1) a) $r(x) = \begin{cases} k & \text{where } |x| \le a/2 \\ \phi & \text{where } |x| > a/2 \end{cases}$

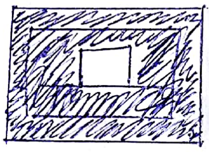
$$F(u) = \int_{x=-\infty}^{\infty} r(x) e^{-2\pi j u x} dx$$

$$= \int_{-\infty}^{a/2} r(x) e^{-2\pi j u x} dx + \int_{a/2}^{\infty} r(x) e^{-2\pi j u x} dx$$

$$= \int_{-\infty}^{a/2} k e^{-2\pi j u x} dx + \cancel{\int_{a/2}^{\infty} \phi e^{-2\pi j u x} dx} \rightarrow \phi$$

$$F(u) = \left[\frac{k e^{-2\pi j u x}}{-2\pi j u} \right]_{-\infty}^{a/2} = \left(\frac{k e^{-2\pi j u (a/2)}}{-2\pi j u} - \phi \right) + \phi$$

b) The original image is a box image like below.



We know this because in the fourier transform, there is a square section of very high response and two major wave patterns running perpendicular to each other. This means there are four strong edges, two horizontal and two vertical, running perpendicular to each other, with the ripples being caused by noise as the fourier transform tries to approximate this.

c) The Dirac delta function. It behaves similarly to an identity function in the context of convolution, therefore the signal will remain unchanged.

D) No. The rows have no common multiples, therefore the filter cannot be separated.

E) yes.

$$\begin{bmatrix} 3 \\ -1 \\ \phi \\ -5 \\ 2 \end{bmatrix} \times [1] \times \begin{bmatrix} -7 & 2 & 1 & 4 & 3 \end{bmatrix} = \begin{bmatrix} -21 & 6 & 3 & 12 & 9 \\ 7 & -2 & -1 & -4 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 35 & -10 & -5 & -20 & -15 \\ -14 & 4 & 2 & 8 & 6 \end{bmatrix}$$