



SEM 2 – 1 (RC 07-08)

F.E. (Semester – II) (Revised 2007 – 08)
Examination, November/December 2015
APPLIED MATHEMATICS – II

Duration : 3 Hours

Total Marks : 100

Instructions : i) Attempt **any five** questions, at least **one** from **each** Module.
ii) Assume suitable data if necessary.

MODULE – I

1. a) Assuming the validity of differentiation under the integral evaluate

$$\int_0^{\infty} e^x \log_e(1 + a^2 e^{-2x}) dx$$

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- b) Find the perimeter of the curve $x^2 + y^2 = 4x$.

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- c) The curve $r = 2a \cos \theta$ is revolved about the x-axis find the surface area of the solid generated.

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2. a) Show that $\vec{r}(t) = Ae^{2t} \vec{i} + Be^{-3t} \vec{j}$, satisfies $\frac{d\vec{r}^2}{dt^2} + \frac{d\vec{r}}{dt} - 6\vec{r} = 0$.

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- b) Find the unit tangent vectors \vec{T} and principal normal \vec{N} for $\vec{r}(t) = \vec{i} \cos^2 2t + \vec{j} \sin 2t + t\vec{k}$ at $t = \pi/2$.

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- c) State and prove Serret-Frenet formula.

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MODULE – II

3. a) Evaluate $\int_0^1 \int_0^1 y e^{xy} dx dy$.

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- b) Write a single integral and evaluate $\int_0^{2\sqrt{y}} \int_0^{\sqrt{y}} 2y + 3 dx dy + \int_{2y-2}^4 \int_{\sqrt{y}}^{\sqrt{y}} 2y + 3 dx dy$.

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- c) Evaluate $\iint r \sin \theta + 3 dr d\theta$ over the region $\{(r, \theta) / r \leq 2 \cos \theta, 0 \leq \theta \leq \pi\}$.

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P.T.O.



4. a) Find the volume of the object generated by the revolution of the region

$$x^2 + y^2 \leq 4x \text{ about the } x\text{-axis.}$$

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- b) Evaluate the cylindrical coordinate integral $\int_0^1 \int_0^\pi \int_0^{1+\cos\theta} 3r \sin\theta + 2 \, dr \, d\theta \, dz$.

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- c) Find the volume of the region bounded by the coordinate planes and the plane $2x + y + 3z = 6$.

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MODULE – III

5. a) Define Divergence of a vector field. Show that divergence of $\frac{\vec{r}}{r^3}$ is zero.

$$\text{Where } \vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \text{ and } r = \sqrt{x^2 + y^2 + z^2}.$$

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- b) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = xy\vec{i} + z\vec{j} + (2y+1)\vec{k}$ and C is the arc of the curve

$$\vec{r} = 2\cos t\vec{i} + 3\sin t\vec{j} + t\vec{k} \text{ from } t = 0 \text{ to } t = \pi/2.$$

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- c) Verify Green's theorem in the plane for $\oint (xy+2) \, dx + (3x^2+y) \, dy$ where C is the boundary of the region enclosed by $x = \sqrt{y}$ and $x = 0$ and $y = 1$.

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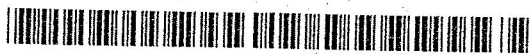
6. a) State Gauss Divergence theorem. Use it to show that $\iiint_S \vec{F} \cdot \vec{n} \, ds = 4\pi$, where S is the surface of the sphere

$$x^2 + y^2 + z^2 = 1, \vec{F}(x, y, z) = (x^2y - 2xz)\vec{i} + (3y - xy^2)\vec{j} + z^2\vec{k} \text{ and } \vec{n} \text{ is the unit normal vector to the surface } S.$$

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- b) Verify Stoke's theorem for $\vec{F} = 2y\vec{i} + (3x^2 + z)\vec{j} + 2yz\vec{k}$, over the surface of the tetrahedron bounded by the coordinate planes and the plane $x + y + z = 1$ above the xy plane.

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MODULE – IV

7. Solve the following differential equations :

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a) $e^y(1+x^2) \frac{dy}{dx} - 2x(1+e^y) = 0$

b) $\frac{dy}{dx} + y \tan x = y^3 \cos x$

c) $\frac{dy}{dx} = \frac{5x - y + 4}{x - 3y + 1}$

d) $(xy + 2x^2y^2) y dx + (xy - x^2y^2) x dy = 0$

8. Solve the following differential equations :

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a) $(D^2 + 4D + 5) y = 3e^{2x} + 5x^2$

b) $(D^3 + 4D^2 + D + 2) y = 3\sin^2 x + 2$

c) $(D^2 + 1) y = \sec x$

d) $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4\cos(\log_e(1+x))$.
