CS F351 Theory of Computation
Tutorial-3

Note: $*$ marked problems can be left to the students to try after the tutorial.

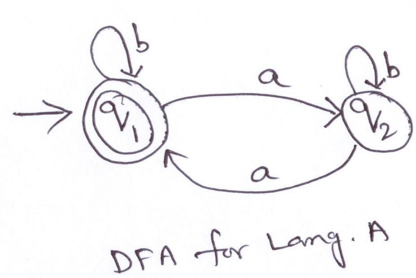
Problem 1 *Design a DFA for the language, over $\Sigma = \{a, b\}$, such that all strings in the language contain even number of a's and each a is followed by at least one b.*

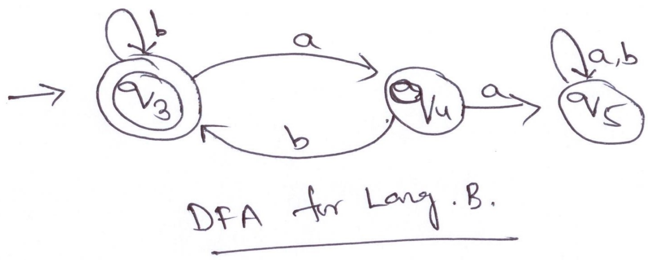
Solution: Consider two sets of the given language:

$A = \{x \in \Sigma^* \mid x \text{ contains even number of } a\text{'s}\}$ and

$B = \{x \in \Sigma^* \mid \text{ each } a \text{ in } x \text{ is followed by at least one } b\}$

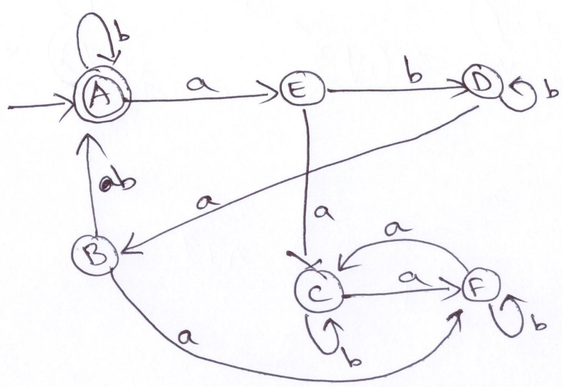
Note that the given language is the same as $A \cap B$. The DFA's for A and B are given below:





Now we construct the DFA for the given language as discussed in the class.

	a	b
A (q_1, q_3)	(q_2, q_4) E	(q_1, q_3) A
B (q_1, q_4)	(q_2, q_5) F	(q_1, q_3) A
C (q_1, q_5)	(q_2, q_5) F	(q_1, q_5) C
D (q_2, q_3)	(q_1, q_4) B	(q_2, q_3) D
E (q_2, q_4)	(q_1, q_5) C	(q_2, q_3) D
F (q_2, q_5)	(q_1, q_5) C	(q_2, q_5) F



Problem 2 Construct NFA for the following languages:

1. $L_1 = \{x \in \{0, 1\}^* \mid x \text{ contains the equal number of occurrences of 01 and 10}\}$.
   ($*$) Also, construct a DFA for the same language.

   Solution:

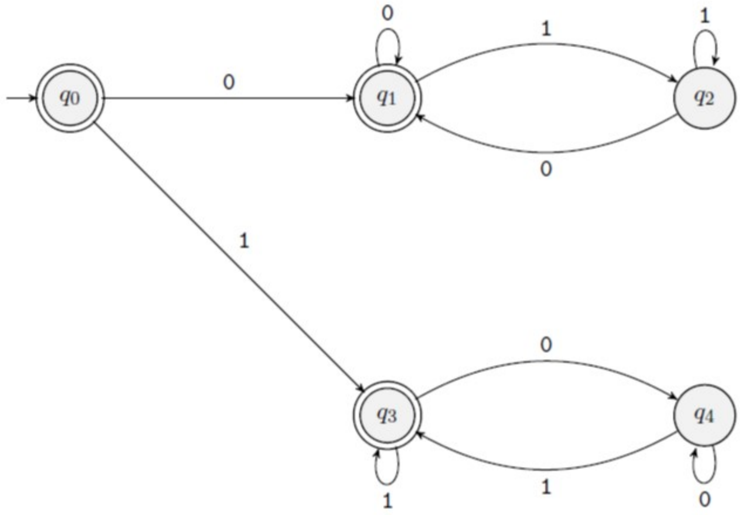


Figure 1: Solution to Problem 2.1.

