Paper / Subject Code: FE1902 / Mathematics-I

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F.E. Semester- I (Revised Course 2019-20)
EXAMINATION FEBRUARY 2022
Mathematics-I

[Duration : Three Hours] **[Total Marks :100]**

Instructions:

1. Attempt five questions, any two questions each from PART-A and PART-B and one from PART-C.
2. Assume suitable data, if necessary.
3. Figures to the right indicate full marks.

PART A

Answer any **TWO** questions from the following: 2 x 20 = 40 Marks

Q.1 a) Evaluate $\int_0^{\infty} \frac{x^4(1+x^5)}{(1+x)^{15}} dx$ (6)

b) Test the nature of the following series (8)

i) $\frac{3}{2} + \frac{5}{10} + \frac{7}{30} + \frac{9}{68} \ldots$

ii) $\sum_1^{\infty} \frac{n^3}{(log2)^n}$

c) Expand the following (6)

$f(x) = \log\tan\left(\frac{\pi}{4} + x\right)$ in powers in of x

Q.2 a) Evaluate (12)

(i) $\lim_{x\to 1} \frac{x^x - x}{x-1-logx}$

(ii) $\lim_{x\to 1}(1-x^2)^{\frac{1}{\log(1-x)}}$

(iii) $\lim_{y\to 0} \frac{tany - y}{y^3}$

b) If $y = pcos(logx) + qsin(logx)$ show that (8)

$$(x^2)y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$$

Q.3 a) Use Taylor's series to expand e^{xcosx} in powers of x (6)

b) Define absolute convergence and conditional convergence and test the series (8)

$1 - \frac{2^2+1}{2^3+1} + \frac{3^2+1}{3^3+1} - \frac{4^2}{4^3} + \cdots$. for absolute convergence and conditional convergence.

c) Evaluate $\int_0^1 (xlogx)^4 dx$ (6)

PART B

Answer any **TWO** questions from the following: 2 x 20 = 40 Marks

Q.4 a) Solve the following differential equations. (12)

i) $x\sqrt{1-y^2}dx + \sqrt{1-x^2}sin^{-1}ydy = 0$

ii) $(1+y^2)dx = (tan^{-1}y - x)dy$

iii) $\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$

b) $If\ z = f(x,y), where\ u = x^2 - y^2 and\ v = 2xy, prove\ that$ (8)

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 4(x^2+y^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$$

Q.5 a) Divide 120 into three parts so that so that sum of their products taken two at a time shall be maximum. (8)

b) Solve the following differential equations (5)
$(xsec^2y - x^2cosy)dy = (tany - 3x^4)dx$

c) If $u = x^3sin^{-1}\frac{x}{y} + x^4tan^{-1}\frac{y}{x}$, find the value of (7)
$x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} + xu_x + yu_y$at x=1 and y=1.

Q.6 a) Use method of Lagrange's multipliers to find the largest product of numbers x, y and z when $x^2 + y^2 + z^2 = 9$ (8)

b) Solve the differential equation $\frac{dy}{dx} = \frac{2x-5y+3}{2x+4y-6}$ (5)

c) If $sin^{-1}(x^2+y^2)^{\frac{1}{5}}$, prove that (7)

$$x^2\frac{\partial^2 u}{\partial x^2} 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{2}{25}tanu(2tan^2u - 3)$$

PART C

Answer any **TWO** questions from the following: 2 x 20 = 40 Marks

Q.7 a) Prove that $\beta(p,q) = \beta(p+1,q) + \beta(p,q+1)$ (6)

b) Verify Euler's theorem for $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ (7)

c) Test the convergence of the series $\frac{1}{1+2}+\frac{2}{1+2^2}+\frac{3}{1+2^3}+\cdots$ (7)

Q.8 a) Find the extreme values of the function $f(x,y)=y^2-4y+x+xy+5+x^2$ (7)

b) Expand $\log(2+\cos x)$ in powers of x. (7)

c) Prove that $\beta(a,b)=\int_0^\infty \frac{x^{a-1}}{(1+x)^{a+b}}dx$ (6)