CS F351 Theory of Computation
Tutorial-5

Note: $*$ marked problems can be left to the students to try after the tutorial.

Problem 1 Prove that the following languages are not regular.

1. $L = \{ss \mid s \in \{0,1\}^*\}$

   Solution: Assume that L is regular. Let p be the pumping length.
   Consider $w = 0^p10^p1 \in L$. Consider $w = xyz$ such that $|xy| \leq p$ and $|y| \geq 1$. Thus, y contains only 0's (in particular, from the leftmost set of 0's).

   Let $y = 0^q$, where $1 \leq q \leq p$ and let $x = 0^r$ $(1 \leq r < p)$. Thus, $z = 0^{p-q-r}10^p1$.

   Consider $xy^2z = 0^r0^{2q}0^{p-q-r}10^p1 = 0^{p+q}10^p1$.

   Since $q \geq 1$, $p + q \neq p$. Thus, $xy^2z = 0^{p+q}10^p1 \notin L$.

   We got a contradiction to the pumping lemma, hence L is not a regular language.

2. $L = \{0^ix \mid i \geq 0, x \in \{0,1\}^* \text{ and } |x| \leq i\}$

   Solution: Assume that L is regular. Let p be the pumping length.
   Let $w = 0^p1^p \in L$ set $i = p$ and $x = 1^p$. Hence, by pumping lemma, there $\exists x, y, z$ such that $|y| > 1$, $0^p1^p = xyz$ with $x = 0^m$, $y = 0^n$ where $m + n \leq p$ and $xy^iz \in L\ \forall i \geq 0$. Setting $i = 0, xz = 0^{n-p}1^p \notin L$. We got a contradiction to the pumping lemma, hence L is not a regular language.

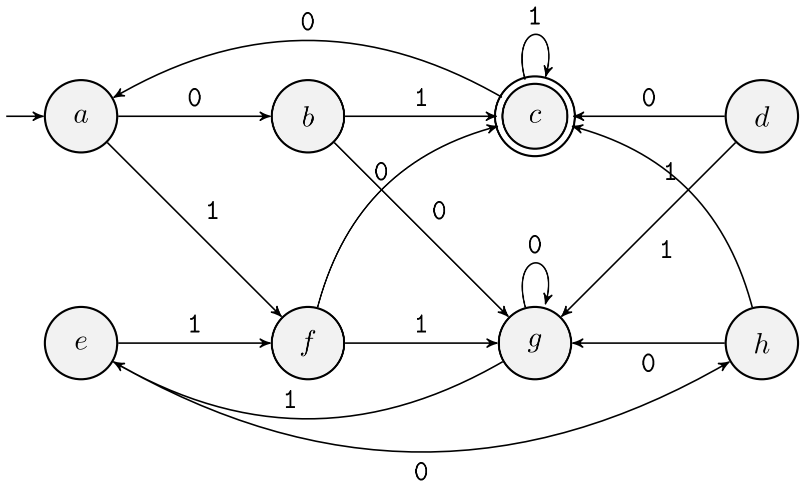
3. $(*)$ $L = \{0^n(12)^m \mid n \geq m \geq 0\}$

   Solution: Let L be a regular language, and p be a pumping length for L given by pumping lemma. Let $w = 0^p(12)^p$ then w can be split into xyz satisfying the conditions of the pumping lemma $|xy| \leq p$, y must contain only $0s$. Pumping lemma states $xy^iz \in L$ even when $i = 0$. So, let us consider the string $xy^0z = xz$, therefore, xz can not have at least as many $0s$ as $(12)s$. Hence $xy \notin L$, a contradiction.

4. $L = \{a^nb^l \mid n \neq l\}$ over $\Sigma = \{a, b\}$

   Solution: Assume that the language is regular. Let m be the pumping length. Let $w = xyz = a^{m!}b^{(m+1)!} \in L$ such that $y = a^k$ where $k < m$. Now pump y, i times then we get $xy^iz = a^{m!-k}(a^k)^ib^{(m+1)!}$. If we pick i such that $m! + (i-1)k = (m+1)!$. Then we will have a contradiction.

Problem 2 Minimize the following DFA.



Solution: We say a state p is distinguishable from a state q if there exist an x such that $\delta(p, x) \in F$ and $\delta(q, x)$ is not, or vice versa.

All the entries (x, y) such that $x \in F$ and $y \notin F$, or vice versa will be filled with $\times$. Hence $(a, c), (b, c), (c, d), (c, e), (c, f), (c, g)$ and (c, h) will be filled with $\times$.

Place $\times$ in the entry (a, b) because $(\delta(a, 1), \delta(b, 1)) = (c, f)$.

