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F.E. (Sem - II) (Revised Course 2016-17)
EXAMINATION Nov/Dec 2019
Engineering Mathematics - II

[Duration : Three Hours]

[Total Marks : 100]

Instructions:

- 1) Attempt **any five** questions, two each from part A and part B and one from part C.
- 2) Figures to the right indicate full marks.
- 3) Assume suitable data if necessary.

Part A

Q.1

- a) Applying differentiation under the integral sign, evaluate (7)

$$\int_0^{\frac{\pi}{2}} \cos \theta \log_e(1 + \alpha \tan^2 \theta) d\theta.$$

- b) Find the perimeter of the asteroid $x^{2/3} + y^{2/3} = 4$ (6)

- c) Change the order of integration and evaluate $\int_0^2 \int_{2x-2}^x y + 2x dx dy$ (7)

Q.2

- a) Evaluate $\int_0^1 (3t \bar{i} + 2\bar{j}) \times (t\bar{i} + 2\bar{j} + \bar{k}) dt$ (5)

- b) Evaluate $\int \int \int \frac{z dx dy dz}{(x^2 + y^2 + z^2)^{3/2}}$ over the region $x^2 + y^2 + z^2 \leq 4$ and $0 \leq z \leq 1$. (8)

- c) Write a single integral and evaluate $\int_0^2 \int_0^{\sqrt{y}} 2x + 1 dx dy + \int_2^4 \int_{y-2}^{\sqrt{y}} 2x + 1 dx dy$ (7)

Q.3

- a) Define curvature of the path of a moving object. A moving object's position vector at time t is $\vec{r}(t) = \cos 2t \bar{i} + \sin 2t \bar{j} + t \bar{k}$. Find its curvature at time t . (7)

- b) Find the area of the surface generated by the revolution of the loop of the curve $9y^2 = x(x-3)^2$ about the x -axis. (7)

- c) Evaluate $\int_{-1}^1 \int_0^x \int_0^{(x+y)} 2z + x dz dy dx$ (6)

Part B

Q.4

- a) Find the directional derivative of $f(x, y, z) = x^2y + xyz + 4$ in the direction of $\hat{i} + \hat{j} - \hat{k}$ At the point $(1, 0, 1)$. (5)

- b) Find the work done in moving a particle in the force field $\vec{F} = (2y + 3)\hat{i} + xz\hat{j} +$ (5)



$(yz - x)\hat{k}$ along the curve $x = 2t^2, y = t$ and $z = t^3$ from $t = 0$ to $t = 1$.

c) Solve the following (10)

i) $\frac{dy}{dx} \oplus e^{3x-2y} + e^{-2y} \cos 2x$ ii) $\frac{d^2y}{dx^2} + y = \cot x$

Q.5 a) Verify Green's Theorem in a plane for $\oint (xy + 4y^2)dx + (x^2 + 3)dy$ along the boundary of the region bounded by $x = y$ and $x = y^2$. (10)

b) Solve the following (10)

i) $Y(2xy + 3)dx + x(1 + 2xy - x^2y^2)dy = 0$ ii) $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 2y = x(\log_e x)^2$

Q.6 a) Verify Stoke's theorem for $\vec{F} = xy\hat{i} - 2yz\hat{j} - zx\hat{k}$ where S is the open surface of the region bounded by the planes $x = 0, x = 2, y = 0, y = 1$ and $z = 0, z = 3$ excluding the face on the XY plane. (10)

b) Solve (i) $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x\cos 2x$ ii) $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$ (10)

Part C

Q.7 a) Find the length of the curve $y = \sqrt{4 - x^2}$ from $x = 0$ to $x = 2$. (5)

b) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{1}{1-x^2+y^2} dx dy$ (5)

c) Find the equation of the line, normal to the surface $2x + 3z^2 = 14$ at the point (1, -2, 2). (5)

d) Solve $(D^2 + D - 12)y = 2 \sin^2 x + 3$ (5)

Q.8 a) If $\vec{r}(t) = 2\sin t\hat{i} + 3\cos 2t\hat{j} + 4t\hat{k}$ is the position vector of a particle in space at time 't' then find its velocity and acceleration vectors at $t = \frac{\pi}{2}$. (5)

b) Evaluate $\int_0^{\pi/2} \int_0^\pi \int_0^{1+\cos\phi} 3\rho \sin\phi d\rho d\theta d\phi$ (6)

c) Prove that for a scalar field ϕ , $\text{Curl}(\nabla\phi) = 0$. (4)

d) Solve $\frac{dy}{dx} = \frac{2y-3x+1}{4y-6x+5}$ (5)