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F.E. Semester-I (Revised Course 2007-08)
EXAMINATION MARCH 2021
Applied Mathematics -I

[Duration : Two Hours] **[Total Marks :60]**

Instructions:-
1) Answer THREE FULL QUESTIONS with ONE QUESTION from ANY THREE MODULES.
2) Assume suitable data, if necessary.

MODULE I

Q1) a) State and prove the Duplication formula for Gamma Function. **(6)**

b) Evaluate **(5)**

$$\int_0^\infty 7^{-4x^2}\,dx$$

c) Show that **(5)**

$$\beta(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$$

d) Prove that **(4)**

$$\mathrm{erf}(\infty)=1$$

Q2) a) Test the convergence of the following series

i) **(4)**

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}$$

ii) **(4)**

$$\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt{n+1}}$$

iii) **(4)**

$$1+\frac{2!}{2^2}+\frac{3!}{3^2}+\cdots$$

b) Define interval of convergence of power series and hence find it for the series. **(8)**

$$x-\frac{x^2}{\sqrt{2}}+\frac{x^3}{\sqrt{3}}-\frac{x^4}{\sqrt{4}}+\cdots$$

MODULE II

Q3) a) Use Dc Moivre's theorem and solve **(6)**

$$x^4 - x^3 + x^2 - x + 1 = 0$$

b) Prove that

$$i^{i^i} = cos\theta + isin\theta$$ **(8)**

where

$$\theta = (2n + 1/2)\pi e^{-(2n+1/2)\pi}$$

c) Separate into real and imaginary part **(6)**

$$\left(1 + i\sqrt{3}\right)^{(1+i\sqrt{3})}$$

Q4: a) Determine the analytic function whose real part is

$$cosx\ cosh\ y$$ **(6)**

b) Show that the function

$$u = e^{-2xy}\sin(x^2 - y^2)$$ **(6)**

is harmonic. Find the conjugate harmonic function.

c) If

$$u - v = \frac{e^y - cosx + sinx}{coshy - cosx}$$

and f(z)=u+iv is an analytic function of z=x+iy, find f(z) in terms of z. **(8)**

MODULE III

Q5) a) If

$$y = \left[x - \sqrt{x^2 - 1}\right]^m, \text{prove that,}$$

$$(x^2 - 1)y_{n+2} + (2n + 1)x\, y_{n+1} + (n^2 - m^2)y_n = 0$$ **(7)**

b) Expand

$$\sec^{-1}\left(\frac{1}{1 - 2x^2}\right)$$

in powers of x. Find the first 4 terms. **(6)**

c) Expand

$tan\left(x+\frac{\pi}{4}\right)$ as far as the term x^4 and evaluate $\tan 46.5°$ **(7)**

Q6) a) Evaluate: **(12)**

i) $\lim_{x\to\pi/2}\frac{\log(x-\pi/2)}{tanx}$

ii) $\lim_{x\to 0}\frac{e^x sinx-x-x^2}{x^2+xlog(1-x)}$

iii) $\lim_{x\to 0}(1/4^x+x)^{\frac{1}{x}}$

b) If $z=f(x,y)$where $u=l\,x+m\,y, v=l\,y-m\,x$ then show that

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=(l^2+m^2)\left(\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}\right)$$ **(8)**

MODULE IV

Q7) a) Form the partial differential equations` by eliminating constants

(i) $z=(x-a)(y-b)$

(ii) $(x-h)^2+(y-k)^2+z^2=a^2$ **(6)**

b) Form the partial differential equations by eliminating functions

$$z=f(z^2-xy,x/z)$$ **(6)**

c) Solve

$$x^2p^2+y^2q^2=z^2$$ **(8)**

Q8) a) If

$u=\tan^{-1}\left[\frac{\sqrt{(x^3+y^3)}}{\sqrt{x}+\sqrt{y}}\right]$, find the value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ **(6)**

b) Obtain the extreme values of the function

$$y^2+4xy+3x^2+x^3$$ **(6)**

c) Find the shortest and longest distances from the point (1,2,-1) to the sphere

$$x^2+y^2+z^2=24$$ **(8)**