SEM 2-1 – (RC 16-17)

F.E. (Semester – II) (Revised in 2016-17) Examination, May/June 2017
ENGINEERING MATHEMATICS – II

Duration : 3 Hours Max. Marks : 100

Instructions : i) *Attempt **five** questions, **any two** questions **each** from Part – **A** and Part – **B** and **one** from Part – **C**.*

ii) *Assume **suitable** data, **if** necessary.*

iii) *Figures to the **right** indicate **full** marks.*

PART – A

Attempt **any two** questions from this section :

1. a) Assuming the validity of differentiation under the integral evaluate

$\int_0^\infty e^x \log_e (1+a^2 e^{-2x})dx.$ **8**

b) Evaluate $\int_0^1\int_0^1 ye^{xy+2}dxdy$. **6**

c) Find the length of the curve $x(t) = 1 - \text{Sint} + \frac{t}{\sqrt{5}}, y(t) = \frac{2}{\sqrt{5}}$ Cost from $t = 0$ to $t = \pi/2$. **6**

2. a) Show that $\bar{r}(t) = Ate^{2t}\bar{i} + Be^{-3t}\bar{j}$, satisfies $\frac{d\bar{r}^2}{dt^2} - \frac{d\bar{r}}{dt} - 6\bar{r} = 0$. **6**

b) Write a single integral and integrate $\int_{-1}^{0}\int_{-1-y}^{1} 2y+1\,dxdy + \int_0^1\int_{y^2}^{1} 2y+1\,dxdy$. **8**

c) State and prove Serret-Fernet formula. **6**

P.T.O.

3\. a) Evaluate $\iint r\sin\theta + 3drd\theta$ over the region $1 \le r \le 1+\cos\theta$. **6**

b) Evaluate the cylindrical coordinate integral $\int_0^{\pi/2}\int_0^1\int_r^{\sqrt{2-r^2}} z\cos\theta \, dz\,dr\,d\theta$. **6**

c) Find the volume of the region bounded by the coordinate planes and the plane 2x + y + 3z = 6. **8**

PART – B

Attempt **any two** questions from this section.

4\. a) Solve the following differential equations. **10**

i) $\frac{dy}{dx} = e^{2x-3y} + e^{-3y}\cos 2x$

ii) $\frac{dy}{dx} = y\tan x = y^3\cos x$.

b) Verify Green theorem in the plane for $\oint_c (xy+2)dx + (3x^2+y)dy$ where C is the triangle having vertices (0, 0) (2, 2) and (1, 0). **10**

5\. a) Solve the following differential equations. **10**

i) $(D^2 + 2D - 15)\,y = 3\cos^2 x + 5$

ii) $(D^3 - 6D + 4)\,y = 2xe^{2x}$

b) For a scalar field φ and a vector field $\overline{F}$, show that

i) $\operatorname{div}(\varphi\overline{F}) = \nabla\phi \bullet \overline{F} + \phi\operatorname{div}(\overline{F})$

ii) $\operatorname{div}(\operatorname{curl}\overline{F}) = 0$. **(5+5)**

6. a) Define divergence of a vector field. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r = \sqrt{x^2 + y^2 + x^2}$ show that div $(r^n\,\bar{r}) = (n+3)\,r^n$. **6**

   b) Use Stoke's theorem to evaluate $\iint_S \nabla \times \bar{F}.\bar{n}\,ds$ where $F = x^2y\bar{i} + (2y+z)\bar{j} + y^2\,\bar{k}, \bar{n}$ is the unit normal vector to S, the surface of the region bounded by x = 0, y = 0, x + 2y + z = 2 and z = 0 which is not included in the xy plane. **8**

   c) Solve the second order homogeneous linear differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log_e x^3$. **6**

PART – C

Attempt **any one** question from this section.

7. a) The curve $r = 2a\mathrm{Cos}\,\theta$ is revolved about the initial line find the surface area of the object generated. **6**

   b) Evaluate the integral $\int_0^{\infty}\int_0^{x} e^{-2(x^2+y^2)}dxdy$ by changing to polar coordinates. **8**

   c) Solve the first order differential equation $\frac{dy}{dx} = \frac{2x - y + 4}{x - 3y + 1}$. **6**

8. a) Find the work done in moving a particle in a force field $\bar{F} = 2x\,\bar{i} + (2xz + y)\bar{j} + z^2\,\bar{k}$ along the curve $x = 3t^2, y = 2t, z = t+1$ from t = 0 to t = 2. **6**

   b) Find the unit tangent vectors $\bar{T}$ and principal normal $\bar{N}$ for $\bar{r}(t) = (2t^2+3)\bar{i} + (5-t^2)\bar{j} + t^2\,\bar{k}$ at t = 1. **8**

   c) Solve the first order differential equation $2\frac{dy}{dx} - \frac{y}{x} = \frac{y^2}{x^2}$. **6**