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F.E. Semester-II (Revised Course 2007-08)
EXAMINATION Aug/Sept 2019
Applied Mathematics-II

[Duration : Three Hours] **[Max. Marks: 100]**

Instructions:-

1. Attempt **any five** question, at least one from each module.
2. Assume suitable data if necessary.

MODULE- I

1.

a) Evaluate $\int_0^{\infty} \frac{\log_e(1+4\alpha t^2)dt}{t^2}$ by applying differentiation under the integral sign. **(7)**

b) Find the length of the curve $y = 2x^{3/2} + 3$ from x=1 to x=3. **(7)**

c) For the curve $x = e^{\theta} Cos\theta, y = e^{\theta} Sin\theta$, show that the length of the curve from $\theta = 0$ to $\theta = \pi/2$ is $(\sqrt{2}\,(e^{\pi/2} - 1)$. **(6)**

2.

a) Find the unit tangent vector, and principal normal of the space curve $\bar{r}(t) = t^3\bar{\imath} + 2t\bar{\jmath} + 4\bar{k}$ at t=1. **(6)**

b) Define Curvature and Torsion of a space curve and prove Serret-Frenet formula. **(8)**

c) Solve $\frac{d\bar{r}^2}{dt^2} = 2\bar{\imath} - \bar{\jmath}, \bar{r}(0) = \bar{\imath} - 2\bar{\jmath}$ and $\frac{d\bar{r}}{dt}|_{t=0} = 5\bar{k}$ **(6)**

MODULE- II

3.

a) Evaluate $\int_0^1 \int_x^1 2y + 3dxdy$ **(7)**

b) Evaluate $\iint y + 2xdxdy$ over the region bounded by $x^2 = y$ and y=1. **(7)**

c) Change to polar co-ordinates and evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{2x}{x^2+y^2} dxdy$ **(6)**

4.

a) The Cardiode $r = a(1 + Cos\theta)$ is revolved about the x-axis. Find the surface area of the solid generated. **(6)**

b) Evaluate the spherical polar coordinates integral $\int_0^{2\pi} \int_0^{\pi} \int_0^{(1-Cos\varphi)/2} rSin\varphi drd\varphi d\theta$ **(6)**

c) Find the volume of the region $\{(x, y, z)/x^2 + y^2 + z^2 \leq 4, x \geq 0, y \geq 0,\ z \geq 0\}$. **(8)**

MODULE- III

5.

a) Find the unit normal vector to the surface $x^2 + 3yz = 1$ at (-2,-1,1) **(5)**

b) If $r = \sqrt{x^2 + y^2 + z^2}$ and $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$, show that $div\left(\frac{\bar{r}}{r^3}\right) = 0$ **(5)**

c) Show that the vector field $\bar{F} = (2xy + 4zCosx)\bar{\imath} + x^2\bar{\jmath} + (4Sinx + 2z)\bar{k}$ is irrotational and find its scalar potential. **(5)**

d) Find the work done in moving a particle in the force field (5)
$\bar{F} = 3z\bar{\imath} + x\,y\bar{\jmath} + y\underline{\bar{k}}$ along the curve $x = y^2$ and $x = z^2$ from (1,1,1) and (4,2,2).

6. a) State and prove Green's theorem in the plane. (6)
b) Use Gauss divergence theorem to evaluate $\int\int_S \bar{F}.\bar{n}ds$ where (7)
$\bar{F} = 2xy\bar{\imath} + zx\bar{\jmath} + z^2\bar{k}$, $\bar{n}$ is the unit normal vector to the surface S, which is the surface of the cube $0 \leq x, y, z \leq 1$.
c) Use Stoke's theorem to evaluate $\iint_S \nabla \times \bar{F}.\bar{n}ds$ where (7)
$F = 2x\bar{\imath} + y^2z\bar{\jmath} + y^2\bar{k}$, $\bar{n}$ is the unit normal vector to S, the surface of the region bounded by coordinate planes x=0,y=0,z=0 and 2x+y+z=2, excluding the surface lying in the xy plane.

MODULE- IV

7. Solve the following differential equations. **20**

a) $(3x^2y + 2xcosy + 4)dx + (x^3 - x^2siny)dy = 0$
b) $Sin2x\,\frac{dy}{dx} + 2yCos2x = 3x^2$
c) $\frac{dy}{dx} = \frac{3y-x+2}{2y-3x+3}$
d) $(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$

8. Solve the following differential equations **20**
a) $(D^2 + 3D + 4)y = Sin2x$
b) $(D^2 - 1)y = 2e^{-2x}$
c) $(D^2 + 5D + 4)y = x^2 + 7x$
d) $(1 + 2x)^2\frac{d^2y}{dx^2} - 6(1 + 2x)\frac{dy}{dx} + 16y = 8(1 + 2x)^2$