

**SEM 2 – 1 (RC 07 – 08)**

**F.E. (Semester – II) (Revised in 2007 – 08) Examination,  
November/December 2017  
APPLIED MATHEMATICS – II**

Duration : 3 Hours

Total. Marks : 100

**Instructions :** 1) Attempt **any five** questions, at least **one** from **each** Module.  
2) Assume suitable data, if necessary.

**MODULE – I**

1. a) Assuming the validity of differentiation under integral sign rule evaluate the following integrals : 7  

$$\int_0^1 \frac{x^m - x^n}{\log x} dx.$$
- b) Find the length of the curve  $x = a(2\cos t - \cos 2t)$  and  $y = a(2\sin t - \sin 2t)$  measured from  $\theta = 0$  to  $\theta = \frac{\pi}{2}$ . 6
- c) The curve  $r = 2a \cos \theta$  is revolved about x-axis, find the surface area of the solid generated. 7
2. a) Evaluate  $\int_0^\pi ((\cos t)\mathbf{i} + (\sin^2 t)\mathbf{j} + k)dt$ . 6
- b) Define Torsion. If  $\vec{r}(t)$  is the position vector of moving object then show that 8  

$$T = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3}$$
- c) Show that  $\vec{r}(t) = Ate^{2t}\mathbf{i} + Be^{3t}\mathbf{j}$  satisfies  $\frac{d^2\vec{r}}{dt^2} - 4\frac{d\vec{r}}{dt} + 4\vec{r} = 0$ . 6

**MODULE – II**

3. a) Evaluate  $\int_1^2 \int_0^y \frac{1}{x^2 + y^2} dx dy$ . 6
- b) Evaluate  $\iint (x^2 + y^2) dx dy$  over the region enclosed by the curves  $y = 4x$ ,  $x + y = 3$ ,  $y = 0$ ,  $y = 2$ . 8

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- c) Change the order of integration and evaluate  $\int_1^2 \int_{1-x}^{x-1} (xy + 5) dx dy$ . 6
4. a) Evaluate  $\iiint \frac{dx dy dz}{x^2 + y^2 + z^2}$  throughout volume of the sphere  $x^2 + y^2 + z^2 = a^2$ . 6
- b) Find the volume of the region enclosed by  $x^2 + y^2 = 4$  and  $x^2 + z^2 = 4$ . 6
- c) Evaluate the spherical coordinate integral  $\int_0^{\frac{\pi}{2}} \int_0^{\pi} \int_0^{1-\cos\phi} 3r^2 \sin\phi + 4 \cos\theta dr d\theta d\phi$ . 8

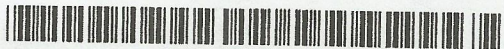
## MODULE – III

5. a) In what direction from the point (1, 3, 2) is the directional derivative of  $\phi = 2xz - y^2$  a maximum? What is its magnitude? 7
- b) Find the total work done in moving a particle in the force field  $\vec{F} = 3xy\mathbf{i} - 5z\mathbf{j} + 10x\mathbf{k}$  along  $x = t^2 + 1$ ,  $y = 2t^2$ ,  $z = t^3$  from  $t = 1$  and  $t = 2$ . 6
- c) Evaluate  $\iiint_S (\nabla r^2) \cdot (\hat{n}) ds = 6V$ , where  $S$  is any closed surface,  $\hat{n}$  is the unit outward normal to  $S$ ,  $V$  is the volume enclosed by  $S$  and  $r^2 = x^2 + y^2 + z^2$ . 7
6. a) Verify Greens theorem in plane for  $\oint [(xy + 4y^2) dx + (x^2 + 3) dy]$  over the boundary of the region bounded by  $x = 1$  and  $y^2 = x$ . 8
- b) Verify Stoke's theorem for  $\vec{F} = (x + y)\mathbf{i} + (2x - z)\mathbf{j} + (y + z)\mathbf{k}$  taken over the triangle ABC cutoff by the plane  $x + 2y + 3z = 6$  on the coordinate axis. 12

## MODULE – IV

7. Solve the following differential equations.
- a)  $(x^2 - 2xy + 3y^2) dx + (4y^3 + 6xy - x^2) dy = 0$ . 5
- b)  $\frac{dy}{dx} + xy = x^3 y^4$ . 5





c)  $\frac{dy}{dx} = \frac{2x+9y-20}{6x+2y-10}$

5

d)  $\frac{dy}{dx} = \frac{y}{x} + \sin \frac{y}{x}$

5

8. Solve the following differential equations.

a)  $(D^2 - 2D + 2)y = e^x + \cos x$

5

b)  $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x^2$

5

c)  $(D^2 - 2D + 1)y = x^2e^{3x}$

5

d)  $x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - 5y = \sin(\log(x))$

5