CS F351 Theory of Computation
Tutorial-6

Note: $*$ marked problems can be left to the students to try after the tutorial.

Problem 1 Prove that the following languages are not regular using Myhill-Nerode theorem.

1. $L_1 = \{ss \mid s \in \{0,1\}^*\}$

   Solution: Let $x = 0^k$ and $y = 0^m$ be two strings in Σ^* such that $k \neq m$. We can show that x and y are distinguishable with respect to L_1 i.e., $(x,y) \notin R_{L_1}$. For $z = 10^k1$ we have $xz = 0^k10^k \in L_1$, but $yz = 0^m10^k1 \notin L_1$ since $k \neq m$. Hence, R_{L_1} has infinite equivalence classes. Thus, L_1 is non-regular.

2. $L_2 = \{w \in \{0,1\}^* \mid w = w^R\}$

   Solution: Let $x = 0^k1$ and $y = 0^m1$ be two strings in Σ^* such that $k \neq m$. We can show that x and y are distinguishable with respect to L_1 i.e., $(x,y) \notin R_{L_2}$. For $z = 10^k$ we have $xz = 0^k110^k \in L_2$, but $yz = 0^m110^k \notin L_2$ since $k \neq m$. Hence, R_{L_2} has infinite equivalence classes. Thus, L_2 is non-regular.

3. $(*)$ $L_3 = \{0^n1^l \mid n \neq l\}$ over $\Sigma = \{0,1\}$

   Solution: Let $x = 0^k$ and $y = 0^m$ be two strings in Σ^* such that $k \neq m$. We can show that x and y are distinguishable with respect to L_3 i.e., $(x,y) \notin R_{L_3}$. For $z = 1^k$ we have $xz = 0^k1^k \notin L_3$, but $yz = 0^m1^k \in L_3$ since $k \neq m$. Hence, R_{L_3} has infinite equivalence classes. Thus, L_3 is non-regular.

Problem 2 Write regular grammars (right-linear grammars) for the following languages:

1. $L = \{w \in \{a,b\}^* \mid w$ has no a's or has no b's$\}$.

   Solution: The productions of the grammar are:

   - $S \to A \mid B$
   - $A \to aA \mid \epsilon$
   - $B \to bB \mid \epsilon$

2. $L = \{w \in \{a,b\}^* \mid w$ has at least one a and at least one $b\}$.

   Solution: The productions of the grammar are:

   - $S \to aX \mid bY$
   - $X \to aX \mid bZ$
   - $Y \to aZ \mid bY$
   - $Z \to aZ \mid bZ \mid \epsilon$.

3. $(*)$ $L = \{w \in \{a,b\}^* \mid |w| \geq 2$ and w starts and ends with the same symbol$\}$.

   Solution: The productions of the grammar are:

   - $S \to aA \mid bB$, $A \to aA \mid bA \mid a$, and $B \to aB \mid bB \mid b$.

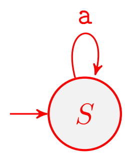
Problem 3 Find an equivalent NFA with ϵ-transitions for the following right-linear grammar:

1. $S \to aS$, $S \to bB$, $S \to cC$, $S \to \epsilon$
   $B \to bB$, $B \to cC$, $B \to \epsilon$, $C \to cC$, and $C \to \epsilon$.

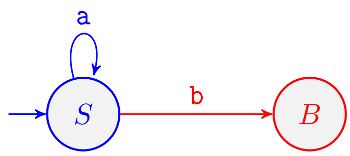
   Solution: An equivalent NFA (with ϵ-transitions) $M = (Q, \Sigma, \delta, s, F)$ is as follows:

   (a) $Q = \{S, B, C, T\}$
   (b) $\Sigma = \{a, b, c\}$
   (c) $s = S$ (start node)
   (d) $F = \{T\}$ and
   (e) δ is as follows:

