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F.E Semester -II (Revised Course 2019-20)
EXAMINATION JANUARY 2022
Mathematics-II

[Duration : Three Hours] **[Total Marks : 100]**

Instructions:
1. Attempt 5 questions, two from Part A, two from Part B and one from Part C
2. Assume missing data, if any.

PART - A
(Attempt any two from this part)

Q.1 **(6)**

a) Find the length of the curve
$x = a(2\cos t - \cos 2t) \& y = a\,(2\sin t - \sin 2t)\ from\ t = 0\ to\ t = \frac{\pi}{2}$

b) Evaluate $\iint (x^2 - y^2)\, dxdy$ over the region bounded by the triangle with vertices (0,1),(1,1) and (1,2) **(7)**

c) Find, by double integration, the volume of the object formed when the region bounded by $y = \sin x, 0 \le x \le \frac{\pi}{2}\ and\ y = 0$ is revolved about the $X-\ axis.$ **(7)**

Q.2 a) Find, by integration, the perimeter of the circle $r = 2a\sin\theta, a > 0$ **(6)**

b) Express the following as one double integral and evaluate **(8)**
$$\int_{-1}^{0}\int_{-y}^{1}(2y+3)\,dxdy + \int_{0}^{1}\int_{y^2}^{1}(2y+3)dxdy$$

c) Find the area common $to\ r = 1\ and\ r = 2\cos\theta$ using double integration. **(6)**

Q.3 a) Convert the following integral to Polar Coordinates and then evaluate **(7)**
$$\int_0^\infty\int_0^\infty \frac{2}{1+(x^2+y^2)^2}\,dxdy$$

b) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}} y e^{\frac{x}{\sqrt{1-y^2}}}\ dxdy$ **(6)**

c) Find the volume of the region bounded by $x^2 + y^2 \leq z^2, x^2 + y^2 + z^2 \leq 1$ *and* $z > 0$ **(7)**

PART-B

(Attempt any two from this part)

Q. 4 a) Find the rate of change of $\varphi = xyz$ in the direction in the direction normal to the surface $x^2y + y^2x + xz^2 = 3$ at the point (1,1,1) **(6)**

b) If $\bar{F} = 2x\hat{\imath} + zy\,\hat{\jmath} + 3xz\,\hat{k}$ *and* $g(x, y, z) = x^2 + 2yz$ then compute $\bar{F}.\nabla g$ **(4)**

c) Solve the following **(10)**

i) $(D^2 - 4D + 4)y = e^x \cos x$

ii) $(D^2 + 4D + 3)y = x^2 + x + 1$

Q. 5 a) Find the work done in moving a particle in the force field **(7)**
$\bar{F} = 3z^2x\,\hat{\imath} + 2x^2y\,\hat{\jmath} + (3z^2 + y)\hat{k}$ along the curve in the plane y =1 and having equation $x^2 = 4z$ *from* $(2,1,1)$ *to* $(4,1,4)$

b) Show that the vector field **(7)**
$\bar{F} = (6xy + \sin z)\,\hat{\imath} + (3x^2 - 4z^2 \sin y)\,\hat{\jmath} + (8\,z\cos y + x\cos z)\,\hat{k}$
Is irrotational and find it's scalar potential

c) Solve **(6)**
$(D^2 - 2D + 1)y = e^x \log x$

Q.6 a) Verify Green's Theorem in the plane for $\oint (x + y)dx + (x - y)dy$ where C is the boundary of the region bounded by $y = 1$ *and* $y = x^2$ **(10)**

b) Solve the following **(10)**

i) $(D^2 + D - 6)y = e^x \cosh x$

ii) $(D^2 - 6D + 9)y = 9^{-x} + \sin x$

PART-C

(Attempt any one from this part)

Q.7 a) Find the area of the surface generated by the revolution of $y = \frac{x^3}{9}$; **(6)**

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$0 \leq x \leq 2$, about the X–axis.

b) Find the volume of the region bounded by the co-ordinate planes and $x + y + z = 1$. (7)

c) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2(1+x)^2$ (7)

Q.8 a) Evaluate $\int_0^1 \int_0^{x^2} e^{y/x}\, dxdy$ (6)

b) Use Gauss Divergence theorem to evaluate $\int\int \bar{F} \,.\, \bar{n} dS$ where $\bar{F} = xy\hat{\imath} + yz\hat{\jmath} + y\hat{k}$ and 'S' is the surface of the region bounded by the coordinate planes and the planes $x = 1, y = 1, z = 1$. (7)

c) Solve $(D^2 - 6D + 8)y = e^{3x} + 8\cos x$ (7)