Computer Vision - Sheet 1

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Question 3

3)
$$((f*g)*h)(x) = \int_{t=\emptyset}^{x} (f*g)(t)\, h(x-t)\, dt$$

$$= \int_{t=\emptyset}^{x} \left(\int_{k=\emptyset}^{t} f(k) g(t-k)\, dk \right) h(x-t)\, dt$$

$$= \int_{k=\emptyset}^{x} \int_{t=k}^{x} f(k) g(t-k) h(x-t)\, dt\, dk$$

$$= \int_{k=\emptyset}^{x} \int_{t=\emptyset}^{x-k} f(k) g(t) h(x-t-k)\, dt\, dk$$

$$= \int_{k=0}^{x} f(k) \left(\int_{t=\emptyset}^{x-k} g(t) h(x-t-k)\, dt \right) dk$$

$$= \int_{k=\emptyset}^{x} f(k) (g*h)(x-k)\, dk$$

$$= (f*(g*h))(x)$$

Question 6

The Fourier Transform of a gaussian is another gaussian:

$$FT(G(x,\sigma) = \frac{\sqrt{2\pi}}{\sigma} G(w,\sigma^{-1}) \tag{1}$$

A convoltion in the space domain is a multiplication is the frequency one:

$$G(x,\sigma) * G(x,\sigma) = \left(\frac{\sqrt{2\pi}}{\sigma} G(w,\sigma^{-1}) \right)^2 \tag{2}$$

Therefore, two convolutions with the standard deviation σ is equivalent to a convolution of standard deviation σ':

$$\frac{\cancel{2\pi}}{\cancel{\sigma^2}} \left(\frac{1}{\cancel{2\pi\sigma^{-2}}} e^{\frac{-x^2}{\sigma^{-2}}} \right) = \frac{\sqrt{2\pi}}{\sigma'} G(w,\sigma'^{-1}) \tag{3}$$

$$e^{\frac{-x^2}{\sigma^{-2}}} = \frac{\cancel{\sqrt{2\pi}}}{\cancel{\sigma'}} \frac{1}{\cancel{\sqrt{2\pi}\,\sigma'^{-1}}} e^{\frac{-x^2}{2(\sigma'^{-1})^2}} \tag{4}$$

$$e^{\frac{-x^2}{\sigma^{-2}}} = e^{\frac{-x^2}{2(\sigma'^{-1})^2}} \tag{5}$$

$$\sigma^{-2} = 2(\sigma'^{-1})^2 \tag{6}$$

$$\sigma^{-1} = \sqrt{2}\sigma'^{-1} \tag{7}$$

$$\sigma\sqrt{2} = \sigma' \tag{8}$$

So, applying two convolutions with std. dev of σ is the same as applying once with $\sigma\sqrt{2}$