



SEM 1 – 1 (RC 16-17)

F.E. (Semester – I) (Revised Course 2016-17) Examination, May/June 2018 ENGINEERING MATHEMATICS – I

Duration : 3 Hours

Total Marks : 100

- Instructions :** 1) Answer **five** questions, atleast **two** from Part – A, two from Part – B and **one** from Part – C.
2) Assume suitable data, **if necessary**.
3) Figures to **right** indicate **full marks**.

PART – A

1. a) Prove that $\int_0^{\infty} \frac{x^5}{5^x} dx = \frac{120}{(\log_e 5)^6}$. 5

b) Prove that $\operatorname{erf}(x) + \operatorname{erfc}(x) = 1$. 5

c) Prove that $\tan \left[i \log \left(\frac{a - ib}{a + ib} \right) \right] = \frac{2ab}{a^2 - b^2}$. 5

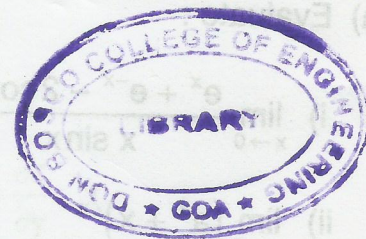
d) If $\log (\log (x + iy)) = p + iq$ then prove that $y = x \tan \left[\tan q \log \sqrt{x^2 + y^2} \right]$. 5

2. a) Test the nature of the following series

i) $\sum_{n=1}^{\infty} \frac{(2n-1)}{n(n+1)(n+2)}$

ii) $\sum_{n=1}^{\infty} \left(2 + \frac{1}{\sqrt{n}} \right)^{-n}$

iii) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{5n+1}$

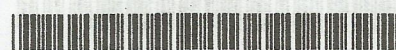


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b) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic function. Find the function $v(x, y)$ such that $u + iv$ is an analytic function. 5

c) Show that $f(z) = \cosh z$ is analytic. 3

P.T.O.



3. a) Find the radius and internal of convergence for the power series

$$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \dots \infty$$

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b) Evaluate $\int_3^7 \sqrt[4]{(x-3)(7-x)} dx$.

6

- c) If $\cosh x = \sec \theta$ then prove that

i) $x = \log (\sec \theta + \tan \theta)$

ii) $\tanh \frac{x}{2} = \tan \frac{\theta}{2}$.

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PART – B

4. a) If $y = a \cos (\log x) + b \sin (\log x)$ where a and b are constants then show that $x^2 y_{n+2} + (2n+1)x y_{n+1} + (n^2+1)y_n = 0$.

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- b) Show that

$$\log (1 - \log (1 - x)) = x + \frac{x^3}{6} + \dots$$

6

- c) If $u = \log \left(\frac{x^4 + y^4}{x + y} \right)$ show that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -3.$$

7

5. a) Evaluate :

i) $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2 \cos x}{x \sin x}$

ii) $\lim_{x \rightarrow 0} (a^x + x)^{\frac{1}{x}}$

iii) $\lim_{x \rightarrow 1} (x^2 - 1) \tan \left(\frac{\pi}{2} x \right)$.

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- b) Solve the partial differential equations :

i) $zx p + zy q = xy$

ii) $x(y-z)p + y(z-x)q = z(x-y)$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$.

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6. a) If $z = \phi(u, v)$, $u = lx + my$, $v = ly - mx$ then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right).$$

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- b) Form the partial differential equations by eliminating the arbitrary constants :

$$Z = (x - a)^2 + (y - b)^2.$$

5

- c) Use method of lagrange multipliers to find the maximum and minimum distances of the point (3, 4, 12) from $x^2 + y^2 + z^2 = 1$.

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PART – C

7. a) Evaluate $\int_0^{\pi/2} \sqrt{\cot \theta} d\theta$.

5

- b) If $\sin(\theta + i\phi) = \tan \alpha + i \sec \alpha$. Prove that $\cos 2\theta \cosh 2\phi = 3$.

5

- c) Use Taylors theorem to express the polynomial $x^5 + 2x^4 - x^2 + x + 1$ in the powers of $(x - 1)$.

5

- d) Find the extreme values of $f(x, y) = 3(x^2 - y^2) - x^3 + y^3$.

5

8. a) Test the convergence of $5 + \frac{10}{3} + \frac{20}{9} + \frac{40}{27} + \dots$

5

- b) Use Demoivre's theorem to solve $x^5 - x^4 + x - 1 = 0$.

5

- c) Verify $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ where $Z = \frac{\log(x^2 + y^2)}{xy}$.

5

- d) Solve the partial differential equation

$$y^2 p - xy q = x(z - 2y) \text{ where } p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}.$$

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