FE1902

Total No. of Printed Pages:02

F.E Semester-I (Revised Course 2019-20)
EXAMINATION AUGUST 2021
Mathematics-I

[Duration : Two Hours] **[Total Marks :60]**

Instructions:

1. Answer THREE FULL QUESTIONS with ONE QUESTION FROM EACH PART.
2. Assume suitable data if necessary.
3. Figures to right indicate full marks.

PART A

Q.1

a) Evaluate $\int_0^{\infty} x^{\frac{3}{2}} e^{-x^2} dx$ **(4)**

b) Use Taylor's theorem to expand $f(x) = \tan^{-1} x$ in powers $(x-1)$ **(4)**

c) Test the convergence of the following series **(12)**

i) $\sum_{n=1}^{\infty} \frac{2n-1}{n(n+1)(n+1)}$

ii) $\sum_{n=1}^{n=\infty} \left(2 + \frac{1}{\sqrt{n}}\right)^{-n}$

iii) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \ldots\ldots$

Q.2

a) If $y = (\sin^{-1} x)^2$ t then proves that **(6)**
$(1-x^2)y_{n+1} - (2n+1)xy_{n+1} - n^2 y_n = 0$

b) Find the interval of convergence of the following series **(8)**

$$\sum_{n=1}^{n=\infty} \frac{5^n x^n}{\sqrt{n}}$$

c) Find the expansion of $\log(1 + \sin x)$ $up\ to\ x^4$ **(6)**

Q.3

a) Evaluate the following : **(12)**

i) $\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$

ii) $\lim_{x \to 1} \frac{x^2-1}{cot\frac{\pi x}{2}}$

iii) $\lim_{x \to 0} \frac{\log(\tan x)}{log^x}$

b) Define absolutely convergent series. Test whether the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} is$ Absolutely or conditionally convergent. **(8)**

PART B

Q.4 a) Solve the following differential equations: **(12)**

i) $e^y(1+x^2)\frac{dy}{dx} - 2x(1+e^y) = 0$

ii) $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$

iii) $\frac{dy}{dx} = \frac{x+2y-3}{y-2x-3}$

b) If$u(x,y,z) = \log(\tan x + \tan y + \tan z)$, *find the value of*

$$\sin 2x\,\frac{\partial u}{\partial x} + \sin 2y\,\frac{\partial u}{\partial y} + \sin 2z\,\frac{\partial u}{\partial z}$$ **(8)**

Q.5 a) Find the largest rectangle that can be inscribed inside $\frac{x^2}{4} + \frac{y^2}{9} = 1$ **(8)**

b) *If* $u = \tan^{-1}(x^2 + 2y^2)$*than evaluate,* **(6)**

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

c) Solve **(6)**

$$\frac{dy}{dx} = \frac{4x+6y+3}{6x+9y+2}$$

Q.6 a) Examine the function $f(x,y) = x^3 - 3x^2 - 4y^2 + 1$ for maxima and minima. **(6)**

b) Verify Euler's Theorem for the function given by, **(8)**

$$u = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right).$$

c) *solve* $x\frac{dy}{dx} + y = y^2 logx$ **(6)**

PART C

Q.7 a) Prove that $\beta(m,n) = 2\int_0^{\pi/2} sin^{2m-1}\theta\, cos^{2n-1}\theta\, d\theta$ **(8)**

b) *If* $= \frac{x^2+y^3}{\sqrt{x+y}}$, *find* $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. **(7)**

c) *Evaluate* $\lim_{x\to a}\frac{\log(x-a)}{\log(e^x - e^a)}$ **(5)**

Q.8 a) Define absolutely convergent and conditionally convergent series and test the absolute convergence and conditional convergence of the following series: **(8)**

$$1 - \frac{1}{5} + \frac{1}{9} - \frac{1}{13} + \cdots ..$$

b) Use Taylor's series expansion to fine the value of f(1.1) where $f(x) = x^3 + 3x^2 + 15x - 10$. **(7)**

c) *solve* $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$ **(5)**