SEM 2 – 1 (RC 07-08)

F.E. (Semester – II) (RC 2007 - 08) Examination, November/December 2018
APPLIED MATHEMATICS – II

Duration : 3 Hours Total Marks : 100

Instructions : *1) Attempt **any five** questions at least **one** from **each** Module.*
*2) Assume suitable data, if **necessary**.*

MODULE – I

1. a) Assuming the validity of differentiation under integral sign rule evaluate the following integrals.

$$\int_0^{\infty} \frac{\tan^{-1} ax}{x(1+x^2)}\, dx .$$ 7

b) Find the length of the curve $y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}}$ measured from x = 0 to x = 3. 6

c) The curve $r = 2a \cos\theta$ is revolved about x-axis, find the surface area of the solid generated. 7

2. a) State and prove Serret-Frenet formula. 8

b) If $\overline{r}$ (t) has constant magnitude. Show that $\overline{r}$ (t) is perpendicular to its tangent $\frac{dr}{dt}$. 6

c) Show that $\overline{r}(t) = Pte^{2t} i + Qe^{3t} j$ satisfies $\frac{d^2r}{dt^2} - 4\frac{dr}{dt} + 4r = 0$. 6

MODULE – II

3. a) Evaluate $\int_0^1 \int_0^{x^2} x(x^2 + y^2)\, dxdy$. 6

b) Change the order of integration and evaluate.

$$\int_1^2 \int_1^{x^2} \frac{x^2}{y}\, dydx .$$ 6

c) Evaluate $\iint (x + y)\, dxdy$ over the region enclosed by the curves x = 0, x = 2, y = x, y = x + 2. 8

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4. a) Evaluate : **6**

$$\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z} x + y + z \, dxdydz.$$

b) Find the volume of the region enclosed by the planes $x = 0$, $y = 0$, $z = 0$,

$x + y + z = a$. **6**

c) Evaluate $\int_{0}^{\frac{\pi}{2}}\int_{0}^{a\cos\theta}\int_{0}^{\sqrt{a^2-r^2}} r\,dz\,dr\,d\theta$. **8**

MODULE – III

5. a) In what direction from the point (1, 2, 1) is the directional derivative of $\phi = x(x-y) + y(y+z)$ is maximum ? What is its magnitude ? **7**

b) Find the total work done in moving a particle in the force field $\overline{F} = 2x^2 i + (2yz - x) j + yk$ along $x = 3t^2$, $y = t$, $z = 3t^2 - t$ from $y = 0$ and $t = 1$. **6**

c) A vector field is given by $\overline{F} = (x^2 - y^2 + x)i - (2xy + y) j$ show that the field is irrotational and find its scalar potential. **7**

6. a) Verify Greens theorem $\overline{F} = (x^2 - xy)i + (x^2 - y^2) j$ over the boundary of the region bounded by $x^2 = 2y$ and $x = y$. **8**

b) Verify Stoke's theorem for $\overline{F} = (x^2 + y^2)i + 4xyj$ taken over the region bounded by the parabola $y^2 = 4x$ and the line $x = 4$. **12**

MODULE – IV

7. Solve the following differential equations

a) $(e^y + 1)\cos x\,dx + e^y \sin x\,dy = 0$. **5**

b) $\frac{dy}{dx} + y\tan x = y^3 \sec x$. **5**

c) $\frac{dy}{dx} = \frac{x-y+3}{2x-2y+5}$. 5

d) $\frac{dy}{dx} - 2xy = x$. 5

8. Solve the following differential equations.

a) $(D^3 - D)\, y = \sin x$. 5

b) $\frac{d^3 y}{dx^3} - 7\frac{dy}{dx} - 6y = xe^{2x}$. 5

c) $(D^2 - 3D + 2)y = e^{3x} + 5$. 5

d) $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - 4y = 2x^2$. 5
