F.E. (Semester – I) (Revised Course 2007-08) Examination, May/June 2014
APPLIED MATHEMATICS – I

Duration : 3 Hours | Total Marks : 100

Instructions : *1) Attempt **any five** questions, at least **one** from **each** Module.*
*2) **Assume** suitable data, **if necessary**.*

MODULE – I

1. a) State the relation between Gamma and Beta function and use it to find the value of $\Gamma\left(\frac{1}{2}\right)$. **4**

b) Show that $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}dx = \frac{1}{a^n b^m}\beta(m,n)$. **6**

c) Evaluate $\int_0^\infty \frac{x^{m-1}}{(1+x^n)^p}dx$ and hence deduce that $\int_0^\infty \frac{x^{m-1}}{1+x^n}dx = \frac{\pi}{n}\operatorname{Cosec}\left(\frac{\pi m}{n}\right)$. **6**

d) $\int_0^\infty e^{-x^2-2bx}dx = \frac{\sqrt{\pi}}{2}e^{b^2}[1-\operatorname{erf}(a)]$. **4**

2. a) Test the convergence of the following series : **12**

i) $\sum_{n=0}^{\infty}\left(\frac{e}{\pi}\right)^n$

ii) $\sum_{n=1}^{\infty}\frac{4}{(4n-3)(4n+1)}$

iii) $\sum_{n=1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$

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b) Define the interval of convergence and find it for the series $\sum_{n=1}^{\infty}\left(\frac{x^2+1}{3}\right)^n$. 6

c) When is a series said to be absolutely convergent ? 2

MODULE – II

3. a) Prove that $\log\left(\tan\left(\frac{\pi}{4}+\frac{xi}{2}\right)\right)=i\tan^{-1}(\sinh x)$. 5

b) If $\tan(\pi/6+i\alpha)=x+iy$. Prove that $x^2+y^2+2x/\sqrt{3}=1$. 5

c) If $\log(\log(x+iy))=p+iq$ then prove that $y=x\tan\left[\tan q\log\sqrt{x^2+y^2}\right]$. 6

d) Separate into real and imaginary part $\cos^{-1}(3i/4)$. 4

4. a) Prove that $\tan\left[i\log\frac{a-ib}{a+ib}\right]=\frac{2ab}{a^2-b^2}$. 5

b) Determine P such that the function $f(x)=\frac{1}{2}\log(x^2+y^2)+i\tan^{-1}(Px/y)$ is an analytic function. 6

c) Show that the function $u=x^3-3xy^2+3x^2-3y^2+1$ is harmonic. Construct the corresponding analytic function $f(z)=u+iv$. 5

d) If n is positive integer then prove that $(\sqrt{3}+i)^n+(\sqrt{3}-1)^n=2^{n+1}\cos\left(\frac{n\pi}{6}\right)$. 4

MODULE – III

5. a) If $y=e^{\tan^{-1}x}$ show that $(1+x^2)y_{n+2}+[2(n+1)x+1]y_{n+1}+n(n+1)y_n=0$. 7

b) Show that $e^{x\cos x}=1+x+\frac{x^2}{2}-\frac{x^3}{3}-\frac{11}{24}x^4+....$ 7

c) Use Taylors theorem to expand $\cos^2 x$ in powers of $\left(x-\frac{\pi}{3}\right)$. 6

6. a) Evaluate :

i) $\lim\limits_{x \to 0} \dfrac{\tan x - x}{x^2 \tan x}$

ii) $\lim\limits_{x \to 0} \dfrac{e^x + e^{-x} - 2\cos x}{x \sin x}$

iii) $\lim\limits_{x \to 1} \dfrac{1 + \log x - x}{1 - 2x + x^2}$. **12**

b) If Z = f(u, v), where u = acoshx cosy and v = asinhx siny, prove that

$$\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} = \frac{a^2}{2}(\cosh 2x - \cos 2y)\left(\frac{\partial^2 Z}{\partial u^2} + xy\frac{\partial^2 Z}{\partial v^2}\right).$$ **8**

MODULE – IV

7. a) Form the partial differential equation eliminating the arbitrary constants

i) $z = a(x + y) + b$

ii) $z = (x^2 + a)(y^2 + b)$. **8**

b) Form the partial differential equation by eliminating arbitrary function from the equation $f(z - xy, x^2 + y^2) = 0$. **6**

c) Solve the partial differential equation $(x - 2z)p + (2z - y)q = y - x$. **6**

8. a) If $u = \tan\left[\dfrac{x^3 + y^3}{x - y}\right]$ prove that $x^2 \dfrac{\partial^2 u}{\partial^2 x} + 2xy\dfrac{\partial^2 u}{\partial y \partial x} + y^2 \dfrac{\partial^2 u}{\partial^2 y} = 2\cos 3u \sin u$. **10**

b) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 1$. **10**