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F.E. (Sem-I) (Revised Course 2016-17)
EXAMINATION Nov/Dec 2019
Engineering Mathematics - I

[Duration : Three Hours] **[Total Marks : 100]**

Instructions:

1) Attempt any two questions from part A, any two from part B and any one from part C.
2) Assume missing data, if any.

PART-A

(Answer Any Two Questions)

Q.1

a) Prove that $\int_0^\infty \frac{e^{-x^2}}{\sqrt{x}}dx \int_0^\infty x^2 e^{-x^4}dx = \frac{\pi}{4\sqrt{2}}$ **(6)**

b) Prove that $\mathrm{erf}(x) + \mathrm{erf}_c(x) = 1$. **(4)**

c) Use DeMoivre's theorem to evaluate all the values of $i^{\frac{1}{4}}$ **(5)**

d) Evaluate $\int_0^\infty \frac{x^3(1-x^4)}{(1+x)^{11}}dx$ using a suitable form of the Beta function. **(5)**

Q.2

a) Test the convergence of the following series **(12)**

(i) $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$

(ii) $\frac{1^2.2^2}{1!} + \frac{2^2.3^2}{2!} + \frac{3^2.4^2}{3!} + \frac{4^2.5^2}{4!} + \cdots$

(iii) $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \cdots$

b) If $'n'$ is a positive integer then prove that **(4)**

$(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos(\frac{n\pi}{6})$

c) Determine the values of a, b, c, d so that the function. **(4)**

$f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$is analytic

Q.3

a) Prove that $i \log\left(\frac{x-i}{\cancel{x+1}}\right) = \pi - 2\tan^{-1} x$ **(6)**

x+i

b) Prove $\sinh^{-1} x = \frac{1}{2}\, cosech^{-1} \frac{1}{2x(1+x^2)^{1/2}}$ **(6)**

c) Find the radius and interval of convergence for the power series **(8)**

$$x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \cdots \infty$$

PART-B **(Answer Any Two Questions)**

Q.4

a) If $u = \cos^{-1}\left(\frac{2x^{\frac{1}{4}} - y^{\frac{1}{4}}}{x-y}\right)$ then evaluate $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy}$ **(6)**

b) If $y = (\sin^{-1} x)^2$ then prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$ **(7)**

c) Apply Taylor's series expansion formula to find the approximate value of $\sqrt[3]{8.01}$ **(7)**

Q.5

a) Evaluate **(12)**

(i) $\lim_{x\to 0} \log_{\sin x}(\sin 2x)$

(ii) $\lim_{x\to 0} \frac{2\cos x - 2 + x^2}{x^4}$

(iii) $\lim_{x\to 0} (\cos x)^{1/x^2}$

b) Form a partial differential equation by eliminating a and b from **(4)**

$$z = (x-a)^2 + (y-b)^2$$

c) Form a partial differential equation by eliminating the arbitrary function from **(4)**

$$z = e^{ay} f(x + by)$$

Q.6

a) If $z = f(u,v)$ *where* $u = lx + my,\ v = ly - mx;\ l$ *and* m *being constants,* then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$ **(8)**

b) Solve the partial differential equation $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$ **(6)**

c) Find and classify the critical points of $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ **(6)**

PART-C
(Answer Any One Question)

Q.7

a) Evaluate $\int_0^1 x(logx)^6 dx$ **(6)**

b) Prove **(8)**

(i) $sin\{i \log(\frac{a-ib}{a+ib})\} = \frac{2ab}{a^2+b^2}$

(ii) $cos\{i \log(\frac{a-ib}{a+ib})\} = \frac{a^2-b^2}{a^2+b^2}$

c) Prove that $\log(1 - \log(1 - x)) = x + \frac{x^3}{6} + \cdots$ **(6)**

Q.8

a) Evaluate $\int_0^1 x^3 (1 - \sqrt{x})^5 \, dx$ using Beta function **(6)**

b) Use Lagrange's method to find the maximum distance of the point $(1,2,-1)$ to the sphere $x^2 + y^2 + z^2 = 24$ **(7)**

c) If $u = \frac{x^3y^3}{e^{x^2+y^2}}$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 4u \log(u)$ **(7)**