SEM 1 – 1 (RC 07-08)

F.E. Semester – I (RC 2007-08) Examination, May/June 2018
APPLIED MATHEMATICS – I

Duration : 3 Hours Total Marks : 100

Instructions : *1) Attempt* ***any five*** *questions atleast* ***one*** *from* ***each*** *Module.*
2) ***Assume*** *suitable data, if necessary.*

MODULE – I

1. a) State and prove duplication formula for gamma function. 6

b) Evaluate : $\int_0^a \frac{1}{\sqrt[n]{a^n - x^n}} dx$ 5

c) Evaluate : $\int_0^\infty \frac{x^4}{4^x} dx$ 5

d) Prove that : $erf_c(x) + erf_c(-x) = 2.$ 4

2. a) Test the convergence of the following series :

i) $\sum_{n=1}^{\infty} \frac{n^3}{e^n}$ 4

ii) $\sum_{n=1}^{\infty} \left(\sqrt{n^2+1} - n\right)$ 4

iii) $1 + \frac{2^2}{3^2} + \frac{2^2.4^2}{3^2.5^2} + \frac{2^2.4^2.6^2}{3^2.5^2.7^2} + ...$ 4

b) Define absolutely convergent and conditionally convergent series. Test whether the following series is absolutely convergent or conditionally convergent series. 8

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$$

P.T.O.

MODULE – II

3. a) Use DeMoivre's theorem and solve $x^3 + 8 = 0$. 6

b) Prove that :

i) $\sinh^{-1}x = \log\left(x+\sqrt{x^2+1}\right)$

ii) $\cosh^{-1}x = \log\left(x+\sqrt{x^2-1}\right)$. 8

c) If $\dfrac{(a+ib)^{x+iy}}{(a-ib)^{x-iy}} = \alpha + i\beta$ find α and β. 6

4. a) Determine the analytic function whose imaginary part is $\tan^{-1}(y/x)$. 6

b) Show that the function $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. Find the conjugate harmonic function. 6

c) If $u - v = \dfrac{\cos x + \sin x - e^{-y}}{2\cos x - 2\cosh y}$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z. 8

MODULE – III

5. a) If $\cos^{-1}\left(\dfrac{y}{b}\right) = \log\left(\dfrac{x}{n}\right)^n$, prove that, $x^2y_{n+2} + (2n+1)\,x\,y_{n+1} + 2n^2y_n = 0$. 7

b) Expand : $\cos^{-1}\left(\dfrac{x^2-1}{x^2+1}\right)$ in powers of x. Find the first 4 terms. 6

c) Calculate the approximate value of $\sqrt{10}$ to four decimal places by taking the first four terms of an appropriate Taylor's expansion. 7

6. a) Evaluate : 12

i) $\lim\limits_{x\to 0}\dfrac{\log x^2}{\cot x^2}$ ii) $\lim\limits_{x\to 0}\dfrac{\tan x - \sin x}{x^3}$

iii) $\lim\limits_{x\to 0}(5^x + x)^{\frac{1}{x}}$

b) If $z = f(x, y)$ where $u = lx + my$, $v = ly - mx$ then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$$

8

MODULE – IV

7. a) Form the partial differential equations by eliminating constants : 6

   i) $2z = (ax + y)^2 + b$

   ii) $(x - h)^2 + (y - k)^2 + z^2 = a^2$.

   b) Form the partial differential equations by eliminating functions $f(x^2 + y^2, z - xy) = 0$. 6

   c) Solve $x^2p^2 + y^2q^2 = z^2$. 8

8. a) If $u = \tan^{-1}\left[\frac{y^2}{x}\right]$, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. 6

   b) Find the extreme values of the function $x^3y^2(1 - x - y)$. 6

   c) Find the largest product of the numbers x, y, z when $x^2 + y^2 + z^2 = 9$. 8