FE211

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F.E. Semester-II (Revised Course 2007-2008)
EXAMINATION Nov/Dec 2019
Applied Mathematics-II

[Duration : Three Hours] **[Total Marks : 100]**

Instructions:
i) Attempt five questions, at least one from each Module.
ii) Assume suitable data, if necessary.
iii) Figures to the right indicate full marks.

MODULE I

QUESTION 1

1. a) Assuming the validity of Differentiating Under the Integral sign (DUIS), show that **(7)**

$$\int_0^1 \frac{x^{\alpha}-1}{\log(x)}\,dx = \log(1+\alpha).$$

b) Find the perimeter of the cardioid $r = a(1+cos\theta), a > 0$. **(6)**

c) Find the surface area of the surface formed by the revolution of the curve $y = \sqrt{x+2}$ from x=0 to x=4 about the X-axis. **(7)**

QUESTION 2

2. a) Find the tangent, normal and binormal for $\overline{r(t)} = sint\,\hat{\imath} + (t+1)\hat{\jmath} + \cos t\hat{k}$ **(6)**

b) If $\overline{r(t)} = 2cost\hat{\imath} + 3sint\hat{\jmath} + 4t\hat{k}$ is the position vector of a particle in space at time 't' then find it's velocity and acceleration vectors at $t = \frac{\pi}{2}$. **(7)**

c) If $\overline{r(t)} = \bar{a}e^{2t} + \bar{b}te^{2t}$ where $\bar{a}$ and $\bar{b}$ are constant vectors then prove that **(7)**

$$\frac{d^2r}{dt^2} - 4\frac{dr}{dt} + 4r = 0$$

MODULE II

QUESTION 3

3. a) Evaluate $\int_1^2 \int_0^y \frac{1}{x^2+y^2}\,dx\,dy$. **(6)**

b) Evaluate $\iint xy\,dxdy$ over the region bounded by $x^2 = y$ and $y = x$. **(7)**

c) Convert to Polar coordinates and hence evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{(x^2+y^2)}\,dxdy$ **(7)**

QUESTION 4

4. a) The region bounded by $y^2 = x$ and $x = 1$ is revolved about the X-axis. As an application of double Integral, find the volume of the object generated. **(7)**

b) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\pi} \int_0^1 3r^2 \sin^2\varphi\, dr d\theta d\varphi$ **(6)**

c) Use triple integration to find the volume of the region bounded by $x^2 + y^2 = 1$, $z = 0$ and $z = 3$. **(7)**

MODULE III

QUESTION 5

5. a) Find the directional derivative of $f(x,y,z) = x^2y + xyz + 4$ in the direction of $\hat{\imath} + \hat{\jmath} - \hat{k}$ at the point (1,0,1) **(6)**

   b) Find the work done in moving a particle in the force field $\bar{F} = (2y+3)\hat{\imath} + xz\hat{\jmath} + (yz - x)\hat{k}$ along the curve $x = 2t^2, y = t$ and $z = t^3$ from $t = 0\ to\ t = 1$ **(6)**

   c) Verify Green's Theorem in a plane for $\oint (xy + 4y^2)dx + (x^2 + 3)dy$ along the boundary of the region bounded by $x = 1$ and $x = y^2$. **(8)**

QUESTION 6

6. a) Verify Stoke's theorem for $\bar{F} = xy\hat{\imath} - 2yz\hat{\jmath} - zx\hat{k}$ where S is the open surface of the region bounded by the planes x=0 , x=1 , y=0, y=2 and z=3 excluding the face on the XY plane. **(12)**

   b) Prove that $\bar{F} = (y^2 - 2xyz^3)\hat{\imath} + (3 + 2xy - x^2z^3)\hat{\jmath} + (6z^3 - 3x^2yz^2)\hat{k}$ is irrotational and hence find it's Scalar Potential. **(8)**

MODULE IV

QUESTION 7

7. a) Solve $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$ **(5)**

   b) Solve $\frac{dy}{dx} = \frac{2y-x+1}{4y-2x+2}$ **(5)**

   c) Solve $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$ **(5)**

   d) Solve $\frac{dy}{dx} - ytanx = y^4secx$ **(5)**

QUESTION 8

8. a) Solve $(D-2)(D+1)^2y = e^{-2x} + 2$ **(5)**

   b) Solve $(D^2 + 5D + 6)y = \sin(x)\cos(x)$ **(5)**

   c) Solve $(D^3 - D^2 - 6D)y = x^2 + 1$ **(5)**

   d) Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2logx$ **(5)**