

**F.E. (Semester – II) (Revised Course 2007-08) Examination,
May/June 2014**

APPLIED MATHEMATICS – II

Duration : 3 Hours

Total Marks : 100

- Instructions :** 1) Attempt 5 questions, atleast one from each Module.
2) Assume suitable data, if necessary.
3) Figures to the right indicate full marks.

MODULE – I

1. a) Assuming the validity of differentiating under the integral sign prove that

$$\int_0^{\infty} \frac{\cos \lambda x}{x} (e^{-ax} - e^{-bx}) dx = \frac{1}{2} \log \left(\frac{b^2 + \lambda^2}{a^2 + \lambda^2} \right) \quad a > 0, b > 0. \quad 7$$

- b) Find the length of the curve $x = a(2 \cos t - \cos 2t)$ and $y = a(2 \sin t - \sin 2t)$ from $t = 0$ to $t = \frac{\pi}{2}$. 6

- c) Find the area of the surface generated by the revolution of $x = \frac{y^3}{3}$; $0 \leq y \leq 1$ about the Y-axis. 7

2. a) Find the unit tangent vector and unit acceleration vector of the curve $x = 2 \cos t$, $y = 2 \sin t + 3$, $z = 4t$ at $t = 1$. 5

- b) State and prove Serret – Frenet formula. 8

- c) Define curvature of a point on a curve. If $\vec{\mu}(t)$ is any vector point function, show that the curvature is given by

$$k = \frac{|\dot{\vec{\mu}} \times \ddot{\vec{\mu}}|}{|\dot{\vec{\mu}}|^3} \quad 7$$



MODULE – II

3. a) Evaluate $\iint_R x + y \, dx dy$ where 'R' is the region bounded by $y = 2x$, $y = x$ and $y = 1$. 7
- b) Write the following as one double integral and evaluate 7
- $$\int_{-1}^0 \int_0^{x+1} 2y + 3 \, dx dy + \int_0^1 \int_0^1 2y + 3 \, dx dy.$$
- c) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \cdot \sqrt{x^2 + y^2} \, dx dy$. 6
4. a) Find the area bounded by $x^2 + y^2 = 4$ and $x + y = 2$ in the 1st quadrant. 6
- b) Evaluate $\iiint_V \frac{dx dy dz}{(1+x+y+z)^3}$ where 'V' is the region bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$. 7
- c) Use triple integration to find the volume of the region bounded by $x^2 + y^2 = 9$, $z = 0$ and $z = 2$. 7

MODULE – III

5. a) Find the rate of change of $f(x, y, z) = x^2y + 3z^2y$ at the point $(1, 2, -1)$ in the direction of $3i + j - k$. 5
- b) Define Curl of a vector field. Prove that $\text{Curl}(\text{grad } f) = 0$. 5
- c) Show that $\vec{F} = (x^2 - y^2 + x) i - (2xy + y) j$ is irrotational and hence find its scalar potential. Also find the line integral of \vec{F} from $(1, 2)$ to $(2, 1)$. 6
- d) Find the unit vector normal to $x^2 + 3y + z^2 = 4$ at $(0, 1, -1)$. 4
6. a) Verify Green's theorem in the plane for $\oint_C [(xy + 4y^2)dx + (x^2 + 3)dy]$ where 'C' is the boundary of the region bounded by $x = 1$ and $y^2 = x$. 8
- b) Verify Stoke's theorem for $\vec{F} = xy i - 2yzj - zxk$ where 'S' is the open surface of the region bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 2$, $z = 3$ above the XOY plane. 12



MODULE – IV

7. Solve the following :

a) $(x + y + 1)^2 \frac{dy}{dx} = 1.$ 5

b) $\left[y \left(1 + \frac{1}{x} \right) + \cos y \right] dx + [x + \log x - x \sin y] dy = 0.$ 5

c) $(xy^2 + 2x^2y^3) dx + (x^2y - x^3y^2) dy = 0.$ 5

d) $\sqrt{1-y^2} dx = (\sin^{-1} y - x) dy.$ 5

8. Solve the following :

a) $(D^3 + D^2 + D)y = e^x \cosh (x).$ 5

b) $(D^2 + 4)y = x^3 + \cos x.$ 5

c) $(D^2 + 1)y = \tan x.$ 5

d) $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = x^2 \log x.$ 5
