F.E. (Semester – II) Examination, May/June 2013
APPLIED MATHEMATICS – II (RC 07-08)

Duration : 3 Hours | Total Marks : 100

Instructions : *1) Attempt **any 5** questions, atleast **one** from **each** Module.*
*2) Assume suitable data **if necessary**.*

MODULE – I

1. a) Assuming the validity of differentiating under the integral sign prove that

$$\int_0^\infty e^{-\beta x}\frac{\sin\alpha x}{x}dx = \tan^{-1}\left(\alpha/\beta\right) \text{ where } \beta > 0.$$ 6

b) Find the length of the curve $x = \frac{1}{3}y^{3/2} - y^{\frac{1}{2}}$ from $y = 1$ to $y = 9$. 7

c) The loop of the curve $x = t^2,\ y = t - \frac{t^3}{3}$ is revolved about the y-axis. Find the surface area of the object generated. 7

2. a) A particle moving in space has constant acceleration $i + 2j$. If its initial displacement and velocity vector is $3\,i$ and $\overline{2i} + \overline{k}$ respectively, find its position vector at any time t. 6

b) Define curvature of a curve. Show that the curvature of any circle is constant and the curvature of a straight line is 0. 8

c) For the space curve $x = 2\cos t$, $y = 2\sin t$ and $z = 3t^2$. Find the principal normal N and unit tangent T. 6

MODULE – II

3. a) Evaluate $\iint\limits_R x + 2ydxdy$, where R is the triangular region with vertices (0, 0) (1, 1) and (1, –1). 7

b) Evaluate $\int_0^{\infty}\int_x^{\infty} e^{-y^2} dydx$. **6**

c) Find the volume of the object generated by the revolution of $r = \cos 2\theta$ about the initial line. **7**

4. a) Evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} y\, e^{x^2+y^2}\, dxdy$ by changing to polar coordinates. **7**

b) Evaluate $\int_0^{1}\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$. **7**

c) Find the volume of the tetrahedron bounded by the co-ordinate plane and $2x + 2y + 3z = 6$. **6**

MODULE – III

5. a) Define gradient of a scaler field. Show that the gradient vector is normal to the level surface of the scaler field. **4**

b) Show that the vector field $F = 4xy\bar{i} + (2x^2 + 4z)\bar{j} + 4y^2\bar{k}$ is irrotational. Find its potential function. **6**

c) Evaluate $\int_c F ds$, where $F = x^2+2yz$ and c is the line from (0, 1, 1) to (1, 1, 2). **4**

d) Use Gauss Divergence theorem to evaluate $\int_s F.nds$ where $F = x^2\bar{i} + y\bar{j} + \bar{k}$, $\bar{n}$ is the unit normal vector to S and S is the surface of the cube $0 \leq x, y, z \leq 1$. **6**

6. a) Verify Green's theorem in the plane for $\oint_c (3x^2 + y^2)dx + 2xydy$ where c is the perimeter of the triangle having vertices (0, 0) (1, 0) and (0, 1). **8**

b) Verify Stoke's theorem for $\bar{F} = x^2\bar{i} + 2yz\bar{j} + x\bar{k}$ and s are the three sides of the tetrahedron, bounded by the co-ordinate plane and the plane $x + 2y + z = 2$, excluding the side in the xy plane. **12**

MODULE – IV

7. Solve the following differential equation : **20**

i) $e^y(1+x^2)\frac{dy}{dx} - 2x(1+e^y) = 0$

ii) $x(1-x^2)\frac{dy}{dx} + (2x^2-1)y = x^3$

iii) $(xy^2+2x^2y^3)\,dx + (x^2y - x^3y^2)\,dy = 0$

iv) $(2x - y + 5)\,dx + (x + 3y + 1)\,dy = 0$

8. Solve the following differential equations : **20**

i) $(D^2+2D+1)y = x\,e^x+2$

ii) $(D^3 + 4D^2 + 3D)\,y = 3e^x \sin 3x$

iii) $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = x^3 \log x$

iv) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$
