**SEM 2 – 1 (RC 07-08)**

**F.E. (Semester – II) Examination, Nov./Dec. 2014**
**(Rev. Course 2007-08)**
**APPLIED MATHEMATICS – II**

Duration : 3 Hours | Total Marks : 100

***Instructions :*** *1) Attempt* ***5*** *questions,* ***atleast one*** *from* ***each Module.***
*2)* ***Assume*** *suitable data, if* ***necessary.***
*3) Figures to the* ***right*** *indicate* ***full*** *marks.*

MODULE – I

1. a) Assuming the validity of differentiation under the integral sign prove that

$$\int_0^{\infty} \frac{\log\left(1+\alpha x^2\right)}{x^2}\,dx = \pi\sqrt{\alpha}\,.$$ 7

b) Find the length of the curve $x = \frac{y^3}{6} + \frac{1}{2y}$ from y = 1 to y = 16. 7

c) Find the area of the surface generated by the resolution of

$y = \sqrt{x+2}\,;\ 0 \le x \le 4$ about the x-axis. 6

2. a) If $\bar{r}(t) = \bar{a}e^{2t} + \bar{b}e^{3t}$; where $\bar{a}$ and $\bar{b}$ are constant vectors, then show that

$$\frac{d^2\bar{r}}{dt^2} - \frac{5\,d\bar{r}}{dt} + 6\bar{r} = 0\,.$$ 6

b) If $\bar{r}$ (t) = 2 cos t i + 3 sint j + 4t k is the position vector of a particle in space at time ‘t’. Find the particle velocity vector and acceleration vector at $t = \pi/2$. 5

c) Define curvature of a curve. Show that the curvature of a circle is constant. 5

d) Evaluate $\int_0^{\pi} \cos t\ \hat{i} + \sin^2 + \hat{j} + \hat{k}\ dt\,.$ 4

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MODULE – II

3. a) Evaluate $\iint_R (x^2 - y^2)\,dxdy$ where 'R' is the triangle with vertices (0, 1), (1, 1) and (1, 2). 6

b) Evaluate $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y}\,dxdy$. 7

c) Change to polar co-ordinates and evaluate $\int_0^a\int_y^a \frac{x^2}{\sqrt{x^2+y^2}}\,dxdy$. 7

4. a) Use double integration to find the area between r = 1 and r = 3 (polar co-ordinates). 6

b) Convert the following to spherical polar co-ordinates and evaluate

$\iiint_V \frac{x^2+y^2}{\sqrt{x^2+y^2+z^2}}\,dxdydz$ where 'V' is the region $x^2+y^2+z^2 \leq 1$ and $z \geq 0$. 7

c) Find the volume of the region bounded by x = 0, y = 0, z = 0 and 3x + 2y + z = 6. 7

MODULE – III

5. a) In what direction is the directional derivative of $f(x, y, z) = x^2y + 3xyz$ at the point (–1, 2, 1) maximum and what is its magnitude ? 5

b) Define divergence of a vector field. Show that div(grad f) = $\nabla^2 f$. 5

c) Find the work done in moving a particle in a force field $\overline{F} = 3y^2\, i + (2yz + x)\, j + xz\, k$ along the curve $x = 3t + 1$, $y = 2t^2$, $z = 2t$ from t = 0 to t = 1. 5

d) Use Gauss divergence theorem to prove $\iint_s \overline{R}\cdot\hat{n}\,ds = 3V$; where 's' is any closed surface, $\hat{n}$ is the unit outward normal to s, 'V' is the volume enclosed by 's' and $\overline{R} = xi + yj + zk$. 5

6. a) Verify Green's theorem in the plane for $\oint\limits_C \left[\left(x^2 + 4y^2\right) dx + (y + 3x)\, dy\right]$

where c is the boundary of the region bounded by $y = 0$, $x = 0$ and $2x + y = 2$. **8**

b) Use Stoke's theorem to evaluate $\iint\limits_S \left(\nabla \times \bar{F}\right) \cdot \hat{n}\, ds$ where

$\bar{F} = (x^2 + 3y)\, i + 2y\, j + (3x + z)\, k$; s is the surface of the tetrahedron having vertices (2, 0, 0), (0, 2, 0), (0, 0, 0), (0, 0, 1) excluding the surface in the XY plane. **12**

MODULE – IV

7. Solve the following :

a) $(\sec x \tan x \tan y - e^x)\, dx + (\sec x \sec^2 y)\, dy = 0$. **5**

b) $\frac{dy}{dx} = \frac{-2x + y - 5}{x + 3y + 1}$. **5**

c) $(x + y + 1)^2 \frac{dy}{dx} = 1$. **5**

d) $\frac{dy}{dx} - y \tan x = y^4 . \sec x$. **5**

8. Solve the following :

a) $(D^2 + 2D + 1)y = xe^x + 2$. **5**

b) $(D^4 + 2D^2 + 1)^2 y = \sin(2x)$. **5**

c) $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$. **5**

d) $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + y = x^2 \log x$. **5**

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