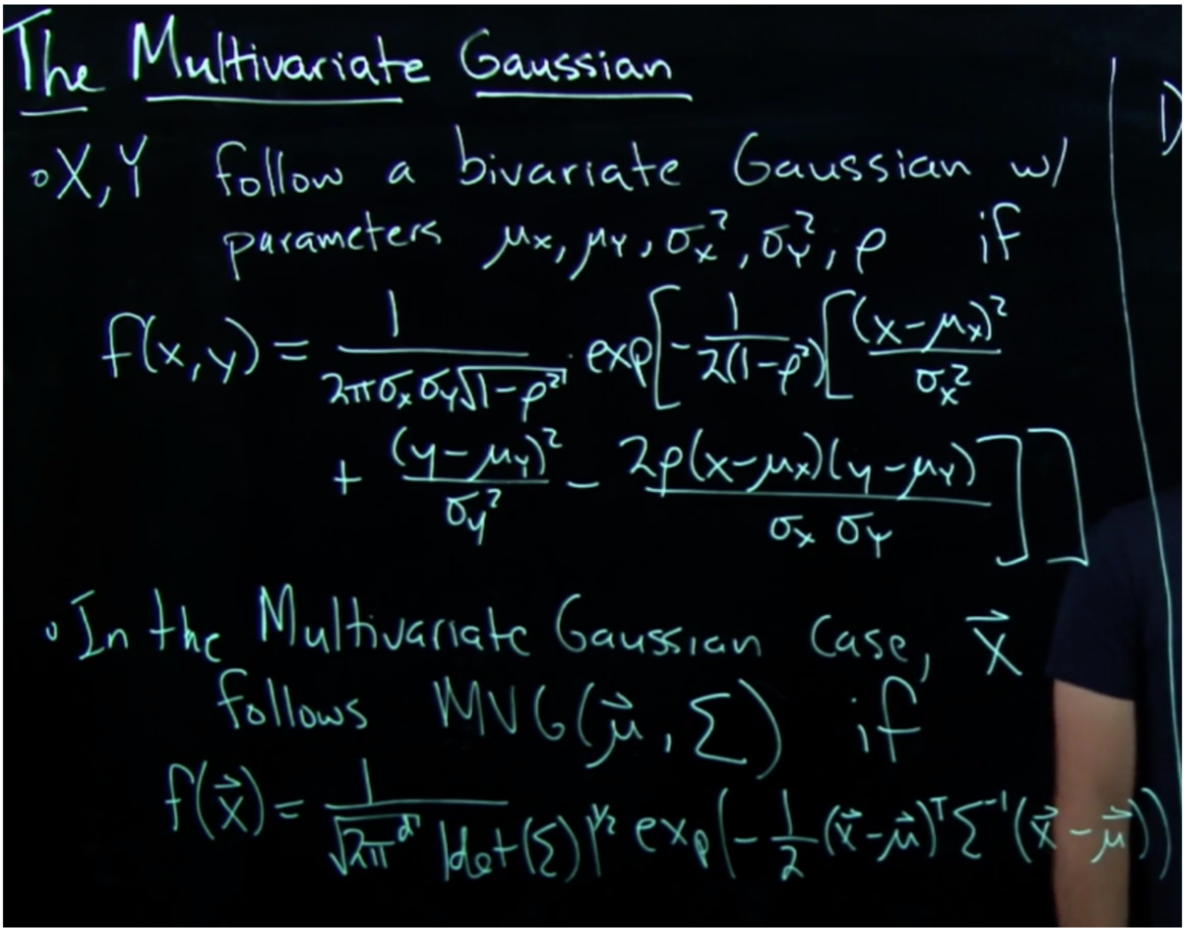
Unit 3 Recitations

Multivariate Gaussian



- f(x,y) is joint pdf
