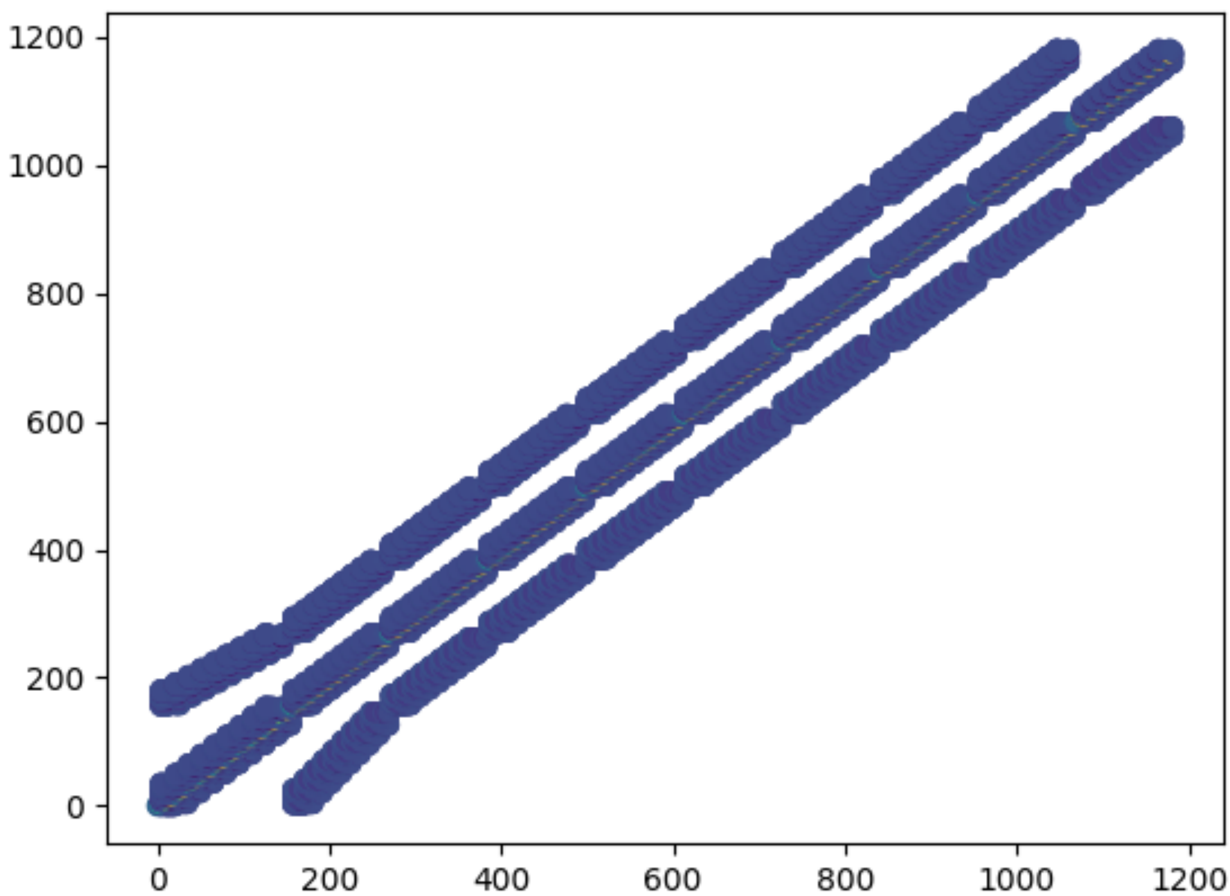
# 利用trsm算法解增广矩阵

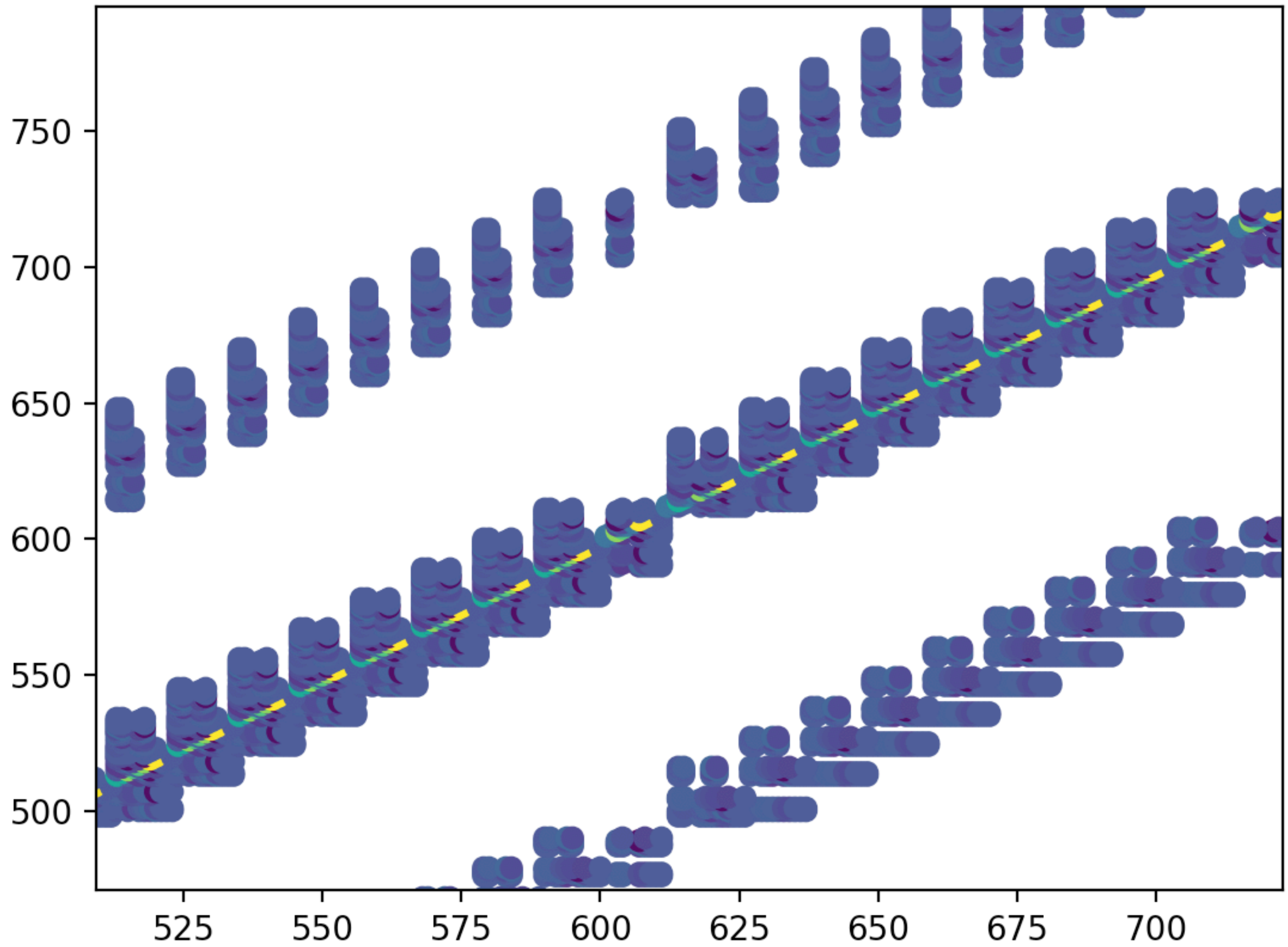
## Ac -- 许秋晗

### 特征描述

- 矩阵图像




- 局部放大




- 对称性: 不对称

## 选择算法

对可能求解成功的算法——尝试

- **Gauss算法**

```
➜ NumberAnalysis $ ./run.py -if input/Ac.txt  1
Use function: gauss
    * Calculate Matrix A with width:1182 and use time: 2.87s.
    * Solution:
        residual: 0.003214
        not match condition
        solution is too long!
```

失败：残差过大

- **Lu_doolittle**

```
➜ NumberAnalysis $ ./run.py -if input/Ac.txt  2
Use function: Lu_doolittle
    * Calculate Matrix A with width:1182 and use time: 1.19s.
    * Solution:
        residual: 0.000053
        match condition
        solution is too long!
```

成功

- **Lu_crout**

```
→ NumberAnalysis $ ./run.py -if input/Ac.txt    3
Use function: Lu_crout
    * Calculate Matrix A with width:1182 and use time: 1.85s.
    * Solution:
        residual: 0.000026
        match condition
        solution is too long!
```

成功

- **Cholesky**

```
→ NumberAnalysis $ ./run.py -if input/Ac.txt  4
Use function: cholesky
    * Calculate Matrix A with width:1182 and use time: 0.70s.
    * Solution:
        residual: 127.019928
        not match condition
