SEM 1 – 1 (RC 16-17)

F.E. (Semester – I) (Revised in 2016-17) Examination, May/June 2017
ENGINEERING MATHEMATICS – I

Duration : 3 Hours | Total Marks : 100

Instructions : 1) *Answer **five** questions. Atleast **two** from Part – **A, two** from Part – **B** and **one** from Part – **C**.*
2) ***Assume*** *suitable data , if necessary.*
3) *Figures to **right** indicate **full** marks.*

PART – A

Answer **any two** questions from the following:

1. a) Evaluate $\int_0^\infty x^{\frac{3}{2}} e^{-x^2}\, dx$. **5**

b) Prove that $erf_c(-x) + erf_c(x) = 2$. **4**

c) Prove that $B(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx$. **6**

d) Use De Moivre's theorem to solve $x^5 + 1 = 0$. **5**

2. a) Test the convergence of the following series **12**

i) $\frac{2}{1^2+1} + \frac{2^2}{2^2+1} + \frac{2^3}{3^2+1} + \ldots\ldots$

ii) $\sum_{n=1}^{\infty}\left(2+\frac{1}{\sqrt{n}}\right)^{-n}$

iii) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{(5n+1)}$.

b) Find the analytic function whose real part is $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. **4**

c) Determine a, b, c, d so that the function
$f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic. **4**

P.T.O.

3. a) If $\cosh x = \sec\theta$, prove that $x = \log(\sec\theta + \tan\theta)$. 6

b) Show that $\log(-\log i) = \log\frac{\pi}{2} - i\frac{\pi}{2}$. 6

c) Define absolutely convergent and conditionally convergent series. Hence test whether following series is absolutely convergent or conditionally convergent $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$. 8

PART – B

Answer **any two** questions from the following :

4. a) If $u = \sin^{-1}\left(\frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{6}} + y^{\frac{1}{6}}}\right)$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. 7

b) If $y = a\cos(\log x) + b\sin(\log x)$ where a and b are constants, then show that $x^2 y_{n+2} + (2n+1)\,x\,y_{n+1} + (n^2+1)y_n = 0$. 7

c) Use Taylor's theorem to expand $f(x) = x^5 + 2x^4 - x^2 + x + 1$ in powers of $(x + 1)$. 6

5. a) Evaluate : 12

i) $\lim_{x\to 0} \frac{e^x - e^{-x} - 2\cos x}{x\sin x}$.

ii) $\lim_{x\to 0}\left[\frac{1}{x^2} - \frac{1}{x\tan x}\right]$.

iii) $\lim_{x\to 0}\left(\cos\sqrt{x}\right)^{\frac{1}{x}}$.

b) Form the partial differential equations by eliminating constants 'a' and 'b'. 4

$z = (x-a)^2 + (y-b)^2$.

c) Form the partial differential equations by eliminating function

$f(x^2 + y^2, z - xy) = 0$. 4

6. a) If $z(x, y) = \phi(u, v)$ where $u = lx + my$, $v = ly - mx$ then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right).$$ 8

b) Solve the partial differential equation $y^2zp + x^2zq = xy^2$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 6

c) Use the method of Lagrange's multipliers to find the maximum and the minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$. 6

PART – C

Answer **any one** question from the following :

7. a) Evaluate $\int_0^\infty \frac{x^4}{4^x} dx$. 4

b) If n is positive integer then prove that $(1 + i)^n + (1 - i)^n = \left(\sqrt{2}\right)^{n+2} \cos\frac{n\pi}{4}$. 5

c) Prove that $\log(1 + \sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \ldots..$ 6

d) Solve the partial differential equation 5

$x(y - z)\, p + y(z - x)\, q = z(x - y)$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

8. a) Test the convergence of the following series 5

$$\frac{1}{1.2.3} - \frac{1}{2.3.4} + \frac{1}{3.4.5} - \ldots$$

b) Prove that $\sin\left\{i \log\left(\frac{a - ib}{a + ib}\right)\right\} = \frac{2ab}{a^2 + b^2}$. 5

c) Use Taylor's series to find the approximate value of $\tan^{-1}(1.003)$. 5

d) Find the extreme values of the function
$f(x.y) = y^2 + 4xy + 3x^2 + x^3$. 5
