FE111

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F.E. Semester-I (Revised Course 2007-2008) EXAMINATION MAY/JUNE 2019
Applied Mathematics-I

[Duration : Three Hours] **[Max.Marks : 100]**

Instructions:

1) Attempt any five questions at least one from each Module.
2) Assume suitable data, if necessary.

MODULE I

Q.1 a) Evaluate 05

$$\int_0^{\pi/2} \sqrt{cot\theta}\, d\theta$$

b) Show that 05

$$\int_0^1 (1 - x^{1/n})dx = \frac{n!\,m!}{(n+m)!}$$

c) Prove that 05

$$\int_0^\infty \frac{x^5}{5^x}dx = \frac{120}{(log5)^6}$$

d) Prove that 05

$$erf(\infty) = 1$$

Q.2 a) Test the convergence of the following series. 04

i)

$$\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$$

ii) 04

$$\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$$

iii) 04

$$\sum_{n=1}^{\infty} (-1)^{n-1}\frac{n}{5n+1}$$

b) Define the interval of convergence and find the interval of convergence of the following series. 08

$$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \cdots$$

MODULE II

Q.3 a) Prove that

$$\sinh^{-1} x = \log(x + \sqrt{x^2 + 1})$$ 06

b) Expand

$\sin^{-1}\theta$ in series of multiples of θ 06

c) If

$$cosh x = sec\theta$$ 08

Prove that

(a)

$$x = \log(sec\theta + tan\theta)$$

(b)

$$\theta = \frac{\pi}{2} - 2\tan^{-1}(e^{-x})$$

(c)

$$tanh\left(\frac{x}{2}\right) = tan\left(\frac{\theta}{2}\right)$$

Q.4 a) Determine 'p' such that 07

$\frac{1}{2}\log(x^2 + y^2) + itan^{-1}\left(\frac{px}{y}\right)$ is ananalytic function

b) Show that the function 07

$$u = e^{-2xy}\sin(x^2 - y^2)$$

is harmonic. Find the conjugate harmonic function.

c) Show that 06

$$tan\left\{ilog\left(\frac{a - bi}{a + bi}\right)\right\} = \frac{2ab}{a^2 - b^2}$$

MODULE III

Q.5 a) If 07

$$y = \sin^{-1} x$$

Prove that

$$(1 - x^2)y_{n+2} - [2n + 1]x\, y_{n+1} - n^2 y_n = 0$$

b) Expand 06

$$\sin^{-1}\left(\frac{2x}{1 + x^2}\right)$$

in powers of x. Find the first 4 terms.

c) Expand 07

$\cot\left(x + \frac{\pi}{4}\right)$ in powers of x and hence find $\cot 46.5°$.

Q.6 a) Evaluate: 12

i)

$$\lim_{x\to 0}\frac{(1+x)^{1/x}-e}{x}$$

ii)

$$\lim_{x\to 0}\frac{1}{x}-cotx$$

iii)

$$\lim_{x\to \pi/2}(cosx)^{\frac{\pi}{2}-x}$$

b) If $z=f(x,y)$ where $u=l\,x+m\,y, v=l\,y-m\,x$ then show that 08

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=(l^2+m^2)\left(\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}\right)$$

MODULE IV

Q.7 a) Form the partial differential equations by eliminating constants 08

(i) $2z=(ax+y)^2+b$

(ii) $(x-h)^2+(y-k)^2+z^2=a^2$

b) Form the partial differential equations by eliminating functions 06

$f(x+y+z, x^2+y^2+z^2)=0$

c) Solve 06

$$z^2(p^2+q^2+1)=1$$

Q.8 a) If 06

$u=\sec^{-1}\left[\frac{x^3-y^3}{x+y}\right]$, find the value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$

b) Find the extreme values of the function 06

$$x^3+3xy^2-15x^2-15y^2+72x$$

c) Using the method of Lagrange's Multipliers, find the largest product of numbers x, y and z when 08

$$x+y+z^2=16$$