        solution is too long!
```

失败：残差过大

- **Jacobi**

```
→ NumberAnalysis $ ./run.py -if input/Ac.txt 5
Use function: jacobi
    * Calculate Matrix A with width:1182 and use time: 4.22s.Iterations:652
    * Solution:
        residual: 0.022785
        not match condition
        solution is too long!
```

失败：残差过大

- **Gauss-Seidel**

```
→ NumberAnalysis $ ./run.py -if input/Ac.txt    6
Use function: GS
    * Calculate Matrix A with width:1182 and use time: 1.35s.Iterations:355
    * Solution:
        residual: 0.022778
        not match condition
        solution is too long!
```

失败：残差过大

- **SOR**

```
→ NumberAnalysis $ ./run.py -if input/Ac.txt    7
Use function: sor
    * Calculate Matrix A with width:1182 and use time: 1.26s.Iterations:248
    * Solution:
        residual: 0.023286
        not match condition
        solution is too long!
```

失败：残差过大

- **Conjugate**

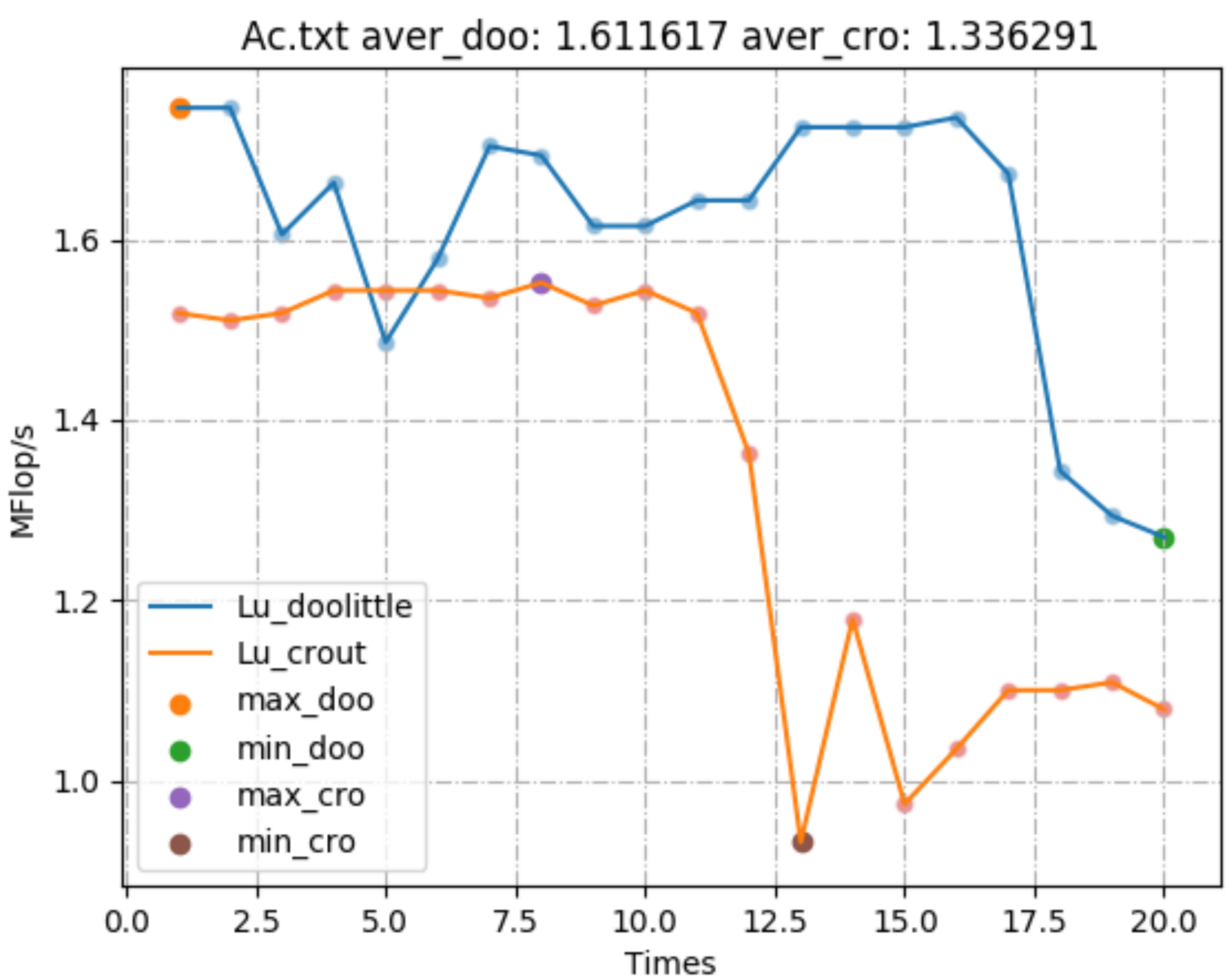
```
➜ NumberAnalysis $ ./run.py -if input/Ac.txt    8
Use function: conjugate
    * Calculate Matrix A with width:1182 and use time: 3.79s.Iterations:1000
    * Solution:
        residual: 200.499054
        not match condition
        solution is too long!
```

