SEM 2 – 1 (RC 07-08)

F.E. (Semester – II) (Revised 07-08) Examination, Nov./Dec. 2016
APPLIED MATHEMATICS – II

Duration : 3 Hours Total Marks : 100

Instructions : *i) Attempt **any five** questions, at least **one** from **each** Module.*
*ii) Assume suitable data **if necessary**.*

MODULE – I

1. a) Evaluate $\int_0^1 \frac{x^a - x^b dx}{\log_e x}$ by applying differentiation under the integral sign. 7

b) Find the length of the loop of the curve $x = 3t^2$, $y = t - 3t^3$. 6

c) The curve $r = 4\text{Cos}\,\theta$ is revolved about the initial line. Find surface area of the object generated. 7

2. a) Define curvature of a curve at a point. Show that the curvature of the parabola $y^2 = 4ax$ is maximum at the vertex. 6

b) Find the unit tangent vector $\vec{T}$ and principal normal $\vec{N}$ for $\overrightarrow{r(t)} = \vec{i}\,\text{Cos}^2(2t) + \vec{j}\,\text{Sin}(2t) + \vec{k}t$ at $t = \pi/2$. 7

c) An object moves with constant acceleration i + 2j. If the initial displacement and velocity is 2i + 3k and 2i + j – 3k respectively, find the position vector of the object at time t. 7

MODULE – II

3. a) Evaluate $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$. 6

b) Write the following as a single integral and evaluate

$\int_0^1\int_0^1 2x + 5dxdy + \int_1^2\int_0^{2-x} 2x + 5dxdy$. 7

c) Evaluate $\iint r + 2\text{Sin}\theta drd\theta$ over the region bounded by $r = 2(1 + \text{Cos}\,\theta)$ above the initial line. 7

4. a) Find by double integration the volume of the solid generated by the revolution of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis. 7

b) Evaluate $\iiint e^{2x+3y+z} dxdydz$. 6

c) Find the volume of the region bounded by the co-ordinate planes and $x + y + z = 2$. 7

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MODULE – III

5. a) Define divergence of a vector field. Show that if $\bar{a}$ is a constant vector and $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ then $\mathrm{div}(\bar{a} \times \bar{r}) = 0$. 6

b) Find the directional derivative of $f(x, y, z) = xz^2 + 3yx$ at the point $(2, 1, -1)$ in direction of the vector $3\bar{i} + \bar{j} - 2\bar{k}$. 4

c) Verify Stoke's theorem for $F = (2x - y)\,\bar{i} - yz^2\bar{j} - y^2z\vec{k}$. Where S is the surface of the tetrahedron bounded by the co-ordinate planes and the plane $x + y + z = 1$ above the xy plane. 10

6. a) Find the work done in moving a particle in a force field $\bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the curve $x = 3t^2$, $y = 2t$, $z = t + 1$ from $t = 0$ to $t = 2$. 6

b) Verify Green theorem in the plane for $\oint_C (x + 3y^2)dx + 2xydy$ where C is the triangle having vertices $(-1, 0)$, $(1, 2)$ and $(1, 0)$. 8

c) Use Gauss divergence theorem to evaluate $\iint_S F.\bar{n}ds$ where $F = xy\bar{i} + yz\bar{j} + y\bar{k}$, $\bar{n}$ is the unit normal vector to S the surface of the region bounded co-ordinate planes and the planes $x = 1$, $y = 1$ and $z = 1$ 6

MODULE – IV

7. Solve the following differential equations : 20

a) $(x^2y^3 + 2y)dx + (2x - 2x^3y^2)dy = 0$.

b) $2\frac{dy}{dx} - \frac{y}{x} = \frac{y^2}{x^2}$

c) $\frac{dy}{dx} = \frac{2y - x + 1}{y + 2x - 5}$

d) $(y^2 + x^2)dx = 2xydy$.

8. Solve the following differential equations : 20

a) $(D - 2)(D + 1)^2\, y = e^{-2x} + 2$

b) $(D^2 + 4)y = x^2 + 2x$

c) $(D^2 - 3D + 2)y = x\mathrm{Sin}x$

d) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = (\log_e x)^2$.
