SEM 1 – 1 (RC 16-17)

F.E. (Semester – I) (Revised Course 2016-17) Examination, May/June 2018
ENGINEERING MATHEMATICS – I

Duration : 3 Hours   Total Marks : 100

Instructions : *1) Answer* ***five*** *questions, atleast* ***two*** *from Part –* ***A, two*** *from Part –* ***B*** *and* ***one*** *from Part –* ***C.***
2) Assume suitable data, ***if necessary.***
3) Figures to ***right*** *indicate* ***full*** *marks.*

PART – A

1. a) Prove that $\int_0^\infty \frac{x^5}{5^x}\,dx = \frac{120}{(\log_e 5)^6}$. **5**

   b) Prove that $\operatorname{erf}(x) + \operatorname{erf}_C(x) = 1$. **5**

   c) Prove that $\tan\left[i\log\left(\frac{a-ib}{a+ib}\right)\right] = \frac{2ab}{a^2-b^2}$. **5**

   d) If $\log(\log(x+iy)) = p + iq$ then prove that $y = x\tan\left[\tan q\log\sqrt{x^2+y^2}\right]$. **5**

2. a) Test the nature of the following series

   i) $\sum\limits_{n=1}^{\infty}\frac{(2n-1)}{n(n+1)(n+2)}$

   ii) $\sum\limits_{n=1}^{\infty}\left(2+\frac{1}{\sqrt{n}}\right)^{-n}$

   iii) $\sum\limits_{n=1}^{\infty}(-1)^{n-1}\frac{n}{5n+1}$. **12**

   b) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic function. Find the function $v(x, y)$ such that $u + iv$ is an analytic function. **5**

   c) Show that $f(z) = \cosh z$ is analytic. **3**

P.T.O.

3. a) Find the radius and internal of convergence for the power series

$$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \ldots \infty$$ 7

b) Evaluate $\int_3^7 \sqrt[4]{(x-3)(7-x)}\, dx$. 6

c) If $\cosh x = \sec\theta$ then prove that

i) $x = \log(\sec\theta + \tan\theta)$

ii) $\tanh\frac{x}{2} = \tan\frac{\theta}{2}$. 7

PART – B

4. a) If $y = a\cos(\log x) + b\sin(\log x)$ where a and b are constants then show that $x^2 y_{n+2} + (2n+1)\, x\, y_{n+1} + (n^2+1)\, y_n = 0$. 7

b) Show that

$$\log(1 - \log(1-x)) = x + \frac{x^3}{6} + \ldots$$ 6

c) If $u = \log\left(\frac{x^4+y^4}{x+y}\right)$ show that :

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -3.$$ 7

5. a) Evaluate :

i) $\lim_{x\to 0}\frac{e^x + e^{-x} - 2\cos x}{x\sin x}$

ii) $\lim_{x\to 0}\left(a^x + x\right)^{1/x}$

iii) $\lim_{x\to 1}\left(x^2-1\right)\tan\left(\frac{\pi}{2}x\right)$. 12

b) Solve the partial differential equations :

i) $zxp + zyq = xy$

ii) $x(y-z)p + y(z-x)q = z(x-y)$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 8

6. a) If $z = \phi(u, v)$, $u = lx + my$, $v = ly - mx$ then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \left(l^2 + m^2\right)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right).$$ 8

b) Form the partial differential equations by eliminating the arbitrary constants :

$Z = (x - a)^2 + (y - b)^2$. 5

c) Use method of lagrange multipliers to find the maximum and minimum distances of the point (3, 4, 12) from $x^2 + y^2 + z^2 = 1$. 7

PART – C

7. a) Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}\, d\theta$. 5

b) If $\sin(\theta + i\phi) = \tan\alpha + i\sec\alpha$. Prove that $\cos 2\theta \cosh 2\phi = 3$. 5

c) Use Taylors theorem to express the polynomial $x^5 + 2x^4 - x^2 + x + 1$ in the powers of $(x - 1)$. 5

d) Find the extreme values of $f(x, y) = 3(x^2 - y^2) - x^3 + y^3$. 5

8. a) Test the convergence of $5 + \frac{10}{3} + \frac{20}{9} + \frac{40}{27} + ...$ 5

b) Use Demoivre's theorem to solve $x^5 - x^4 + x - 1 = 0$. 5

c) Verify $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ where $Z = \frac{\log\left(x^2 + y^2\right)}{xy}$. 5

d) Solve the partial differential equation

$y^2 p - xy\, q = x(z - 2y)$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 5