

Total No. of Printed Pages:2

F.E. Semester –II (Revised Course 2016-17)
EXAMINATION AUGUST 2021
Engineering Mathematics-II

[Duration : Two Hours]

[Total Marks : 60]

Instructions:

- 1) Answer THREE FULL QUESTIONS with ONE QUESTION FROM EACH PART.
- 2) Assume suitable data, if necessary
- 3) Figures to the right indicate full marks

PART-A

- Q.1
- a) Evaluate $\int_0^{\infty} \frac{\log_e(1+at^2)dt}{t^2}$ by applying differentiation under the integral sign. 7
 - b) Find the length of the Cycloid $x = (\theta + \sin \theta)$, $y = (1 - \cos \theta)$ between $\theta = 0$ and $\theta = \pi$. 6
 - c) Change the order of integration and evaluate $\int_0^1 \int_x^{2-x} 2x + 1 dy dx$ 7
- Q.2
- a) Evaluate $\int_0^1 (3t\bar{i} + 2\bar{k}) \times (t\bar{i} + 5\bar{j}) dt$ 5
 - b) Evaluate $\iiint \frac{dx dy dz}{(x^2 + y^2 + z^2)}$ over the region $x^2 + y^2 + z^2 \leq 4$, and $y \geq 0, z \geq 0$. 8
 - c) Write a single integral and evaluate $\int_{-1}^0 \int_0^{x+1} 3y + 2 dx dy + \int_0^1 \int_0^{1-x} 3y + 2 dx dy$ 7
- Q.3
- a) Define curvature. Show that the curvature of $\vec{r}(t) = 2\cos t \bar{i} + 2\sin t \bar{j}$ is constant. 6
 - b) Find the area of the surface generated by revolution of the curve $y = \sqrt{x+1}$, $0 \leq x \leq 4$ about the x-axis. 7
 - c) Evaluate $\int_{-1}^1 \int_0^x \int_0^{(x+y)} 2y + x dz dy dx$ 6

PART-B

- Q.4
- a) Find the directional derivative of $f(x, y, z) = x^2y + 2yz$ in the direction of $\hat{i} - \hat{j} + \hat{k}$ at the point $(1, -1, 0)$. 5
 - b) Find the work done in moving a particle in the force field $\vec{F} = (2y + 3)\hat{i} + xz\hat{j} + (yz - 2)\hat{k}$ along the curve $x = 2t^2$, $y = t$ and $z = t^2$ from $t=0$ to $t=1$. 5
 - c) Solve the following 10
 - i) $\frac{dy}{dx} = e^{2x-y} + 2xe^{-y}$
 - ii) $(1+x)\frac{dy}{dx} + y = x^2$

- Q.5 a) Verify Green's Theorem in a plane for $\oint (x + 4y^2)dx + (x^2 + 3)dy$ along the boundary of the region $y \geq 0, y^2 \leq x$ and $x \leq 1$. 10
- b) Solve the following 10
- i) $(D^2 - 6D + 9)y = 2e^{3x}$
- ii) $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = (\log_e x)^2$
- Q.6 a) Define curl of vector field. Find the curl of $\phi(x, y, z) = 2xy\bar{i} + z\bar{j} + y^2\bar{k}$. 5
- b) Show that for a scalar field f and vector field ϕ $\text{div}(f\phi) = f\text{div } \phi + \nabla f \cdot \phi$ 5
- c) Solve 10
- i) $(D^2 + 4D + 3)y = xe^{2x}$
- ii) $(D^2 + 4)y = 3\sin 2x$

PART-C

- Q.7 a) Find the length of $x = a(2\cos t - \cos 2t)$ and $y = a(2\sin t - \sin 2t)$ from $t = 0$ to $t = \frac{\pi}{2}$. Where a is a constant. 5
- b) Evaluate $\int_0^1 \int_0^x xe^{y/x} dy dx$ 5
- c) Find the equation of the line normal to the surface $2x + 3z^2 - y^2 = 10$ at the point $(1, -2, 2)$. 5
- d) Solve $-(D^2 + 4D - 12)y = \sin^2 x$ 5
- Q.8 a) If $\vec{r}(t) = 2\cos t \hat{i} + 3t \hat{j} + \sin t \hat{k}$ is the position vector of a particle in space at time 't' then find its velocity and acceleration vectors at $t = \frac{\pi}{2}$. 5
- b) Evaluate $\int_0^{\pi/2} \int_0^\pi \int_0^{1+\cos\phi} 3\rho \sin \phi \, d\rho d\theta d\phi$ 6
- c) Prove that for a scalar field ϕ , $\text{Curl}(\nabla\phi) = 0$. 4
- d) Solve $\frac{dy}{dx} - y \tan x = y^4 \sec x$ 5