失败：残差过大

**经过比较可得，LU分解算法更好。**

## 选择原因

- 针对Lu_doolittle算法和Lu_crout算法，测评其综合性能并择优。




| doolittle平均性能 | 关系 | crout平均性能 |
| --- | --- | --- |
| 1.6116 MFlop/s | > | 1.3363 MFlop/s |
| **doolittle最大性能** | - | **crout最大性能** |
| 1.7464 MFlop/s | > | 1.5523 MFlop/s |
| **doolittle最小性能** | - | **crout最小性能** |
| 1.2701 MFlop/s | > | 0.9314 MFlop/s |

## 效率分析

| doolittle平均性能 |
| --- |
| 1.6116 MFlop/s |
| **doolittle最大性能** |
| 1.7464 MFlop/s |
| **doolittle最小性能** |
| 1.2701 MFlop/s |

## 代码

```
void LU_doolittle_2017011321(const float *A, float *x, const float *b, int
n)
{
    float *At = (float *)malloc(sizeof(float) * n * n);
    memcpy(At, A, sizeof(float) * n * n);
    float *bt = (float *)malloc(sizeof(float) * n);
    memcpy(bt, b, sizeof(float) * n);

    float *L = (float *)malloc(sizeof(float) * n * n);
    float *U = (float *)malloc(sizeof(float) * n * n);
    memset(L, 0, sizeof(float) * n * n);
    for (int i = 0; i < n; i++)
    {
        L[i * n + i] = 1.0;
    }
    memset(U, 0, sizeof(float) * n * n);

    for (int i = 0; i < n; i++)
    {
        float pivot = At[i * n + i];
        for (int j = i + 1; j < n; j++)
        {
            float scale = -At[j * n + i] / pivot;
            At[j * n + i] = -scale;
            for (int k = i + 1; k < n; k++)
            {
                At[j * n + k] += scale * At[i * n + k];
            }
        }
    }

    float *y = (float *)malloc(sizeof(float) * n);

    for (int i = 0; i < n; i++)
        for (int j = 0; j < i; j++)
            L[i * n + j] = At[i * n + j];
    for (int i = 0; i < n; i++)
```

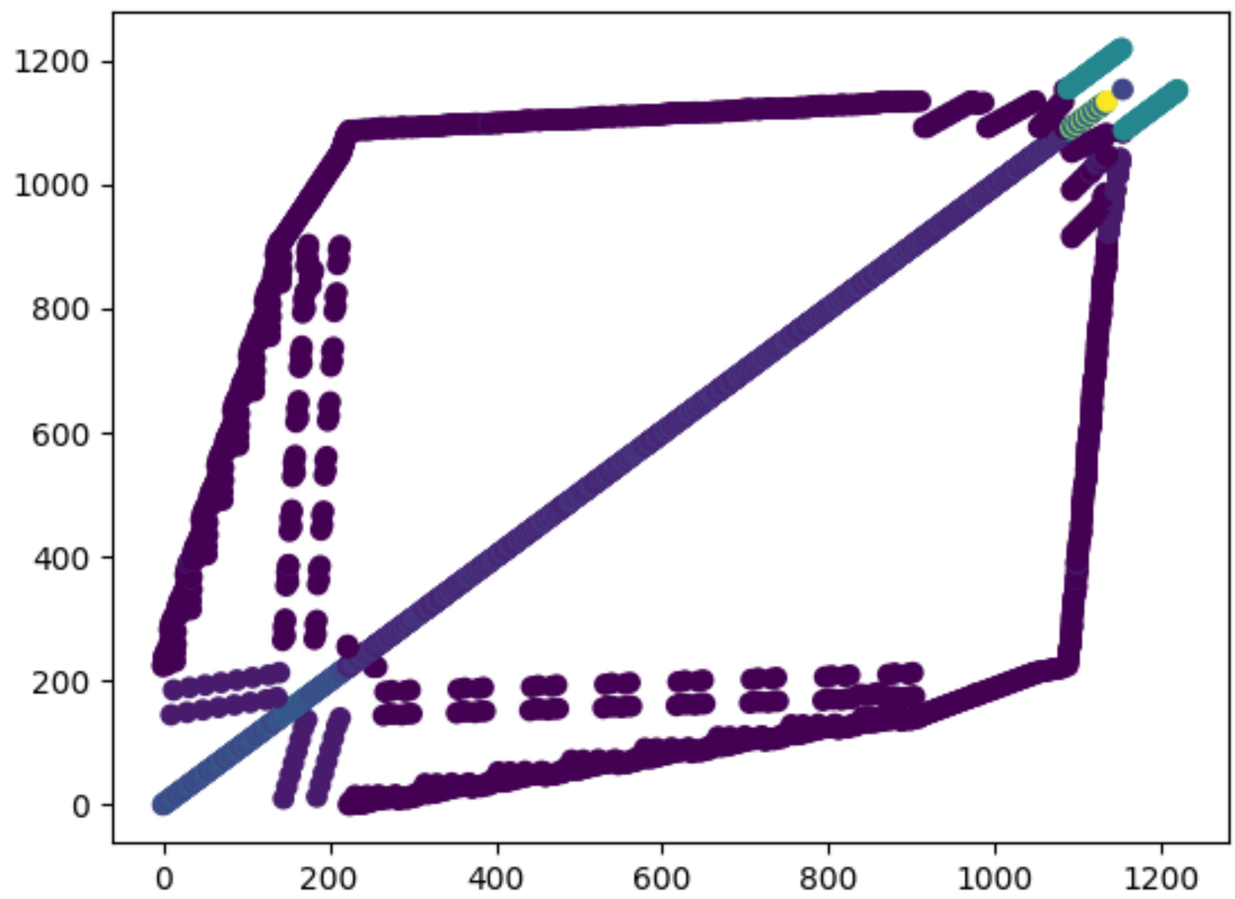
```
        for (int j = i; j < n; j++)
            U[i * n + j] = At[i * n + j];

    for (int i = 0; i < n; i++)
    {
        float sum = 0;
        for (int j = 0; j < i; j++)
            sum += L[i * n + j] * y[j];
        y[i] = (bt[i] - sum) / L[i * n + i];
    }
    for (int i = n - 1; i >= 0; i--)
    {
        float sum = 0;
        for (int j = i + 1; j < n; j++)
        {
            sum += U[i * n + j] * x[j];
        }
        x[i] = (y[i] - sum) / U[i * n + i];
    }
    free(At);
    free(bt);
    free(L);
    free(U);
    free(y);
}
```