Place $\times$ in the entry (a, d) because $(\delta(a, 0), \delta(d, 0)) = (b, c)$.

Entry (a, e) will wait for the entry (b, h) because $(\delta(a, 0), \delta(e, 0)) = (f, f)$ and $(\delta(a, 1), \delta(e, 1)) = (b, h)$.

Entry (a, g) will wait for the entry (b, g) because $(\delta(a, 0), \delta(g, 0)) = (b, g)$. It can also wait for $(\delta(a, 1), \delta(g, 1)) = (f, e)$.

Entry (b, g) will be filled with $\times$ because $(\delta(b, 1), \delta(g, 1)) = (c, e)$.

Entry (a, g) will receive $\times$ because (b, g) has received $\times$. The string 01 distinguishes pair (a, g).

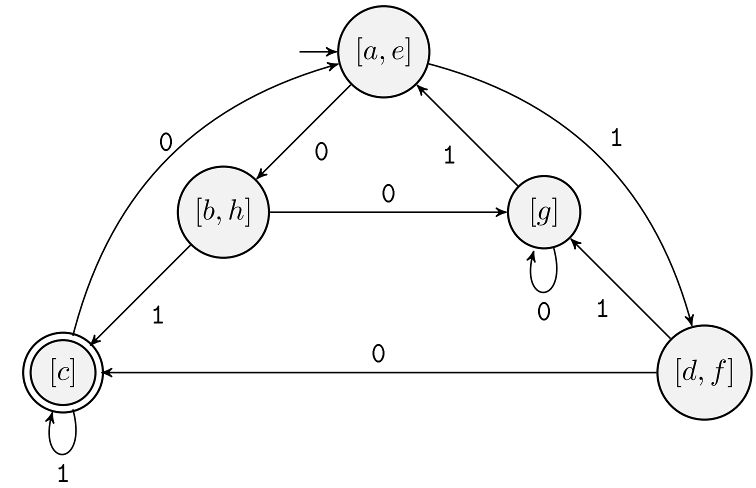
Similarly all the entries will be filled.

The final table has been shown above. In the above table the pairs $(a, e), (b, h)$ and (d, f) do not receive $\times$. Hence the states a and e will be equivalanet, b and h will be equivalent and d and f will be equivalent. These states will be merged in the minimized DFA.

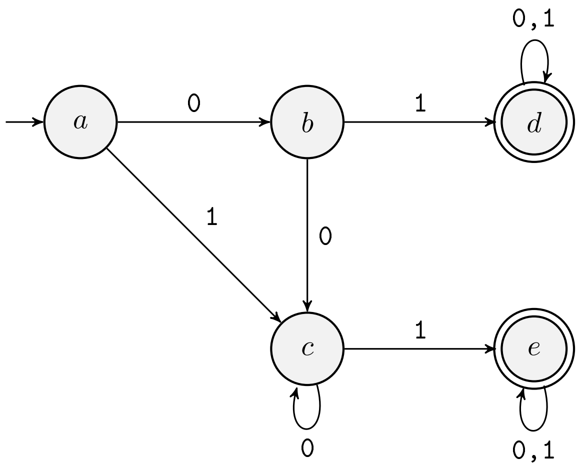
b	$\times$						
c	$\times$	$\times$					
d	$\times$	$\times$	$\times$				
e	E	$\times$	$\times$	$\times$			
f	$\times$	$\times$	$\times$	E	$\times$		
g	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	
h	$\times$	E	$\times$	$\times$	$\times$	$\times$	$\times$
	a	b	c	d	e	f	g

The states corresponding to cells with E are equivalent.

The minimized DFA will look like as follows.



Problem 3 $(*)$ Minimize the following DFA.



Solution:

Mark all the cells $(a, d), (a, e), (b, d), (b, e), (c, d),$ and (c, e) with $\times$ since in each of these pairs one state is a final state and other is a non-final states (see the table below).

b				
c				
d	$\times$	$\times$	$\times$	
e	$\times$	$\times$	$\times$	
	a	b	c	d

Next, check (a, b):

$\delta(a, 0) = b$ and $\delta(b, 0) = c$ and (b, c) cell is unmarked.

$\delta(a, 1) = c$ and $\delta(b, 1) = d$ and (c, d) cell is marked with $\times$. Hence mark the cell (a, b) with $\times$.

Check (a, c):

$\delta(a, 1) = c$ and $\delta(c, 1) = e$ and (c, e) cell is marked with $\times$. Hence, mark the cell (a, c) with $\times$.

You can proceed in the same way, no other pairs are marked with $\times$. The final table is given below:

	a	b	c	d
b	×			
c	×	E		
d	×	×	×	
e	×	×	×	E

The states corresponding to cells with E are equivalent.

The equivalent minimized DFA is: