F.E. (Semester – I) (RC 2007 – 08) Examination, November/December 2017

APPLIED MATHEMATICS – I

Duration : 3 Hours

Total Marks : 100

Instructions : *1) Attempt **any five** questions at least **one** from **each** Module.*

*2) Assume suitable data, if **necessary**.*

MODULE – I

1. a) Prove that $\int_0^\infty \frac{x^{l-1}}{(1+x)^{l+m}}dx = B(l, m)$. 5

b) Evaluate $\int_0^\infty \frac{x^3}{3^x}dx$. 5

c) Evaluate $\int_0^1 x^4 \left(\log\frac{1}{x}\right)^3 dx$. 5

d) Prove that error function is an odd function. 5

2. a) Test the convergence of the following series.

i) $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$. 4

ii) $\sum_{n=2}^{\infty} \frac{1}{n\log n}$. 4

iii) $1 - \frac{2}{3} + \frac{3}{3^2} - \frac{4}{3^3} + \ldots$ 4

b) Find the interval of convergence of the following series $\frac{x}{1^2} - \frac{x^2}{3^2} + \frac{x^3}{5^2} - \frac{x^4}{7^2} + \ldots$ 8

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MODULE – II

3. a) Use De Moivre's Theorem and solve $x^6 - i = 0$. 6

b) Prove that $i^{i^{i\cdots}} = \cos\theta + i\sin\theta$ where $\theta = (2n+1/2)\,\pi e^{-(2n+1/2)\pi}$. 8

c) Separate into real and imaginary $\cos^{-1}\left(\frac{3i}{4}\right)$. 6

4. a) Determine the analytic function whose real part is $e^{-x}(\cos y + \sin y)$. 6

b) Show that the function $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. Find the conjugate harmonic function. 6

c) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z. 8

MODULE – III

5. a) If $y = e^{\tan^{-1}x}$, prove that, $(1+x^2)\,y_{n+2} + [2(n+1)x - 1]\,y_{n+1} + (n^2+n)\,y_n = 0$. 7

b) Expand $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in powers of x. Find the first 4 terms. 6

c) Use Taylor's theorem to expand $\tan^{-1}x$ in powers of $\left(x - \frac{\pi}{4}\right)$ and hence find $\tan^{-1}(2)$. 7

6. a) Evaluate : 12

i) $\lim\limits_{x \to 0} \frac{\log x}{\operatorname{cosec} x}$.

ii) $\lim\limits_{x \to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$.

iii) $\lim\limits_{x \to 0} (a^x + x)^{\frac{1}{x}}$.

b) If $z = f(x, y)$ where $u = lx + my$, $v = ly - mx$ then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right).$$ 8

MODULE – IV

7. a) Form the partial differential equations by eliminating constants

i) $2z = (ax + y)^2 + b$

ii) $(x - h)^2 + (y - k)^2 + z^2 = a^2$. **6**

b) Form the partial differential equations by eliminating functions $f(x^2 + y^2 + z^2, x + y + z) = 0$. **6**

c) Solve $z^2 (p^2 + q^2 + 1) = 1$. **8**

8. a) If $u = \sin^{-1}\left[\dfrac{x^{1/4} + y^{1/4}}{x^{1/6} + y^{1/6}}\right]$, find the value of $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2}$. **6**

b) Find the extreme values of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. **6**

c) Find the maximum and minimum distances of the point (3, 4 , 12) from the sphere $x^2 + y^2 + z^2 = 1$. **8**
