SEM 2-1 (RC 07 – 08)

F.E. (Sem. – II) (Revised 2007 – 08) (Course) Examination, Nov./Dec. 2012
APPLIED MATHEMATICS – II

Duration : 3 Hours

Total Marks : 100

Instructions : *i) Attempt **any five** questions, at least **one** from **each** module.*
*ii) Assume suitable data **if necessary**.*

MODULE – I

1. a) Evaluate $\int_0^{\infty} \frac{\cos \lambda x}{x} (e^{-ax} - e^{-bx})\, dx$ applying differentiation under the integral sign. 6

b) Find the length of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ between two cusp. 6

c) Find the curved surface area of the solid generated by the revolution about x – axis of $x(t) = 1 - \sin t + \frac{t}{\sqrt{5}}$, $y(t) = \frac{2}{\sqrt{5}} \cos t$, from $t = 0$ to $t = \pi/2$. 8

2. a) The position vector of a moving object is $\overline{r}(t) = 2 \sin t\ \overline{i} + 2 \cos t\ \overline{j} + 3t\ \overline{k}$. Show that velocity and acceleration vectors at $t = \pi/2$ are perpendicular. 4

b) Find the principal normal N and the binomial vector B of $\overline{r}(t) = 2 \sin t\ \overline{i} + 2 \cos t\ \overline{j} + 3t\ \overline{k}$ at $t = 0$. 6

c) Evaluate $\int_0^{\pi} \overline{\cos t\, i + \sin^2 t\, j + k}\, dt$. 5

d) Define Curvature. If $x = \cos t$, $y = \sin t$, $z = 2t$. Find the curvature at $t = \pi/2$. 5

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MODULE – II

3. a) Evaluate $\int_0^{\infty}\int_x^{\infty} \frac{e^{-2y}}{y}\, dxdy$. **6**

   b) Evaluate $\iint 3x + 2dxdy$ over the region enclosed by $x^2 = y$ and $y - x = 2$. **8**

   c) Change the order of integration of $\int_1^2\int_0^x 2y + xdxdy$ and then evaluate. **6**

4. a) The region bounded by $x^2 = 4y$ and $y = 1$ is revolved about the x – axis. Find the volume of the object generated. **6**

   b) Evalute the Spherical Polar coordinates integral $\int_0^{\pi/2}\int_0^{\pi}\int_0^1 3r^3 \sin^3\varphi \, dr d\theta d\varphi$. **6**

   c) Find the volume of the region enclosed $x^2 + y^2 = 4$ and $x^2 + z^2 = 4$. **8**

MODULE – III

5. a) Define Curl of a vector field. Show that Curl $(\nabla \phi) = 0$ where ϕ is a scalar point function. **6**

   b) What is the greatest rate of change of $f(x, y, z) = 2x^2 + 3z + y^2$ at the point $(1, -2, 2)$? **4**

   c) Evalute $\iint_S \nabla \times \overrightarrow{F} \cdot \overrightarrow{n}\, ds$. Where S is the triangle having vertices $(1, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 2)$ $\overrightarrow{n}$ is the unit normal vector to the S and $\overrightarrow{F} = (x + yz)\overrightarrow{i} + (3z + x^2)\overrightarrow{j} + yx\overrightarrow{k}$. **10**

6. a) Verify Green's theorem in the plane for $\oint_C (x + 3y^2)dx + xydy$ where C is the boundary of the region enclosed by $y^2 = x$ and $x = 1$. **8**

   b) Verify Gauss divergence theorem for $F = (z^2 + 2x)\overrightarrow{i} + (x + 2z^2)\overrightarrow{j} - (y^2 + 3z)\overrightarrow{k}$, over the surface of the tetrahedron enclosed by the coordinate planes and the plane $x + y + z = 1$. **12**

MODULE – IV

7. Solve the following differential equations : **20**

a) $\frac{dy}{dx} - x^2 e^y = e^{2x + y}$.

b) $(x^2y^3 + 2y)dx + (2x - 2x^3y^2)dy = 0$

c) $\frac{dy}{dx} = \frac{2y - x + 1}{4y - 2x + 2}$

d) $(\sec x \tan x \tan y - e^{2x})dx + \sec x \sec^2 y\, dy = 0$.

8. Solve the following differential equations : **20**

a) $(D^2 + 2D - 15)y = 2 \sin^2 x + 3$

b) $(D^2 + 4)\ y = 4 \tan 2x$

c) $(D^3 - 6D + 4)y = 5xe^{2x}$

d) $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$.