- multivariate has vector multiplication
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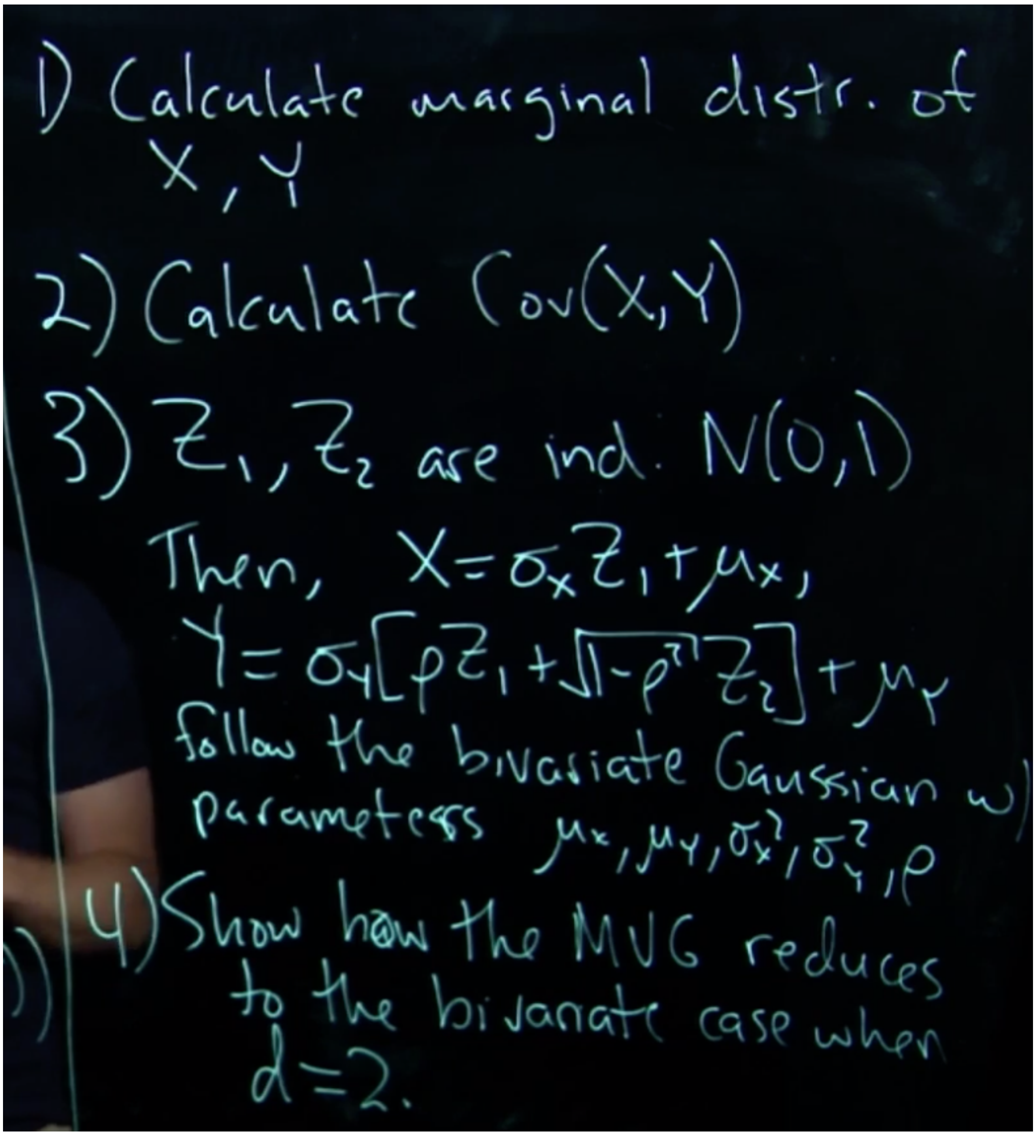
