



SEM 2 – 1 (RC 07-08)

F.E. (Semester – II) (RC 2007-08) Examination, May/June 2018 APPLIED MATHEMATICS – II

Duration : 3 Hours

Total Marks : 100

- Instructions :** 1) Attempt **any five** questions, at least **one** from **each** Module.
2) Assume suitable data, **if** necessary.

MODULE – I

1. a) Assuming the validity of differentiation under integral sign rule evaluate the following integrals : $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$.

7

- b) Find the length of astroid $x = a \cos^3 t$, $y = a \sin^3 t$.

6

- c) Find the area of lemniscate $r^2 = a^2 \cos 2\theta$.

7

2. a) Show that $\vec{r}(t) = A(e^{2t})\mathbf{i} + B e^{-3t}\mathbf{j}$ satisfies $\frac{d^2\vec{r}}{dt^2} + \frac{d\vec{r}}{dt} - 6\vec{r} = 0$.

6

- b) For the following curve $\vec{r} = \cos t \mathbf{i} + \sin t \mathbf{j} + t \mathbf{k}$, find curvature and torsion.

6

- c) The acceleration of a particle at any time is given by $\mathbf{a} = e^{-t} \mathbf{i} - 6(t+1)\mathbf{j} + 3 \sin t \mathbf{k}$. Find velocity \mathbf{V} and displacement \mathbf{r} at any time 't' given that $\mathbf{V} = 0$ when $t = 0$ and $\mathbf{r} = 0$ when $t = 0$.

8

MODULE – II

3. a) Evaluate the integral $\int_0^1 \int_0^x x x^2 + y^2 dx dy$.

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- b) Change the order of integration and evaluate $\int_3^5 \int_0^{\frac{4}{x}} (xy) dy dx$.

6

- c) Using double integration find the area bounded by the parabolas $x = y^2$, $x = 2y - y^2$.

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4. a) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x dz dx dy$.

6

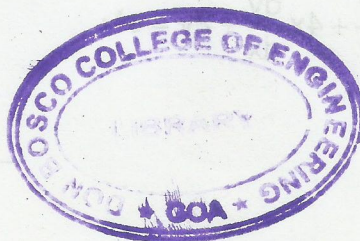
- b) Find the volume of the solid under the plane $3x + 2y + z = 12$ and above the rectangle $R = (x, y)/0 \leq x \leq 1, -2 \leq y \leq 3$ by the planes $x = 0, y = 0, z = 0, x + y + z = a$.

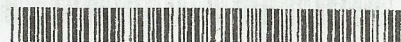
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- c) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{a \cos \theta} \int_0^{\sqrt{a^2 - r^2}} r dz dr d\theta$.

8

P.T.O.





MODULE – III

5. a) Find the maximum directional derivative of $xy(x - y + 2z)$ at $(1, 1, 0)$. 7
- b) If $\vec{F} = (4x + 3y + az)\mathbf{i} + (bx - y + z)\mathbf{j} + (2x + cy + z)\mathbf{k}$ is irrotational, find constants a, b, c . 6
- c) Find the work done in moving a particle from $A(1, 0, 1)$ to $B(2, 1, 2)$ along a straight line AB in the force field $\vec{F} = x^2\mathbf{i} + (x - y)\mathbf{j} + (y + z)\mathbf{k}$. 7
6. a) Verify Greens theorem in plane for $\oint \left[\left(\frac{1}{y} \right) dx + \left(\frac{1}{x} \right) dy \right]$ where C is the region bounded by $x = 1, x = 4, y = \sqrt{x}$. 8
- b) Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\mathbf{i} + 2xy\mathbf{j}$ in the xy plane and region bounded by $y = 0, x = 2, y = x$. 12

MODULE – IV

7. Solve the following differential equations.
- a) $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$. 5
- b) $x \sin\left(\frac{y}{x}\right) dy = \left[y \sin\left(\frac{y}{x}\right) - x \right] dx$. 5
- c) $\frac{dy}{dx} = \frac{2y - x - 4}{y - 3x + 3}$. 5
- d) $\frac{dy}{dx} + \tan x = \cos y \cos^3 x$. 5
8. Solve the following differential equations.
- a) $\frac{d^2y}{dx^2} + 9y = x \cos 2x$. 5
- b) $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = x^2$. 5
- c) $(D^2 - 3D + 2)y = e^{4x} + \sin 3x + x^2$. 5
- d) $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$. 5

