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F.E. Semester-II (Revised Course 2016-17)
EXAMINATION OCTOBER 2020
Engineering Mathematics –II

[Duration : Two Hours] **[Total Marks : 60]**

Instructions:
1) Answer THREE FULL QUESTIONS with ONE QUESTION FROM EACH PART.
2) Assume suitable data, if necessary.
3) Figures to the right indicate full marks.

Part- A

Q.1
a) Evaluate $\int_0^{\pi} \sec x\, log_e(1+\alpha Cosx\,)dx$ by applying differentiation under the integral sign. 7

b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{dxdy}{1-x^2+y^2}$ 6

c) Find the perimeter of the asteroid $x = aCos^3 t, y = a\, Sin^3 t$ 7

Q.2
a) Show that $\bar{r}(t) = Ate^{-2t}\,\bar{\imath} + Be^{3t}\,\bar{\jmath}$, satisfies $\frac{d\bar{r}^2}{dt^2} + \frac{d\bar{r}}{dt} - 6\bar{r} = 0.$ where A and B are any constants. 6

b) Change the order of integration and evaluate $\int_1^3 \int_{x-1}^{(2x-2)} x + 3\; dxdy$ 8

c) Define Curvature. Show that the curvature of a circle is the same at every point. 6

Q.3
a) Evaluate $\int\int 2r\, Sin\theta + 4drd\theta$ over the region $1 + Cos\theta \le r \le 1$ 6

b) Evaluate the cylindrical coordinate integral $\int_0^{\pi} \int_0^1 \int_{2r}^{\sqrt{5-r^2}} 2zSin\theta dzdrd\theta$ 6

c) Find the volume of the region $\{(x,y,z)/x^2 + y^2 \le 4, 0 \le z \le 3\}$ 8

Part- B

Q.4
a) Solve the following differential equations 10

i) $x\frac{dy}{dx} + y = y^2 \log_e x$ ii) $\frac{dy}{dx} = \frac{y-x-1}{2y-3x+3}$

b) Verify Green theorem in the plane for $\oint (y^2 + 2x\,)dx + (5 + xy)dy$ where C is the boundary of the region enclosed by $y^2 = 4x \;and\; y = 2x$ 10

Q.5
a) Solve the following differential equations 10

i) $(D^2 + 4D - 5)y = 3e^{-2x} + 5x$

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ii) $(D^3 - 4D + 3)y = 3Sinx$

b) If $r = \sqrt{x^2 + y^2 + z^2}$ and $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ then show that $\mathrm{Curl}(r^n \bar{r}) = 0$ **5**

c) Find the direction along which the directional derivative of $F(x, y, z) = x^2 + 2yx + z^2$ at (2,0,-1) is maximum. What is the magnitude of the maximum? **5**

Q.6 a) Define divergence of a vector field. If $\bar{r} = x\bar{\imath} + y\bar{\imath} + z\bar{k}$ and $\bar{a}$ is a constant vector. Then show that div $(\bar{a}x\bar{r}) = 0$ **6**

b) Use Stoke's theorem to evaluate $\int_s \int \nabla \times F.\bar{n}ds$ $where$ $F = xy\bar{\imath} + 3y^2\bar{\jmath} - 2yx^2\bar{k}, \bar{n}$ is the unit normal vector to S, the surface of the region boundary by x= 0, y=0, 2x+y+z =2 and z=0 which is not included in the xy plane. **8**

c) Solve the second order homogeneous linear differential equation $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^{2x}$ **6**

Part- C

Q.7 a) The ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is revolved about the x-axis find the volume area of the object generated. **6**

b) Evaluate the integral $\int_0^\infty \int_x^\infty e^{-(x^2+y^2)}dxdy$ by changing to polar coordinates **8**

c) Solve the first order differential equation $y(xy + 2x^2 y^2)dx + x(yx - x^2 y^2)dy = 0$ **6**

Q.8 a) Find the work done in moving a particle in a force field $\bar{F} = (2x + 3)\bar{\imath} + (xz + y)\bar{\jmath} + z\bar{k}$ along the curve $x = t,\ y = 2t^2,\ z = t + 2\ from\ t = 0\ to\ t = 3.$ **6**

b) Find the area of the surface generated by the revolution of the curve $y = \sqrt{x + 2}\quad 0 \le x \le 4$ about the x-axis **7**

c) Solve the first order differential equation $\frac{dy}{dx} + \frac{xy}{1-x^2} = xy^{1/2}$ **7**