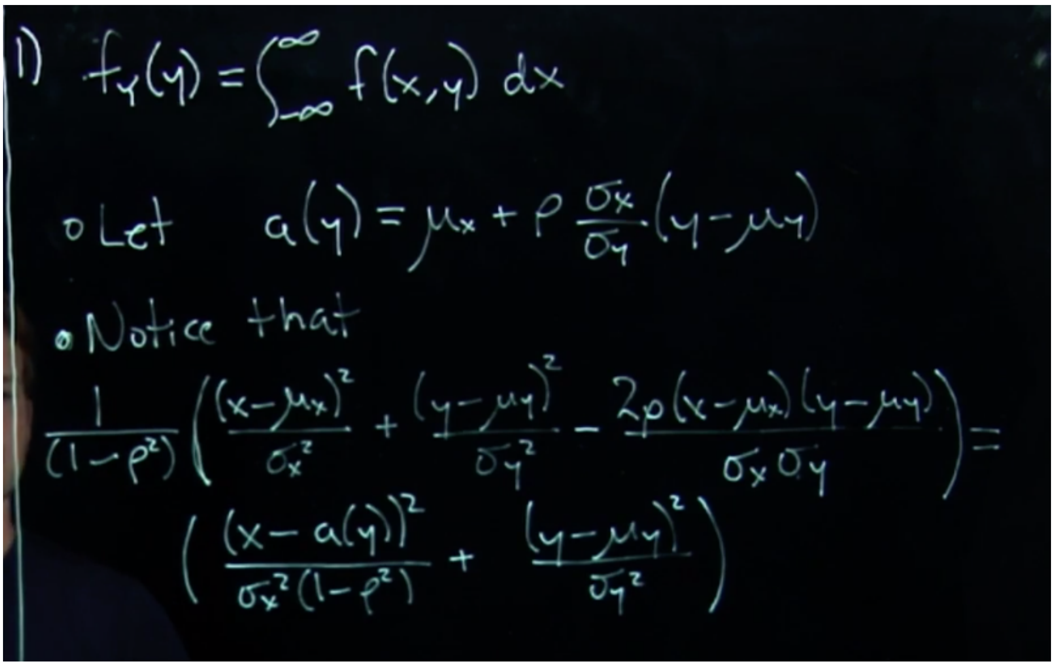
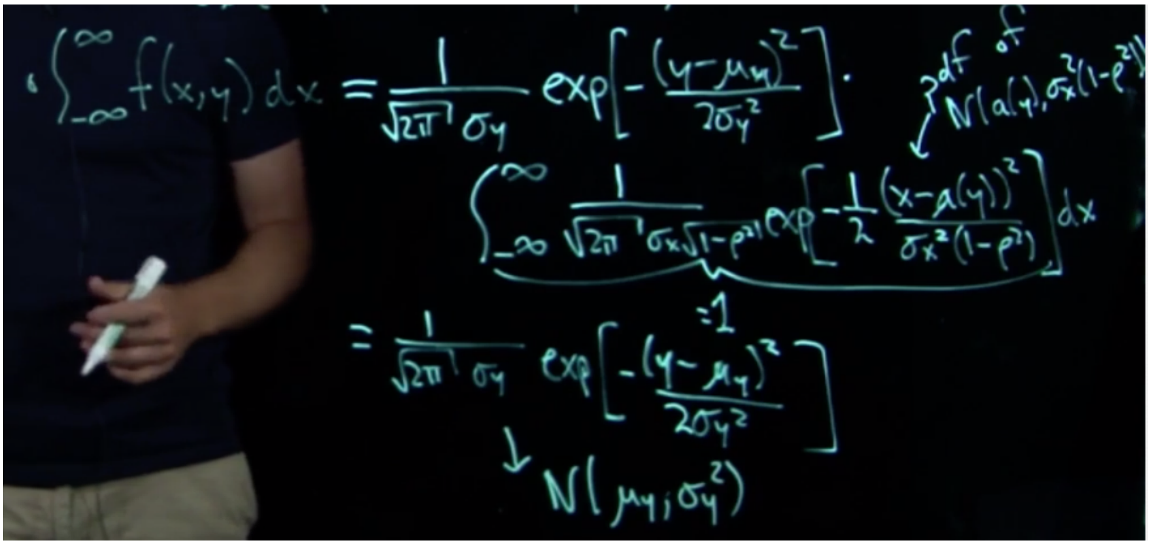
Going to cover



