FE111

Total No. of Printed Pages:2

F.E. Semester-I (Revised Course 2007-2008)
EXAMINATION Nov/Dec 2019
Applied Mathematics-I

[Duration : Three Hours] **[Total Marks : 100]**

Instructions:
1) Attempt any five questions, at least one from each module.
2) Assume suitable data, if necessary.

Module I

1. a) Show that $\int_0^\infty x^n e^{-a^2x^2} dx = \frac{1}{2a^{n+1}} \Gamma\left(\frac{n+1}{2}\right)$. Hence deduce that $\int_0^\infty e^{-a^2x^2} dx = \frac{\sqrt{\pi}}{2a}$. **(6)**

b) Evaluate $\int_0^\infty \frac{x^3 dx}{3^x}$ using gamma function. **(4)**

c) show that $\int_0^1 (1 - x^{1/n})^m dx = \frac{n!m!}{(m+n)!}$ **(5)**

d) Prove that $erf(-x) = -erf(x)$ **(5)**

2. a) Test the convergence of the following series **(12)**

i) $\frac{3}{4} + \frac{3.4}{4.6} + \frac{3.4.5}{4.6.8} + \cdots \infty$ ii) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ iii) $\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$

b) Define the interval of convergence and find it for the following series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{10^n}$ **(6)**

c) State D'Almbert ratio test for the convergence of a series. **(2)**

Module II

3. a) Use De Moivre's theorem and solve $x^9 - x^5 + x^4 - 1 = 0$ **(6)**

b) If $\sin(\alpha + i\beta) = x + iy$ prove that : **(8)**

$$\frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1$$
$$\frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1$$

c) If $\frac{u-1}{u+1} = Sin(x + iy)$. Find u. **(6)**

4.

a) Considering the principal value only, prove that the real part of $(1+i\sqrt{3})^{1+i\sqrt{3})}$ is $2e^{-\pi/\sqrt{3}}Cos(\pi/3+\sqrt{3}Log_e 2))$ **(7)**

b) Show that the function $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. Find the conjugate harmonic function. **(7)**

c) Find the analytic function for which Sinhx Cosy is the imaginary part **(6)**

Module III

5.

a) If $y = e^{Tan^{-1}x}$ show that $(1+x^2)y_{n+2}+[2(n+1)x-1]y_{n+1}+n(n+1)y_n = 0$ **(7)**

b) Explain Log(1+Sin×) in powers of ×. Find the first 5 terms. **(7)**

c) Use Taylors theorem to expand Sin^2 × in powers of $(\times - \pi)$ **(6)**

6. Evaluate : a)

i) $\text{Lim}_{x\to 0}\frac{2Cosx-2+x^2}{x^4}$ ii) $\text{Lim}_{x\to 0}\frac{Log_e(1-x^3)}{Sin^3x}$ iii) $\text{Lim}_{x\to 0}\frac{Sinx-Log(e^xCosx)}{xSinx}$ **(12)**

b)) If Z=f(u,v), where u=aCoshx Cosy and v=aSinhx Siny, prove that

$$\frac{\partial^2 Z}{\partial x^2}+\frac{\partial^2 Z}{\partial y^2}=\frac{a^2}{2}(Cosh2x-Cos2y)\left(\frac{\partial^2 Z}{\partial u^2}+xy\frac{\partial^2 Z}{\partial v^2}\right)$$ **(8)**

Module IV

7.

a) Form the partial differential equation eliminating the arbitrary constants. **(8)**

i) z=axy+b ii) $z = ax^2+by^2+ab$

b) Form the partial differential equation by eliminating arbitrary function from the equation $z=e^yf(x+y)$ **(6)**

c) Solve the partial differential equation $(a-x)\frac{\partial z}{\partial x}+(b-y)\frac{\partial z}{\partial y}=c-z$ **(6)**

8.

a) If $u = Sin^{-1}\left[\frac{x^{\frac{1}{4}}+y^{\frac{1}{4}}}{x^{\frac{1}{6}}+y^{\frac{1}{6}}}\right]$. Evaluate $x^2\frac{\partial^2 z}{\partial^2 x}+2xy\frac{\partial^2 z}{\partial y\partial x}+y^2\frac{\partial^2 z}{\partial^2 y}$ **(6)**

b) Find the maximum and minimum of $f(x,y)=x^3+y^3-3x-12y+20$ **(7)**

c) Use the method of Lagrange's multiplier to find the point on the curve $xy^2=52$ nearest to the origin. **(7)**