



SEM 1 – 1 (RC 07-08)

F.E. Semester – I (RC 2007-08) Examination, May/June 2018 APPLIED MATHEMATICS – I

Duration : 3 Hours

Total Marks : 100

- Instructions :**
- 1) Attempt **any five** questions atleast **one** from **each** Module.
 - 2) **Assume** suitable data, if necessary.

MODULE – I

1. a) State and prove duplication formula for gamma function.

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b) Evaluate : $\int_0^a \frac{1}{\sqrt[n]{a^n - x^n}} dx$

5

c) Evaluate : $\int_0^\infty \frac{x^4}{4^x} dx$

5

d) Prove that : $\operatorname{erf}_c(x) + \operatorname{erf}_c(-x) = 2$.

4

2. a) Test the convergence of the following series :

i) $\sum_{n=1}^{\infty} \frac{n^3}{e^n}$

4

ii) $\sum_{n=1}^{\infty} (\sqrt{n^2+1} - n)$

4

iii) $1 + \frac{2^2}{3^2} + \frac{2^2 \cdot 4^2}{3^2 \cdot 5^2} + \frac{2^2 \cdot 4^2 \cdot 6^2}{3^2 \cdot 5^2 \cdot 7^2} + \dots$

4

b) Define absolutely convergent and conditionally convergent series. Test whether the following series is absolutely convergent or conditionally convergent series.

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$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$$

P.T.O.



MODULE – II

3. a) Use DeMoivre's theorem and solve $x^3 + 8 = 0$. 6

b) Prove that :

i) $\sinh^{-1}x = \log \left(x + \sqrt{x^2 + 1} \right)$

ii) $\cosh^{-1}x = \log \left(x + \sqrt{x^2 - 1} \right)$. 8

c) If $\frac{(a+ib)^{x+iy}}{(a-ib)^{x-iy}} = \alpha + i\beta$ find α and β . 6

4. a) Determine the analytic function whose imaginary part is $\tan^{-1}(y/x)$. 6

b) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic. Find the conjugate harmonic function. 6

c) If $u - v = \frac{\cos x + \sin x - e^{-y}}{2 \cos x - 2 \cosh y}$ and $f(z) = u + iv$ is an analytic function of

$z = x + iy$, find $f(z)$ in terms of z . 8

MODULE – III

5. a) If $\cos^{-1} \left(\frac{y}{b} \right) = \log \left(\frac{x}{n} \right)^n$, prove that, $x^2 y_{n+2} + (2n+1) x y_{n+1} + 2n^2 y_n = 0$. 7

b) Expand : $\cos^{-1} \left(\frac{x^2 - 1}{x^2 + 1} \right)$ in powers of x . Find the first 4 terms. 6

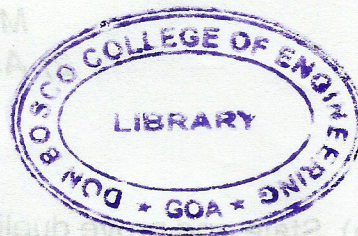
c) Calculate the approximate value of $\sqrt{10}$ to four decimal places by taking the first four terms of an appropriate Taylor's expansion. 7

6. a) Evaluate : 12

i) $\lim_{x \rightarrow 0} \frac{\log x^2}{\cot x^2}$

ii) $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$

iii) $\lim_{x \rightarrow 0} (5^x + x)^{\frac{1}{x}}$





b) If $z = f(x, y)$ where $u = lx + my$, $v = ly - mx$ then show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2) \left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} \right)$$

8

MODULE – IV

7. a) Form the partial differential equations by eliminating constants :

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i) $2z = (ax + y)^2 + b$

ii) $(x - h)^2 + (y - k)^2 + z^2 = a^2$.

b) Form the partial differential equations by eliminating functions

$f(x^2 + y^2, z - xy) = 0$.

6

c) Solve $x^2 p^2 + y^2 q^2 = z^2$.

8

8. a) If $u = \tan^{-1} \left[\frac{y^2}{x} \right]$, find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$

6

b) Find the extreme values of the function $x^3 y^2 (1 - x - y)$.

6

c) Find the largest product of the numbers x, y, z when $x^2 + y^2 + z^2 = 9$.

8