## 优化策略

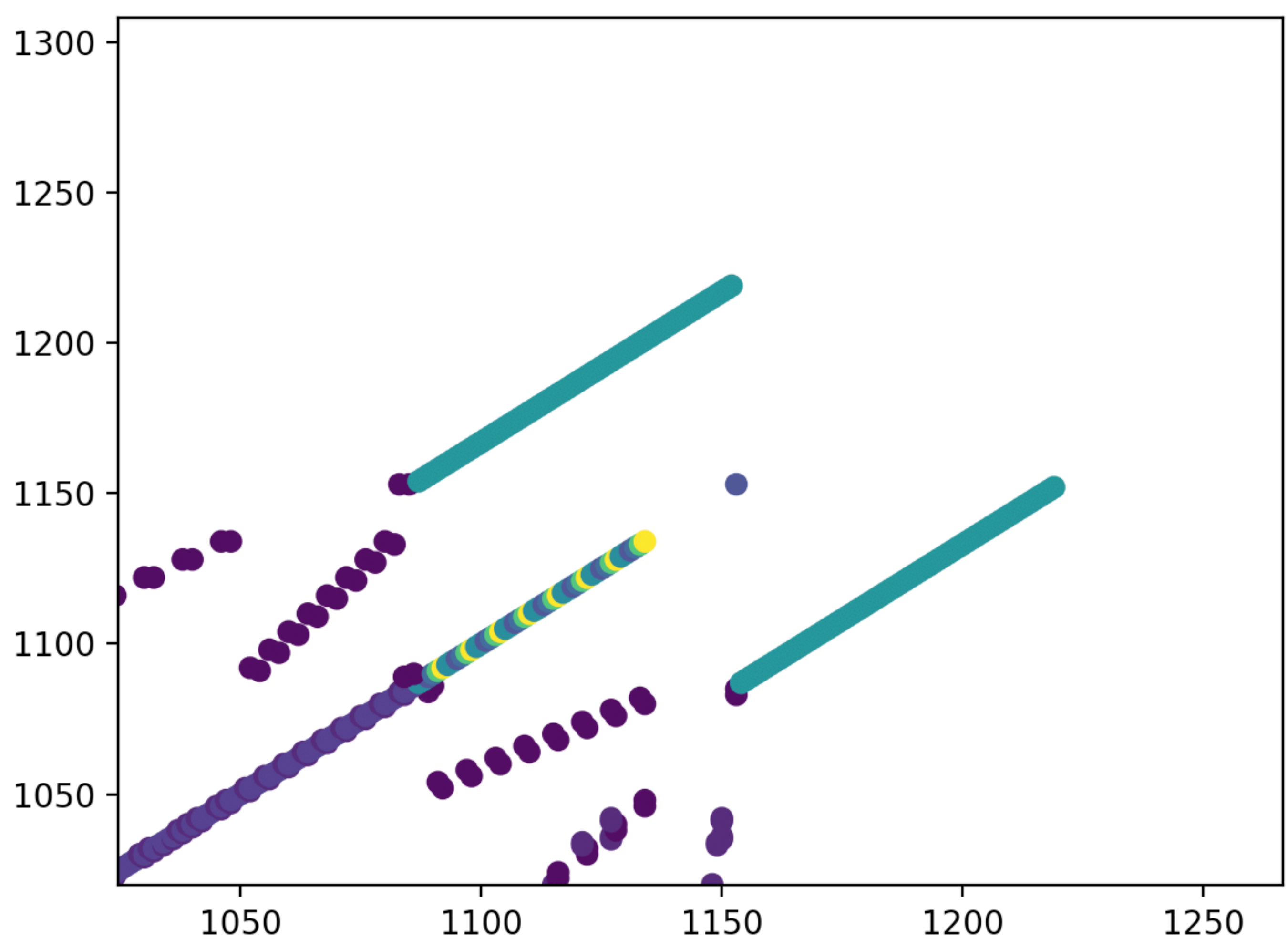
# Af -- 沈湘洁

## 特征描述

- 矩阵图像




- **对称性**: 不对称
- **对角元素分布**




从完整矩阵看，为稀疏矩阵，大部分集中在对角线上且连续分布，对角线两侧分布有间断元素。

## 选择算法

由矩阵的非对称性，可排除cholesky算法；

对于除cg以外的迭代法，由于矩阵右下角存在连续0元素位于对角线上，使用时需要对于矩阵元素进行整理，行交换，再进行迭代。

由于零散分布的较小元素，高斯消元法由于浮点运算误差，极易出现错误的解，为避免不稳定现象的发生，舍弃此法。

- Lu算法
  - Doolittle 算法

```
→ NumberAnalysis $ ./run.py -if input/Af.txt 2
Use function: Lu_doolittle
    * Calculate Matrix A with width:1220 and use time: 1.88s.
    * Solution:
        residual: 6.055333
        not match condition
        solution is too long!
```