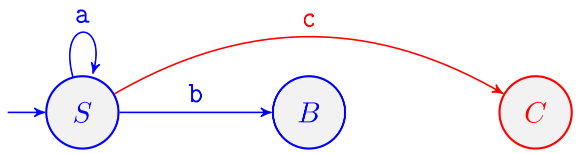
   - $S \to aS$ gives



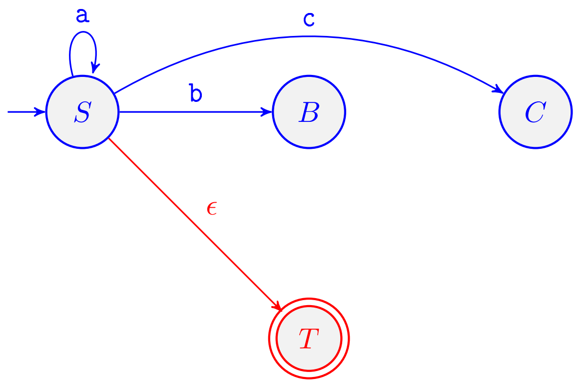
   - $S \to bB$ gives



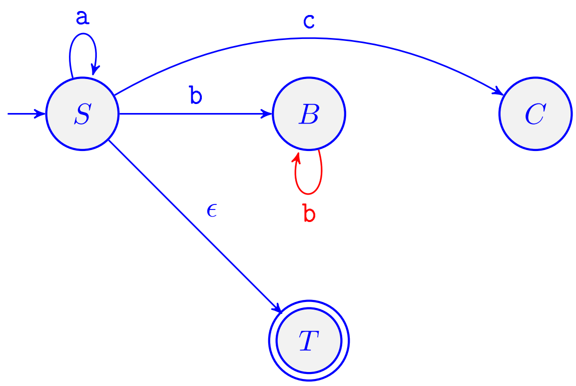
   - $S \to cC$ gives



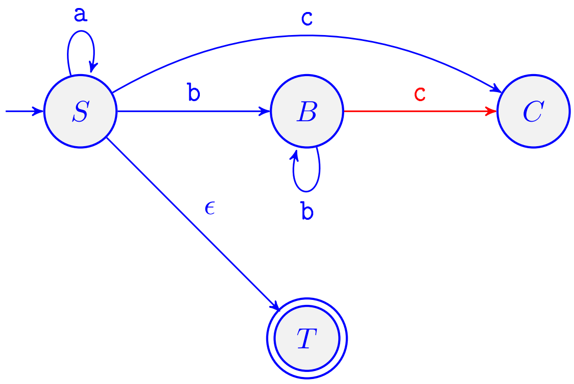
   - $S \to \epsilon$ gives



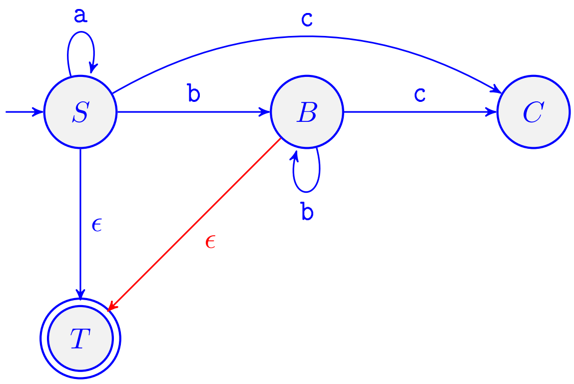
   - $B \to bB$ gives



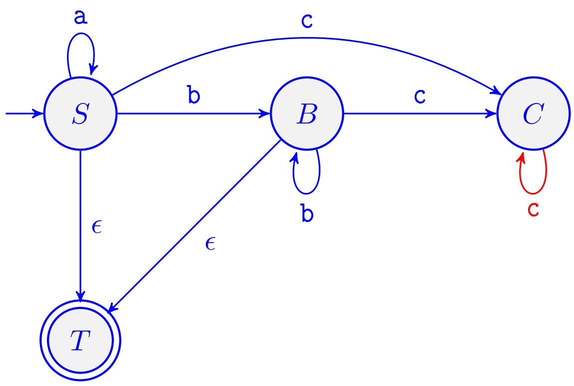
- $B \to cC$ gives



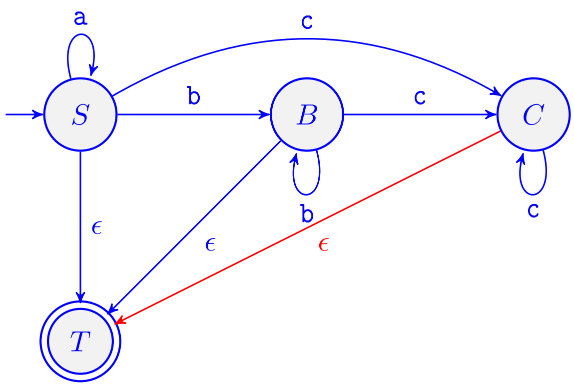
- $B \to \epsilon$ gives



- $C \to cC$ gives



- Finally, $C \to \epsilon$ gives



2. $(*)$ $S \to 0A$, $A \to 10A$ and $A \to \epsilon$.

