NONDIM-ISH $\tau_w, u_\tau, \delta_\nu$ | #3004 | 26 Nov 2013 | 1

$\tau_w \equiv \mu \partial_y u|_w$, WITH DIMENSIONS $\mu_0 u_0 / l_0 \equiv \tau_0$

$u_\tau \equiv \sqrt{\frac{\tau_w}{\rho}}$ WITH u_0

$\delta_\nu \equiv \frac{\nu}{u_\tau} = \frac{\mu}{\rho u_\tau}$ WITH l_0

$Re \equiv \frac{\rho_0 u_0 l_0}{\mu}$

$\partial_y = \frac{1}{l_0} \partial_y^*$

Q: CONNECT $\tau_w^*, u_\tau^*, \delta_\nu^*$ WITH $\tau_w, u_\tau, \delta_\nu$ ACCOUNTING FOR NONDIMENSIONALIZATION

$$\tau_w^* \equiv \mu^* \partial_y^* u^*|_w$$

$$= \frac{\mu}{\mu_0} l_0 \partial_y \frac{u}{u_0} \qquad \text{SUPPRESSING "}|_w\text{"}$$

$$= \frac{l_0}{\mu_0 u_0} \mu \partial_y u = \frac{l_0}{\mu_0 u_0} \tau_w$$

$$\boxed{\text{A: } \tau_w = \frac{\mu_0 u_0}{l_0} \tau_w^* = \tau_0 \tau_w^*}$$

$$u_\tau^* \equiv \sqrt{\frac{\tau_w^*}{\rho^*}} = \sqrt{\frac{\rho_0 \tau_w}{\rho \tau_0}} = \sqrt{\frac{\rho_0}{\tau_0}} \sqrt{\frac{\tau_w}{\rho}} = \sqrt{\frac{\rho_0}{\tau_0}} u_\tau$$

NOW $$\sqrt{\frac{\rho_0}{\tau_0}} = \sqrt{\frac{\rho_0 l_0}{\mu_0 u_0}} = \sqrt{\frac{u_0^2}{u_0^2} \frac{\rho_0 l_0}{\mu_0 \cancel{u_0}}} = \frac{1}{u_0} \sqrt{Re}$$

SO $$u_\tau^* = \frac{1}{u_0} \sqrt{Re}\, u_\tau$$ THUS

$$\boxed{\text{A: } u_\tau = \frac{u_0 u_\tau^*}{\sqrt{Re}}}$$

$$\delta_\nu^* \equiv \frac{\mu^*}{\rho^* u_\tau^*} = \frac{\mu}{\mu_0} \frac{\rho_0}{\rho} \frac{u_0}{\sqrt{Re}\, u_\tau} = \frac{1}{\sqrt{Re}} \frac{\rho_0 u_0}{\mu_0} \frac{\mu}{\rho u_\tau}$$

$$= \frac{1}{\sqrt{Re}} \frac{\rho_0 u_0}{\mu_0} \delta_\nu = \frac{1}{\sqrt{Re}} \left(\frac{\rho_0 u_0 l_0}{\mu_0 l_0}\right) \delta_\nu = \frac{Re}{\sqrt{Re}} \frac{1}{l_0} \delta_\nu$$

$$= \frac{\sqrt{Re}}{l_0} \delta_\nu$$

SO

$$\boxed{\text{A: } \delta_\nu = \frac{l_0 \delta_\nu^*}{\sqrt{Re}}} \Rightarrow Re_\tau \equiv \frac{\delta}{\delta_\nu} = \frac{l_0 \delta^*}{l_0 \delta_\nu^*} \sqrt{Re} = \sqrt{Re} \frac{\delta^*}{\delta_\nu^*}$$

FOR CHANNEL HALF-WIDTH δ