Mathematical Modeling and Simulation

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1. In the pig problem, perform a sensitivity analysis based on the cost per day of keeping the pig. Consider the effect both on the best time to sell and on the resulting profit.

Sensitivity Analysis on the Cost

Assume that instead of the cost of 0.45 cent per day, the cost of keeping the pig is uncertain. Hence, let $C = ct$, then $P = 130 + 1.25t - 0.05t^2 - ct$, where `P` is the revenue from selling the pig and `t` is the time to sell the pig. To get the maximum profit, we use the concept of maxima in calculus. So, we find the derivative of `P`, set it to zero, then solve for `t`. `t` is the best time to sell the pig to get the maximum profit.

$$P' = 1.25 - 0.1t - c$$

$$0 = 1.25 - 0.1t - c$$

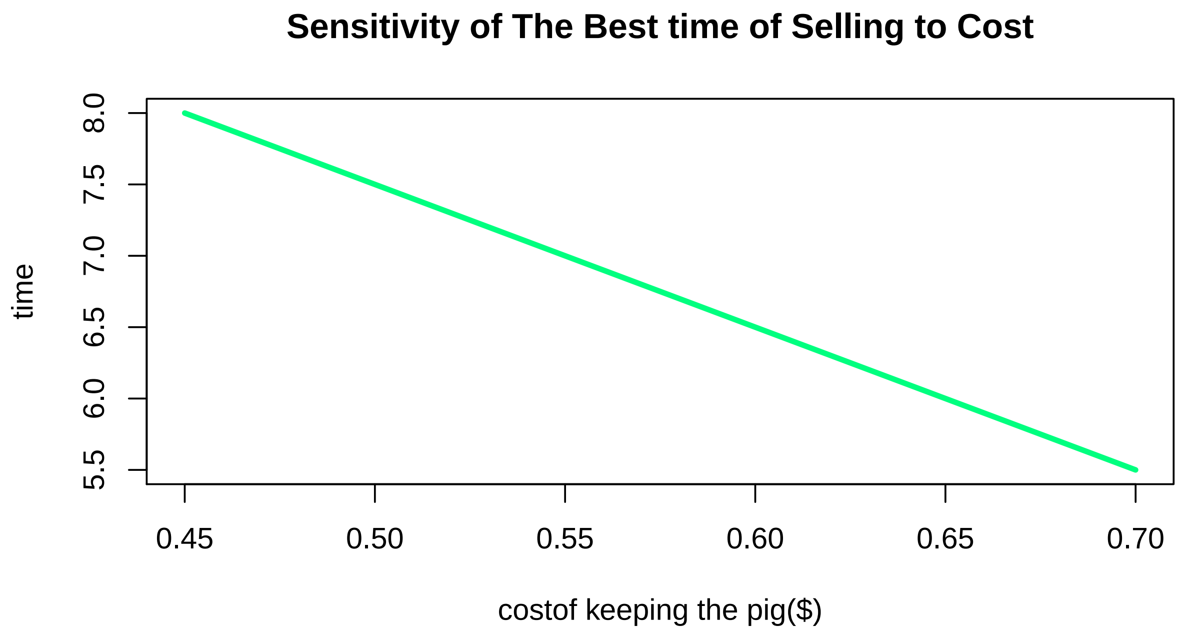
$$0.1t = 1.25 - c$$

$$t = (1.25 - c)/0.1$$

$$t = (25 - 20c)/2$$

Since $t >= 0$, then $(25 - 20c)/2 >= 0$. Solve for c and we get $0 <= c <= 1.25$. Hence, to keep `t` positive, the cost should be between 0 and 1.25.

costperday	time
0.45	8.0
0.50	7.5
0.55	7.0
0.60	6.5
0.65	6.0
0.70	5.5



Based on the table, we can find the ratio of the changes in time and cost or use the derivative of `t`. Thus,

