2+n^2)y_n = 0$ 7

b) Expand $\cos^{-1}\left(\dfrac{x^2-1}{x^2+1}\right)$ in powers of x. Find the first 4 terms. 6

c) Expand

$\cot\left(x + \dfrac{\pi}{4}\right)$ in powers of x and hence find cot 46.5°. 7

6. a) Evaluate 12

i) $\lim\limits_{x\to 0} \dfrac{(1+x)^{1/x} - e}{x}$

ii) $\lim\limits_{x\to 0} \dfrac{a}{x} - \cot\left(\dfrac{x}{a}\right)$

iii) $\lim\limits_{x\to \pi/2} (\cos x)^{\frac{\pi}{2}-x}$

b) If $z = f(x, y)$ where $u = lx + my$, $v = ly - mx$ then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$$ 8

MODULE – IV

7. a) Form the partial differential equations by eliminating constants

i) $2z = (ax + y)^2 + b$

ii) $(x - h)^2 + (y - k)^2 + z^2 = a^2$. 6

b) Form the partial differential equations by eliminating functions

$f(x + y + z, x^2 + y^2 + z^2) = 0$. 6

c) Solve $x^2p^2 + y^2q^2 = z^2$. 8

8. a) If $u = \tan^{-1}\left[\frac{y^2}{x}\right]$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. 6

b) Find the extreme values of the function $x^3y^2(1 - x - y)$. 6

c) Find a point in the plane $x + 2y + 3z = 13$ nearest to the point (1, 1, 1) using Lagrange's Multipliers. 8
