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F.E. Semester- I (Revised Course 2019-20)
EXAMINATION SEPTEMBER 2020
Mathematics- I

[Duration : Two Hours] **[Total Marks : 60]**

Instructions:

1) Answer THREE FULL QUESTIONS with ONE QUESTION FROM EACH PART.
2) Assume suitable data, if necessary.
3) Figures to right indicate full marks.

PART A

Q.1

a) Evaluate $\int_0^{\infty} 5^{-4x^2}\, dx$ (4 marks)

b) Use Taylor's theorem evaluate $\sqrt{1.02}$ upto 4 places of decimals. (4 marks)

c) Test the convergence of the following series (12 marks)

i) $\sum_{n=1}^{n=8} \frac{1}{\sqrt{n}} \tan \frac{1}{n}$

ii) $\sum_{n=1}^{n=\infty} \frac{2n^3}{n!}$

iii) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$

Q.2

a) $y = e^{m\cos^{-1}x}$ then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$ (6 marks)

b) Find the interval of convergence of the following series $\sum_{n=1}^{\infty} \frac{n(x+2)^n}{3^n}$ (8 marks)

c) Evaluate $\int_0^1 x^2 \left[\log \frac{1}{x}\right]^5 dx$ (6 marks)

Q.3

a) Evaluate (12 marks)

i) $\lim_{x\to 1} \sec\left(\frac{\pi}{2x}\right) \log x$

ii) $\lim_{x\to\infty} (e^x + x)^{\frac{1}{x}}$

iii) $\lim_{y\to 0} \left[cosec^3 y - \frac{1}{y^3}\right]$

b) Prove that $\beta(m,n) = \frac{1}{2}\int_0^\infty \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\, dx.$ (4 marks)

c) Find the expansion of $\tan^{-1} x$ in powers of (x-1). (4 marks)

PART B

Q.4 a) Solve the following differential equations (12 marks)

i) $(4xy + 3y^2 - x)dx + x\,(x + 2y)dy = 0$

ii) $\frac{dy}{dx} = \frac{6x-4y+3}{3x-2y+1}$

iii) $e^x \frac{dy}{dx} + 3y = x^2 y$

b) If $u = z\tan^{-1}\frac{y}{x}$, find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ (8 marks)

Q.5 a) $\frac{3}{x} + \frac{4}{y} + \frac{5}{z} = 6$ find the values of x,y,z such that $x + y + z$ is minimum using Lagrange's method. (8 marks)

b) If $y = \sec^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then evaluate $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ (6 marks)

c) Solve $secxdy = (y + \sin x)\, dx$ (6 marks)

Q.6 a) Find the extreme values of function $x^2y^3(1 - x - y)$ (6 marks)

b) Verify Euler's Theorem for $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ (8 marks)

c) Solve $\frac{dy}{dx} - \frac{y}{x} = x\,\sin\frac{y}{x}$ (6 marks)

PART C

Q.7 a) Find $\int_0^3 \frac{x^{\frac{3}{2}}}{\sqrt{3-x}}\, dx\ \int_0^1 \frac{dx}{\sqrt{1-x}}$ (8 marks)

b) If $u = \sec^{-1}\left(\frac{x^2+y^2}{x-y}\right)$. Prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\cot u\,(2 + \cot^2 u)$ (7 marks)

c) Evaluate $\lim_{x\to 0}\frac{\sqrt{x}\tan x}{(e^x-1)^{\frac{3}{2}}}$ (5 marks)

Q.8

a) Define absolutely convergent and conditionally convergent series and test the absolute convergence and conditional convergence of the following series. (8 marks)

$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{3\sqrt{n}}$

b) Prove that $\sec^2 x = 1 + x^2 + \frac{2x^4}{3} + \cdots$ (7 marks)

c) Solve $\frac{dy}{dx} + y = y^2(\cos x - \sin x)$ (5 marks)