Total No. of Printed Pages:3

F.E. Semester- I (Revised Course 2016-17)
EXAMINATION JANUARY 2021
Engineering Mathematics-I

[Duration : Two Hours] **[Total Marks : 60]**

Instructions:

1) Answer THREE FULL QUESTIONS with ONE QUESTION FROM EACH PART.
2) Assume suitable data, if necessary.
3) Figures to the **right** indicate full **marks**.

PART-A

Q.1

a) Evaluate $\int_0^{\infty} 5^{-4x^2}\, dx$ **(7)**

b) Prove that $\boldsymbol{\beta(m,n)} = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\, \boldsymbol{dx}$ **(7)**

c) State De Moivre's Theorem and find the values of $(1+i)^{2/3}$ **(6)**

Q.2

a) Test the following series for convergence **(12)**

i. $\frac{1}{6} - \frac{2}{11} + \frac{3}{16} - \frac{4}{21} + \cdots$

ii. $\sum_1^{\infty} n sin^2 \frac{1}{n}$

iii. $\sum_1^{\infty} \frac{2n-1}{n(n+1)(n+2)}$

b) If $\tan\left(\frac{\pi}{6} + i\alpha\right) = x + iy, prove\ that\ x^2 + y^2 + \frac{2x}{\sqrt{3}} = 1$ **(8)**

Q.3

a) Define absolute convergence and conditional convergence and test $\sum_1^{\infty} \frac{(-1)^{n-1}}{2n-1}$ for absolute convergence and conditional convergence. **(6)**

b) Prove that $\tan\left[i \log\left(\frac{a-ib}{a+ib}\right)\right] = \frac{2ab}{a^2-b^2}$ **(6)**

c) Show that $v = \log(x^2+y^2) + x - 2y$ is harmonic function. Here determine analytic function $f(z) = u + iv$. **(8)**

PART-B

Q.4

a) If $u = cosec^{-1}\left(\frac{\sqrt{x}+\sqrt{y}}{x^{1/3}+y^{1/3}}\right)$, find $x^2u_{xx} + 2xyu_{xy} + y^2u_{yy}$ **(8)**

b) If $y = \log(x + \sqrt{1 + x^2})$ show that **(7)**

$(1 + x^2)y_{n+2} + (2n + 1)x\, y_{n+1} + n^2y_n = 0$

c) Use Taylor's series to expand the polynomial $2x^3 + 7x^2 + x - 6$ in powers of $x - 2$ **(5)**

Q.5

a) Evaluate **(12)**

(i) $\lim_{x\to\pi/2} Sin\, x^{\tan^2 x}$

(ii) $\lim_{x\to 0} \frac{\tan x - \sin x}{x^3}$

(iii) $\lim_{x\to 0} \frac{e^x + e^{-x} - 2\cos x}{x \sin x}$

b) Find partial differential equation by eliminating constants m and n. **(4)**
$z = (x - m)^2(y - n)^2$

c) Find partial differential equation by eliminating arbitrary function. **(4)**
$f(x + y + z,\ \ x^2 + y^2 + z^2) = 0$

Q.6

a) If $z = f(x, y)$, where $u = x^2 - y^2$ and $v = 2xy$, prove that **(8)**

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 4(x^2 + y^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$$

b) Solve the partial differential equation **(6)**
$y^2zp + x^2zp = xy^2$ where $p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$

c) Use method of Lagrange's multipliers to find greatest and smallest values that the function $f(x, y) = x\,y$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$ **(6)**

Part-C

Q.7

a) Prove that $\int_0^\infty \frac{x^{a-1}}{(1+x)^{a+b}}dx = \int_0^1 \frac{x^{a-1}+x^{b-1}}{(1+x)^{a+b}}dx$ **(8)**

b) Prove that $\sin^{-1}(cosec\theta) = \frac{\pi}{2} + ilog(\cot\frac{\theta}{2})$ **(6)**

c) Expand the following functions in powers of x upto the fourth term $f(x) = e^x\, Sec\, x$ **(6)**

Q.8

a) Test the convergence of the series $\frac{2}{1} + \frac{2^2}{2} + \frac{2^3}{3} + \cdots$ **(6)**

b) Determine α such that $f(z) = \log(x^2 + y^2) + itan^{-1}\left(\frac{\alpha x}{y}\right)$ is an analytic function **(6)**

c) Obtain maxima and minima of the function $x^3 + y^3 - 63(x + y)$ **(8)**