F.E. (Semester – II) (Revised 2007 – 08) Examination, November/December 2015
APPLIED MATHEMATICS – II

Duration : 3 Hours   Total Marks : 100

Instructions : *i) Attempt* ***any five*** *questions, at least* ***one*** *from* ***each*** *Module.*
ii) Assume suitable data if necessary.

MODULE – I

1. a) Assuming the validity of differentiation under the integral evaluate

$\int_0^{\infty} e^x \log_e(1+a^2e^{-2x})dx$. **8**

b) Find the perimeter of the curve $x^2 + y^2 = 4x$. **6**

c) The curve $r = 2a\text{Cos}\theta$ is revolved about the x-axis find the surface area of the solid generated. **6**

2. a) Show that $\bar{r}(t) = Ae^{2t}\,\bar{i} + Be^{-3t}\,\bar{j}$, satisfies $\frac{d\bar{r}^2}{dt^2} + \frac{d\bar{r}}{dt} - 6\bar{r} = 0$. **6**

b) Find the unit tangent vectors $\vec{T}$ and principal normal $\vec{N}$ for

$r(t) = \vec{i}\,\text{Cos}^2 2t + \vec{j}\text{Sin}2t + t\vec{k}$ at $t = \pi/2$. **8**

c) State and prove Serret-Fernet formula. **6**

MODULE – II

3. a) Evaluate $\int_0^1\int_0^1 ye^{xy}dxdy$. **6**

b) Write a single integral and evaluate $\int_0^2\int_0^{\sqrt{y}} 2y + 3dxdy + \int_2^4\int_{y-2}^{\sqrt{y}} 2y + 3dxdy$. **8**

c) Evaluate $\iint r\text{Sin}\theta + 3drd\theta$ over the region $\{(r, \theta)/ r \le 2\text{Cos}\theta, 0 \le \theta \le \pi\}$. **6**

P.T.O.

4. a) Find the volume of the object generated by the revolution of the region $x^2 + y^2 \leq 4x$ about the x-axis. **8**

b) Evaluate the cylindrical coordinate integral $\int_0^1 \int_0^{\pi} \int_0^{1+\cos\theta} 3r\sin\theta + 2\,dr\,d\theta\,dz$. **6**

c) Find the volume of the region bounded by the coordinate planes and the plane $2x + y + 3z = 6$. **6**

MODULE – III

5. a) Define Divergence of a vector field. Show that divergence of $\frac{\bar{r}}{r^3}$ is zero. Where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r = \sqrt{x^2 + y^2 + z^2}$. **6**

b) Evaluate $\int_C F.dr$ where $\bar{F} = xy\bar{i} + z\bar{j} + (2y+1)\bar{k}$ and c is the arc of the curve $\bar{r} = 2\cos t\,\bar{i} + 3\sin t\,\bar{j} + t\bar{k}$ from $t = 0$ to $t = \pi/2$. **6**

c) Verify Green's theorem in the plane for $\oint (xy+2)\,dx + (3x^2 + y)\,dy$ where C is the boundary of the region enclosed by $x = \sqrt{y}$ and $x = 0$ and $y = 1$. **8**

6. a) State Gauss Divergence theorem. Use it to show that $\iint_S F.\bar{n}\,ds = 4\pi$, where S is the surface of the sphere

$x^2 + y^2 + z^2 = 1$, $F(x, y, z) = (x^2y - 2xz)\,\bar{i} + (3y - xy^2)\,\bar{j} + z^2\bar{k}$ and $\bar{n}$ is the unit normal vector to the surface S. **8**

b) Verify Stoke's theorem for $F = 2y\bar{i} + (3x^2 + z)\bar{j} + 2yz\bar{k}$, over the surface of the tetrahedron bounded by the coordinate planes and the plane $x + y + z = 1$ above the xy plane. **12**

MODULE – IV

7. Solve the following differential equations : **20**

a) $e^{y}\left(1+x^{2}\right)\frac{dy}{dx}-2x\left(1+e^{y}\right)=0$

b) $\frac{dy}{dx}+y\text{Tan}x=y^{3}\text{Cos}x$

c) $\frac{dy}{dx}=\frac{5x-y+4}{x-3y+1}$

d) $\left(xy+2x^{2}y^{2}\right)ydx+\left(xy-x^{2}y^{2}\right)xdy=0$

8. Solve the following differential equations : **20**

a) $(D^2+4D+5)\,y=3e^{2x}+5x^2$

b) $(D^3+4D^2+D+2)\,y=3\text{Sin}^2x+2$

c) $(D^2+1)\,y=\text{Sec}x$

d) $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\text{Cos}(\log_e(1+x))$.