2. (*) $L_2 = \{x \in \{0,1\}^* \mid x$ contains two 0s separated by a substring whose length is a multiple of $3\}$.

Solution:

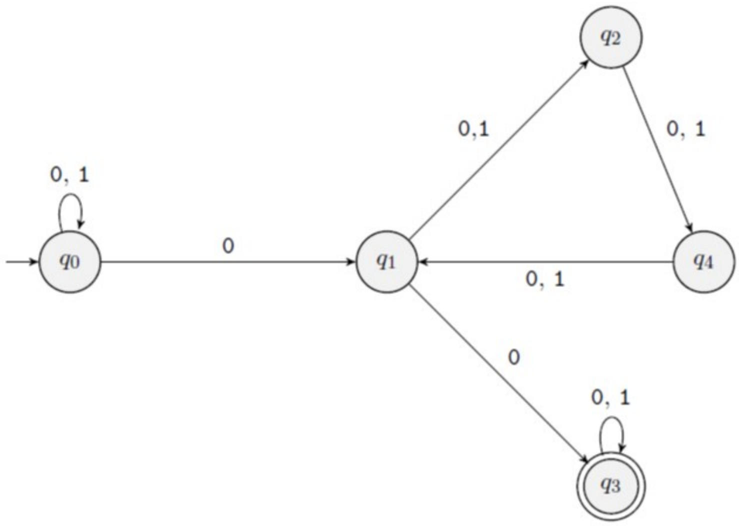


Figure 2: Solution to Problem 2.2.

Problem 3 ($*$) Say that string x is a **prefix** of y if a string z such that $y = xz$ and that x is a **proper prefix** of y if $x \neq y$. Let A be a regular language.

Show that $NOPREFIX(A) = \{w \in A \mid$ no proper prefix of w is a member of $A\}$ is also a regular language.

Solution:

Let $M = (Q, \Sigma, \delta, q_0, F)$ be an DFA such that $L(M) = A$. We now construct $M' = (Q', \Sigma, \delta', q_0', F')$ such that $L(M') = NOPREFIX(A)$ where

1. $Q' = Q$.
2. For $r \in Q'$ and $a \in \Sigma$ define $\delta'(r, a) = \begin{cases} \delta(r,a) & \text{if } r \notin F \\ \emptyset & \text{if } r \in F \end{cases}$
3. $q_0' = q_0$.
4. $F' = F$.

Problem 4 Construct an equivalent DFA for the following NFA's.

1. $(\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta, q_0, \{q_3\})$ where δ is given in the below table.

δ	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$
q_2	$\{q_3\}$	$\emptyset$
q_3	$\{q_3\}$	$\{q_3\}$

Solution: The equivalent DFA is given below. The states marked with $*$ are the final states.

δ	0	1
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_2, q_3\}$
$*\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_2, q_3\}$
$*\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_3\}$
$*\{q_0, q_3\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_3\}$

2. ($*$) $(\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \delta, q_0, \{q_1, q_3\})$ where δ is given in the below table.

δ	0	1
$\rightarrow q_0$	$\{q_1, q_3\}$	$\{q_1\}$
q_1	$\{q_2\}$	$\{q_1, q_2\}$
q_2	$\{q_3\}$	$\{q_0\}$
q_3	$\emptyset$	$\{q_0\}$

Solution: The equivalent DFA is given below. The states marked with $*$ are the final states.

δ	0	1
$\rightarrow \{q_0\}$	$\{q_1, q_3\}$	$\{q_1\}$
$*\{q_1, q_3\}$	$\{q_2\}$	$\{q_0, q_1, q_2\}$
$*\{q_1\}$	$\{q_2\}$	$\{q_1, q_2\}$
$\{q_2\}$	$\{q_3\}$	$\{q_0\}$
$*\{q_0, q_1, q_2\}$	$\{q_1, q_2, q_3\}$	$\{q_0, q_1, q_2\}$
$*\{q_1, q_2\}$	$\{q_2, q_3\}$	$\{q_0, q_1, q_2\}$
$*\{q_3\}$	$\emptyset$	$\{q_0\}$
$*\{q_1, q_2, q_3\}$	$\{q_2, q_3\}$	$\{q_0, q_1, q_2\}$
$*\{q_2, q_3\}$	$\{q_3\}$	$\{q_0\}$
$\emptyset$	$\emptyset$	$\emptyset$