  - Crout 算法

```
→ NumberAnalysis $ ./run.py -if input/Af.txt 3
Use function: Lu_crout
    * Calculate Matrix A with width:1220 and use time: 2.05s.
    * Solution:
        residual: 6.000025
        not match condition
        solution is too long!
```

- cg迭代法

```
→ NumberAnalysis $ ./run.py -if input/Af.txt 8
Use function: conjugate
    * Calculate Matrix A with width:1220 and use time: 1.17s.Iterations:299
    * Solution:
        residual: 0.000098
        match condition
        solution is too long!
```

## 选择原因

- **仅有cg迭代法在误差的允许范围之内。**
- Af为稀疏矩阵，Lu对于Af缺乏优势；

  在元素的数值大小较为零散，差距稍大的情况下，高斯消元法不在适用；

  在使用迭代法时，由于对角线元素有连续为0的情况，换行较为繁琐在使用cg迭代法后，误差在允许范围内，且效率最高。

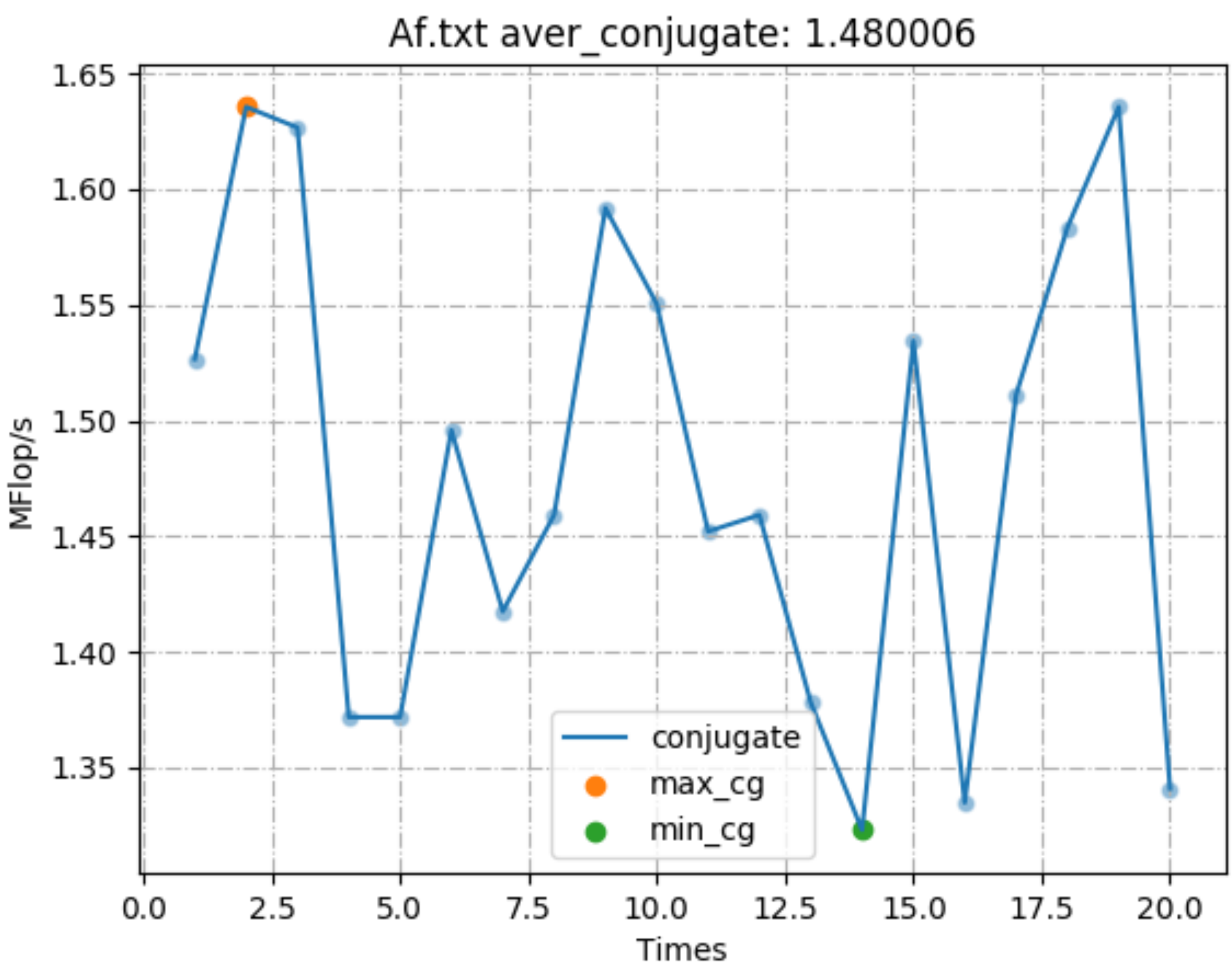
  由于此矩阵非对称正定矩阵，因此所求得二次泛函的极小值未必为该线性方程的解，但在尝试中，所解出的结果正确。

- **代码**

```
void cg(float *A,float *x,float *b,int n,int iter,int maxiter,float threshold)
{
```

```
    memset(x,0,sizeof(float)*n);
    float *residual=(float*)malloc(sizeof(float)*n);
    float *y=(float*)malloc(sizeof(float)*n);
    float *p=(float*)malloc(sizeof(float)*n);
    float *q=(float*)malloc(sizeof(float)*n);
    iter=0;
    float norm=0;
    float rho=0;
    float rho_1=0;

    matvec (A, x, y, n) ;
    for(int i=0;i<n;i++)
        residual[i]=b[i]-y[i];
    do
    {
        rho=dotproduct(residual,residual,n);
        if(iter==0)
        {
            for(i=0;i<n;i++)
                p[i]=residual[i];
        }
        else
        {
            float beta=rho/rho_1;
            for(i=0;i<n;i++)
                p[i]=residual[i]+beta*p[i];
        }
        matter(A, p, q, n) ;

        float alpha=rho/dotproduct (p,q,n);
        for(int i=0;i<n;i++)
            x[i]+=alpha*p[i];
        for(int i=0;i<n;i++)
            residual[i]+=-alpha*q[i];
        rho_1=rho;
        float error=vec2norm(residual,n)/vec2norm(b,n);
        if(error<threshold){
            printf("\niter=%d\n",iter);
            printf("\nerror=%f\n",error);
            break;
        }
    }while(iter++<maxiter);
    free(residual);
    free(y);
    free(p);
    free(q);
}
```

