Pattern Matching and Machine Learning for Audio Signal Processing
Exercise 1

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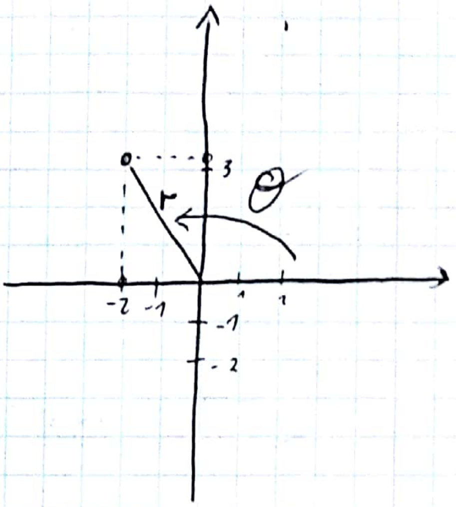
a) $z = (1+i\sqrt{3})^2 = (1+i\sqrt{3})(1+i\sqrt{3}) = 1 + i\sqrt{3} + 3i^2 =$

$= 1 + i\sqrt{3} + i\sqrt{3} + 3(-1)$



$\cancel{z} =$

$i^2 = -1$

$z = -2 + 2\sqrt{3}\, i$

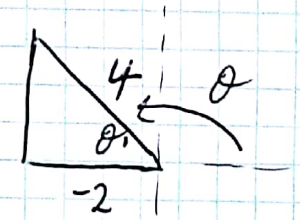
$z = r\cos\theta + (r\sin\theta)i$

$r^2 = (-2)^2 + (2\sqrt{3})^2 =$

$= 4 + (4 \cdot 3) =$

$= 16$

$r = 4$



$\cos\theta' = \frac{-2}{4} = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$

$z = 4\cos\left(\frac{2\pi}{3}\right) + \left(4\sin\left(\frac{2\pi}{3}\right)\right)i$

b) $z = \dfrac{e^{\frac{\pi}{3}i}}{e^{\frac{2}{3}\pi i} \cdot 2e^{\frac{\pi}{6}i}} = \dfrac{e^{-\frac{1}{3}\pi i} \cdot e^{-\frac{1}{6}\pi i}}{2} =$

$= \dfrac{e^{-\frac{\pi}{2}i}}{2}$

Euler's Formula

$e^{i\alpha} = \cos\alpha + i\sin\alpha$

$e^{-i\alpha} = \cos(-\alpha) + i\sin(-\alpha) = \cos\alpha - i\sin\alpha$

$z = \dfrac{e^{-\frac{\pi}{2}i}}{2} = \dfrac{\cos\left(\frac{\pi}{2}\right) - i\sin\left(\frac{\pi}{2}\right)}{2} = \dfrac{0 - 1i}{2}$

$z = \dfrac{-i}{2} = \left(-\frac{1}{2}\right)i = -\frac{1}{2}i$

c)

$$5e^{\frac{\pi}{2}i} + \sqrt{2}\, e^{\frac{\pi}{4}i}$$

$$5\left(\overset{0}{\cos}\left(\frac{\pi}{2}\right) + i\,\overset{1}{\sin}\left(\frac{\pi}{2}\right)\right) + \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$$

$$5i + \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$5i + \frac{2}{2} + i\frac{2}{2}$$

$$5i + 1 + i$$

$$6i + \underline{1}$$

$$Re = \underline{1}$$

d) $\sin(z)^2 + \cos(z)^2 = 1 \qquad z \in \mathbb{C}$

Euler formula

$$\boxed{\sin x = \frac{e^{ix} - e^{-ix}}{2i} \qquad \cos x = \frac{e^{ix} + e^{-ix}}{2}}$$

$$\left(\frac{e^{iz} - e^{-iz}}{2i}\right)^2 + \left(\frac{e^{iz} + e^{-iz}}{2}\right)^2 = 1$$

$$\frac{1}{4i^2}\left(e^{2iz} - e^{iz-iz} - e^{-iz+iz} + e^{-2iz}\right) +$$

$$+ \frac{1}{4}\left(e^{2iz} + e^{iz-iz} + e^{-iz+iz} + e^{-2iz}\right) = 1$$

$$\frac{1}{4i^2}\left(e^{2iz} - e^0 - e^0 + e^{-2iz}\right) + \frac{1}{4}\left(e^{2iz} + e^0 + e^0 + e^{-2iz}\right) = 1$$

$$\frac{e^{2iz} + e^{-2iz} - 2}{4i^2} + \frac{e^{2iz} + e^{-2iz} + 2}{4} = 1$$

$$\frac{-\cancel{e^{2iz}} - \cancel{e^{-2iz}} + 2}{4} + \frac{\cancel{e^{2iz}} + \cancel{e^{2iz}} + 2}{4} = 1$$

$$1 = 1$$

Exercise 1.2

C)

When the two spectrograms are compared, one can see that the wave patterns in the cos and sin spectrograms are inverted with respect to each other. Where a high point occurs in sin, a low point occurs in cos and vise versa.