1)a) $f(x) = 100(x_2 - x_1^2)^2 + (1-x_1)^2$

$$= 100(x_2 - x_1^2)(x_2 - x_1^2) + (1-x_1)(1-x_1)$$
$$= 100(x_2^2 - x_2x_1^2 - x_2x_1^2 + x_1^4) + (1 - x_1 - x_1 + x_1^2)$$
$$= 100(x_2^2 - 2x_2x_1^2 + x_1^4) + (1 - 2x_1 + x_1^2)$$

$$\frac{df}{dx_1} = 100(-4x_2x_1 + 4x_1^3) - 2(1-x_1)$$

$$\frac{df}{dx_2} = 200(x_2 - x_1^2)$$

$$\boxed{\nabla f(x) = \frac{df}{dx_1} + \frac{df}{dx_2} = 200(x_2 - x_1^2) + 100(-4x_2x_1 + 4x_1^3) - 2(1-x_1)}$$

$$\boxed{H = \begin{bmatrix} \frac{d^2f}{d^2x_1} & \frac{d^2f}{dx_1dx_2} \\ \frac{d^2f}{dx_2dx_1} & \frac{d^2f}{d^2x_2} \end{bmatrix} = \begin{bmatrix} -400x_2 + 1200x_1^2 + 2 & -400x_1 \\ -400x_1 & 200 \end{bmatrix}}$$

For function $f(x) = 100(x_2 - x_1^2)^2 + (1-x_1)^2$

$(x_2 - x_1^2)^2$ will always be $\geq \varnothing$

$(1-x_1)^2$ will always be $\geq \varnothing$

$\therefore$ min value is $\varnothing$

$(1-x_1)^2 = \varnothing$ iff $x_1 = 1$

$(x_2 - x_1^2)^2 = \varnothing$ iff $x_2 = x_1^2$

$\therefore$ in order to reach the minimum value of $\varnothing$, $X^*$ must equal $(1,1)^T$

$\boxed{\therefore X^* = (1,1)^T \text{ is the only local minimizer}}$

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b) $f(x) = 8x_1 + 12x_2 + x_1^2 - 2x_2^2$

$$\frac{d f(x)}{dx_1} = 8 + 2x_1 = \varnothing \text{ iff } x_1 = -4$$

$$\frac{d f(x)}{dx_2} = 12 - 4x_2 = \varnothing \text{ iff } x_2 = 3$$

$$\nabla f(x) = 8 + 2x_1 + 12 - 4x_2$$
$$= 20 + 2x_1 - 4x_2$$

The only coordinates that satisfy ~~this~~ is

$x_1 = -4,\ x_2 = 3$

$\therefore$ Only one ~~saddle~~ stationary point

- check values of gradient on either side of stationary point

$x_1 = -5$
$x_2 = 3$
$\nabla f(x) = 8 - 10 + 12 - 12$ $\boxed{= -2}$

$x_1 = -4$
$x_2 = 2$
$\nabla f(x) = 8 - 8 + 12 - 8$ $\boxed{= 4}$

$x_1 = -3$
$x_2 = 3$
$\nabla f(x) = 8 - 6 + 12 - 12$ $\boxed{= 2}$

$x_1 = -4$
$x_2 = 4$
$\nabla f(x) = 8 - 8 + 12 - 16$ $\boxed{= -4}$

$x_1 < -4$ will result in a lower than $\varnothing$ gradient value

$x_1 > -4$ " " greater than $\varnothing$ " "

$x_2 < 3$ " " greater than $\varnothing$ " "

$x_2 > 3$ " " lower than $\varnothing$ " "

$\boxed{\therefore \text{The point is a saddle point.}}$