CS F351 Theory of Computation
Tutorial-10

Problem 1 Show that the following languages are not context-free.

1. $L = \{a^i b^j c^k \mid 0 \leq i \leq j \leq k\}$

   For a solution, please refer *Michael Sipser, Theory of Computation.*

2. $L = \{ww \mid w \in \{0,1\}^*\}$

   For a solution, please refer *Michael Sipser, Theory of Computation.*

3. $L = \{a^{n!} \mid n \geq 0\}$

   Solution: Suppose L is context free. Pick m as pumping length and $s = a^{m!} = uvxyz$, whatever the decomposition it must be of the form $v = a^k, y = a^l$. Then for $i = 0$, $|uxz| = m! - (k+l)$. In order to $uxz \in L$, $m! - (k+l) = j!$ for some j. Since $k + l \leq m$, $m! - (k+l) > (m-1)!$. Therefore it is impossible to find such a j.

4. $L = \{wtw^R \mid w, t \in \{0,1\}^*, |w| = |t|\}$

   Solution: Suppose L is context free. Pick p, and let $s = 0^{2p}1^p0^p0^{2p} \in L$. Now, $uvxyz = 0^{2p}1^p0^p0^{2p}$. With the condition $|vxy| \leq p$ we have three cases to see.

   Case 1: vy contains only 0's and it is choosen from 0^{2p}. Let i be a number with $7p > |vy| \times (i+1) \geq 6p$. Then either the length of uv^ixy^iz is not multiple of 3, or this string is of the form wtw' such that $|w| = |t| = |w'|$ with w' is all 0's and w is not all 0's($w' \neq w$).

   Case 2: vy does not contain any 0's in the last 0^{2p} of s.Then either the length of uv^2xy^2z is not multiple of 3, or this string is of the form wtw' such that $|w| = |t| = |w'|$ with w' is all 0's and w is not all 0's($w' \neq w$).

   Case 3: vy is not all 0's and some 0s are form the last 0^{2p} of s.As $|vxy| \leq p$, vxy is in the case must be substring of 1^p0^p. Then either the length of uv^2xy^2z is not multiple of 3, or this string is of the form wtw' such that $|w| = |t| = |w'|$ with w' is all 0's and w is not all 0's($w' \neq w$).

   There is no way s can satisfy pumping lemma.

Problem 2 Use CYK-algorithm to show that string (()()) can be generated by the grammar with the productions:

$$\begin{gathered} S \rightarrow SS \mid OS_1 \mid OC \\ S_1 \rightarrow SC \\ O \rightarrow (\\ C \rightarrow) \end{gathered}$$

V

$j \backslash i$	1	2	3	4	5	6
1	O	O	C	O	C	C
2	ϕ	S	ϕ	S	ϕ	
3	ϕ	ϕ	ϕ	S_1		
4	ϕ	S	ϕ			
5	ϕ	S_1				
6	S					