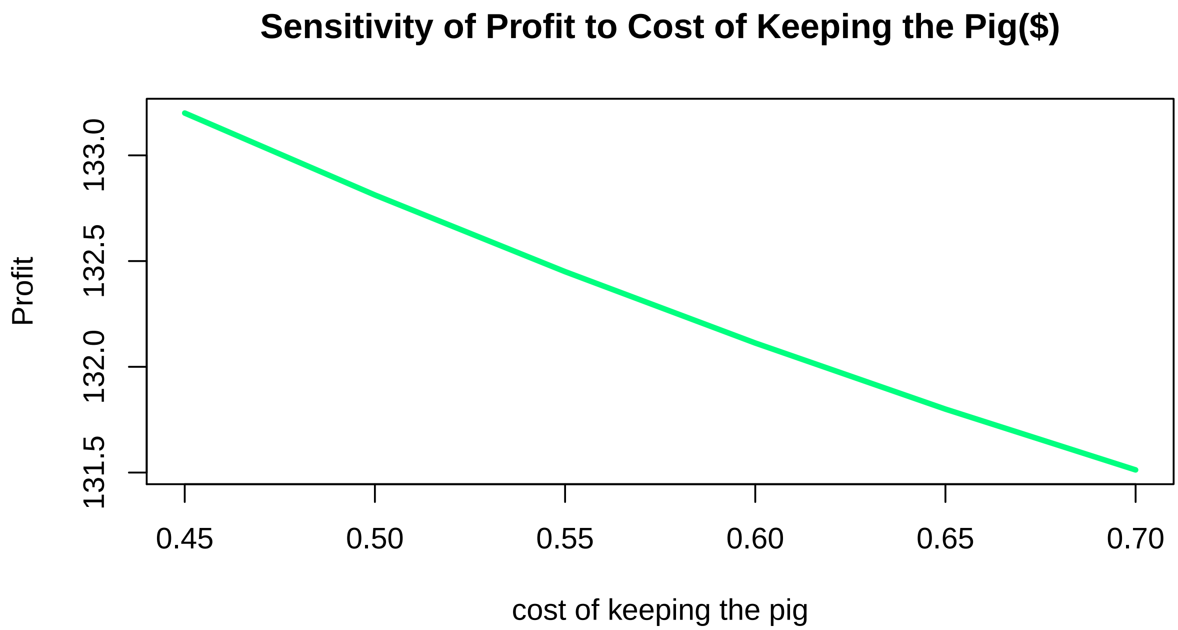
$$dt/dc = -10$$

The sensitivity of the best time to sell to cost is then -0.5625. Hence, a percent increase in the cost of keeping the pig will led to a shorter time, i.e. 0.56% shorter, of selling the pig to get the maximum profit.

To determine the sensitivity of the profit to cost, substitute the equation of `t` in `P`. We then get profit in terms of the cost.

$$P = 5c^2 - 12.5c + 137.8125$$

costperday	time	profit
0.45	8.0	133.2000
0.50	7.5	132.8125
0.55	7.0	132.4500
0.60	6.5	132.1125
0.65	6.0	131.8000
0.70	5.5	131.5125



Using the table above, the ratio of the changes in profit and cost is

$$dP/dc = (133.20 - 132.81) - (0.45 - 0.5)$$
$$dP/dc = -7.8$$

Thus, the sensitivity of the profit to cost of keeping the pig is

$$S(P, c) = -7.8 * (0.45/133.2)$$
$$S(P, c) = -0.027$$

Hence, a percent increase in the cost of keeping the pig will decrease maximum profit by 0.27%.

