Paper / Subject Code: FE1902 / Mathematics-I

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F.E Sem-I (Revised Course 2019-2020)
EXAMINATION NOV/DEC 2019
Mathematics-I

[Duration : Three Hours] **[Total Marks : 100]**

Instructions:-

1. Answer five questions. At least two from part-A, two from part –B and one from part – C.
2. Assume suitable data, if necessary.
3. Figures to right indicate full marks.

Part A

Answer any two questions of the following.

Q.1

a) Evaluate $\int_0^{\infty} x^{\frac{3}{2}} e^{-x^2}\, dx$ **(4 marks)**

b) Use Taylor's theorem to expand $f(x) = \sqrt{1 + x + 2x^2}$ in powers of $(x-1)$ using Taylor's series. **(4 marks)**

c) Test the convergence of the following series **(12 marks)**

i. $\sum_{n=1}^{n=\infty} n\sin^2\left(\frac{1}{n}\right)$

ii. $\sum_{n=1}^{n=\infty}\left(1+\frac{1}{n}\right)^{-n^2}$

iii. $\frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \frac{5}{26} - \ldots\ldots\ldots\ldots$

Q.2

a) If $y = \left(x + \sqrt{p^2 + x^2}\right)^2$ then prove that **(6 Marks)**

$$(p^2 + x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - 4)y_n = 0$$

b) Find the interval of convergence of the following series **(8 Marks)**

$$\sum_{n=1}^{n=\infty} \frac{3^n x^n}{\sqrt{n}}$$

c) Evaluate $\int_5^8 \sqrt[3]{(x-5)(8-x)}\,dx$ **(6 Marks)**

Q.3

a) Evaluate **(12 Marks)**

i) $\lim_{x\to 0}(\cos\sqrt{x})^{\frac{1}{x}}$

ii) $\lim_{x\to 1}\frac{x^2-1}{\cot\frac{\pi x}{2}}$

iii) $\lim_{y\to -4} \frac{\sin(\pi y)}{y^2-16}$

b) Evaluate $\int_0^\infty \frac{x^5+x^7}{(1+x)^{14}}dx$ **(4 Marks)**

c) Find the expansion of $e^{\cos x}$ up to x^5 **(4 Marks)**

Part B

Solve the two questions of the following

Q.4 a) Solve the following differential equations **(12 Marks)**

i. $\sec x dy = (y + \sin x)dx$

ii. $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$

iii. $\frac{dy}{dx} = \frac{x+2y-3}{2x+y-3}$

b) $If\ u\ (x, y, z) = \log(\tan x\ + \tan y + \tan z), find\ the\ value\ of$ **(8 Marks)**

$$\sin 2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z}$$

Q.5 a) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube use Lagrange's method of undetermined multipliers. **(08 Marks)**

b) $If\ u = \tan^{-1}(x^2 + 2y^2)\ then\ evaluate$ **(06 Marks)**

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

c) Solve * assume the equation of sphere

Tan $y\frac{dy}{dx} + \tan x = \cos y \cos^2 x$ **(06 Marks)**

Q.6 a) Find the values of $x\ and\ y$ for which $x^2 + y^2 + 6x = 12$ has a minimum value and find this minimum value. **(06 Marks)**

b) Verify Euler's Theorem for **(08 Marks)**

$$u = x^2 tan^{-1}\left(\frac{y}{x}\right) - y^2 tan^{-1}\left(\frac{x}{y}\right)$$

c) Solve $(y - 2x^3)dx - x(1 - xy)dy = 0$ **(06 Marks)**

Part C

Answer any one question of the following.

Q.7 a) Prove that $\beta\ (m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx$ **(08 Marks)**

b) $If\ u = \sin^{-1}\left(\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{6}}+y^{\frac{1}{6}}}\right), find\ x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ **(07 Marks)**

c) Evaluate $\lim_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)}$ **(05 Marks)**

Q.8 a) Define absolutely convergent and conditionally convergent series and test the absolute convergence and conditional convergence of the following series. **(08 Marks)**

$$1 - \frac{1}{5} + \frac{1}{9} - \frac{1}{13} + \ \dots\dots$$

b) Prove that $\log \tan\left(\frac{\pi}{4} + x\right) = 2x + \frac{4x^3}{3} + \ \dots\dots$ **(07 Marks)**

c) Solve $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$ **(05 Marks)**