SEM 2 – 1 (RC 16-17)

F.E. (Semester – II) (RC 2016-17) Examination, Nov./Dec. 2017
ENGINEERING MATHEMATICS – II

Duration : 3 Hours Total Marks : 100

Instructions : *i) Attempt **five** question, **any two** questions **each** from Part **A** and **B** and **one** from Part **C**.*
*ii) Assume suitable data, if **necessary**.*
*iii) Figures to the **right** indicate **full** marks.*

PART – A

Attempt **any two** questions from this Part.

1. a) Evaluate $\int_0^\infty \frac{\text{Cos}\alpha x}{x}\left(e^{-ax}-e^{-bx}\right)dx$ by applying differentiation under the integral sign. 7

b) Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dxdy}{\sqrt{1+x^2-y^2}}$. 6

c) Find the length of the cycloid $x = a(\theta + \text{Sin}\theta), y = a(1-\text{Cos}\theta)$ between two cusp. 7

2. a) Solve the vector differential equations $\frac{d^2\overline{r}}{dt^2} = 3t\,\overline{i} + 2\text{Cost}\,\overline{j} + \overline{k}$ given that $\left.\frac{d\overline{r}}{dt}\right|_{t=0} = \overline{i} - \overline{k}$ and $\overline{r}(0) = \overline{j} - 2\overline{k}$. 6

b) Change the order of integration and evaluate $\int_0^2 \int_{y^2}^{y+2} 3x + ydxdy$. 8

c) Define Curvature of a curve at a point. If x(t) = Cos(2t), y(t) = Sin(2t) and z(t) = 2t, find the curvature at $t = \pi/4$. 6

P.T.O.

3. a) Evaluate $\iint 3r\text{Sin}\theta\,dr\,d\theta$ over the region $\{(r,\theta)/r \le 1,\ r \le 1+\text{Cos}\theta,\ 0 \le \theta \le \pi\}$. **6**

b) Evaluate the cylindrical coordinate integral $\int_0^{\pi}\int_0^{1}\int_0^{\sqrt{4-r^2}} 2z\text{Sin}\theta\,dz\,dr\,d\theta$. **6**

c) Find the volume of the region $\{(x, y, z)/x^2 + y^2 + z^2 \le 4, 0 \le z \le 1\}$. **8**

PART – B

Attempt **any two** questions from this Part.

4. a) Solve the following differential equations. **10**

i) $\frac{dy}{dx} - x^2e^y = e^{2x+y}$ ii) $\frac{dy}{dx} = \frac{2y-x+1}{4y-2x+2}$

b) Verify Green theorem in the plane for $\oint\left(y^2+2x\right)dx + (5+xy)dy$ where C is the boundary of the region enclosed by $y^2 = 4x$ and $y = 2x$. **10**

5. a) Solve the following differential equations. **10**

i) $(D^2 + 4D - 5)\,y = 3e^{2x} + 5\text{Cos}x$

ii) $(D^3 - 5D + 4)y = 2x^2$.

b) Define Curl of a vector field. Show that if $\bar{a}$ is any constant vector and $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ then Curl $(\bar{a} \times \bar{r}) = 2\bar{a}$. **5**

c) Find the direction along which the directional derivative of F(x, y, z) = $x^2 + 3zy + z^2$ at (2, –1, 0) is maximum. What is the magnitude of the maximum ? **5**

6. a) Define divergence of a vector field. If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $\bar{a}$ is a constant vector then shown that $\text{div}(\bar{a} \times \bar{r}) = 0$. **6**

b) Use Stoke's theorem to evaluate $\iint_S \nabla \times F.\bar{n}ds$ where $F = xy\,\bar{i} + 3y^2\,\bar{j} - 2yx\,\bar{k}$, $\bar{n}$ is the unit normal vector to S, the surface of the region bounded by x = 0, y = 0, x + 3y + 2z = 6 and z = 0 which is not included in the xy plane. **8**

c) Solve the second order homogeneous linear differential equation

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = (\log_e x)^2.$$ **6**

PART – C

Attempt **any one** question from this Part.

7. a) The cardiode $r = 1 + \cos\theta$ is revolved about the initial line find the surface area of the object generated. **6**

   b) Evaluate the integral $\int_0^{\infty}\int_y^{\infty} e^{-3\left(x^2+y^2\right)}\,dxdy$ by changing to polar coordinates. **8**

   c) Solve the first order differential equation $(x^2y^3 + 2y)dx + (2x - 2x^3y^2)dy = 0$. **6**

8. a) Find the work done in moving a particle in a force field $\overline{F} = (2y+1)\overline{i} + x^2\overline{j} + 3z\overline{k}$ along the curve $x = z^2$ and $y = z - 2$ from $z = 0$ to $z = 2$. **6**

   b) Find the unit tangent and acceleration vector of the curve $x = 3\cos t$, $y = 3\sin t$ and $z = 4t^2$ at $t = 1$. **7**

   c) Solve the first order differential equation $\tan x \frac{dy}{dx} + y = 2\cos x$. **7**

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