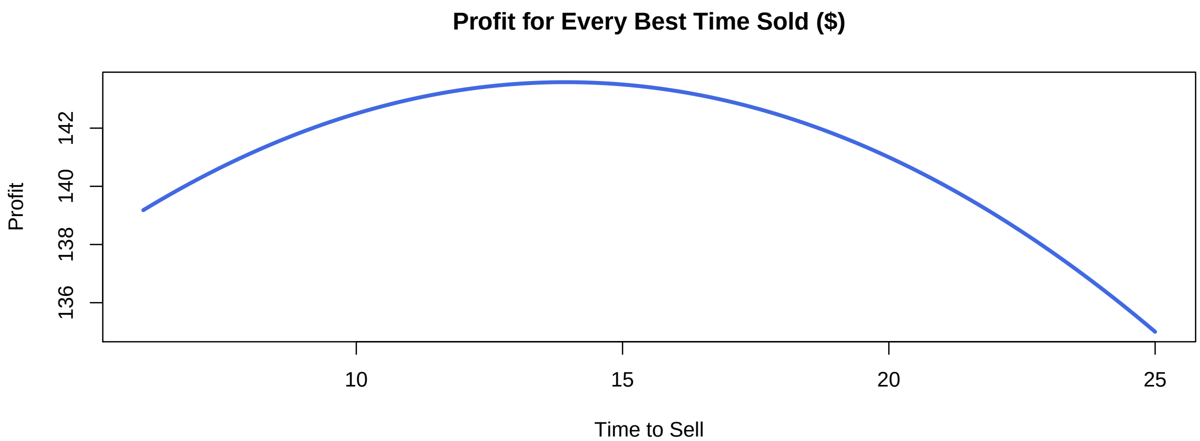
If a new feed costing 60 cents/day would let the pig growth at the rate of 7 lbs/day, would it be worth switching feed?

The new feed has a different cost and it makes the growth rate of the pig increase. This will led to a new equation for the profit.

$$C = 0.60t$$
$$w = 200 + 7t$$
$$P = 130 + 1.95t - 0.07t^2$$

time	profit
6	139.18
8	141.12
10	142.50
12	143.32
14	143.58

time	profit
16	143.28
18	142.42
20	141.00
22	139.02
24	136.48



With the new profit equation, we can get a new maximum profit.

$$P' = 1.95 - 0.14t$$

$$0 = 1.95 - 0.14t$$

$$t = 13.93$$

This is approximately 14 days. With $t = 14days$, $P = 143.58dollars$. Hence, the new feeds can give a better maximum profit. Therefore, it is better to change the feeds.

What is the minimum improvement in growth rate that would make this new feed worthwhile?

To find the minimum improvement in growth rate, we need to take the relationship of the growth rate to time using the profit equation and setting growth rate as a parameter.

$$P = 130 + 0.65gt - 2.6t - 0.01gt^2$$

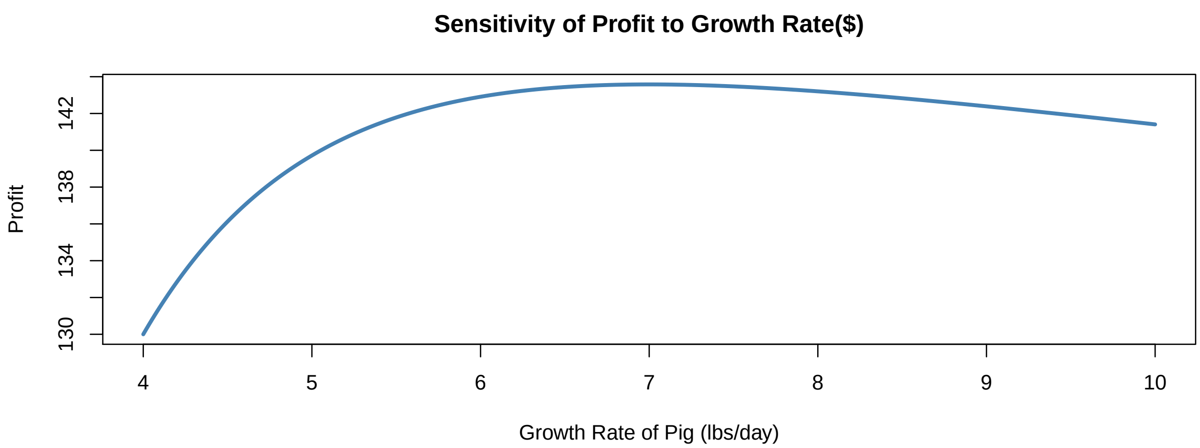
$$P' = 0.65g - 2.6 - 0.02gt$$

$$0 = 0.65g - 2.6 - 0.02gt$$

$$t = (65g - 260)/2g$$

Solve for `g` when $t >= 0$, `0 <= g <= 4`.

