Total No. of Printed Pages:2

F.E. Semester-II (Revised Course 2019-20)
EXAMINATION AUGUST 2021
Mathematics-II

[Duration: Two Hours] **[Total Marks: 60]**

Instructions:
i) Answer THREE FULL QUESTIONS with ONE QUESTION FROM EACH PART.
ii) Assume suitable data,if necessary
iii) Figures to the right indicate full marks

Part A

Q.1 a) The asteroid $x^{2/3} + y^{2/3} = 4$ is revolved about the x-axis. Find the surface area of the object generated. 7

b) Find the length of the cycloid $x = (\theta + sin\theta), y = (1 - cos\theta) between\ \theta = 0\ and\ \theta = \pi.$ 6

c) Change the order of integration and evalute $\int_0^1 \int_{y-1}^1 2x + 1dxdy$ 7

Q.2 a) Change to polar coordinates and evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{y}{x^2+y^2} dxdy$ 6

b) Evalute $\iiint \frac{zdxdydz}{(1+x^2+y^2)}$ over the region $x^2 + y^2 \leq 4$ and $y \geq 0, 0 \leq z \leq 2$ 7

c) Write a single integral and evaluate $\int_0^1 \int_{1-x}^1 3x + 1dydx + \int_1^2 \int_{x-1}^1 3x + 1dydx$ 7

Q.3 a) Evaluate $\iint 2xy + 3dxdy$ over the region $x^2 \leq y, x \geq 0, y \leq x + 2$ 6

b) Find the area of the surface generated by the revolution of the curve $y = \sqrt{x+1},$ $0 \leq x \leq 4$ about the axis. 7

c) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x + y\ dzdxdy$ 6

Part B

Q.4 a) Find the directional derivative of $f(x, y, z) = z^2x + 2yx$ in the direction of $\hat{\imath} - \hat{\jmath} + 2\hat{k}$ at the point (-1,1,0). 6

b) Find the work done inmoving aparticle in the force $\overline{F} = 3y\hat{\imath} + xz\hat{\jmath} + (yz - 2)\hat{k}$ along the curve $x = 2t^2, y = 3t$ and $z = t^2$ from $t = 0$ to $t = 1$. 7

c) Solve the differential equation $(D^2 + 2D + 1)y = x^3 + 2x^2$ 7

Q.5 a) Verify Green's Theorem in a plane for $\oint (y + 2x^2)dx + yx^2dy$ along the boundry of the 8 region $y \geq 0, y^2 \leq 4x$ and $x \leq 1$.

b) Solve the following 12
(i) $(D^2 - 5D + 6)y = 3e^{3x} + e^{2x}$

(ii) $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 2x^3$

Q6 a) Define gradient of a scaler field. Show that if f and g are scalar fields then $grad(fg) = f(gradg) + g(gradf)$ 6

b) Use Gauss Divergence Theorem to evaluate $\int\int_S \bar{F}.\widehat{nds}$ where $\bar{F} = xy\hat{\imath} + yz\hat{\jmath} + y\hat{k}$ and S is the surface of the closed region bounded by the coordinate planes and the planes $x = 1, y = 1$ and $z = 1$. 8

c) Solve the differential equation $(D^2 - 4D + 5)y = xe^x$ 6

Part C

Q.7 a) Find the length of $x = \frac{y^3}{6} + \frac{1}{2y}$ from $y = 1$ to $y = 16$. 7

b) Find the equation of the line, normal to the surface $2x + 3y^2 - z^2 = 10$ at the point (1,-2,2). 6

c) Solve $D^2 + 4D - 12)y = \sin^2 x$ 7

Q.8 a) Show that the vector field $\bar{f} = 2xy\bar{\imath} + (x^2 + 2z)\bar{\jmath} + (2y + 2z)\,\bar{k}$ is irrational and find its scalar potential. 7

b) Evaluate $\int_0^{\pi/2}\int_0^{\pi}\int_0^{1+cos\varphi} 3\rho \sin\varphi \; d\rho d\theta d\varphi$ 6

c) The region $y^2 \leq x, x \leq 1$ is revolved about the y-axis, find the volume of the object generated. 7