SEM 1-1 (RC 07-08)

F.E. (Semester – I) (Revised in 2007-08) Examination, May/June 2017

APPLIED MATHEMATICS – I

Duration : 3 Hours　　　　Max. Marks : 100

Instructions : *1) Attempt **five** questions, atleast **one** from **each** Module.*
*2) Assume missing data if **any**.*

MODULE – I

1. a) Prove $\int_0^\infty \sqrt{x}.e^{-x^2}dx \int_0^\infty e^{-x^2}\frac{1}{\sqrt{x}}dx = \frac{\pi}{2\sqrt{2}}$. **7**

b) Evaluate $\int_0^1 x^3\left(1-\sqrt{x}\right)^5 dx$. **5**

c) Evaluate $\int_0^{\pi/2} \sqrt{\cot\theta}.d\theta$. **5**

d) Prove that $\mathrm{erf}(\infty) = 1$. **3**

2. a) Test the following series for convergence : **12**

i) $\sum_{n=1}^{\infty} \frac{2n-1}{n(n+1)(n+2)}$

ii) $\sum_{n=1}^{\infty} \frac{1}{(\log n)^n}$

iii) $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$.

b) Use Leibnitz test to test the convergence of series $\sum_{n=0}^{\infty} (-1)^n.\frac{1}{(3n+1)^2}$. **5**

c) Define interval of convergence for a power series. **3**

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MODULE – II

3. a) Prove that $(1+\cos\theta+i\sin\theta)^n + (1+\cos\theta-i\sin\theta)^n = 2^{n+1}.\cos^n(\theta/2)\cos\left(\frac{n\theta}{2}\right)$. **7**

b) If $\log(\log(x+iy)) = p+iq$, then prove that $y = x\tan\left(\tan q.\log\sqrt{x^2+y^2}\right)$. **7**

c) If $\tan\left(\pi/6 + i\alpha\right) = x + iy$, prove that $x^2 + y^2 + \frac{2x}{\sqrt{3}} = 1$. **6**

4. a) Test for analyticity for the following : **7**

i) $f(z) = z.|z|^2$

ii) $f(z) = z.\bar{z}$

iii) $f(z) = \sin z$

b) Determine p such that $f(z) = \frac{1}{2}\log\left(x^2+y^2\right) + i\tan^{-1}\left(\frac{px}{y}\right)$ is an analytic function. **7**

c) Show that $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic and find the analytic function $f(z) = u + iv$. **6**

MODULE – III

5. a) If $y = a\cos(\log_e x) + b\sin(\log_e x)$, then show that

$x^2y_{n+2} + 2(n+1)xy_{n+1} + (n^2+1)y_n = 0$. **7**

b) Show that $\log_e(1-\log_e(1-x)) = x + \frac{x^3}{6} + ...$ **7**

c) Use Taylor's theorem to express the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $x - 2$. **6**

6. a) Evaluate : **12**

i) $\lim_{x\to 1}\left(1-x^2\right)^{\frac{1}{\log_e(1-x)}}$

ii) $\lim_{x\to a}\log_e\left(2-\frac{x}{a}\right)^{\cot(x-a)}$

iii) $\lim_{x\to 0}\left[\frac{1}{x^2}-\frac{1}{x\tan x}\right]$

b) If $z = f(u, v)$, where $u = x^2 - y^2$ and $v = 2xy$, prove that

$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=4\left(x^2+y^2\right)\left(\frac{\partial^2 z}{\partial u^2}+\frac{\partial^2 z}{\partial v^2}\right)$$ **8**

MODULE – IV

7. a) From the partial differential equation by eliminating the arbitrary functions : **8**

i) $z = f(x^2 + y^2) + x + y$

ii) $f(x + y + z, x^2 + y^2 + z^2) = 0$

b) Solve the partial differential equations : **12**

i) $x\left(z^2-y^2\right)\frac{\partial z}{\partial x}+y\left(x^2-z^2\right)\frac{\partial z}{\partial y}=z\left(y^2-x^2\right)$

ii) $\frac{y^2 z}{x}\frac{\partial z}{\partial x}+xz\frac{\partial z}{\partial y}=y^2$

8. a) If $u=\sin^{-1}\left(\sqrt{x^2+y^2}\right)$, find the value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$. **8**

b) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$. **6**

c) Examine the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ for extreme values. **6**

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