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SEM 1 – 1

F.E. (Semester – I) (Revised Course) Examination, Nov./Dec. 2013
APPLIED MATHEMATICS – I

Duration : 3 Hours | Total Marks : 100

Instructions : *1) Attempt **any five** questions, at least **one** from **each** Module.*
*2) **Assume** suitable data, if necessary.*

MODULE – I

1. a) Show that $\int_0^\infty x^n e^{-a^2x^2}\,dx = \frac{1}{2a^{n+1}}\Gamma\left(\frac{n+1}{2}\right)$. Hence deduce that $\int_0^\infty e^{-a^2x^2}\,dx = \frac{\sqrt{\pi}}{2a}$. **6**

b) Evaluate $\int_0^1 x^3\left(1-\sqrt{x}\right)^5 dx$. **4**

c) Show that $\int_0^1 \left(1-x^{1/n}\right)^m dx = \frac{n!m!}{(m+n)!}$. **5**

d) Prove that $\frac{\beta(m+1,n)}{m} = \frac{\beta(m,n+1)}{n} = \frac{\beta(m,n)}{m+n}$. **5**

2. a) Test the convergence of the following series : **12**

i) $\frac{3}{4}+\frac{3\cdot 4}{4\cdot 6}+\frac{3\cdot 4\cdot 5}{4\cdot 6\cdot 8}+\ldots\infty$

ii) $\sum_{n=1}^{\infty}\frac{n^2}{e^n}$

iii) $\frac{1}{1\cdot 2\cdot 3}+\frac{3}{2\cdot 3\cdot 4}+\frac{5}{3\cdot 4\cdot 5}+\ldots$

b) Define interval of convergence and find it for the series $\frac{1}{2}x+x^2+\frac{9}{8}x^3+x^4+\frac{25}{32}x^5+\ldots$ **6**

c) State D'Almbert ratio test for the convergence of a series. **2**

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MODULE – II

3. a) If $z = \cos\theta + i\sin\theta$ prove that :

i) $\dfrac{2}{1+z} = 1 - i\tan\dfrac{\theta}{2}$

ii) $\dfrac{1+z}{1-z} = i\cot\dfrac{\theta}{2}$. **5**

b) If $\tan z = \dfrac{i}{2}(1-i)$ prove that $z = \dfrac{\tan^{-1} z}{2} + \dfrac{i}{4}\log_e 5$. **5**

c) If $\tan(a + i\beta) = x + iy$. Prove that $x^2 + y^2 + 2x\cot 2\alpha = 1$. **6**

d) Use DMT to solve the equation $x^5 + x^4 + x^3 + x^2 + x + 1 = 0$. **4**

4. a) Considering the principal value only, prove that the real part of $\left(1+i\sqrt{3}\right)^{\left(1+i\sqrt{3}\right)}$ is $2e^{-\pi/\sqrt{3}}\cos\left(\pi/3 + \sqrt{3}\log_e 2\right)$. **7**

b) If $\dfrac{(1+i)^{(x+iy)}}{(1-i)^{(x-iy)}} = a + ib$ then considering the principal value only, prove that

$\tan^{-1}\left(\dfrac{b}{a}\right) = \dfrac{\pi x}{2} + y\log 2$. **6**

c) Show that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic. Construct the corresponding analytic function $f(z) = u + iv$. **7**

MODULE – III

5. a) If $y = a\cos(\log x) + b\sin(\log x)$ where 'a' and 'b' are constants then show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$. **7**

b) Expand $\log(1 + 2\sin x)$ in powers of x. Find the first 5 terms. **7**

c) Use Taylors theorem to expand $\mathrm{Sec}^2 x$ in powers of $(x - \pi/4)$. **6**

6. a) Evaluate :

i) $\lim_{x\to 1} \dfrac{x^x - x}{x - 1 - \log x}$

ii) $\lim_{x\to 0} \log_{\tan^2(x)} \tan^2(2x)$

iii) $\lim\limits_{x\to 0} \dfrac{\sin x - \log\left(e^x \cos x\right)}{x \sin x}$. 12

b) If $z = f(x, y)$ where $x = e^u + e^{-v}$ and $y = e^{-u} - e^v$ then prove that $z_u - z_v = xz_x - yz_y$. 8

MODULE – IV

7. a) Form the partial differential equation eliminating the arbitrary constants :

i) $z = axy + b$

ii) $z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$. 8

b) If $z = x^n f\left(\dfrac{y}{x}\right) + y^{-n} f\left(\dfrac{x}{y}\right)$ then prove that

$x^2z_{xx} + 2xyz_{xy} + y^2z_{yy} + xz_x + yz_y = n^2z$. 6

c) Solve the partial differential equation $zx\dfrac{\partial z}{\partial x} + zy\dfrac{\partial z}{dy} = xy$. 6

8. a) If $u = \sin^{-1}\left[\dfrac{x^2\sqrt{y} + y^2\sqrt{x}}{3x^2 - 4y^2}\right]$ then prove :

i) $2(xu_x + yu_y) = \tan u$

ii) $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy} = \dfrac{-\sin u \cos 2u}{4\cos^3 u}$. 8

b) Find the maximum and minimum distance of the point (3, 4, 12) to the sphere $x^2 + y^2 + z^2 = 1$. 6

c) Discuss $u(x, y) = x^2y - 3x^2 - 2y^2 - 4y + 3$ for extreme values. 6
