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SEM 2-1 (RC 07-08)

F.E. (Sem. – II) Examination, May/June 2008
(Revised 2007-08 Course)
APPLIED MATHEMATICS – II

Duration : 3 Hours

Total Marks: 100

Instructions: i) Attempt **any five** questions, at least **one** from **each** Module.
ii) Assume suitable data if **necessary**.

MODULE – I

1. a) Assuming the validity of differentiation under the integral sign show that

$$\int_0^1 \frac{x^a - x^b}{\log_e x} dx = \log_e \left(\frac{a+1}{b+1} \right).$$

6

- b) Find the length of the curve $x = a(2\cos t - \cos 2t)$, $y = a(2\sin t - \sin 2t)$ from $t = 0$ to $t = \pi/2$.

8

- c) Find the perimeter of the curve $\theta = \frac{1}{2} \left(r + \frac{1}{r} \right)$. For $r = 1$ to $r = 4$.

6

2. a) Show that if $\vec{r} = \vec{a}\sin\omega t + \vec{b}\cos\omega t$, where \vec{a} and \vec{b} are constant vectors then

$$\frac{d^2\vec{r}}{dt^2} = -\omega^2 \vec{r}.$$

5

- b) For the space curve $x = 3\cos t$, $y = 3\sin t$ and $z = 4t$ find the vectors \vec{T} , \vec{B} at $t = 0$.

6

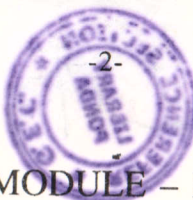
- c) Evaluate $\int_0^{\pi/2} \cos^2 t \vec{i} + \sin t \vec{j} + 5\vec{k} dt$.

4

- d) If $\vec{r}(t)$ is a vector field having a constant magnitude, show that $\vec{r}(t) \cdot \frac{d\vec{r}}{dt} = 0$.

5

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MODULE – II

3. a) Evaluate $\int_0^2 \int_x^{2x} 2y^2 \sin(xy) dx dy$. 6

b) Change the order of integration and evaluate $\int_0^2 \int_{y-2}^{2y} 2x + y dx dy$. 8

c) Change to polar coordinates and evaluate $\int_0^\infty \int_0^\infty \frac{2}{1 + (x^2 + y^2)^2} dx dy$. 6

4. a) Find the volume of the solid generated by the revolution of region bounded by $y^2 = 4x$ and $x = 1$ about the y-axis. 6

b) Evaluate $\int_0^1 \int_0^{1-x^2} \int_0^{4-x^2-y} x dx dy dz$. 6

c) Find the volume of the region bounded by the cylinder $x^2 + y^2 = 4$ the plane $z = 0$ and $x + 2z = 6$. 8

MODULE – III

5. a) Define divergence of a vector field. If f is a scalar point function and \vec{u} is a vector point function show that $\text{div}(f\vec{u}) = f\text{div}\vec{u} + \text{grad}f \cdot \vec{u}$. 6

b) Show that the vector field

$\vec{f}(x, y, z) = (6x\sin z - 4y\sin x)\vec{i} + 4\cos x\vec{j} + (3x^2\cos z + 6z)\vec{k}$ is irrotational. Find its scalar potential. 6

c) Find the unit vector, normal to the surface $x^2 + 3y + z^2 = 4$ at the point $(0, 1, -1)$. 4

d) In what direction is the directional derivative of $f = 3x^2 + 4yz$ at the point $(2, 1, 0)$ maximum. 4



6. a) Verify Green theorem in the plane for $\oint (x^2 + 4y^2)dx + (y + 3x)dy$ where C is the boundary of the region bounded by $y = 0$, $x = 0$ and $2x + y = 2$. 8

b) Use Stoke's theorem to evaluate $\iint_S \nabla \times F \cdot \bar{n} ds$ where $F = (x + 2z)$

$\bar{i} + 3xy\bar{j} + yx^2\bar{k}$, \bar{n} is the unit normal vector to S, and S the surface of the tetrahedron bounded by the coordinate planes and the plane $x + 2y + z = 2$, excluding the surface in the xy plane. 8

c) Use Gauss divergence theorem to show that $\iiint_S \bar{r} \cdot \bar{n} ds = 3V$ where S is any

closed surface, \bar{n} is the unit normal vector to surface S, V is the volume enclosed in surface S and $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$. 4

MODULE - IV

7. Solve the following differential equations : 20

a) $(x + y + 1)^2 \frac{dy}{dx} = 1$

b) $\frac{dy}{dx} + \frac{2}{x}y = \frac{y^3}{x^3}$

c) $\frac{dy}{dx} = \frac{2x - y + 5}{x - 3y + 3}$

d) $(2x^2y^2 + y)dx + (3x - x^3y)dy = 0$

8. Solve the following differential equations : 20

a) $(D^2 + 2D + 1)y = \sin 2x + 5x$

b) $(D^4 - 2D^3 + D^2)y = x^3$

c) $(D^2 + 1)y = \sec x$

d) $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log_e x$