## 效率分析



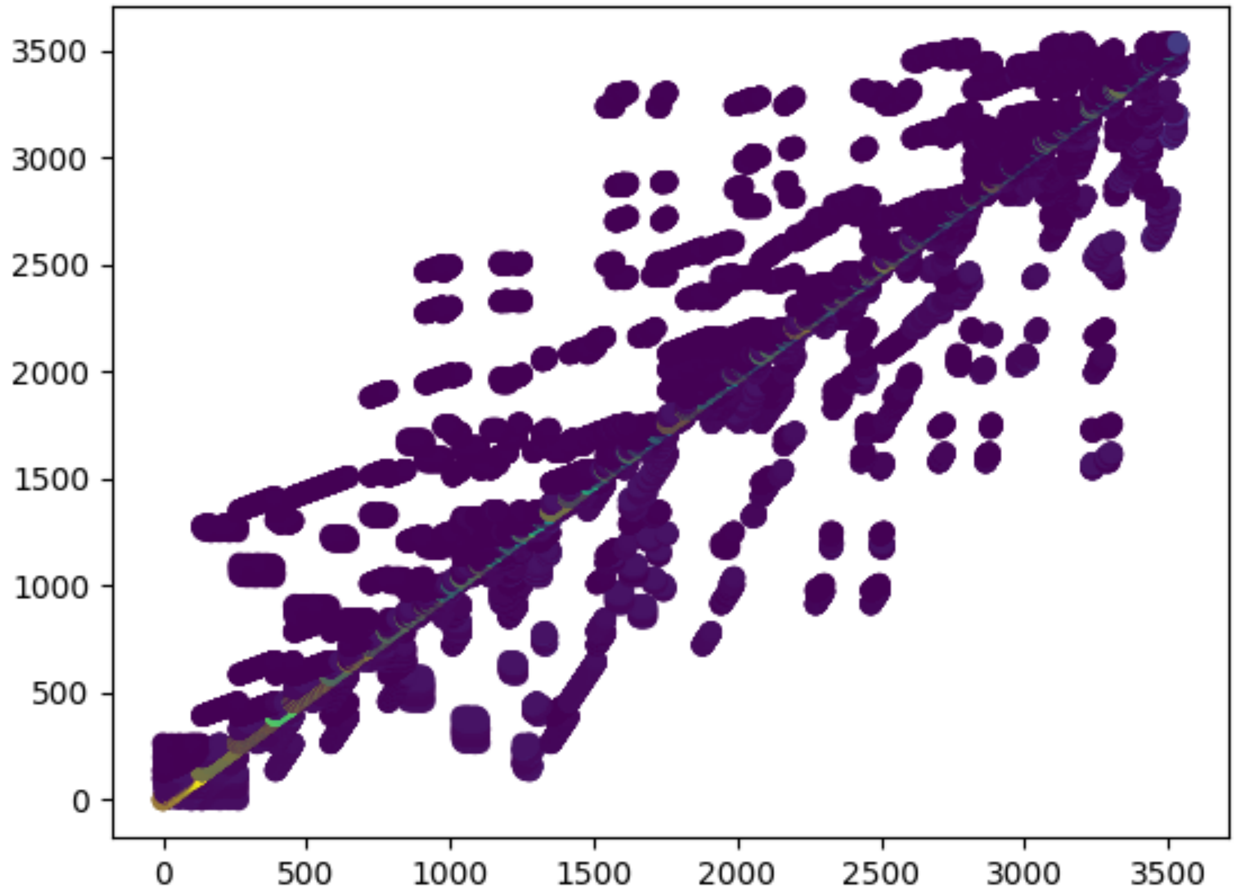

| **Conjugate 平均效率** |
| --- |
| 1.4800 MFlop/s |
| **Conjugate 最大效率** |
| 1.6356 MFlop/s |
| **Conjugate 最小效率** |
| 1.3230 MFlop/s |