growth	time	profit
4	0.00000	130.0000
5	6.50000	139.7175
6	10.83333	142.9097
7	13.92857	143.5804
8	16.25000	143.2031
9	18.05556	142.3881
10	19.50000	141.4075

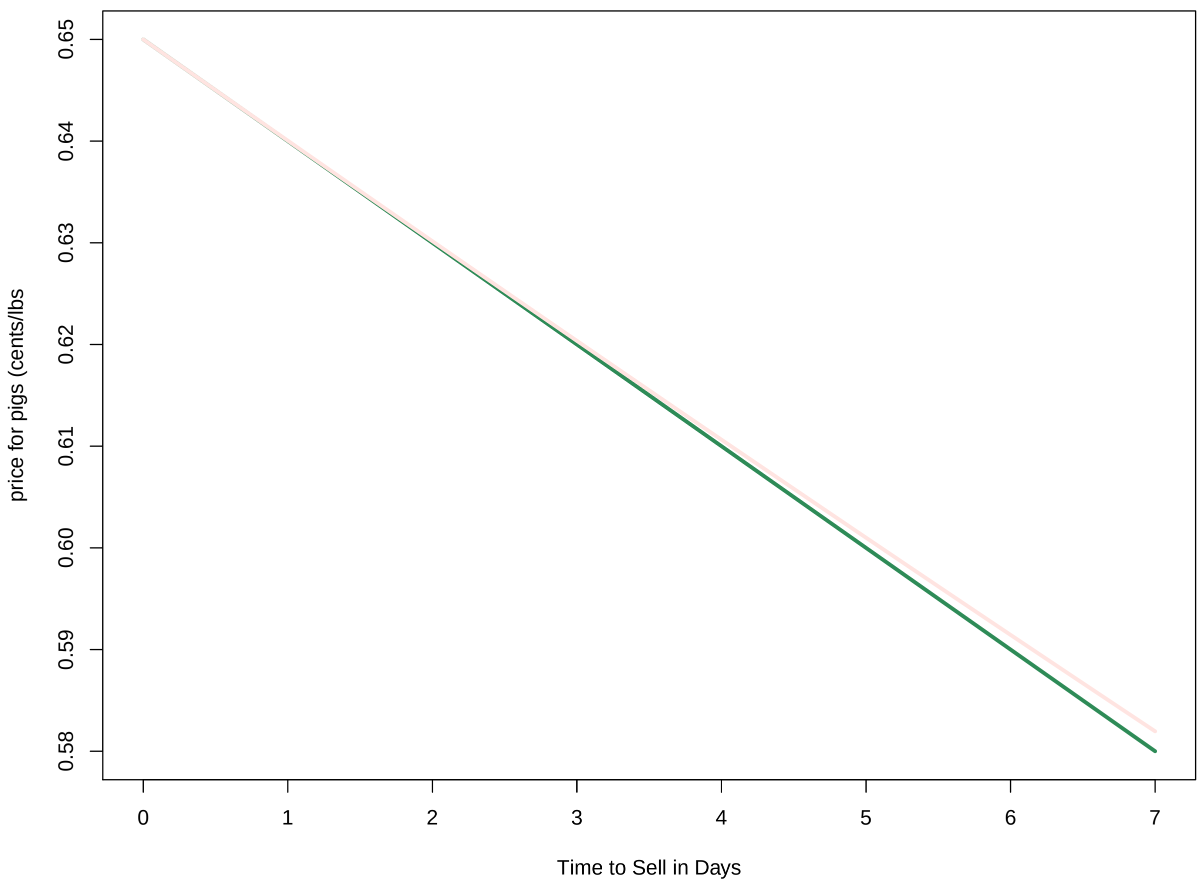

Sensitivity of Profit to Growth Rate($)
Profit
130
134
138
142
4
5
6
7
8
9
10
Growth Rate of Pig (lbs/day)

