# SEM 1 – 1 (RC 2016 – 17)

**F.E. (Semester – I) (RC 2016-17) Examination, Nov./Dec. 2016**
**ENGINEERING MATHEMATICS – I (New)**

Duration : 3 Hours — Total Marks : 100

***Instructions*** : *1) Answer **five** questions. At least **two** from Part – **A, two** from Part – **B** and **one** from Part – **C**.*
*2) Assume suitable data, **if necessary**.*
*3) Figures to **right** indicate **full** marks.*

PART – A

Answer **any two** questions from the following :

1. a) Evaluate $\int_0^{\infty} \frac{x^4}{4^x} dx$. **5**

   b) Prove that $\operatorname{erf}(x) + \operatorname{erf}_c(x) = 1$. **4**

   c) Solve $x^6 - i = 0$. **6**

   d) Prove that $\beta(m, n) = \beta(m, n + 1) + \beta(m + 1, n)$. **5**

2. a) Test the convergence of the following series : **12**

   i) $\sum_{n=1}^{n=\infty} \left(\frac{e}{\pi}\right)^n$

   ii) $\frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \frac{5}{26} - \text{........}$

   iii) $\sum_{n=1}^{n=\infty} \frac{n!\,2^n}{n^n}$.

   b) Show that the function $u(x, y) = x^3 - 3xy^2$ is a harmonic function. Find the function $v(x, y)$ such that $u + iv$ is an analytic function. **4**

   c) Determine a, b, c, d so that the function :

   $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic. **4**

P.T.O.

3. a) If $\dfrac{(1+i)^{a+ib}}{(1-i)^{a-ib}} = x + iy$, then considering the principal value only prove that

$\tan^{-1}\dfrac{y}{x} = \dfrac{\pi}{2}a + b\log 2$. **7**

b) If $\tan\dfrac{x}{2} = \tanh\dfrac{u}{2}$. Prove that $u = \log_e \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)$. **5**

c) Define the interval of convergence and find it for the following series :

$x - \dfrac{x^2}{\sqrt{2}} + \dfrac{x^3}{\sqrt{3}} - \dfrac{x^4}{\sqrt{4}} + \ldots\ldots$ **8**

PART – B

Answer **any two** questions from the following :

4. a) If $y = e^{p\sin^{-1}x}$, prove that,

$(1 - x^2)y_{n+2} - (2n + 1)x\,y_{n+1} - (n^2 + p^2)y_n = 0.$ **7**

b) Show that $e^{x\cos x} = 1 + x + \dfrac{x^2}{2} - \dfrac{x^3}{3} - \ldots\ldots$ **6**

c) If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$, find the value of $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2}$. **7**

5. a) Evaluate : **12**

i) $\lim\limits_{x \to 0} \dfrac{x\cos x - \log(1+x)}{x^2}$

ii) $\lim\limits_{x \to 0} (a^x + x)^{\frac{1}{x}}$

iii) $\lim\limits_{x \to 1} (x^2 - 1)\tan\left(\dfrac{\pi}{2}x\right)$.

b) Form the partial differential equations by eliminating constants 'a' and 'b' : **4**
$z = a(x + y) + b$.

c) Form the partial differential equations by eliminating function. **4**
$f(x + y + z, x^2 + y^2 + z^2) = 0$.

6. a) If $z = f(u, v)$ where $u = x^2 - y^2$, $v = 2xy$ then show that **8**

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 4(x^2 + y^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right).$$

b) Solve the partial differential equation : **6**

$$x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2) \text{ where } p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}.$$

c) Use the method of Lagrange's Multipliers to find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$. **6**

PART – C

Answer **any one** question from the following :

7. a) Evaluate $\int_0^1 x^3(1 - \sqrt[3]{x})^4 dx$. **4**

b) If n is positive integer then prove that **5**

$$(1 + \sqrt{3}i)^n + (1 - \sqrt{3}i)^n = 2^{n+1}\cos\frac{n\pi}{3}.$$

c) Use Taylor's theorem to expand $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of $(x - 1)$. **6**

d) Solve the partial differential equation : **5**

$$x^2(y - z)p + y^2(z - x)q = z^2(x - y) \text{ where } p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}$$

8. a) Test the convergence of the following series : **5**

$$1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+.....$$

b) Prove that $\tan\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\}=\frac{2ab}{a^2-b^2}$. **5**

c) Use Taylor's series to find the approximate value of $\sqrt{36.5}$. **5**

d) Find the extreme values of the function :

$f(x, y) = 4 + 3xy^2 - 3x^2 - 3y^2 + x^3$. **5**

---