### 优化策略

## Am -- 连浩丞

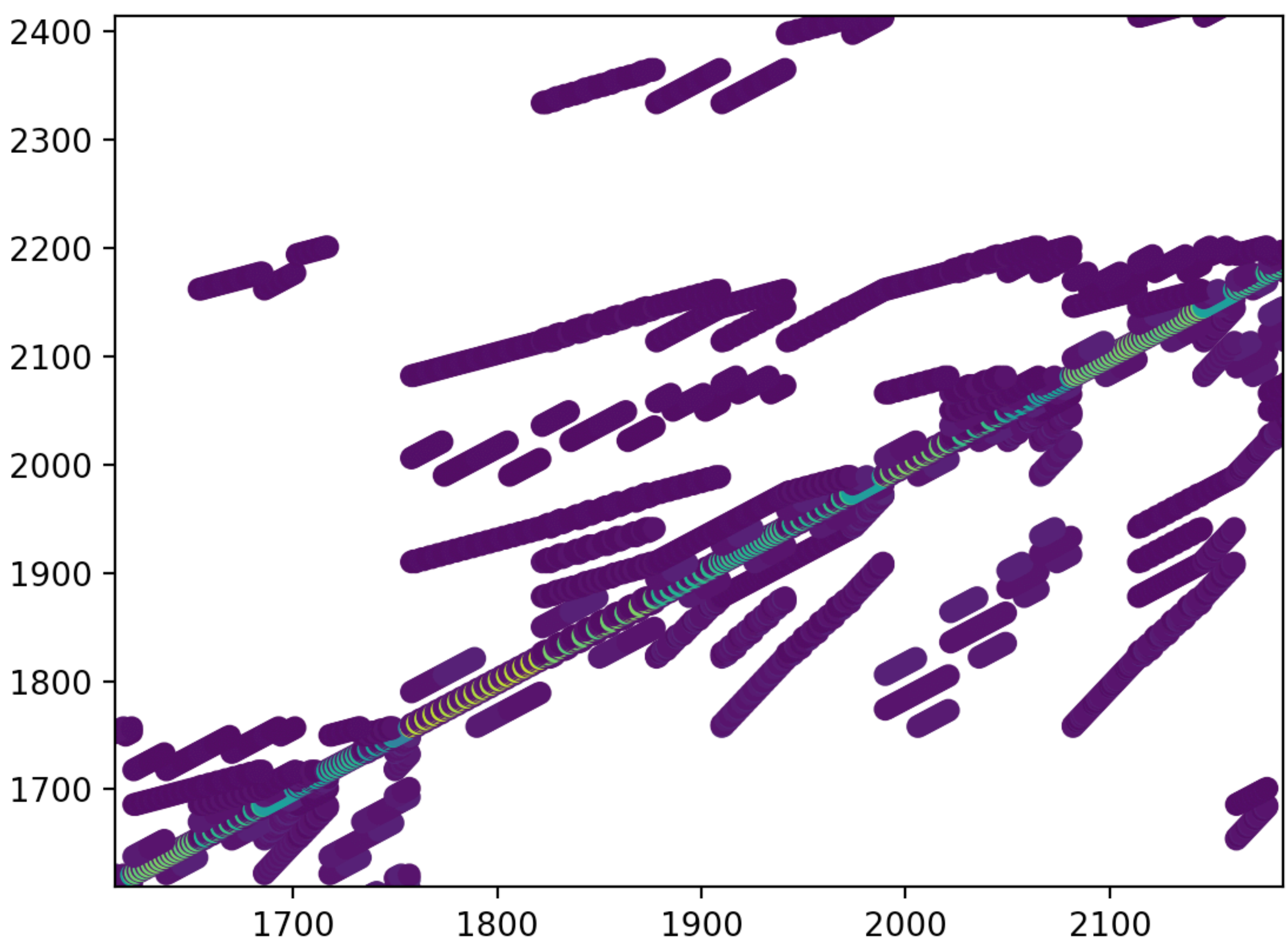
### 特征描述

- 矩阵图像（惊了）




- **对称性**: 结构对称，数值不对称
- 对角元素分布

  从小图上看，Am矩阵中元素多为0元素，属稀疏矩阵。经过**局部放大**后：




有结论:

  - 对角线元素远大于非对角线元素；
  - 对角线两侧相应元素，上三角元素值稍大一些。

## 选择算法

- 优先考虑针对对称矩阵的算法如**Cholesky算法** 。
- 其次考虑迭代法，针对Am矩阵特点，迭代法的解可能收敛速度较快。
- 最后考虑使用其他直接法求解。

### 对可能求解成功的算法一一尝试：

- **Cholesky算法**

```
➜ NumberAnalysis $ ./run.py -if input/Am.txt   4
Use function: cholesky
    * Calculate Matrix A with width:3534 and use time: 51.64s.
    * Solution:
        residual: 7.642481
        not match condition
        solution is too long!
```

  失败: 超出误差

- **迭代法:**
  - **Jacobi算法**

    ```
    ➜ NumberAnalysis $ ./run.py -if input/Am.txt   5
    Use function: jacobi
        * Calculate Matrix A with width:3534 and use time: 9.30s.Iterations:266
        * Solution:
            residual: 0.000046
            match condition
            solution is too long!
    ```

    成功

  - **GS算法**

    ```
    ➜ NumberAnalysis $ ./run.py -if input/Am.txt   6
    Use function: GS
        * Calculate Matrix A with width:3534 and use time: 0.35s.Iterations:10
        * Solution:
            residual: 0.000019
            match condition
            solution is too long!
    ```

    成功

  - **sor算法**

    ```
    ➜ NumberAnalysis $ ./run.py -if input/Am.txt   7
    Use function: sor
        * Calculate Matrix A with width:3534 and use time: 0.33s.Iterations:9
        * Solution:
            residual: 0.000019
            match condition
            solution is too long!
    ```

    成功

  - **Conjugate算法**

```
→ NumberAnalysis $ ./run.py -if input/Am.txt    8
Use function: conjugate
    * Calculate Matrix A with width:3534 and use time: 1.57s.Iterations:29
    * Solution:
        residual: 0.000086
        match condition
        solution is too long!
```

成功

- 直接法
  - Gauss算法

```
→ NumberAnalysis $ ./run.py -if input/Am.txt    1
Use function: gauss
    * Calculate Matrix A with width:3534 and use time: 78.18s.
    * Solution:
        residual: 0.000047
        match condition
        solution is too long!
```

成功

  - Lu_doolittle算法

```
→ NumberAnalysis $ ./run.py -if input/Am.txt    2
Use function: Lu_doolittle
    * Calculate Matrix A with width:3534 and use time: 43.24s.
    * Solution:
        residual: 0.000010
        match condition
        solution is too long!
```

成功

  - Lu_crout算法

```
→ NumberAnalysis $ ./run.py -if input/Am.txt    3
Use function: Lu_crout
    * Calculate Matrix A with width:3534 and use time: 120.94s.
    * Solution:
        residual: 18.117893
        not match condition
        solution is too long!
```

失败

**最终，我选择了sor算法**

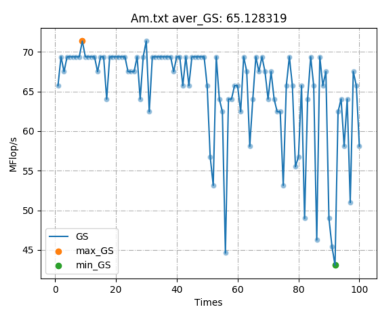
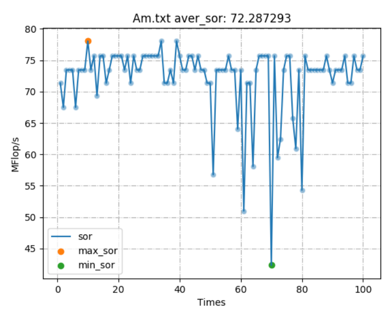
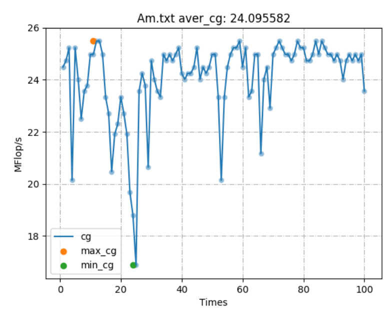
- 代码:

```
void sor_2017011344(const float *A, float *x, const float *b, int n,
int *iter, int maxiter, float threshold) {
    size_t sz = sizeof(float) * n;
    float w = 1.0f;
    memset(x,0,sz);
    for (*iter = 0; *iter < maxiter; ++*iter) {
        float norm = 0;
```

