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SEM 2-1 (RC 07-08)

F.E. (Sem. – II) Examination, May/June 2008
(Revised 2007-08 Course)
APPLIED MATHEMATICS – II

Duration : 3 Hours | Total Marks: 100

*Instructions: i) Attempt **any five** questions, at least **one** from **each** Module.*
*ii) Assume suitable data if **necessary**.*

MODULE – I

1. a) Assuming the validity of differentiation under the integral sign show that

$$\int_0^1 \frac{x^a - x^b}{\log_e x} dx = \log_e\left(\frac{a+1}{b+1}\right).$$ 6

b) Find the length of the curve $x = a(2\text{Cos}t - \text{Cos}2t)$, $y = a(2\text{Sin}t - \text{Sin}2t)$ from $t = 0$ to $t = \pi/2$. 8

c) Find the perimeter of the curve $\theta = \frac{1}{2}\left(r + \frac{1}{r}\right)$ For $r = 1$ to $r = 4$. 6

2. a) Show that if $\bar{r} = \bar{a}\text{Sin}wt + \bar{b}\text{Cos}wt$, where $\bar{a}$ and $\bar{b}$ are constant vectors then

$$\frac{d\bar{r}^2}{dt^2} = -w^2\,\bar{r}.$$ 5

b) For the space curve $x = 3\text{Cos}t$, $y = 3\text{Sin}t$ and $z = 4t$ find the vectors $\bar{T}, \bar{B}$ at $t = 0$. 6

c) Evaluate $\int_0^{\pi/2} \text{Cos}^2 t\bar{i} + \text{Sin}t\,\bar{j} + 5\bar{k}dt$. 4

d) If $\bar{r}(t)$ is a vector field having a constant magnitude, show that $\bar{r}(t) \circ \frac{d\bar{r}}{dt} = 0$. 5

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MODULE – II

3. a) Evaluate $\int_0^2\int_x^2 2y^2 \sin(xy)\,dxdy$. 6

b) Change the order of integration and evaluate $\int_0^2\int_{y-2}^{2y} 2x + y\,dxdy$. 8

c) Change to polar coordinates and evaluate $\int_0^\infty\int_0^\infty \frac{2}{1+(x^2+y^2)^2}\,dxdy$. 6

4. a) Find the volume of the soild generated by the revolution of region bounded by $y^2 = 4x$ and $x = 1$ about the y-axis. 6

b) Evaluate $\int_0^1\int_0^{1-x^2}\int_3^{4-x^2-y} x\,dxdydz$. 6

c) Find the volume of the region bounded by the cylinder $x^2 + y^2 = 4$ the plane $z = 0$ and $x + 2z = 6$. 8

MODULE – III

5. a) Define divergence of a vector field. If f is a scaler point function and $\bar{u}$ is a vector point function show that $\operatorname{div}(f\bar{u}) = f\operatorname{div}\bar{u} + \operatorname{grad}f \circ \bar{u}$. 6

b) Show that the vector field

$\bar{f}(x, y, z) = (6x\sin z - 4y\sin x)\bar{i} + 4\cos x\bar{j} + (3x^2\cos z + 6z)\bar{k}$ is irrotational. Find its scalar potential. 6

c) Find the unit vector, normal to the surface $x^2 + 3y + z^2 = 4$ at the point (0, 1, –1). 4

d) In what direction is the directional derivative of $f = 3x^2 + 4yz$ at the point (2, 1, 0) maximum. 4

6. a) Verify Green theorem in the plane for $\oint(x^2+4y^2)dx+(y+3x)dy$ where C is the boundary of the region bounded by y = 0, x = 0 and 2x + y = 2. **8**

b) Use Stoke's theorem to evaluate $\iint_S \nabla\times F.\bar{n}ds$ where F = (x + 2z) $\bar{i}+3xy\bar{j}+yx^2\bar{k}$, $\bar{n}$ is the unit normal vector to S, and S the surface of the tetrahedron bounded by the coordinate planes and the plane x + 2y + z = 2, excluding the surface in the xy plane. **8**

c) Use Gauss divergence theorem to show that $\iint_S \bar{r}\circ\bar{n}ds=3V$ where S is any closed surface, $\bar{n}$ is the unit normal vector to surface S, V is the volume enclosed in surface S and $\bar{r}=x\bar{i}+y\bar{j}+z\bar{k}$. **4**

MODULE – IV

7. Solve the following differential equations : **20**

a) $(x+y+1)^2\dfrac{dy}{dx}=1$

b) $\dfrac{dy}{dx}+\dfrac{2}{x}y=\dfrac{y^3}{x^3}$

c) $\dfrac{dy}{dx}=\dfrac{2x-y+5}{x-3y+3}$

d) $(2x^2y^2+y)dx+(3x-x^3y)dy=0$

8. Solve the following differential equations : **20**

a) $(D^2+2D+1)y=\text{Sin}2x+5x$

b) $(D^4-2D^3+D^2)y=x^3$

c) $(D^2+1)y=\text{Sec}x$

d) $x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}-3y=x^2\log_e x$
