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F.E. Semester- I (Revised Course 2007-08)
EXAMINATION Aug/Sept 2019
Applied Mathematics-I

[Duration : Three Hours]

[Max. Marks : 100]

Instructions:

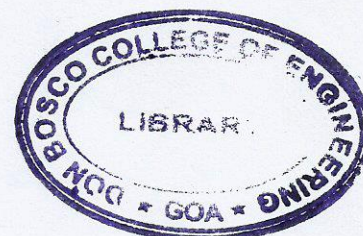
1. Attempt any five questions at least one from each Module.
2. Assume suitable data, if necessary.

MODULE- I

- Q.1
- a) Evaluate $\int_0^1 x^3(1-x^2)^{1/2} dx$ (5)
 - b) Show that $\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$ (5)
 - c) Evaluate $\int_3^7 ((x-3)(7-x))^{1/4} dx$ (5)
 - d) Prove that the error function is an odd function. (5)
- Q.2
- a) Test the convergence of the following series.
 - i) $\sum_{n=1}^\infty \frac{2n-1}{n(n+1)(n+2)}$ (4)
 - ii) $\sum_{n=1}^\infty (1)^{n-1} \frac{1}{4n-3}$ (4)
 - iii) $\sum_{n=1}^\infty \left(2 + \frac{1}{\sqrt{n}}\right)^{-n}$ (4)
 - b) Define absolutely convergent and conditionally convergent series. Test whether the following series is absolutely convergent or conditionally convergent series:// $\sum_{n=1}^\infty \frac{(1)^{n-1}}{2n-1}$ (8)

MODULE II

- Q.3
- a) Use De Moivre's theorem and solve $x^6 - i - 0$ (6)
 - b) If $\alpha + i\beta = \tanh(x + i\pi/4)$, find $(\alpha^2 + \beta^2)$ (6)
 - c) Prove that $\sin^{-1}(\operatorname{cosec}\theta) = \frac{\pi}{2} + i \log(\cot\theta/2)$ (8)



- Q.4 a) Determine a, b, c, d, so that the function $(x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic (6)
- b) Show that the function $i^i = \cos\theta + i\sin\theta$; where $\theta = (2n + 1/2)\pi e^{-(2n+1/2)\pi}$ (6)
- c) If $u - v = \frac{e^y - \cos x + \sin x}{\cosh y - \cos x}$ (8)
- And $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z .

MODULE III

- Q.5 a) If $y = a\cos(\log x) + b\sin(\log x)$. prove that $x^2 y_{n+2} + (2n + 1)x y_{n+1} + (n^2 + 1)y_n = 0$ (7)
- b) Expand $\sec^{-1}\left(\frac{1}{1-2x^2}\right)$ in powers of x . Find the first 4 terms. (6)
- c) Expand $\cot\left(x + \frac{\pi}{4}\right)$ in powers of x and hence find $\cot 46.5^\circ$. (7)
- Q.6 a) Evaluate : (12)
- 1) $\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x}$
- 2) $\lim_{x \rightarrow 0} \frac{1}{x} - \cot x$
- 3) $\lim_{x \rightarrow \pi/2} (\cos x)^{\cos^2 x}$
- b) If $z = f(x, y)$ where $u = lx + my$, $v = ly - mx$ then show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right)$ (8)

MODULE IV

- Q.7 a) Form the partial differential equations by eliminating constants (6)
- i) $z = (x - a)(y - b)$
- ii) $(x - h)^2 + (y - k)^2 + z^2 = a^2$
- b) Form the partial differential equations by eliminating functions $f(x + y + z, x^2 + y^2 + z^2) = 0$ (6)

c) Solve

$$x^2 p^2 + y^2 q^2 = z^2 \quad (8)$$

Q.8

a) If

$$u = \sin^{-1} \left[\frac{x^{1/4} + y^{1/4}}{x^{1/6} + y^{1/6}} \right]. \text{ Find the value of } x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} \quad (6)$$

b) Obtain the extreme values of the function.

$$y^2 + 4xy + 3x^2 + x^3 \quad (6)$$

c) Using Lagrange's Multiplier's method find the shortest distance from the point (1, 2, 2) to the sphere. (8)

$$x^2 + y^2 + z^2 = 36$$