DON BOSCO COLLEGE OF ENGINEERING GOA LIBRARY

SEM 1-1 (RC 07 – 08)

F.E. (Semester – I) (Revised Course 2007-08) Examination, May/June 2015
APPLIED MATHEMATICS – I

Duration : 3 Hours Total Marks : 100

Instructions : 1) *Attempt **any five** questions, atleast **one** from **each** Module.*
2) ***Assume** suitable data, if **necessary**.*

Module – I

1. a) State the relation between Gamma and Beta function and use it to find the value of $\Gamma(1/2)$. **5**

b) Evaluate $\int_0^{\infty} e^{-\alpha^2 x^2}\, dx$. **5**

c) Evaluate $\int_0^1 x^3 (1-\sqrt{x})^5\, dx$. **5**

d) Show that $\int_0^{\infty} e^{-x^2-2ax}\, dx = \frac{\sqrt{\pi}}{2} e^{a^2} (1-\text{erf}(a))$. **5**

2. a) State D'Alembert Ratio test for the convergence of a series. **2**

b) Test the convergence of the following series. **12**

i) $\frac{1}{1+3}+\frac{2}{1+3^2}+\frac{3}{1+3^3}+\ldots$

ii) $\sum_{n=1}^{\infty} \frac{n!2^2}{n^n}$

iii) $\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\ldots$

c) Define the radius of convergence of a series and find it for the series

$x-\frac{x^2}{\sqrt{2}}+\frac{x^3}{\sqrt{3}}-\frac{x^4}{\sqrt{4}}+\ldots$ **6**

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Module – II

3. a) Prove that $\text{Sin}^{-1}\left(e^{i\theta}\right) = \text{Cos}^{-1}\left[\sqrt{\text{Sin}\theta} + i\log\left(\sqrt{\text{Sin}\theta} + \sqrt{1+\text{Sin}\theta}\right)\right]$. **6**

b) Use De Moivres theorem to prove that $\text{Cos}4\theta = \text{Cos}^4\theta - 6\text{Cos}^2\theta\ \text{Sin}^2\theta + \text{Sin}^4\theta$. **5**

c) Prove that $\text{Sinh}^{-1} x = \log_e\left(x + \sqrt{x^2+1}\right)$. **4**

d) Prove that the real part of the principal value of

$i^{\log_e(1+i)}$ is $e^{-\frac{\pi^2}{8}}\ \text{Cos}\left(\frac{\pi}{4}\log_e 2\right)$. **5**

4. a) Solve the equation $x^7 + 1 = 0$. **5**

b) Show that the function Sinhz is analytic and find its derivative. **5**

c) Show that $u(x, y) = x^3 - 3xy^2$ is a harmonic function. Find function $v(x, y)$, such that $u + iv$ is an analytic function. **5**

d) If $\text{Sin}\alpha + \text{Sin}\beta + \text{Sin}\gamma = \text{Cos}\alpha + \text{Cos}\beta + \text{Cos}\gamma = 0$, prove that $\text{Cos}\,3\alpha + \text{Cos}\,3\beta + \text{Cos}\,3\gamma = 3\,\text{Cos}\,(\alpha+\beta+\gamma)$. **5**

Module – III

5. a) If $y = \frac{\text{Sin}^{-1}x}{\sqrt{1-x^2}}$ then show that $(1 - x^2)\, y_{n+2} - (2n + 3)xy_{n+1} - (n + 1)^2\, y_n = 0$. **7**

b) Show that $(1 + x)^{1+x} = 1 + x + x^2 + x^3/2 + \ldots$ **7**

c) Use Taylor's theorem to expand $\text{Tan}^2 x$ in powers of $(x - \pi/4)$. **6**

6. Evaluate : **12**

a) i) $\lim_{x\to 0}\left(\text{Cos}\sqrt{x}\right)^{1/x}$ ii) $\lim_{x\to 1}(x^2 - 1)\,\text{Tan}\left(\frac{\pi x}{2}\right)$

iii) $\lim_{x\to\frac{\pi}{2}}(\text{Cosec}\,x)^{\text{Tan}^2 x}$

b) If $z = f(x, y)$ where $x = u + v$ and $y = u.v$ prove that

$$\frac{\partial^2 z}{\partial x^2} - y\frac{\partial^2 z}{\partial y^2} = \frac{1}{u-v}\left[u\frac{\partial^2 z}{\partial u^2} - v\frac{\partial^2 z}{\partial v^2}\right].$$ **8**

Module – IV

7. a) Form the partial differential equations by eliminating arbitrary functions of the equations given below. **8**

   i) $z = f(x + at) + g(x - at)$

   ii) $z = f(x) + e^y g(x)$.

   b) Solve the partial differential equation. **12**

   i) $(a - x) \dfrac{\partial z}{\partial x} + (b - y) \dfrac{\partial z}{\partial y} = c - z$

   ii) $(z - y) \dfrac{\partial z}{\partial x} + (x - z) \dfrac{\partial z}{\partial y} = y - x$.

8. a) If $u = \sin^{-1}\left(\dfrac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{6}} + y^{\frac{1}{6}}}\right)$. Prove that $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2} = \dfrac{1}{144}$

   $\tan u (\tan^2 u 11)$ **7**

   b) Find the extreme values of the function $f(x, y) = y^2 + 4xy + 3x^2 + x^3$. **6**

   c) Find the minimum value of $x^2 + y^2 + z^2$ given that $ax + by + cz = p$. **7**
