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SEM 1 – 1

F.E. (Semester – I) (Revised Course 2007-08) Examination, May/June 2013

APPLIED MATHEMATICS – I

Duration : 3 Hours — Total Marks : 100

Instructions: *1) Attempt **any five** questions, at least **one** from **each** Module.*
*2) **Assume** suitable data, **if necessary**.*

MODULE – I

1. a) Show that $\int_0^\infty \frac{x^5}{5^x}dx = \frac{120}{(\log_e 5)^6}$. **4**

b) Evaluate $\int_0^1 \frac{x}{\sqrt{1-x^5}}dx$ **4**

c) Express $\int_0^1 x^m(1-x^n)^p dx$ in terms of Gamma function. **6**

d) Prove that $\frac{d}{dx}(\text{erf}_c(ax)) = \frac{-2a}{\sqrt{\pi}}e^{-a^2x^2}$. **6**

2. a) Test the convergence of the following series : **12**

i) $\frac{3}{4} + \frac{3.4}{4.6} + \frac{3.4.5}{4.6.8} + \ldots \infty$

ii) $\sum_{n=1}^{\infty} n\text{Sin}^2\left(\frac{1}{n}\right)$

iii) $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(3n+1)^2}$

P.T.O.

b) Define the interval of convergence and find it for the series : **8**

$$x-\frac{x^2}{\sqrt{2}}+\frac{x^3}{\sqrt{3}}-\frac{x^4}{\sqrt{4}}+\ldots\ldots$$

MODULE – II

3. a) Find all values of $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{3/4}$ and show that their product is 1. **6**

b) If $x + iy = \text{Cosh}\,(u + iv)$ show that **6**

i) $\frac{x^2}{\text{Cosh}^2 u}+\frac{y^2}{\text{Sinh}^2 u}=1$

ii) $x^2\,\text{Sec}^2 v - y^2\,\text{Co sec}^2\,v = 1$

c) If $\text{Cosh}x = \text{Sec}\theta$ prove that **8**

i) $x = \text{Log}\,(\text{Sec}\theta + \text{Tan}\,\theta)$

ii) $\theta=\frac{\pi}{2}-2\text{Tan}^{-1}(e^{-x})$.

4. a) Prove that $\text{Cos}\left[i\log\left(\frac{a-ib}{a+ib}\right)\right]=\frac{a^2-b^2}{a^2+b^2}$. **6**

b) Show that the function $f(z) = \text{Cosh}z$ is analytic and find its derivative. **6**

c) Determine the analytic function whose real part is $e^{2x}\,(x\,\text{Cos}2y - y\,\text{Sin}2y)$. **8**

MODULE – III

5. a) If $y = e^{\text{Tan}^{-1}x}$ prove that $(1 + x^2)\,y_{n+2} + [2\,(n + 1)\,x - 1]\,y_{n+1} + n\,(n + 1)\,y_n = 0$. **7**

b) Prove that $\log_e(1 - x + x^2) = -x+\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{1}{4}x^4-\frac{1}{5}x^5+\ldots.$ **7**

c) Expand $\text{Sin}^2 x$ in powers of $\left(x-\frac{\pi}{2}\right)$. **6**

6. Evaluate : **12**

a) i) $\lim_{x\to 0} \dfrac{\log_e(1-x^2)}{\log_e(\mathrm{Cos}x)}$

ii) $\lim_{x\to a} \log_e\left(2-\dfrac{x}{a}\right)\mathrm{Cot}(x-a)$

iii) $\lim_{x\to 0} (\mathrm{Cos}x)^{1/x^2}$

b) If $x = u + v + w$, $y = uv + vw + wu$, $z = uvw$ & ϕ is a function of x, y, z, **8**

prove that $x\dfrac{\partial\phi}{\partial x} + 2y\dfrac{\partial\phi}{\partial y} + 3z\dfrac{\partial\phi}{\partial z} = u\dfrac{\partial\phi}{\partial u} + v\dfrac{\partial\phi}{\partial v} + w\dfrac{\partial\phi}{\partial w}$

MODULE – IV

7. a) Form the partial differential equation eliminating the arbitrary functions from $z = f(x + 7y) + g(x - 7y)$. **5**

b) Form the partial differential equation by eliminating arbitrary constants from

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1.$ **4**

c) Solve the following partial differential equation :

i) $p^2 + 10pq + 25q^2 = 0$ **5**

ii) $px^2 + qy^3 - x\,\mathrm{Cot}z = 0$ **6**

where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$ and z is a function of x and y.

8. a) If $u = \mathrm{Cot}^{-1}\left[\dfrac{2x^{\frac{1}{4}} - y^{\frac{1}{4}}}{x-y}\right]$ then find the value of $x^2\dfrac{\partial^2 u}{\partial^2 x} + 2xy\dfrac{\partial^2 u}{\partial y\partial x} + y^2\dfrac{\partial^2 u}{\partial^2 y}$. **7**

b) Find the maximum and minimum values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$. **6**

c) Find the largest rectangle that can be inscribed inside $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$. **7**

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