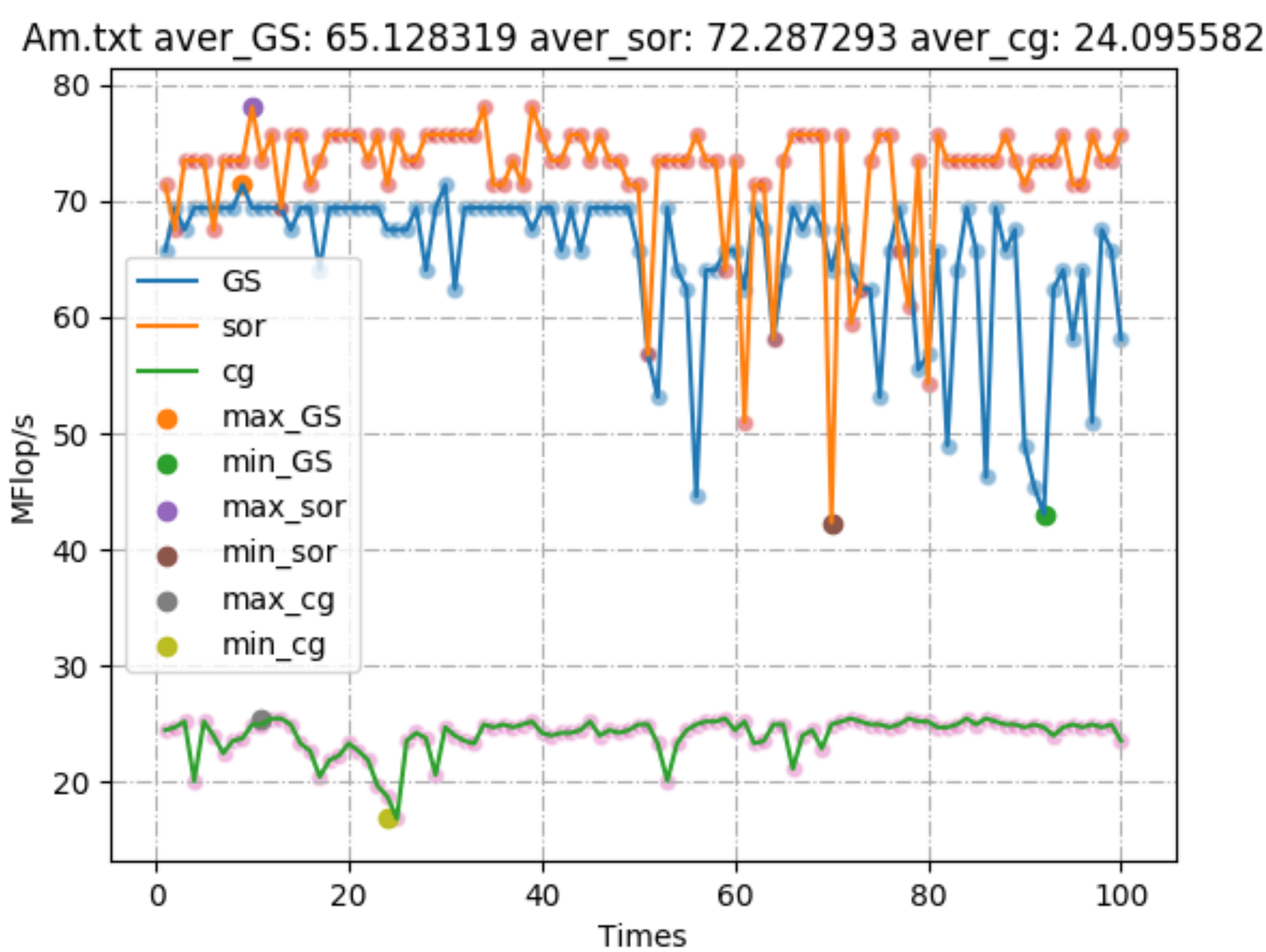
```
        for (int i = 0; i < n; ++i) {
            float xx = x[i], sum = 0;
            for (int j = 0; j < n; ++j)if (i != j) sum += A[pos(i, j)]
* x[j];
            float tmp = (1 - w) * xx + (b[i] - sum) / A[pos(i, i)];
            if(isnormal(tmp))x[i] = tmp;
            norm = fabsf(xx-x[i]) > norm?fabsf(xx-x[i]):norm;
        }
        if (norm < threshold)break;
    }
}
```

## 选择原因

由于直接法与jacobi算法效率明显低于迭代法，所以只针对**迭代法**测试其综合性能并**择优**。

| GS | Sor | cg |
| --- | --- | --- |
|  |  |  |

合并上表：



| **Sor平均性能** | **关系** | **Gs平均性能** | **关系** | **cg平均性能** |
|---|---|---|---|---|
| 72.29 MFlop/s | > | 65.13 MFlop/s | > | 24.10 MFlop/s |
| **Sor最大性能** | **关系** | **Gs最大性能** | **关系** | **cg最大性能** |
| 78.05 MFlop/s | > | 71.36 MFlop/s | > | 25.48 MFlop/s |
| **Sor最小性能** | **关系** | **Gs最小性能** | **关系** | **cg最小性能** |
| 42.33 MFlop/s | < | 43.06 MFlop/s | > | 16.87 MFlop/s |

## 效率分析

提取Sor算法性能表：

| **Sor平均性能** |
|---|
| 72.29 MFlop/s |
| **Sor最大性能** |
| 78.05 MFlop/s |
| **Sor最小性能** |
| 42.33 MFlop/s |

## 优化策略

- 从语言特性上尽可能提升效率：
  - `for(int i=0;i<n;i++)` -> `for(int i=0;i<n;++i)`
  - `malloc(sizeof(float)*n)` ->

    ```
    size_t sz = sizeof(float) * n;
    float *L = malloc(sz), *U = malloc(sz), *y = malloc(sz / n);
    ```

  - 宏函数（直接法可提升三倍效率）

    ```
    int pos(int i,int j,int n){
      return i*n+j;
    }
    ```

    ->

    ```
    #define pos(i,j) i*n+j
    ```