FE111

Total No. of Printed Pages:2

F.E. Semester- I (Revised Course 2007-08)
EXAMINATION SEPTEMBER 2020
Applied Mathematics-I

[Duration : Two Hours] **[Total Marks : 60]**

Instructions:
1) Answer THREE FULL QUESTIONS with ONE QUESTION from ANY THREE MODULES.
2) Assume missing data if any.

Module -I

Q.1 a) Prove that $\int_0^1 x(\log x)^6\, dx = \frac{\sqrt{7}}{128}$ (5)

b) Express $\int_0^\infty e^{-x^n} dx, \quad n > o$ as Gamma function. (5)

c) Prove that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\, dx$ (6)

d) Prove that $erfc(n) + \ erfc(x) = 1$ (4)

Q.2 a) Test the following series for convergence. (12)

i) $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots$

ii) $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$

iii) $\sum_{n=1}^{\infty} \frac{3^n.\ n!}{n^n}$

b) Define conditionally convergent series and give one example. (4)

c) Find the radius of convergence for the series $\frac{1}{2}x + x^2 + \frac{9}{8}x^3 + x^4 + \frac{25}{32}x^5 + \cdots$ (4)

Module -II

Q.3 a) As an application of De-Moivers Theorem solve $x^3 + 1 = 0$ (5)

b) If $x = cos\,\alpha + i\,sin\,\alpha,\ y = cos\,\beta + i\sin\beta,\ z = \cos\gamma + i\sin\gamma$, and $x + y + z = 0$, then prove that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$ (5)

c) Prove $sin\left\{i\log\left(\frac{a-ib}{a+ib}\right)\right\} = \frac{2ab}{a^2+b^2}$ (5)

d) Prove $\cos h^{-1}(\sqrt{1+x^2}) = \sin h^{-1} x$ (5)

Q.4 a) Test if $f(z) = z^3 + 1 - i\,z^2$ is analytic and also find $f^1(z)$ in terms of z. (7)

b) Determine the analytic function whose real part is $e^{2x}(x\cos 2y - y\sin 2y)$ (6)

c) Find a and b if $f(z) = \cos x(\cos h\,y + a\sin hy) + i\sin x(\cos hy + b\sin hy)$ (7)

Module -III

Q.5 a) $if\quad y = e^{\tan^{-1} x}$, Show that $(1+x^2)y_{n+2} + [2(n+1)x+1]y_{n+1} + n(n+1)y_n = 0$ (7)

b) Show that $e^{x\cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} - \frac{11}{24}x^4 + \cdots$ (7)

c) Use Taylor theorem to expand $\cos^2 x$ is powers of $(x - \pi/_3)$ (6)

Q.6 a) Evaluate (12)

i) $\lim\limits_{x \to 0} \frac{\tan x - x}{x^2 \tan x}$

ii) $\lim\limits_{x \to 0} \frac{e^x + e^{-x} - 2\cos x}{x \sin x}$

iii) $\lim\limits_{x \to 1} \frac{1 + \log x - x}{1 - 2x + x^2}$

b) If $z = f(u,x)$, where $u = a\cos hx.\cos y$ and $x = a\sin hx\sin y$, prove that (8)
$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{a^2}{2}(\cos h2x - \cos 2y)\left[\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right]$

Module -IV

Q.7 a) Form the partial differential equation by eliminating the arbitrary constants (8)

i) $z = a(x+y) + b$

ii) $z = (x^2 + a)(y^2 + b)$

b) Form the partial differential equations by eliminating arbitrary function from the equation $f(z - xy, x^2 + y^2) = 0$ (6)

c) Solve the partial differential equation $(x - 2z)p + (2z - y)q = y - x$ (6)

Q.8 a) If $u = \tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$ (10)

b) Find the minimum value of $x^2 + y^2 + z^2$ subject the condition $x + y + z = 1$ (10)