SEM 1-1 (RC 07 – 08)

F.E. (Semester – I) (Revised Course 2007-08) Examination, Nov./Dec. 2012

APPLIED MATHEMATICS – I

Duration : 3 Hours Total Marks : 100

Instructions : *1) Attempt **any five** questions, at least one from each Module.*

*2) Assume suitable data, **if necessary**.*

MODULE – I

1. a) Show that $\int_0^\infty \frac{x^4}{4^x}dx = \frac{24}{(\log_e 4)^5}$. **4**

b) Evaluate $\int_0^1 x^3\left(1-\sqrt{x}\right)^5 dx$. **4**

c) Show that $\int_a^b (x-a)^m(b-x)^n dx$. **6**

d) Prove that $\frac{1}{x}\frac{d}{da}\text{erf}_c(ax) = \frac{1}{a}\frac{d}{dx}\text{erf}(ax)$. **6**

2. a) Test the convergence of the following series : **12**

i) $\frac{2}{1}+\frac{2^2}{2}+\frac{2^3}{3}+\ldots.\infty$

ii) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}\text{Tan}\left(\frac{1}{n}\right)$

iii) $\frac{1}{2}-\frac{2}{5}+\frac{3}{10}+\ldots\ldots$

b) Define the interval of convergence and find it for the series $\sum_{n=1}^{\infty}\frac{(x+2)^n}{3^n n}$. **8**

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MODULE – II

3. a) Solve the equation $x^7 + x^4 + x^3 + 1 = 0$ by using De Moivre's theorem. **6**

b) Separate into real and imaginary parts $\log_e(\text{Sin}(x + iy))$. **7**

c) Prove that $\text{Tanh}^{-1}(\text{Sin}\theta) = \text{Cosh}^{-1}(\text{Sec}\theta)$. **7**

4. a) Prove that $\text{Sin}\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\} = \frac{2ab}{a^2+b^2}$. **6**

b) Show that real and imaginary parts of the function $w = \log_e z$, satisfies the Cauchy -Reiman equations when $z \neq 0$. **6**

c) Show that the function $u(x, y) = e^x \cos y$ is harmonic. Find the analytic function $f(z) = u + iv$ and hence find the harmonic conjugate $v(x, y)$. **8**

MODULE – III

5. a) If $y = 2x\sqrt{1-x^2}$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-4)y_n = 0$. **7**

b) Expand $\log_e(1 + \text{Sinx})$ in powers of x. **7**

c) Use Taylors theorem to expand $1/x^3$ at $x = 1$. **6**

6. a) Evaluate : **12**

i) $\lim\limits_{x \to 0} \frac{\text{Tanx} - x}{x^2 \text{Tanx}}$

ii) $\lim\limits_{\theta \to \frac{\pi}{2}} \frac{\log_e\left(\theta - \pi/2\right)}{\text{Tan}\theta}$

iii) $\lim\limits_{x \to \frac{\pi}{2}} (\text{Cosx})^{\left(\frac{\pi}{2} - x\right)}$.

b) If $z = f(u, v)$ where $u = x - y$ and $v = xy$, prove that **8**

$$x\frac{\partial^2 z}{\partial x^2} + y\frac{\partial^2 z}{\partial y^2} = (x+y)\left(\frac{\partial^2 z}{\partial u^2} + xy\frac{\partial^2 z}{\partial v^2}\right)$$

MODULE – IV

7. a) Form the partial differential equation eliminating the arbitrary functions from the equations : 5

   i) $z = f(x + 5y) + g(x - 5y)$

   ii) $z = f(x)\, g(y)$. 4

   b) Solve the partial differential equations :

   i) $2(p + q) = 3(x^4 + y^6)$ 5

   ii) $px^2 + q\text{Coty} - z^2 = 0$. 6

   Where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ and z is a function of x and y.

8. a) If $u = \text{Sec}^{-1}\left[\frac{\sqrt{x} - 2\sqrt{y}}{y^3\sqrt{x}}\right]^{-\frac{1}{3}}$ Find the value of $x^2\frac{\partial^2 u}{\partial^2 x} + 2xy\frac{\partial^2 u}{\partial y \partial x} + y^2\frac{\partial^2 u}{\partial^2 y}$. 7

   b) Find the maximum and minimum value of $x^3 + y^3 - 3xy$. 6

   c) Find the point on the curve $xy = 16$ nearest to the origin. 7
