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SEM 1-1 (RC 07-08)

F.E. (Sem. – I) (Revised 2007-08 Course) Examination, May/June 2012
APPLIED MATHEMATICS – I

Duration : 3 Hours | Total Marks : 100

Instructions : *1) Attempt **any five** questions. Atleast **one** from **each** Module.*
*2) **Assume** suitable data, **if necessary.***

MODULE – I

1. a) Prove that $\int_0^1 \frac{dx}{\sqrt{-\log x}} = \sqrt{\pi}$. **5**

b) Evaluate $\int_0^\infty \frac{p^{a-1}}{(1+p)^{a+b}} dp$. **5**

c) Prove that $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}} dx = \frac{\beta(m,n)}{a^n(1+a)^m}$. **6**

d) Show that $\int_0^\infty e^{-x^2-2bx}\, dx = \frac{\sqrt{\pi}}{2} e^{b^2}[1-\text{erf}(b)]$. **4**

2. a) Test the convergence of the following series. **12**

1) $\left[\frac{2^2}{1^2} - \frac{2}{1}\right]^{-1} + \left[\frac{3^3}{2^3} - \frac{3}{2}\right]^{-2} + \left[\frac{4^4}{3^4} - \frac{4}{3}\right]^{-3} + \ldots \infty$

2) $\sum \frac{3^n . n!}{n^n}$

3) $\frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \frac{5}{26} \ldots \infty$

b) Define absolutely convergent and conditionally convergent series. Find the type of the following series $\sum_1^\infty \frac{(-1)^{n-1}}{2n-1}$. **8**

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MODULE – II

3. a) Solve the following equation by using De Moivre's theorem $x^9 - x^5 + x^4 - 1 = 0$. **4**

b) If $\cosh x = \sec\theta$. Prove that $\tanh\frac{x}{2} = \tan\frac{\theta}{2}$. **5**

c) If $\sinh(\theta + i\theta) = \cos\alpha + i\sin\alpha$ prove that $\cos^2\alpha = \cos^4\phi$. **6**

d) If $\tan\log(x + iy) = a + ib$, $a^2 + b^2 \neq 1$. Prove that $\tan\log(x^2 + y^2) = \frac{2a}{1 - a^2 - b^2}$. **5**

4. a) Prove that $\cosh^{-1}\sqrt{1+x^2} = \tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$. **6**

b) If the following function is analytic find values of a, b.
$f(z) = \cos x(\cosh y + a\sinh y) + i\sin x(\cosh y + b\sinh y)$ **6**

c) If f(z) is analytic function find it in term of z if $U - V = \frac{e^y - \cos x + \sin x}{\cosh y - \cos x}$. **8**

MODULE – III

5. a) If $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$ prove that $(1 - x^2)y_{n+2} - (2n + 3)xy_{n+1} - (n + 1)^2 y_n = 0$.

Hence deduce $y_n(0)$. **9**

b) Prove that $(1 + x)^x = 1 + x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \frac{3}{4}x^5 + \ldots$ **6**

c) Calculate the approximate value of cos 32° using Taylor's theorem. **5**

6. a) If $W = f(u, v)$, where $u = a\cosh x\cos y$, $v = a\sinh x\sin y$.

Prove that $\frac{\partial^2 w}{\partial\lambda^2} + \frac{\partial^2 w}{\partial y^2} = \frac{a^2}{2}(\cosh 2x - \cos 2y)\left(\frac{\partial^2 w}{\partial u^2} + \frac{\partial^2 w}{\partial v^2}\right)$. **8**

b) Evaluate : **12**

1) $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$

2) $\lim_{x \to 0} \left(\frac{1}{x}\right)^{1-\cos x}$

3) $\lim_{x \to 0} \frac{\log(x-a)}{\log(e^x - e^a)}$

MODULE – IV

7. a) If $u = \tan^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{y}}\right)$ find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. **6**

b) Examine the function $u = xy + a^3\left(\frac{1}{x}+\frac{1}{y}\right)$ for extreme values. **7**

c) Find the area of a greatest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **7**

8. a) Form a partial differential equation by eliminating arbitrary constants. **8**

1) $ax^2 + by^2 + z^2 = 1$

2) $x^2 + y^2 + (z-a)^2 = b^2$

b) Solve the following partial differential equation. **12**

1) $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$

2) $P(1+q^2) = q(z-a)$.
