SEM 2 – 1 (RC 07-08)

F.E. (Semester – II) (Revised 07-08) Examination, May/June 2016
APPLIED MATHEMATICS – II

Duration : 3 Hours

Total Marks : 100

Instructions : *i) Attempt **any five** questions, at least **one** from **each** module.*
*ii) Assume suitable data **if necessary**.*

MODULE – I

1. a) Evaluate $\int_0^1 \frac{x^{\alpha}-1}{\log_e x}dx$ by applying differentiation under the integral sign ($\alpha > 0$). **7**

b) Find the length of the curve $x = 1 - \cos t + \frac{t}{\sqrt{10}}$, $y = \frac{3}{\sqrt{10}}\sin t$ from $t = 0$ to $t = \pi/2$. **6**

c) Find the perimeter of the cardiode $r = a(1 + \cos\theta)$. **7**

2. a) Define curvature of a curve at a point. Show that the curvature of the ellipse $\frac{x^2}{4} + y^2 = 1$ is maximum at $(\pm 2, 0)$. **8**

b) If $\overline{r(t)} = \bar{a}e^{2t} + \bar{b}te^{2t}$, where $\bar{a}$ and $\bar{b}$ are constant vectors, then show that $\frac{d^2\bar{r}}{dt^2} - 4\frac{d\bar{r}}{dt} + 4\bar{r} = 0$. **6**

c) Find the unit tangent vector $\vec{T}$ and binomial vector $\vec{B}$ for $x = 3t + t^2$, $y = 2t$, $z = 2t - 1$ at the point $t = 1$. **6**

MODULE – II

3. a) Evaluate $\iint 3x + y\,dxdy$ over the region bounded by $y^2 = 4x$ and $2x + y = 4$. **7**

b) Write the following as a single integral and evaluate

$$\int_0^1\int_{1-x}^1 2x + 1\,dxdy + \int_1^2\int_{x-1}^1 2x + 1\,dxdy.$$ **7**

c) Change to Polar coordinates and evaluate $\int_0^2\int_0^{\sqrt{4-y^2}} \frac{1}{1+\sqrt{x^2+y^2}}dxdy$. **6**

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4. a) Find the volume of the object generated by the revolution of $r = 2 + \cos\theta$ about the initial line. **6**

b) Evaluate the cylindrical coordinates integral $\int_0^1 \int_0^{\pi/2} \int_0^{2\cos\theta} 2r\sin\theta + 1 \, dr\,d\theta\,dz$. **6**

c) Evaluate $\iiint x + yz\,dx\,dy\,dz$ over the region bounded by $y^2 = 4x$, $z = 0$, $z = 1$ and $x = 1$. **8**

MODULE – III

5. a) Find the directional derivative of $f(x, y, z) = 3xy + 2z^2 + 1$ at the point $(-1, 1, 3)$ in the direction of the vector $\bar{i} + 2\bar{j} - 2\bar{k}$. In what direction at the given point is the directional derivative maximum ? **6**

b) Find the work done in moving a particle in a force field. **6**
$\bar{F} = 3yx\bar{i} + 2z^2x\bar{j} + 3y\bar{k}$, along the curve in the plane $y = 2$ and having equation $x^2 = 2z$ from (2, 2, 2) to (4, 2, 8).

c) Verify Green theorem in the plane for $\oint_C (3yx + 4x^2)\,dx + (5x + y^2)\,dy$ where C is the triangle having vertices (1, 0), (0, 1) and (1, 1). **8**

6. a) Define curl of a vector field. If f is a scalar point function and $\bar{q}$ is a vector point function prove that $\text{curl}\,(f\bar{q}) = f\nabla \times \bar{q} + \nabla f \times \bar{q}$. **8**

b) Verify Gauss divergence theorem for the vector field $\bar{F} = x\bar{i} + y\bar{j} - (x+y)\bar{k}$ over the surface of the tetrahedron bounded by the coordinate planes and $x + y + z = 1$. **12**

MODULE – IV

7. Solve the following differential equations : **20**

a) $y(xy + 2x^2y^2)dx + x(yx - x^2y^2)dy = 0$

b) $e^y(1 + x^2)\dfrac{dy}{dx} - 2x(1 + e^y) = 0$

c) $(x + 2y + 3)dx - (2x - y + 1)dy = 0$

d) $\text{Sin}x\frac{dy}{dx} + 3y = \text{Cos}x.$

8. Solve the following differential equations : **20**

a) $(D^2 - 4D - 3)y = e^x \text{Cos}2x$

b) $(D^2 + 5D + 4)y = x^2 + 7x + 9$

c) $(D + 2)(D - 1)^2 y = e^{2x} + 2\,\text{Sin}x$

d) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \text{Sin}(\log_e x).$