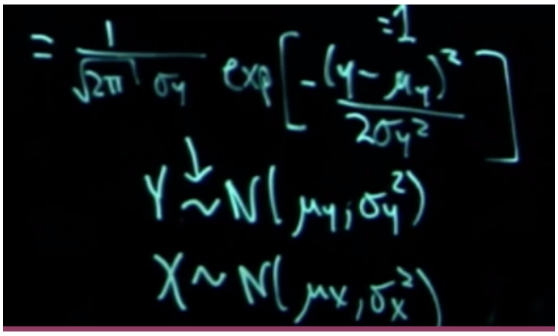
1. Calculate marginal distribution



- integrate out x from the joint pdf
- trying to extract all the x's from the function to make it easier to integrate and then complete the square
- simplify the equation by using a trick to set a = ...

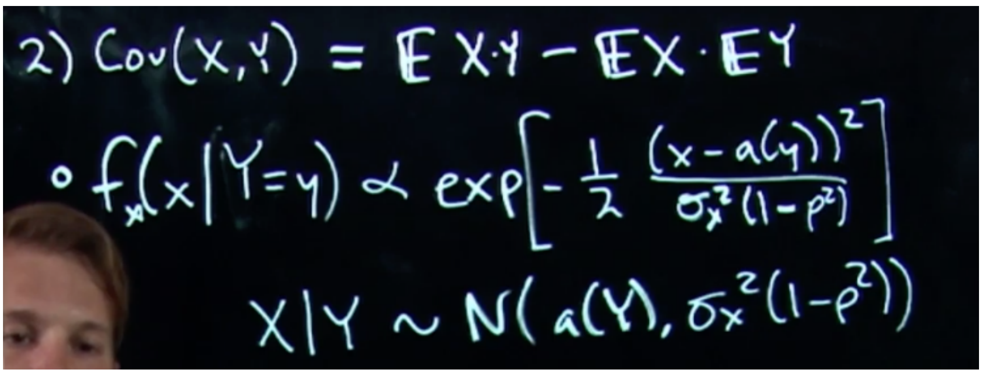


- pulled out the terms that depend on just y and not x
- the second part is now the pdf of a normal exponential which integrates to 1

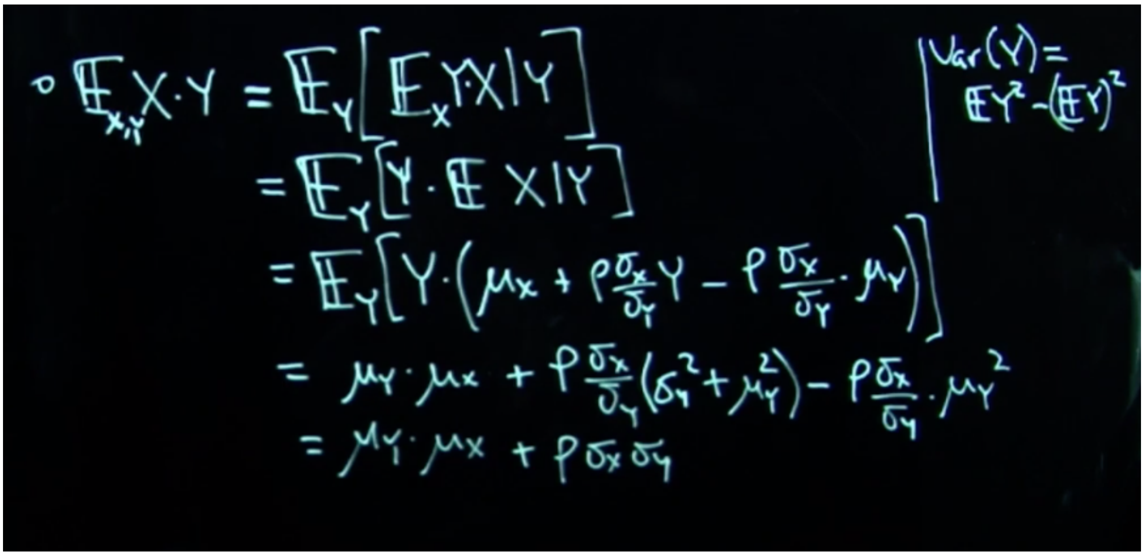


- so in a bivariate case we have to normal distributions

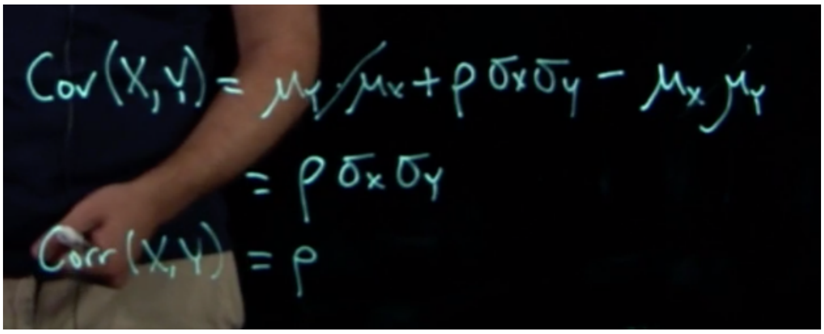
2. Calculate covariance



- what is the distribution of X at a fixed Y (Y=y) proportional to



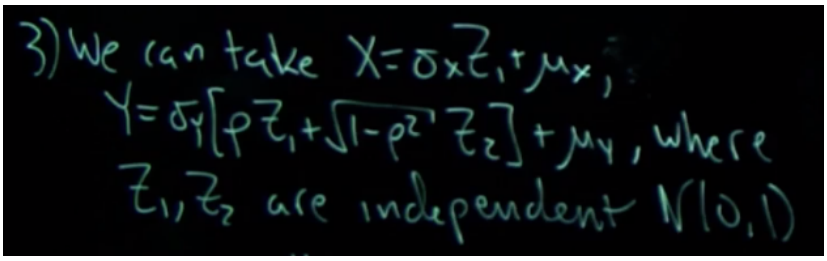
- use law of total expectations
- E(X|Y) = a(Y) (take from last section)
- sigma_y^2 + mu_y^2 for E[Y^2], from Var(Y) equation on side



now we have all the variables defined:

- mu_x = mean of x
- mu_y = mean of y
- sigma_y^2 = marginal variance of y
- sigma_x^2 = marginal variance of x
- roe = correlation between x and y

3. Generate a bivariate normal distribution from 2 independent standard normals



3) ∘ Know X & Y have a bivariate Gaussian distribution.

