SEM 1 – 1 (RC 16-17)

F.E. (Semester – I) (Revised in 2016 – 2017)
Examination, November/December 2017
ENGINEERING MATHEMATICS – I

Duration : 3 Hours | Max. Marks : 100

Instructions : *1) Answer **five** questions, atleast **two** from Part – **A**, **two** from Part – **B** and **one** question from Part – **C**.*
*2) Assume suitable data, if **necessary.***
*3) Figures to **right** indicate **full** marks.*

PART – A

1. a) Prove that $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$. **6**

b) Prove that $\int_0^1 x^3\left(1-\sqrt{x}\right)^5 dx = \frac{1}{5148}$. **6**

c) If $(1+i)^{x+iy} = a + ib$ then considering the principal value only prove that

$$2\tan^{-1}\left(\frac{b}{a}\right) = \frac{\pi}{2}x + y\log 2$$ **8**

2. a) Test the nature of the following series.

i) $\sum_{n=1}^{\infty}\left(2+\frac{1}{n}\right)^n$

ii) $\sum_{n=1}^{\infty}\frac{2^n n!}{n^n}$

iii) $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{4n-3}$ **12**

b) Find the value of P such that $f(z) = \frac{1}{2}\log\left(x^2+y^2\right)+i\tan^{-1}\left(\frac{Px}{y}\right)$ is an analytic function. **4**

c) Find the analytic function whose real part is $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$. **4**

P.T.O.

3. a) Evaluate $\int_0^\infty \frac{dx}{2^{3x^2}}$. 5

b) Prove that $n\,\beta(m+1, n) = m\beta(m, n+1)$. 5

c) If $\tan\left(\frac{\pi}{6}+i\alpha\right)=x+iy$ prove that $x^2 + y^2 + \frac{2x}{\sqrt{3}} = 1$. 5

d) Prove that $\sinh^{-1} x = \frac{1}{2}\operatorname{cosech}^{-1}\left(\frac{1}{2x(1+x^2)^{1/2}}\right)$. 5

PART – B

4. a) If $y = e^{\tan^{-1}x}$ show that $(1 + x^2)\, y_{n+2} + [2(n + 1)\, x - 1]\, y_{n+1} + n(n+1)y_n = 0$. 7

b) Use Taylor's theorem to expand $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$ in the powers of $(x - 1)$. 6

c) If $u = x^3 \sin^{-1}\left(\frac{y}{x}\right) + x^4 \tan^{-1}\left(\frac{y}{x}\right)$ find the value of

$$\frac{x^2\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}.$$ 7

5. a) Evaluate :

i) $\lim\limits_{x\to 0}\frac{e^x - e^{-x} - 2\log(1+x)}{x\sin x}$

ii) $\lim\limits_{x\to a}\log\left(2-\frac{x}{a}\right)\cot(x-a)$

iii) $\lim\limits_{x\to 0}(\cos x)^{1/x^2}$. 12

b) Solve the partial differential equations.

i) $y^2 z\, p + x^2 z\, q = xy^2$

ii) $(z - y)\, p + (x - z)\, q = (y - x)$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 8

6. a) If z = f (x, y)

$x = u + v, y = u.v$

Prove that $\frac{\partial^2 z}{\partial x^2} - y\frac{\partial^2 z}{\partial y^2} = \frac{1}{u-v}\left[\frac{u\partial^2 z}{\partial u^2} - \frac{v\partial^2 z}{\partial v^2}\right]$. 8

b) Form the partial differential equation by eliminating function 'f'
$z = e^{ay} f(x + by)$. 6

c) Use method of lagrange multipliers to find the largest product of the numbers x, y and z when $x^2 + y^2 + z^2 = 9$. 6

PART – C

7. a) Prove that $\mathrm{erf}_c(x) + \mathrm{erf}_c(-x) = 2$. 5

b) Prove that $\sin^{-1}(\operatorname{cosec}\theta) = \frac{\pi}{2} + i\log\left(\cot\frac{\theta}{2}\right)$. 5

c) Prove that $\sqrt{1+\sin x} = 1 + \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{48} + \frac{x^4}{384} + \ldots\infty$. 5

d) Determine and classify extreme points of $f(x, y) = y^2 + 4xy + 3x^2 + x^3$. 5

8. a) Prove that $\int_0^1 \frac{x^2}{\sqrt{1-x^2}}dx \int_0^1 \frac{dx}{(1+x^4)^{1/2}} = \frac{\pi}{4\sqrt{2}}$. 6

b) Use De Moivre's theorem to solve $x^4 + 1 = 0$. 4

c) Use Taylor's series expansion to find the approximate value of $\sqrt{25.15}$. 5

d) Solve the partial differential equation

$(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 5
