CS F351 Theory of Computation
Tutorial-2

Problem 1 Let L_1 and L_2 be two languages. Prove that $(L_1L_2)^R = L_2^R L_1^R$.

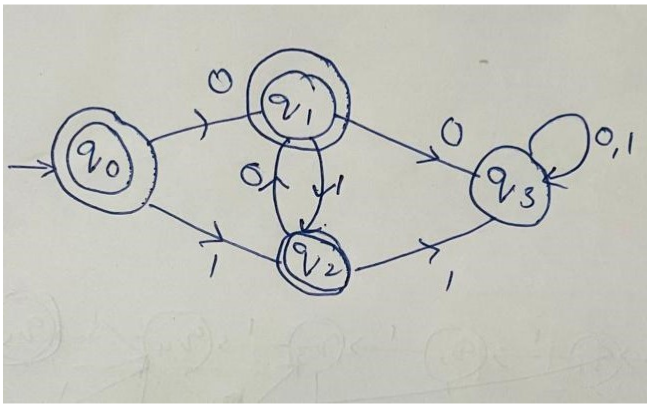
Solution: (i) Let $w \in (L_1L_2)^R \implies w^R \in L_1L_2$
$\implies \exists x$ and y such that $w^R = xy \in L_1L_2$ and $x \in L_1$ and $y \in L_2$.
Then $(xy)^R = y^R x^R \in L_2^R L_1^R$. Thus, $(L_1L_2)^R \subseteq L_2^R L_1^R$.

(ii) Let $w \in L_2^R L_1^R \implies \exists x$ and y such that $w = xy$ and $x \in L_2^R$ and $y \in L_1^R$.
Then $(xy)^R = y^R x^R \in L_1L_2$. Hence $w = xy \in (L_1L_2)^R$. Thus, $L_2^R L_1^R \subseteq (L_1L_2)^R$.

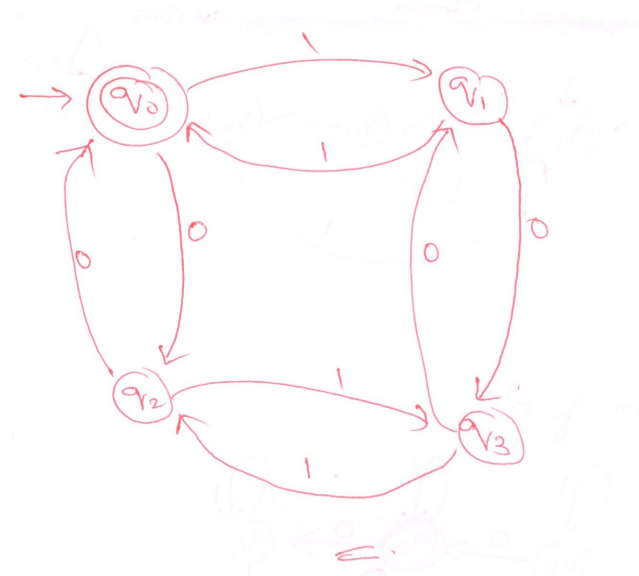
Therefore, $(L_1L_2)^R = L_2^R L_1^R$.

Problem 2 Give DFA's on $\Sigma = \{0, 1\}$ accepting the following strings.

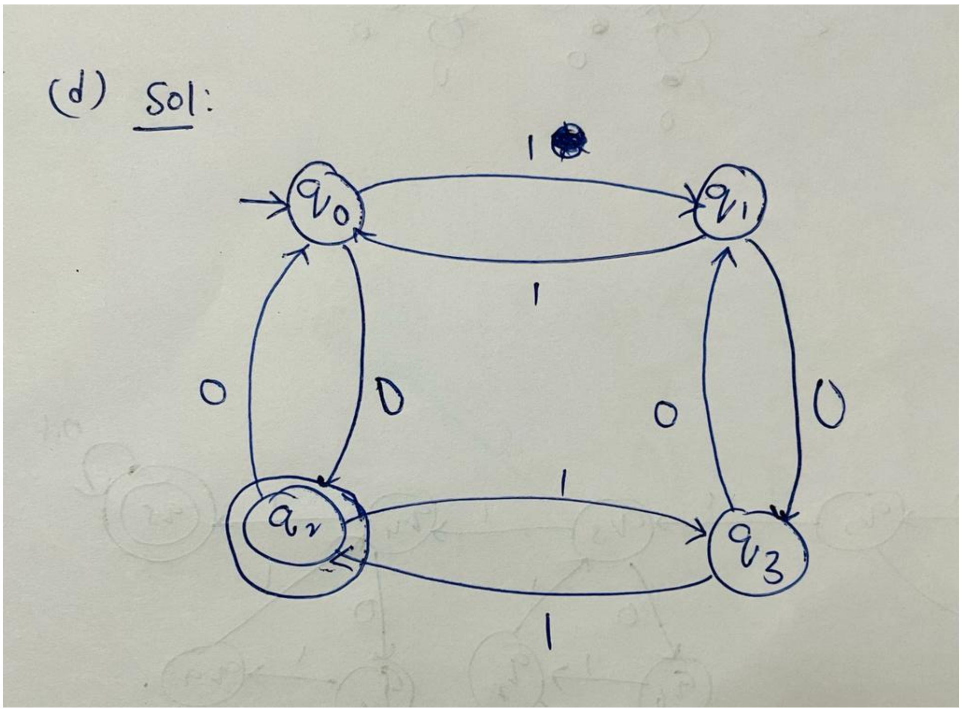
(a) The set of all strings such that each string has neither 00 nor 11 as a substring.



