SEM 2 – 1 (RC 07 – 08)

F.E. (Semester – II) (Revised in 2007 – 08) Examination,
November/December 2017
APPLIED MATHEMATICS – II

Duration : 3 Hours Total. Marks : 100

Instructions : *1) Attempt **any five** questions, at least **one** from **each** Module.*
*2) Assume suitable data, **if** necessary.*

MODULE – I

1. a) Assuming the validity of differentiation under integral sign rule evaluate the following integrals : 7

$$\int_0^1 \frac{x^m - x^n}{\log x} dx.$$

b) Find the length of the curve $x = a(2\cos t - \cos 2t)$ and $y = a(2\sin t - \sin 2t)$ measured from $\theta = 0$ to $\theta = \frac{\pi}{2}$. 6

c) The curve $r = 2a\cos\theta$ is revolved about x-axis, find the surface area of the solid generated. 7

2. a) Evaluate $\int_0^\pi ((\cos t)i + (\sin^2 t)j + k)dt$. 6

b) Define Torsion. If $\bar{r}(t)$ is the position vector of moving object then show that 8

$$T = \frac{|r' r'' r'''|}{|r' \times r''|^2}$$

c) Show that $\bar{r}(t) = Ate^{2t}\, i + Be^{3t}\, j$ satisfies $\frac{d^2r}{dt^2} - 4\frac{dr}{dt} + 4r = 0$. 6

MODULE – II

3. a) Evaluate $\int_1^2 \int_0^y \frac{1}{x^2 + y^2} dxdy$. 6

b) Evaluate $\iint (x^2 + y^2)dxdy$ over the region enclosed by the curves $y = 4x$, $x + y = 3$, $y = 0$, $y = 2$. 8

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c) Change the order of integration and evaluate $\int_1^2 \int_{1-x}^{x-1} (xy+5)\,dxdy$. **6**

4. a) Evaluate $\iiint \frac{dxdydz}{x^2+y^2+z^2}$ throughout volume of the sphere $x^2+y^2+z^2=a^2$. **6**

b) Find the volume of the region enclosed by $x^2+y^2=4$ and $x^2+z^2=4$. **6**

c) Evaluate the spherical coordinate integral $\int_0^{\frac{\pi}{2}} \int_0^{\pi} \int_0^{1-\cos\phi} 3r^2\sin\phi + 4\cos\theta\, dr d\theta d\phi$. **8**

MODULE – III

5. a) In what direction from the point (1, 3, 2) is the directional derivative of $\phi = 2xz - y^2$ a maximum ? What is its magnitude ? **7**

b) Find the total work done in moving a particle in the force field $\bar{F} = 3xyi - 5zj + 10xk$ along $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from t = 1 and t = 2. **6**

c) Evaluate $\iint_s (\nabla r^2).(\hat{n})ds = 6V$, where S is any closed surface, $\hat{n}$ is the unit outward normal to S, V is the volume enclosed by S and $r^2 = x^2 + y^2 + z^2$. **7**

6. a) Verify Greens theorem in plane for $\oint \left[(xy+4y^2)dx + (x^2+3)dy \right]$ over the boundary of the region bounded by x = 1 and $y^2 = x$. **8**

b) Verify Stoke's theorem for $\bar{F} = (x + y)i + (2x - z)j + (y + z)k$ taken over the triangle ABC cutoff by the plane $x + 2y + 3z = 6$ on the coordinate axis. **12**

MODULE – IV

7. Solve the following differential equations.

a) $(x^2 - 2xy + 3y^2)\,dx + (4y^3 + 6xy - x^2)\,dy = 0$. **5**

b) $\frac{dy}{dx} + xy = x^3y^4$. **5**

c) $\frac{dy}{dx} = \frac{2x+9y-20}{6x+2y-10}$. **5**

d) $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$. **5**

8. Solve the following differential equations.

a) $(D^2 - 2D + 2)y = e^x + \cos x$. **5**

b) $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x^2$. **5**

c) $(D^2 - 2D + 1)y = x^2e^{3x}$. **5**

d) $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - 5y = \sin(\log(x))$. **5**