∘ $\mathbb{E}X = \mathbb{E}(\sigma_x Z_1 + \mu_x) = \mathbb{E}\sigma_x Z_1 + \mathbb{E}\mu_x = \mu_x$

- by plugging in formula

∘ $$\mathbb{E}Y = \mathbb{E}\left(\sigma_Y\left(\rho Z_1 + \sqrt{1-\rho^2}\, Z_2\right) + \mu_Y\right)$$
$$= \mathbb{E}\sigma_Y \rho Z_1 + \mathbb{E}\sigma_Y\sqrt{1-\rho^2}\, Z_2 + \mathbb{E}\mu_Y = \mu_Y$$

- Z_1 and Z_2 have means of 0 because they are normal

∘ $$\mathrm{Var}(X) = \mathrm{Var}(\sigma_x Z_1 + \mu_x) = \mathrm{Var}(\sigma_x Z_1)$$
$$= \sigma_x^2 \mathrm{Var}(Z_1) = \sigma_x^2$$

∘ $$\mathrm{Var}(Y) = \mathrm{Var}\left(\sigma_Y \rho Z_1 + \sigma_Y\sqrt{1-\rho^2}\, Z_2\right)$$
$$= \mathrm{Var}(\sigma_Y \rho Z_1) + \mathrm{Var}\left(\sigma_Y\sqrt{1-\rho^2}\, Z_2\right)$$
$$= \sigma_Y^2\rho^2 + \sigma_Y^2(1-\rho^2) = \sigma_Y^2$$

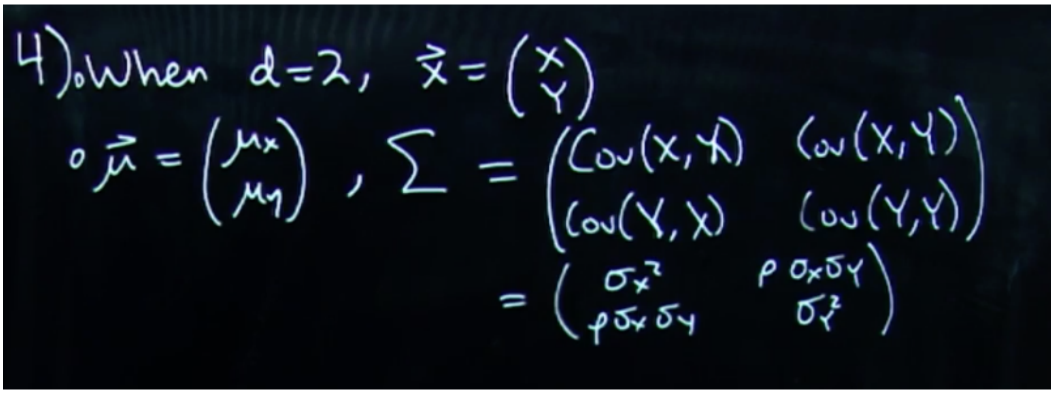
- can pull the constant out of the variance and square it (sigma because it's multiplying the random variable)
- didn't include or removed the mu constant because it's adding to the random variable so does not affect the variance

∘ $$\mathrm{Cov}(X,Y) = \mathrm{Cov}\left(\sigma_x Z_1 + \mu_x, \sigma_Y\left(\rho Z_1 + \sqrt{1-\rho^2}\, Z_2\right) + \mu_Y\right)$$
$$= \mathrm{Cov}\left(\sigma_x Z_1, \sigma_Y \rho Z_1 + \sigma_Y\sqrt{1-\rho^2}\, Z_2\right)$$
$$= \mathrm{Cov}(\sigma_x Z_1, \sigma_Y \rho Z_1) + \mathrm{Cov}\left(\sigma_x Z_1, \sigma_Y\sqrt{1-\rho^2}\, Z_2\right)$$
$$= \sigma_x\sigma_Y\rho\, \mathrm{Cov}(Z_1, Z_1) + \sigma_x\sigma_Y\sqrt{1-\rho^2}\, \underbrace{\mathrm{Cov}(Z_1, Z_2)}_{=0}$$
$$= \sigma_x\sigma_Y\rho$$