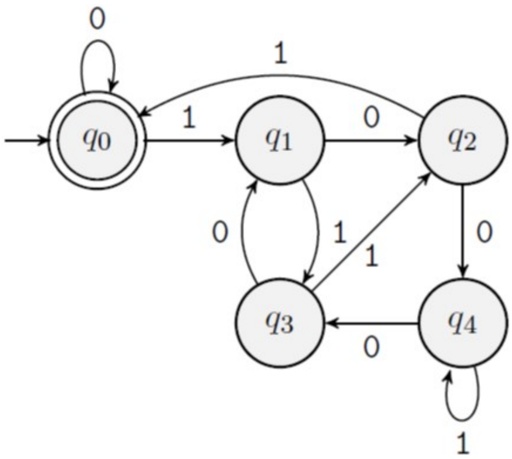
(b) The set of all strings such that each string has an even number of 0's and even number of 1's.



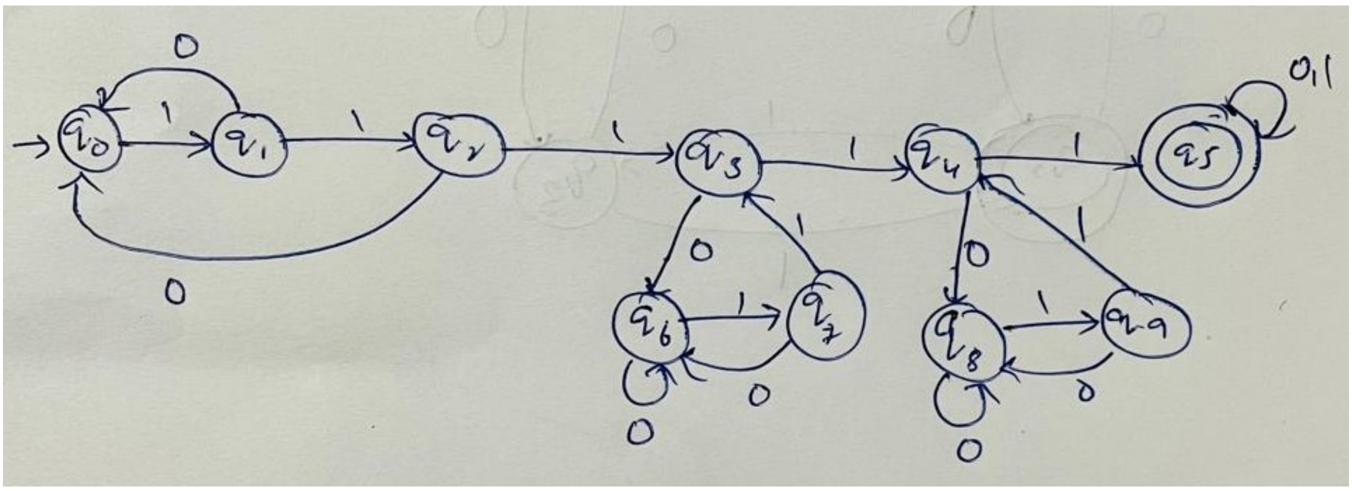
(c) (∗) The set of all strings such that each string has an odd number of 0's and even number of 1's.



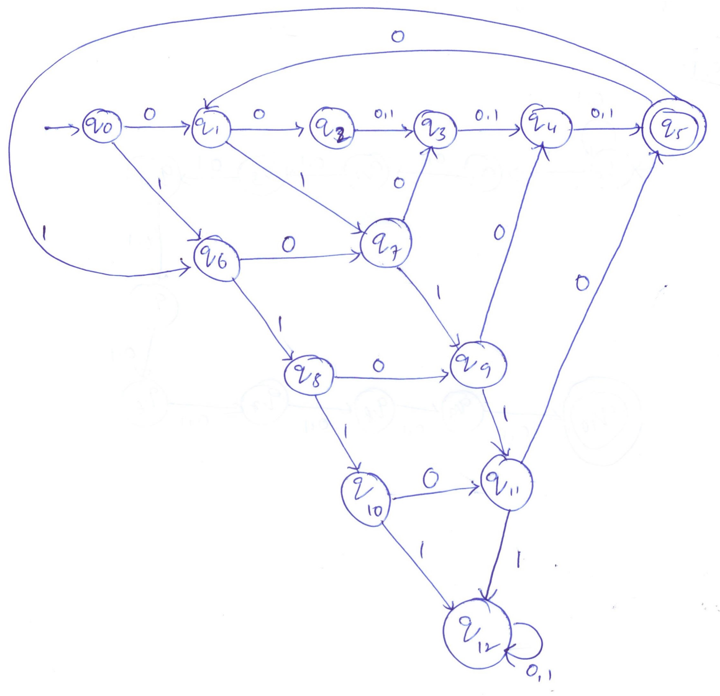
(d) (∗) The set of all binary strings whose decimal equivalent is divisible by 5.



(e) The set of all strings containing at least three occurrences of three consecutive 1's, overlapping is permitted e.g., the string 11111 should be accepted.



(f) (*) The set of all strings such that every block of five consecutive symbols contains at least two 0's.
Solution:



Problem 3 Design a DFA for the following set: the set of strings, over an alphabet $\Sigma = \{0\}$, whose length is divisible by 2 or 7.