

SEM 1 – 1 (RC 07-08)

F.E. (Semester – I) (RC 2007-08) Examination, Nov./Dec. 2016 APPLIED MATHEMATICS – I

Duration : 3 Hours

Total Marks : 100

- Instructions :** 1) Attempt **any five** questions, atleast **one** from **each** Module.
2) **Assume** suitable data, if **necessary**.

MODULE – I

1. a) Evaluate $\int_0^1 x^6 (1-x^2)^{\frac{1}{2}} dx$. 5

b) Evaluate $\int_0^1 (x \log x)^3 dx$. 5

c) Show that $B(m, n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$. 5

d) Prove that $\frac{d}{dx} [\operatorname{erf}(ax)] = \frac{2a}{\sqrt{\pi}} e^{-a^2 x^2}$. 5

2. a) Test the convergence of the following series.

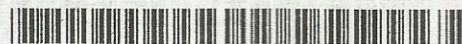
i) $\sum_{n=1}^{\infty} \left(\frac{n!}{n^n} \right)^n$. 4

ii) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$. 4

iii) $2 - \frac{3}{2} + \frac{4}{3} - \frac{5}{4} + \dots$ to ∞ . 4

b) Define interval of convergence of power series and hence find it for the series.

$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \dots$ 8



MODULE – II

3. a) Use De Moivre's theorem and solve $x^9 - x^5 + x^4 - 1 = 0$. 6
- b) If $\sin(\alpha + i\beta) = x + iy$ prove that :
- i) $\frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1$
- ii) $\frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1$. 8
- c) Separate into real and imaginary part $\sqrt{i}^{\sqrt{i}}$. 6
4. a) Determine the analytic function whose real part is $\cos x \cosh y$. 6
- b) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Find the conjugate harmonic function. 6
- c) Show that $i^i = \cos \theta + i \sin \theta$; $\theta = \left(2n + \frac{1}{2}\right)\pi e^{-\left(2n + \frac{1}{2}\right)\pi}$. 8

MODULE – III

5. a) If $y = \log \left(x + \sqrt{x^2 + p^2}\right)^2$, prove that,
 $(x^2 + p^2)y_{n+2} + (2n + 1)x y_{n+1} + n^2 y_n = 0$. 7
- b) Expand $\log(1 + e^x)$ in powers of x . Find the first 5 terms. 6
- c) Use Taylor's theorem to show that $\sqrt{1+x+2x^2} = 1 + \frac{x}{2} + \frac{7}{8}x^2 - \frac{7}{16}x^3 + \dots$. 7
6. a) Evaluate : 12
- i) $\lim_{x \rightarrow a} \frac{\log(x-a)}{\log(e^x - e^a)}$ ii) $\lim_{x \rightarrow a} \left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ iii) $\lim_{x \rightarrow 0} \frac{e^x - e^{\sin x}}{x - \sin x}$
- b) If $x = p \cos \theta - q \sin \theta$, $y = p \sin \theta + q \cos \theta$ and u is function of x and y , then show that
- $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial p^2} + \frac{\partial^2 u}{\partial q^2}$$
- 8



MODULE – IV

7. a) Form the partial differential equations by eliminating constants

i) $z = (x - a)(y - b)$.

ii) $(x - h)^2 + (y - k)^2 + z^2 = a^2$.

6

b) Form the partial differential equations by eliminating functions

$z = f(z^2 - xy, x/z)$.

6

c) Solve $x^2p^2 + y^2q^2 = z^2$.

8

8. a) If $u = \sec^{-1} \left[\frac{\sqrt{x} - 2\sqrt{y}}{y^3\sqrt{x}} \right]$, find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$.

6

b) Obtain the maxima and minima of the function $x^3 + y^3 - 63(x + y)$.

6

c) Use the method of Lagrange's multipliers to find the greatest and smallest

values that the function $f(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

8
