F.E. (Semester – II) (RC 2016-17) Examination, May/June 2018
ENGINEERING MATHEMATICS – II

Duration : 3 Hours   Total Marks : 100

Instructions : *i) Attempt **five** questions, two **each** from Part – **A** and Part – **B** and **one** from Part – **C**.*
*ii) Assume suitable data, **if necessary**.*
*iii) Figures to the **right** indicate **full marks**.*

PART – A

1. a) Evaluate $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)}dx$ by applying differentiation under the integral sign. 8

   b) Find the perimeter of the curve $r = 2a\sin\theta$. 6

   c) Evaluate $\int_0^1 \int_{y^2}^1 \sin\left(\frac{\pi y}{\sqrt{x}}\right) dxdy$. 6

2. a) Change the order of integration and evaluate $\int_0^1 \int_{x-1}^{x+1} 3y + x\,dxdy$. 8

   b) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x + y\,dzdxdy$. 6

   c) For the curve $\overline{r}(t) = a\cos t\,\overline{i} + a\sin t\,\overline{j} + bt\overline{k}$ $a \geq 0, b \geq 0$ and $a^2 + b^2 \neq 0$ find the curvature. 6

3. a) Find by double integration the volume of the solid generated by the revolution of the ellipse $x^2 + \frac{y^2}{4} = 1$ about the x-axis. 6

   b) The velocity of a particle at time t is given by $\overline{v} = \sin 2t\,\overline{i} + 2\cos 2t\,\overline{j} + t\overline{k}$ was at origin at t = 0. Find the position vector and acceleration of the particle for t ≥ 0. Also find the times for which the position and acceleration are orthogonal. 7

   c) Evaluate $\iiint 2x + z\ \ dxdydz$ over the region $\{(x, y, z)/y^2 \leq x, x \leq 1, 0 \leq z \leq 1\}$. 7

P.T.O.

PART – B

4. a) In which direction is the rate of change of $f(x, y, z) = 2x^2 + 3z + y^2$ at $(1, -2, 2)$ maximum ? Find the magnitude of this maximum. **4**

b) Show that $\bar{F} = 4xy\hat{i} + (2x^2 + 4z)\hat{j} + 4y\hat{k}$ is irrotational and find its scalar Potential. **6**

c) Solve the following : **10**

i) $(x + y + 1)^2 \dfrac{dy}{dx} = 1$

ii) $\dfrac{dy}{dx} - y\tan x = y^4 \sec x$.

5. a) Use Stoke's theorem to evaluate the surface integral $\iint (\nabla \times \bar{F}) \cdot \hat{n} dS$ where $\bar{F} = (x^2y)\hat{i} + (2y + z)\hat{j} + y^2\hat{k}$ and $\hat{n}$ is the unit outward normal vector to S, S being the surface of the region bounded by $x = 0$, $y = 0$, $x + 2y + z = 2$ excluding the surface in the xy plane. **8**

b) Prove for any scalar fields f and g **6**

i) $\text{div}(\text{grad } f) = \nabla^2 f$

ii) $\text{grad}(f\,g) = f(\text{grad} g) + g(\text{grad} f)$.

c) Solve $x^2 \dfrac{d^2y}{dx^2} + 5x\dfrac{dy}{dx} + 4y = x\log_e x$. **6**

6. a) Verify Green's Theorem in the plane for $\oint (2x + y^2)dx + (5 + xy)dy$ along the boundary of the region bounded by $y^2 = 4x$ and $y = 2x$. **10**

b) Solve : **10**

i) $(D^3 - D^2 - 6D)y = x^2 + 1$

ii) $(D^2 + 5D + 6)y = \sinh(x)$.

PART – C

7. a) If $\bar{r}(t) = \bar{a}e^{2t} + \bar{b}e^{2t}$, then prove that $\frac{d^2\bar{r}}{dt^2} - 4\frac{d\bar{r}}{dt} + 4\bar{r} = 0$. Where $\bar{a}$ and $\bar{b}$ are constant vectors. 7

b) If $f = x^2\bar{i} - 2xy\bar{j} + yz\bar{k}$, compute $\int_C f \cdot dr$ between (2, 1, 2) and (6, 9, 8) where C is the path with parametric equations $x = 2t$, $y = t^2$, $z = 3t - 1$. 7

c) Solve $(1+x)\frac{dy}{dx} + 1 = 2e^{-y}$. 6

8. a) The curve $r = 2a\cos\theta$ is revolved about the X-axis. Use integration to find the surface area of the object generated. 7

b) Find the volume of the region $\{(x, y, z)/x^2 + y^2 \leq z^2, x^2 + y^2 + z^2 \leq 4\}$. 7

c) Solve $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\log_e x$. 6