Paper / Subject Code: FE211 / Applied Mathematics-II

FE211

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F.E. (Semester- II) (Revised Course 2007-08)
EXAMINATION OCTOBER 2020
Applied Mathematics- II

[Duration : Two Hours] **[Total Marks : 60]**

Instructions:

1) Answer THREE FULL QUESTIONS with ONE QUESTION from ANY THREE MODULES.
2) Assume suitable data if necessary.

MODULE- I

Q.1

a) Evaluate $\int_0^{\pi/2} Log_e\,(\alpha^2 Sin^2 x + \beta^2 Cos^2 x)dx$ by applying differentiation under the integral. (7)

b) Find the length of the curve $x = \frac{y^3}{6} + \frac{1}{2y}$ from $y = 1\ to\ y = 16$. (6)

c) The cardiode $r = 2 + 2\ Cos\theta$ is revolved about the initial line, compute the surface area of the object generated. (7)

Q.2

a) Define curvature. If $\bar{r}(t)$ is any vector point function, Show that curvature $K = \frac{|\dot{\bar{r}} \times \ddot{\bar{r}}|}{|\dot{\bar{r}}|^3}$. Use it to find the curvature of $\frac{x^2}{4} + \frac{y^2}{9} = 1$ at point (0,3) . (8)

b) Solve the vector differential equation $\frac{d\bar{r}}{dt} = t\bar{i} + S\ int\,\bar{j} + 5\bar{k}$ with initial condition $\bar{r}(0) = 2\bar{i} + 3\bar{j} - \bar{k}$ (6)

c) Find the unit Tangent vector T and principal normal N of the curve $x = 3t^2 + 2, y = 2t + 3, z = 2t^2 - 1$ at the point t= 1 (6)

MODULE- II

Q.3

a) Evaluate $\int_0^1 \int_0^{\sqrt{x}} \frac{2x^2 y}{x^2+y^4} dxdy$ (6)

b) Evaluate $\iint 2x + 3dxdy$ over the region enclosed by $y^2 = x$ and $x - y = 2$ (6)

c) Write the following as a single integral and evaluate $\int_0^1 \int_0^{\sqrt{x}} 2xydxdy + \int_0^2 \int_0^{2-x} 2xydxdy$ (8)

Q.4

a) Evaluate $\iiint \frac{z}{\sqrt{x^2+y^2}}\, dxdydz$ over the region $\{(x, y, z)/x^2 + y^2 + z^2 \leq 4, x \geq 0, y \geq 0, z \geq 0\}$ (6)

b) Evaluate $\iiint 3y + xzdxdydz$ over the region enclosed by $y^2 = 4x, z = 0, z = 1\ and\ x = 1$. (6)

c) Find the volume of the tetrahedron bounded by the coordinate planes and plane 2x+y +3z=6 (8)

MODULE- III

Q.5

a) If $r = \sqrt{x^2 + y^2 + z^2}$ and $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$ then show that $div\left(\frac{\vec{r}}{r^3}\right) = 0$ (6)

b) Find the work done in moving a particle in a force field $\overline{F} = zx\overline{\imath} + 2x^2y\,\overline{\jmath} + (z^2 + 2y)\overline{k}$, along the curve in the plane x=2 and having equation $y^2 = 4z$ from (2,2,1) to (2,4,4) (6)

c) Using Stoke's theorem or otherwise evaluate $\iint_S \nabla \times \vec{F}.nds$ where $\vec{F} = (x^2 + 3y)\vec{\imath} + 2y\vec{\jmath} + (3x + z)\vec{k}$ and S is section of the plane x+ 2y+z =4 which is in the first octant. (8)

Q.6

a) State and prove Green's theorem in the plane. (8)

b) Verify Gauss divergence theorem for $\vec{F} = (2x + y)\vec{\imath} + (y^2 + z)\vec{\jmath} + z^2\vec{k}$ and S is the surface of the cube $0 \leq x, y, z \leq 1$. (12)

MODULE- IV

Q.7 Solve the following differential equations (20)

a) $(x^2y^2 + 2xy + 1)\, ydx + (y^2x^2 - xy + 3)xdy = 0$

b) $(1 + x)\frac{dy}{dx} + 1 = 2e^{-y}$

c) $(x + 2y + 3)dx - (2x - y + 1)dy = 0$

d) $Sinx\ \frac{dy}{dx} + 3y = Cos\ x$

Q.8 Solve the following differential equations (20)

a) $(D^2 + 4D - 8)y = e^{2x}\,Cos\ x + x$

b) $(D^2 + 3D - 4)\ y = x\ Sin2x$

c) $(D + 2)(D - 1)^2y = e^{2x} + x^2 + 3x$

d) $x^2\frac{d^2y}{dx^2} - 2x\ \frac{dy}{dx} - 4y = 3x^2 + \log_e x$