   Solution: We first modify the grammar such that the right hand side of each production contains exactly one terminal symbol.

   Here, $A \to 10A$ can be replaced with $A \to 1B$ and $B \to 0A$.

   Thus, the new grammar is:

- $S \to 0A$,
- $A \to 1B$,
- $B \to 0A$ and
- $A \to \epsilon$.

An equivalent NFA (with ϵ-transitions) $M = (Q, \Sigma, \delta, s, F)$ is as follows:

(a) $Q = \{S, A, B, T\}$

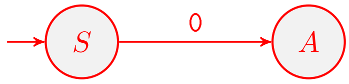
(b) $\Sigma = \{0, 1\}$

(c) $s = S$ (start node)

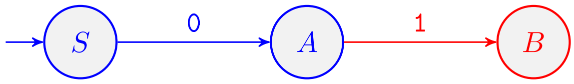
(d) $F = \{T\}$ and

(e) δ is as follows:

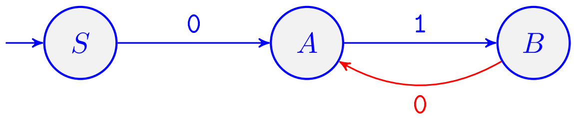
- $S \to 0A$ gives



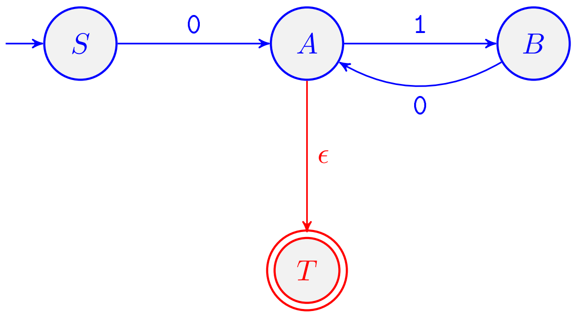
- $A \to 1B$ gives



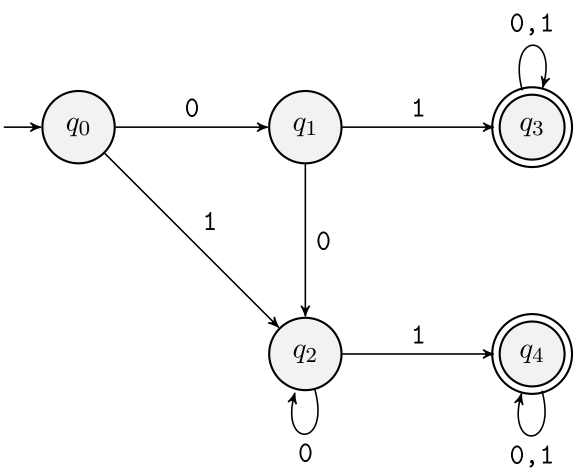
- $B \to 0A$ gives



- Finally, $A \to \epsilon$ gives



Problem 4 Find a right linear grammar for the following DFA:



Solution: $G = (V, T, P, S)$ where

1. $V = \{q_0, q_1, q_2, q_3, q_4\}$
2. $T = \{0, 1\}$
3. $S = q_0$ and
4. the set P of productions are given below:
   - $q_0 \to 0q_1$ (due to $\delta(q_0, 0) = q_1$)
   - $q_0 \to 1q_2$ (due to $\delta(q_0, 1) = q_2$)
   - $q_1 \to 0q_2$ (due to $\delta(q_1, 0) = q_2$)
   - $q_1 \to 1q_3$ (due to $\delta(q_1, 1) = q_3$) and $q_1 \to 1$ since q_3 is a final state.
   - $q_2 \to 0q_2$ (due to $\delta(q_2, 0) = q_2$)
   - $q_2 \to 1q_4$ (due to $\delta(q_2, 1) = q_4$) and $q_2 \to 1$ since q_4 is a final state.
   - $q_3 \to 0q_3$ (due to $\delta(q_3, 0) = q_3$) and $q_3 \to 0$ since q_3 is a final state.
   - $q_3 \to 1q_3$ (due to $\delta(q_3, 1) = q_3$) and $q_3 \to 1$ since q_3 is a final state.
   - $q_4 \to 0q_4$ (due to $\delta(q_4, 0) = q_4$) and $q_4 \to 0$ since q_4 is a final state.
   - $q_4 \to 1q_4$ (due to $\delta(q_4, 1) = q_4$) and $q_4 \to 1$ since q_4 is a final state.
