SEM 2 – 1 (RC 07-08)

F.E. (Semester – II) (RC 2007-08) Examination, May/June 2017
APPLIED MATHEMATICS – II

Duration : 3 Hours | Total Marks : 100

Instructions : *1) Attempt **any five** questions, at least **one** from **each** Module.*
*2) Assume suitable data, **if necessary**.*

MODULE – I

1. a) Assuming the validity of differentiation under integral sign rule evaluate the following integrals : $\int_0^\infty \frac{e^{-x} - e^{-\alpha x}}{x \sec x} dx$ **7**

b) Find the total length of the loop of the curve $9y^2 = (x + 7)(x + 4)^2$. **6**

c) Find the area of the surface generated by the revolution of $y = \sqrt{x+2}, 0 \le x \le 4$ about the x-axis. **7**

2. a) Evaluate $\int_0^{\frac{\pi}{2}} ((\cos^2 t) i + (\sin t) j + k) dt$ **6**

b) State and prove Serret-Frenet formula. **8**

c) If $\bar{r}(t) = (2 \cos^2 t)\, i + (3 \sin^t)\, j + (4t)\, k$ is the position vector of a particle in the space at time t. Find the particle velocity vector and acceleration vector at $t = \frac{\pi}{2}$. **6**

MODULE – II

3. a) Evaluate $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\, dy dx$. **6**

b) Change to polar coordinates and evaluate. **8**

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{y e^{\sqrt{x^2+y^2}}}{x^2+y^2}\, dy dx.$$

c) Evaluate $\iint \frac{1}{x^4 + y^2}\, dx dy$ over the region $y \ge x^2, x \ge 1$. **6**

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4. a) Evaluate $\iiint \frac{dxdydz}{x^2+y^2+z^2}$ throughout volume of the sphere $x^2 + y^2 + z^2 = a^2$. **6**

   b) Find the volume of the region enclosed by $x^2+y^2 = 4$ and $x^2 + z^2 = 4$ **6**

   c) Evaluate $\int_0^{\frac{\pi}{2}}\int_0^{x}\int_0^{1-\cos\phi} 3r^2 \sin\phi + 4\cos\theta \, dr d\theta d\phi$. **8**

MODULE – III

5. a) Find the rate of change of $\phi = xyz$ in the direction normal to the surface $x^2y + y^2x + yz^2 = 3$ at the point (1,1,1). **7**

   b) Show that the vector field $F = 4xyi + (2x^2 + 4z^2y) j + 4y^2zk$ is irrotational. Find its potential function. **6**

   c) Find the total work done in moving a particle in the force field $\overline{F} = 3xyi - 5zj + 10xk$ along $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from t =1 and t = 2. **7**

6. a) Verify Greens theorem in plane for $\oint [(xy + y^2) dx + x^2 dy]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. **8**

   b) Verify Stoke's theorem for $\overline{F} = 4xzi - y^2j + yzk$ over the area in the plane $z = 0$ bounded by $x = 0$, $y = 0$ an $x^2 + y^2 = 1$. **12**

MODULE – IV

7. Solve the following differential equations.

   a) $x \cos^2 y dx - y \cos^2 x dy = 0$. **5**

   b) $\sin x \frac{dy}{dx} + y \cos x = 2\sin^2 x \cos x$. **5**

   c) $\frac{dy}{dx} = \frac{4x+6y+3}{6x+9y+2}$. **5**

   d) $(x^2 + 2xy - y^2) dx+(y^2 + 2xy - x^2) = 0$. **5**

8. Solve the following differential equations.

   a) $(D^2 - 4)y = e^x + \sin 2x$. **5**

   b) $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1 + x^2$ **5**

   c) $(D^2 - 4D + 3) y = e^x \cos 2x$. **5**

   d) $x^2 \frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y = x\log(x)$. **5**

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