CS F351 Theory of Computation
Tutorial-11

Problem 1 Design Turing machine, with single tape, to accept the following languages.

1. The set of strings with an equal number of 0's and 1's.

   Solution: On input string w:

   (1) Scan the tape and mark the first 0 that has not been marked. If no unmarked 0 is found, go to step 4. Otherwise, move the head back to the front of the tape.

   (2) Scan the tape and mark the first 1 that has not been marked. If no unmarked 1 is found, *reject*.

   (3) Move the head back to the front of the tape and go to the step 1.

   (4) Move the head back to the front of the tape. Scan the tape to see if any unmarked 1s remain. If none are found, *accept*; otherwise *reject*.

2. $\{a^n b^n c^n \mid n \geq 1\}$

   Solution:

   (1) Mark the leftmost unmarked a then move right.

   (2) Mark the leftmost unmarked b then move right

   (3) Mark the leftmost unmarked c then move left

   (4) Go to far left till we get the leftmost unmarked a.

   (5) Repeat above steps till all as, bs and cs are marked

   (6) At last if everything is marked that means string is *accepted*, else *reject*.

3. $\{a^i b^j c^k \mid i \times j = k \text{ and } i, j, k \geq 0\}$

   Solution: On input string w:

   (1) Scan the input from left to right to determine whether it is a member of $a^*b^*c^*$ and *reject* if it isn't.

   (2) Return the head to the left-hand end of the tape.

   (3) Cross of an a and scan to the right until a b occurs. Shuttle between the b's and the c's, crossing off one of each until all b's are gone. If all c's have been crossed off and some b's remain, *reject*.

   (4) Restore the crossed off b's and repeat step if there is another a to cross off. If all a's have been crossed off, determine whether all c's also have been crossed off. If yes, *accept*; otherwise, *reject*.

4. $\{w\#w \mid w \in \{0,1\}^*\}$

   Solution: See a solution in *Michael Sipser, Introduction to Theory of Computation*.

Problem 2 Proper subtraction $m - n$ is defined to be $m - n$ if $m \geq n$, and zero otherwise. Design a Turing machine which computes the proper subtraction $m - n$ for given $m \geq 1$ and $n \geq 1$.

Hint: Store the input on the tape in the form $0^m 1 0^n$ and output the result in the form 0^{m-n}.

Solution: See a solution in Hopcroft and Ullman, Introduction to Automata theory, languages, adn computation.