

SEM 1 – 1 (RC 07 – 08)

F.E. (Semester – I) (Revised Course 2007-08) Examination, Nov./Dec. 2014 APPLIED MATHEMATICS – I

Duration : 3 Hours

Total Marks : 100

Instructions : 1) Attempt **any five** questions, at least **one** from **each** Module.
2) **Assume** suitable data, if necessary.

MODULE – I

1. a) Show that $\int_0^{\infty} x^n e^{-a^2 x^2} dx = \frac{1}{2a^{n+1}} \Gamma\left(\frac{n+1}{2}\right)$ and hence deduce that

$$\int_0^{\infty} e^{-a^2 x^2} dx = \frac{\sqrt{\pi}}{2a} \quad 6$$

b) Evaluate $\int_0^{\infty} e^{-\sqrt{x}} \sqrt[4]{x} dx$. 4

c) Prove that $\int_0^1 \left(1 - x^{\frac{1}{n}}\right)^m dx = \frac{m!n!}{(m+n)!}$. 5

d) Show that $\operatorname{erf}_c(x) + \operatorname{erf}_c(-x) = 2$. 5

2. a) State Cauchy's Root test for the convergence of a series. 2

b) When is a series said to be absolutely convergent ? Give an example of absolutely convergent series. Also show that every absolutely convergent series is convergent. 6

c) Test the convergence of the following series : 12

i) $\sum_{n=0}^{\infty} \frac{2^n n!}{n^n}$

ii) $\sum_{n=1}^{\infty} \left(2 + \frac{1}{\sqrt{n}}\right)^{-n}$

iii) $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \dots$

P.T.O.



MODULE – II

3. a) If $\tan(\theta + \phi) = e^{i\alpha}$. Prove that $\theta = \frac{n\pi}{2} + \frac{\pi}{4}$ and $\phi = \frac{1}{2} \log_e \tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)$. 6
- b) If $\tan\left(\frac{x}{2}\right) = \tanh\left(\frac{u}{2}\right)$. Prove that $u = \log_e \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$. 4
- c) Express $\sin 5\theta$ in terms of multiples of $\sin \theta$. 5
- d) Prove that $\cos^{-1} x = \log_e \left(x + \sqrt{x^2 - 1}\right)$. 5
4. a) Prove that $\log_e (-\log_e i) = \log_e \frac{\pi}{2} - i \frac{\pi}{2}$. 5
- b) Considering principal value only, separate $(\sqrt{i})^i$ into real and imaginary parts. 5
- c) Solve the equation $x^5 + 1 = 0$. 5
- d) Show that $f(z) = \log_e z$ is analytic everywhere in the complex plane except at origin and find its derivative. 5

MODULE – III

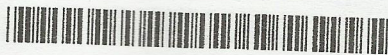
5. a) If $y = a \cos(\log_e x) + b \sin(\log_e x)$ then show that $x^2 y_{n+2} + 2(n+1)xy_{n+1} + (n^2 + 1)y_n = 0$. 7
- b) Show that $\log_e(1 - \log_e(1 - x)) = x + \frac{x^3}{6} + \dots$ 7
- c) Use Taylor's theorem to express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $(x - 2)$. 6

6. Evaluate :

a) i) $\lim_{x \rightarrow 1} (1 - x^2)^{\frac{1}{\log_e(1-x)}}$

ii) $\lim_{x \rightarrow a} \log_e \left(2 - \frac{x}{a}\right) \cot(x - a)$

iii) $\lim_{x \rightarrow 0} \left[\frac{1}{x^2} - \frac{1}{x \tan x} \right]$



- b) If $z = f(u, v)$, where $u = x^2 - y^2$ and $v = 2xy$, prove that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 4(x^2 + y^2) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right).$$

8

MODULE – IV

7. a) Form the partial differential equation eliminating the arbitrary functions

i) $z = f(x^2 + y^2) + x + y$

ii) $f(x + y + z, x^2 + y^2 + z^2) = 0$.

8

- b) Solve the partial differential equation

i) $x(z^2 - y^2) \frac{\partial z}{\partial x} + y(x^2 - z^2) \frac{\partial z}{\partial y} = z(y^2 - x^2)$

ii) $\frac{y^2 z}{x} \frac{\partial z}{\partial x} + xz \frac{\partial z}{\partial y} = y^2$.

12

8. a) If $u = \sin^{-1}(\sqrt{x^2 + y^2})$, find the value of $x^2 \frac{\partial^2 u}{\partial^2 x} + 2xy \frac{\partial^2 u}{\partial y \partial x} + y^2 \frac{\partial^2 u}{\partial^2 y}$.

8

- b) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$.

6

- c) Examine the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ for extreme values.

6