2. Reconsider the pig problem, but now assume that the price for pigs is starting to level off. Let

$$p = 0.65 - 0.01t + 0.00004t^2$$

Represent the price for pigs (cents/lbs) after `t` days.

a. Graph along with our original price equation. Explain why our original price equation could be considered as an approximation to `p` *for values of* `t` *near zero.*



The original equation is a good approximation to the new price equation for `t` near zero because as `t` approaches zero, $0.00004t^2$ approaches zero and thus making the new equation equivalent to the original equation.

b. Find the best time to sell the pig. Use the Five Step Method and model as a one variable optimization problem.

Step 1: **Variables**

$$t = time(days)$$

$$w = weight of the pig(lbs)$$

$$p = price of the pig(dollars/lbs)$$

$$c = cost of keeping the pig t days(dollars)$$

$$R = revenueobtainedbysellingthepig(dollars)$$

$$P = profitfromsaleofpig(dollars)$$

Assumptions

$$w = 200 + 5t$$

$$p = 0.65 - 0.01t + 0.00004t^2$$

$$c = 0.45t$$

$$R = pw$$

$$P = R - c$$

$$t >= 0$$

Objective: Maximize `P`.

Step2: **Modeling Approach**

"In Calculus, a local maximum point on a function is a point (x,y) on the graph of the function whose y coordinate is larger than all other y coordinates on the graph of points"close to" (x,y). More precisely, (x,f(x)) is a local maximum if there is an interval (a,b) with a= f(z) for every z in (a,b).

Fermat's Theorem states that if f(x) has a local extremum at x = a and f is differentiable at a, then f'(a) = 0.

Any valye of x for which f'(x) is zero or undefined is called a critical value for f.

The critical points of a cubic equation are those values of x where the slope of the cubic function is zero. They are found by setting derivative of cubic equation equal to zero obtaining f'(x)= $3ax^2 + 2bx + C = 0$. The solutions of that equation are the critical points of the cubic equation" - wikipedia

Step 3: **Formulate the Model**

$$P = (0.65 - 0.01t + 0.00004t^2)(200 + 5t) - 0.45t$$

$$P = 130 + 0.8t - 0.042t^2 + 0.0002t^3$$

let f(x) = P; x = t

x >= 0.

