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F.E. Semester I (Revised course 2016-17)
EXAMINATION JANUARY 2022
Engineering Mathematics-I

[Duration : Three Hours] **[Total Marks :100]**

Instructions:

1. Attempt **five** questions, any two questions each from **PART-A** and **PART-B** and **one** from **PART-C**
2. Assume suitable data, if necessary.
3. Figures to the **right** indicate full **marks**

PART A

Q.1 Answer any **TWO** questions from the following 2 × 20=40Marks

a) Show that $\int_0^\infty x^n\, e^{-a^2x^2}dx = \frac{1}{2a^{n+1}}\Gamma\left(\frac{n+1}{2}\right)$. (7)

Hence deduce that $\int_0^\infty e^{-a^2x^2}\,dx = \frac{\sqrt{\pi}}{2a}$.

b) Show that $\int_0^1 \left(1 - x^{1/n}\right)^m dx = \frac{n!m!}{(m+n)!}$ (6)

c) Separate into real and imaginary part z^z where $z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$. (7)

Q.2 a) Test the following series for convergence (12)

i) $\frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \cdots$

ii) $\sum_{n=1}^{\infty} \frac{n^2}{e^2}$

iii) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$

b) If $\tan\left(\frac{\pi}{6} + i\alpha\right) = x + iy,\ prove\ that\ x^2 + y^2 + \frac{2x}{\sqrt{3}} = 1$ (8)

Q.3 a) Define the interval of convergence and find it for the following series (6)

$$\frac{1}{2}x + x^2 + \frac{9}{8}x^3 + x^4 + \frac{25}{32}x^5 + \cdots$$

b) Prove that $\tan\left[i\log\left(\frac{a-ib}{a+ib}\right)\right] = \frac{2ab}{a^2-b^2}$ (6)

c) Show that $u = \frac{1}{2}\log(x^2 + y^2)$ is harmonic function. Find the analytic function for which u is the real part. (8)

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PART B

Q.4 Answer any **TWO** questions from the following: 2 × 20=40 Marks

a) If $u = cosec^{-1}\left(\frac{\sqrt{x}+\sqrt{y}}{x^{1/3}+y^{1/3}}\right)$, find $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$ (8)

b) If $y = (x^2 - 1)^2$, then show that (7)
$(x^2 - 1)y_{n+2} + 2x\, y_{n+1} - n(n + 1)y_n = 0$

c) Use Taylor's theorem to expand $\frac{1}{x^2}$ in powers of x-1. (5)

Q.5 a) Evaluate (12)

(i) $\lim_{x\to\pi/2}(secx)^{tanx}$

(ii) $\lim_{x\to 0}(coosx)^{\frac{1}{x^2}}$

(iii) $\lim_{x\to 0}\frac{e^x+e^{-x}-2\cos x}{x\sin x}$

b) Form the partial differential equation by eliminating the arbitrary constants (8)

i) $z = (x - a)^2 + (y - b)^2$

ii) $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$

Q.6 a) If $Z=f(u,v)$ where $u=x-y$ and $v-xy$, prove that (8)
$x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial y^2} = (x + y)\left(\frac{\partial^2 z}{\partial u^2} + xy\frac{\partial^2 z}{\partial v^2}\right)$

b) Solve the partial differential equation (6)
$y^2zp + x^2zp = xy^2 \; where \; p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$

c) Find the maximum and minimum of $f(x, y) = x^3 + y^3 - 3axy$ (6)

PART C

Q.7 Answer any **ONE** questions from the following 1 × 20=20 Marks

a) Prove that $\int_0^{\infty}\frac{x^{a-1}}{(1+x)^{a+b}}dx = \int_0^{1}\frac{x^{a-1}+x^{b-1}}{(1+x)^{a+b}}dx$ (7)

b) If $\tan z = \frac{i}{2}(1 - i)$, then prove that $z = \frac{tan^{-1}z}{2} + \frac{i}{4}log_e 5$ (6)

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c) Prove that $\sin(e^x - 1) = x + \frac{1}{2}x^2 - \frac{5}{24}x^4 + \cdots$ (7)

Q.8

a) Test the convergence of the series $\frac{2}{1} + \frac{2^2}{2} + \frac{2^3}{3} + \cdots$ (6)

b) If $e^z = \sin(u+iv)$ and $z = x+iy$, then prove that (6)

$2e^{2x} = \cosh 2v - \cos 2u$.

c) use the method of Lagrange's multiplier to find the point on $x^2 + y^2 + z^2 = 25$ where $f(x,y,z) = x+2y+3z$ has its maximum and minimum. (8)

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