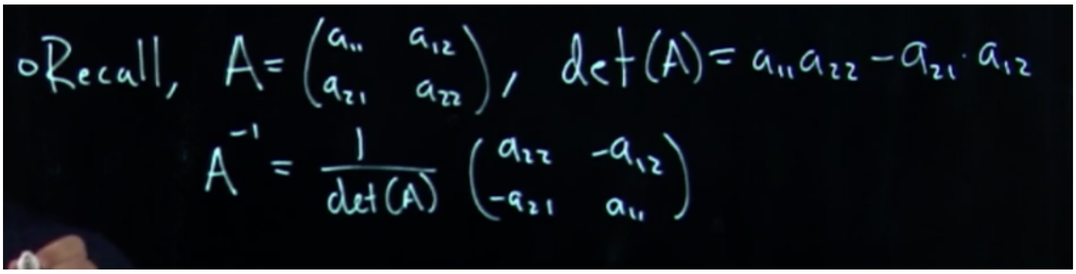
- removed mu_x and mu_y again as they do not affect the covariance since they're constants
- break up covariance of all of it into a sum of covariances
- last term is 0 because Z_1 and Z_2 are independent
- so we can correctly generate the bivariate x and y case from 2 independent standard normals and they'll have the correct marginal parameters and

covariance

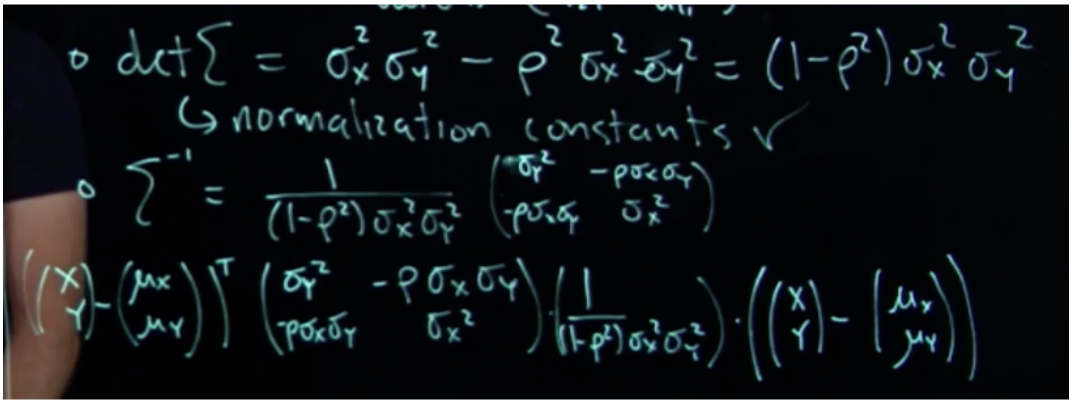
4. Reduce multivariate case to bivariate case when dimensions =2



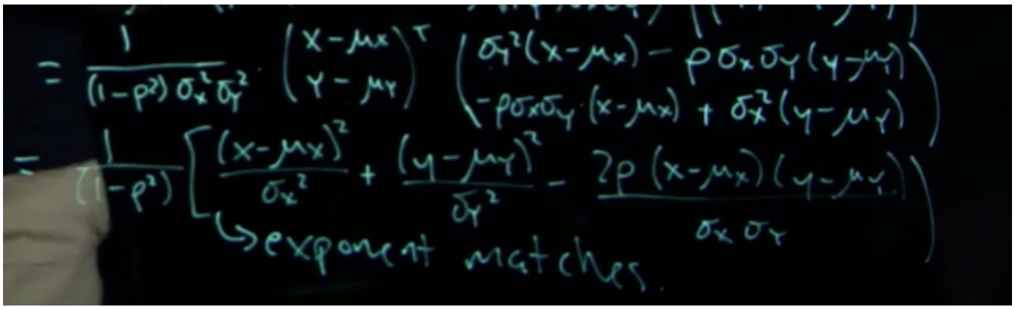
- the order of the rv's in the covariance doesn't matter as they're the same



trying to show that the 2 sections of the bivariate and multivariate pdf are equal to each other
- the 1/2pi part and the exponential part



then pull the constant out and multiply the matrices from left to right



when simplifying we end up with the same parameter