Total No. of Printed Pages:2

F.E. Semester-I (Revised Course 2016-17)
EXAMINATION JULY 2021
Engineering Mathematics -I

[Duration : Two Hours] **[Total Marks : 60]**

Instructions:
1) Answer THREE FULL QUESTIONS with ONE QUESTION FROM EACH PART.
2) Assume missing data, if any.

PART-A

Q.1 a) Evaluate $\int_0^1 x\,(logx)^6 dx$. (5)

b) Prove that $\text{erf}(\infty) = 1$ (4)

c) Use DeMoivre's theorem to evaluate all the values of $i^{\frac{3}{4}}$ (5)

d) Evaluate $\int_0^2 (4-x^2)^{3/2}\, dx$ using Beta function. (6)

Q.2 a) Test the convergence of the following series (12)

(i) $\Sigma \frac{3^n n! n!}{(2n)!}$

(ii) $\frac{2^2}{3} + \frac{3^2}{3^2} + \frac{4^2}{3^3} + \frac{5^2}{3^4} + \cdots$

(iii) $\frac{1}{1.2.3} - \frac{3}{2.3.4} + \frac{5}{2.4.5} - \cdots$

b) Prove that $f(z) = sinz$ is analytic. (4)

c) Determine the value of P so that $f(z) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\frac{py}{x}$ is analytic. (4)

Q.3 a) If $\log[log(x+iy)] = p + iq$ then prove that $y = x\, tan\left[\tan q \log\sqrt{x^2+y^2}\right]$ (6)

b) Prove $\cosh^{-1}\sqrt{1+x^2} = \sinh^{-1} x$ (6)

c) Find the radius and interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{3^n x^n}{\sqrt{n}}$ (8)

PART – B

Q.4 a) If $u = \tan^{-1}\left(\frac{x^5+y^5}{x-y}\right)$ then evaluate $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy}$ (7)

b) If $y = a\cos(logx) + bsin(logx)$ then prove that $x^2y_{n+2}(2n+1)xy_{n+1} + (n^2 + 1)y_n = 0$ (6)

c) Use Taylor's series expansion to expand the polynomial $x^5 + 2x^4 - x^2 + x + 1$ in powers of $(x-1)$. (7)

Q.5 a) Evaluate (12)

(i) $\lim_{x\to 1} \frac{1+logx-x}{1-2x+x^2}$

(ii) $\lim_{x\to 0} \frac{e^x-e^{-x}-2x}{x^2sinx}$

(iii) $\lim_{x\to 0}(\cos x)^{1/x^2}$

b) Form a partial differential equation by eliminating $a\ and\ b$ from $z = (x^2+a)(y^2+b)$ (4)

c) Form a partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2 + z^2, x + y + z) = 0$ (4)

Q.6 a) If $z = f(u,v)$ where $u = lx + my$, $v = ly - mx$; l and m eing constants, then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v})$ (8)

b) Solve the partial differential equation $(z-y)p + (x-z)q = (y-x)$ (6)

c) Find and classify the critical points of $f(x,y) = x^3 + y^3 - 63\,(x+y) + 12xy$ (6)

PART – C

Q.7 a) Evaluate $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} \cdot \int_0^{\pi/2} \sqrt{sin\theta}\, d\theta$ (6)

b) Use Lagrange's method to find the minimum value of $f(x,y,z) = x^2 + y^2 + z^2$ subject to the condition $x + y + z = 1$. (8)

c) Use Taylor's Series expansion to find the approximate value of $\tan^{-1}(1.003)$ (6)

Q.8 (a) Prove $\sin\left\{ilog\left(\frac{a-ib}{a+ib}\right)\right\} = \frac{2ab}{a^2+b^2}$ (6)

(b) Define absolute convergence and conditional convergence. Check if the following series is absolutely or conditionally convergent $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)}$ (8)

(c) Use Taylor's series to find approximate value of $\sqrt{25.15}$ (6)