F.E. (Semester – I) (RC 2016-17) Examination, November/December 2018
ENGINEERING MATHEMATICS – I

Duration : 3 Hours Total Marks : 100

Instructions : *1) Answer **five** questions. At least **two** from Part – **A**, **two** from Part – **B** and **one** from Part – **C**.*
2) Assume suitable data, if necessary.
*3) Figures to **right** indicate **full** marks.*

PART – A

Answer **any two** questions from the following :

1. a) Evaluate $\int_0^\infty x^2 e^{-x^8} dx$. **5**

b) Show that $\frac{d}{dx}\operatorname{erf}(ax) = \frac{2a}{\sqrt{\pi}} e^{-a^2x^2}$. **4**

c) Evaluate $\int_0^1 x^{m-1}(1-x^2)^{n-1} dx$. **6**

d) Use De Moivre's theorem to find all values of $\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{\frac{3}{4}}$ and show that their continued product is 1. **5**

2. a) Test the convergence of the following series. **12**

i) $\frac{1^2.2^2}{1!}+\frac{2^2.3^2}{2!}+\frac{3^2.4^2}{3!}+\frac{4^2.5^2}{4!}+\ldots$

ii) $\sum_{n=1}^{n=\infty}\left(\frac{n+1}{3n}\right)^n$

iii) $\sum_{n=1}^{n=\infty}\frac{(-1)^{n-1}}{5n-3}$.

b) Show that the function $u(x, y) = x^3 - 3xy^2$ is a harmonic function. Find the function $v(x, y)$ such that $u + iv$ is an analytic function. **4**

c) Determine p, q, r so that the function.

$f(z) = (x^3 + px^2y + qxy^2 + ry^3)$ is analytic. **4**

P.T.O.

3. a) Prove that $\log \tan\left(\frac{\pi}{4}+i\frac{\pi}{2}\right)=i\tan^{-1}(\sinh x)$. 6

b) If $\sinh(x - iy) = e^{i\frac{\pi}{3}}$ show that $3\sinh^2 x + \cosh^2 x = 4\sinh^2 x \cosh^2 x$. 6

c) Find the interval of convergence for the following series $\sum_{n=1}^{\infty}\frac{x^n}{5^n n^2}$. 8

PART – B

Answer **any two** questions from the following :

4. a) If $u=\operatorname{cosec}^{-1}\sqrt{\left(\frac{x^{\frac{1}{5}}+y^{\frac{1}{5}}}{x^{\frac{1}{7}}+y^{\frac{1}{7}}}\right)}$, find the value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ 7

b) If $y = \log\left(x+\sqrt{x^2+p^2}\right)^2$, prove that $(x^2 + p^2)y_{n+2} + (2n + 1)x\, y_{n+1} + n^2 y_n = 0$. 7

c) Expand $\log(1 + e^x)$ in powers of x. Find the first 5 terms. 6

5. a) Evaluate : 12

i) $\lim_{x\to 1}\frac{x^x - x}{x-1-\log x}$

ii) $\lim_{x\to 0}\left[\cot x-\frac{1}{x}\right]$

iii) $\lim_{x\to\infty}\frac{\log(1+e^{3x})}{x}$

b) Form the partial differential equations by eliminating constants 'm' and 'n' 4

$z = m\log(x^2 + y^2) + n$

c) Form the partial differential equations by eliminating function

$z=y^2+2f\left(\frac{1}{x}+\log y\right)$. 4

6. a) If $u = f(x, y)$ where $x = p\cos\theta - q\sin\theta$, $y = p\sin\theta + q\cos\theta$ then show that

$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\frac{\partial^2 u}{\partial p^2}+\frac{\partial^2 u}{\partial q^2}$. 8

b) Solve the partial differential equation. **6**

$(x^2 - z^2 - y^2)p + 2xyq = 2xz$ where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$.

c) Find the greatest and least values of the function f(x, y) = xy takes on the ellipse $\dfrac{x^2}{8} + \dfrac{y^2}{2} = 1$ using the method of Lagrange's Multipliers. **6**

PART – C

Answer **any one** questions from the following :

7. a) Evaluate $\int_6^8 \sqrt[5]{(x-6)(8-x)}\,dx$. **4**

b) Prove that $(1 + i)^{100} + (1 - i)^{100} = -2^{51}$. **5**

c) Prove that $\log(1 + \tan x) = x - \dfrac{x^2}{2} + \dfrac{2x^3}{3} - \ldots$ **6**

d). Solve the partial differential equation **5**

$(x - 2z)p + (2z - y)q = y - x$ where $p = \dfrac{\partial z}{\partial x}$ and $q = \dfrac{\partial z}{\partial y}$

8. a) Test the convergence of the following series $\sum_{n=1}^{\infty} \dfrac{1}{\sqrt{n^3+1}}$. **5**

b) If $\tan\left(\dfrac{x}{2}\right) = \tanh\left(\dfrac{u}{2}\right)$, prove that $u = \log\tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)$. **5**

c) Use Taylors series to find the approximate value of $\sqrt{1.02}$. **5**

d) Find the extreme values of the function $f(x, y) = y^2 + 4xy + 3x^2 + x^3$. **5**
