Paper / Subject Code: FE111 / Applied Mathematics-I

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F.E. Semester- I (Revised Course 2007-08)
EXAMINATION Aug/Sept 2019
Applied Mathematics-I

[Duration : Three Hours] **[Max. Marks : 100]**

Instructions:
1. Attempt any five questions at least one from each Module.
2. Assume suitable data, if necessary.

MODULE- I

Q.1 a) Evaluate **(5)**
$\int_0^1 x^3(1-x^2)^{1/2}dx$

b) Show that **(5)**
$\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx$

c) Evaluate **(5)**
$\int_3^7 \left((x-3)(7-x)\right)^{1/4} dx$

d) Prove that the error function is an odd function. **(5)**

Q.2 a) Test the convergence of the following series.

i) $\sum_{n=1}^{\infty} \frac{2n-1}{n(n+1)(n+2)}$ **(4)**

ii) $\sum_{n=1}^{\infty} (1)^{n-1} \frac{1}{4n-3}$ **(4)**

iii) $\sum_{n=1}^{\infty} \left(2+\frac{1}{\sqrt{n}}\right)^{-n}$ **(4)**

b) Define absolutely convergent and conditionally convergent series. Test whether the following series is absolutely convergent or conditionally convergent series:// **(8)**
$\sum_{n=1}^{\infty} \frac{(1)^{n-1}}{2n-1}$

MODULE II

Q.3 a) Use De Moivre's theorem and solve **(6)**
$x^6 - i - = 0$

b) If **(6)**
$\alpha + i\beta = \tanh(x + i\pi/4)$, find $(\alpha^2+\beta^2)$

c) Prove that **(8)**
$\sin^{-1}(cosec\theta) = \frac{\pi}{2} + i log\, (cot\theta/2)$

Q.4 a) Determine a, b, c, d, so that the function **(6)**
$(x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic

b) Show that the function **(6)**
$i^{i^i} = cos\theta + isin\theta$; where $\theta = (2n + 1/2)\pi e^{-(2n+1/2)\pi}$

c) If **(8)**
$u - v = \frac{e^y - cosx + sinx}{coshy - cosx}$
And $f(z) = u + iv$ is an analytic function of $z = x + iy$, find f(z) in terms of z.

MODULE III

Q.5 a) If **(7)**
$y = a\cos(logx) + bsin(\log x)$. prove that
$x^2 y_{n+2} + (2n + 1)x\ y_{n+1} + (n^2 + 1)y_n = 0$

b) Expand **(6)**
$\sec^{-1}\left(\frac{1}{1-2x^2}\right)$ in powers of x. Find the first 4 terms.

c) Expand **(7)**
$\cot\left(x + \frac{\pi}{4}\right)$ in powers of x and hence find $\cot 46.5°$.

Q.6 a) Evaluate : **(12)**

1) $\lim_{x\to 0} \frac{(1+x)^{1/x} - e}{x}$

2) $\lim_{x\to 0} \frac{1}{x} - cotx$

3) $\lim_{x\to \pi/2} (cosx)^{\cos^2 x}$

b) If $z = f(x,y)$ where $u = l\,x + m\,y,\ v = l\,y - m\,x$ then show that **(8)**
$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$

MODULE IV

Q.7 a) Form the partial differential equations by eliminating constants **(6)**
i) $z = (x - a)(y - b)$
ii) $(x - h)^2 + (y - k)^2 + z^2 = a^2$

b) Form the partial differential equations by eliminating functions **(6)**
$f(x + y + z\,, x^2 + y^2 + z^2) = 0$

c) Solve **(8)**

$$x^2p^2 + y^2q^2 = z^2$$

Q.8 a) If **(6)**

$u = \sin^{-1}\left[\frac{x^{1/4}+y^{1/4}}{x^{1/6}+y^{1/6}}\right]$. Find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

b) Obtain the extreme values of the function. **(6)**

$$y^2 + 4xy + 3x^2 + x^3$$

c) Using Lagrange's Multiplier's method find the shortest distance from the point (1, 2, 2) to the sphere. **(8)**

$$x^2 + y^2 + z^2 = 36$$