DON BOSCO COLLEGE OF ENGINEERING
LIBRARY
GOA

SEM 1 – 1 (RC 07-08)

F.E. (Semester – I) (RC 2007 – 08) Examination, May/June 2016
APPLIED MATHEMATICS – I

Duration : 3 Hours Total Marks : 100

Instructions : *1) Attempt **any five** questions, at least **one** from **each** Module.*
*2) Assume suitable data if **necessary**.*

MODULE – I

1. a) State and prove Duplication formula for Gamma function. 6

b) Evaluate $\int_0^2 x^3 \sqrt{8-x^3}\,.dx$. 5

c) Evaluate $\int_3^7 \sqrt[4]{(x-3)(7-x)}\,dx$. 5

d) Prove that $\text{erf}_c\,(x) + \text{erf}_c\,(-x) = 2$. 4

2. a) Test the convergence of the following series.

i) $\sum_{n=1}^{\infty}\left(\frac{n}{(3n+1)}\right)^n$. 4

ii) $\sum_{n=1}^{\infty}\left(\sqrt{n^2+1}-n\right)$. 4

iii) $1-\frac{1}{5}+\frac{1}{9}-\frac{1}{13}+\ldots.$ to ∞. 4

b) Define absolutely convergent and conditionally convergent series. Test whether the following series is absolutely convergent or conditionally convergent series. 8

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}.$$

P.T.O.

MODULE – II

3. a) Use De Moivre's theorem and solve $x^3 + 8 = 0$. 6

b) Prove that $\sin^{-1}(\operatorname{cosec}\theta) = \frac{\pi}{2} + i\log\left(\cot\frac{\theta}{2}\right)$. 8

c) If $\frac{(a+ib)^{x+iy}}{(a-ib)^{x-iy}} = \alpha + i\beta$ find α and β. 6

4. a) Determine the analytic function whose imaginary part is $\tan^{-1}(y/x)$. 6

b) Show that the function $u = e^{-2xy}\sin(x^2 - y^2)$ is harmonic. Find the conjugate harmonic function. 6

c) Show that $i^i = \cos\theta + i\sin\theta;\ \theta = \left(2n+\frac{1}{2}\right)\pi e^{-\left(2n+\frac{1}{2}\right)\pi}$ 8

MODULE – III

5. a) If $y = e^{p\cos^{-1}x}$, prove that, $(1 - x^2)y_{n+2} - (2n + 1)x\,y_{n+1} - (n^2 + p^2)\,y_n = 0$. 7

b) Expand $\sec^2 x$ in powers of x. Find the first 4 terms. 6

c) Use Taylor's theorem to expand $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$ in powers of $(x - 1)$ and hence find $f(0.99)$. 7

6. a) Evaluate : 12

i) $\lim_{x\to 0}\frac{e^{\sin x} - 1 - x}{x^2}$

ii) $\lim_{x\to 1}(1-x).\tan\left(\frac{\pi x}{2}\right)$

iii) $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$.

b) If $z = f(x, y)$ where $x = e^u\sec v$, $y = e^u\tan v$ then show that

$$\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 = e^{-2u}\left[\left(\frac{\partial z}{\partial u}\right)^2 - \cos^2 v\left(\frac{\partial z}{\partial v}\right)^2\right].$$ 8

MODULE – IV

7. a) Form the partial differential equations by eliminating constants 6

i) $2z = (ax + y)^2 + b$

ii) $(x - h)^2 + (y - k)^2 + z^2 = a^2$.

b) Form the partial differential equations by eliminating functions $f(x^2 + y^2, z - xy) = 0$. 6

c) Solve $x^2p^2 + y^2q^2 = z$. 8

8. a) If $u = \sec^{-1}\left[\frac{\sqrt{x} - 2\sqrt{y}}{y^3\sqrt{x}}\right]$, find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$. 6

b) A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of the box requiring least material for its construction. 6

c) Use the method of Lagrange's Multipliers to find the maximum and minimum distance of the point (5, 3) from the circle $x^2 + y^2 = 1$. 8
