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Pattern Matching and Machine Learning for Audio Signal Processing
Exercise sheet 3

To be uploaded in eCampus till: 04-05-2018 22:00 (strict deadline)

Exercise 3.1

[$3 + 3 = 6$ points]

Let $\mathrm{DFT}_N = (\omega_N^{kj})_{j,k\in[0:N-1]}$ with $\omega_N := \exp(-2\pi i/N)$ be the DFT matrix of size N. For $x, y, z \in \mathbb{C}^N$ let $X := \mathrm{DFT}_N \cdot x$, $Y := \mathrm{DFT}_N \cdot y$, and $Z := \mathrm{DFT}_N \cdot z$.

(a) Show that if $x \in \mathbb{R}^N$, $N = 2M$, is a real signal, then X_0 and X_M are real, too, and $\forall k \in [1 : M-1]$ it holds that $X_{N-k} = \overline{X_k}$, i.e. $\mathrm{Re}(X_{N-k}) = \mathrm{Re}(X_k)$ and $\mathrm{Im}(X_{N-k}) = -\mathrm{Im}(X_k)$.

(b) For $x, y \in \mathbb{R}^N$, show how one can reconstruct the Fourier transforms X and Y from the Fourier transform Z of the complex signal $z := x + iy$. In other words: One can calculate the Fourier transform of two real signals using one complex Fourier transform.

Exercise 3.2

[3 points]

Calculate by hand the spectrum of the vector $x \in \mathbb{C}^4$ defined by $x := [2, 3, -1, 1]^T$.

Exercise 3.3 (Bonus)

[6 bonus points]

Let p denote an odd prime and $\boldsymbol{x}$ the corresponding quadratic residue sequence, defined by $x_n := \exp(2\pi i n^2/p)$. In Theorem 7 (see slides) we have shown that $\mathrm{ACF}[\boldsymbol{x}] = (p, 0, \ldots, 0)^\top \in \mathbb{C}^p$. For $m \in [1 : p-1]$ define $\boldsymbol{x}^m$ by $\boldsymbol{x}^m(n) := x_n^m$. Prove that $\mathrm{ACF}[\boldsymbol{x}^m] = \mathrm{ACF}[\boldsymbol{x}]$.

Exercise 3.4

[6 points]

Implement in Matlab the decimation-in-time algorithm of Cooley-Tukey which has been presented in the lecture. Test your function with a signal of your choice.