Step 4: **Solve the Model**

$$f(x) = 130 + 0.8x - 0.042x^2 + 0.0002x^3$$

$$f'(x) = 0.8 - 0.084x + 0.0006x^2$$

$$0 = 0.8 - 0.084x + 0.0006x^2$$

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## [1]  10.27842+0i 129.72158+0i
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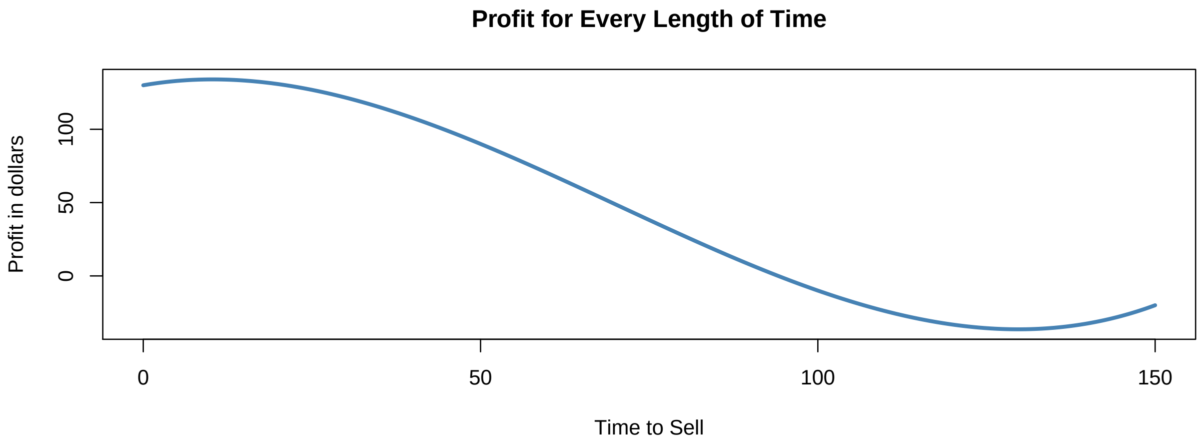

time	profit
5	132.975
10	134.000
50	90.000
100	-10.000
130	-36.400

There are two possible values of `x` and these are the critical points in our graph. Looking at the graph, we can see that approximately 10 days is the local maximum.

Step 5: Thus, the best time to sell the pig is after 10 days with maximum profit of 134 dollars.

c. The parameter 0.00004 represents the rate at which price is leveling off. Conduct a sensitivity analysis on this parameter. Consider both the optimal time to sell and the resulting profit.

Sensitivity analysis on the parameter 0.00004

We will now consider the different value of this parameter, so we set it to `k` making the `p` as

$$p = 0.65 - 0.01x + kx^2$$
$$f(x) = (0.65 - 0.01x + kx^2)(200 + 5x) - 0.45x$$
$$f(x) = 130 + 0.8x - 0.05x^2 + 200kx^2 + 5kx^3$$
$$f'(x) = 0.8 - 0.1x + 400kx + 15kx^2$$
$$0 = 0.8 - 0.1x + 400kx + 15kx^2$$

This is inconvenient to solve, so we will consider different values of `k`.

parameter	time	profit
4e-06	8.1707897+ 0.00000i	133.84174+ 0.0000i
4e-06	1631.8292103+ 0.00000i	758663.75826+ 0.0000i
4e-05	10.2784238+ 0.00000i	134.00278+ 0.0000i
4e-05	129.7215762+ 0.00000i	-36.40278+ 0.0000i
4e-04	-5.0000000+10.40833i	129.80000+12.6288i
4e-04	-5.0000000-10.40833i	129.80000-12.6288i
4e-03	-0.5452241+ 0.00000i	129.55130+ 0.0000i
4e-03	-24.4547759+ 0.00000i	82.39370+ 0.0000i

Looking at the table, we can actually see which values profit we can drop so we can have one value for every parameter k.

	parameter	time	profit
1	4e-06	8.1707897+ 0.00000i	133.8417+ 0.0000i
3	4e-05	10.2784238+ 0.00000i	134.0028+ 0.0000i
5	4e-04	-5.0000000+10.40833i	129.8000+12.6288i
7	4e-03	-0.5452241+ 0.00000i	129.5513+ 0.0000i

Sensitivity Analysis: Parameter k and Optimal Time to Sell

This time, sensitivity is calculated using difference quotient.

$$S(x, k) = (dx/dk)(k/x)$$

$$S(x, k) = (8.1707897 - 10.2784238)/(0.000004 - 0.00004) * (0.00004/10.2784238)$$

The sensitivity of time to parameter `k` is 0.227838. Hence, for every 1 percent increase in parameter k, we will wait 0.23% longer to sell the pig.

Sensitivity Analysis: Parameter k and Profit

We take the profit sensitivity similarly.

$$S(f(x), k) = (df(x)/dk)(k/f(x))$$

$$S(f(x), k) = (133.8417 - 134.00278)/(0.000004 - 0.00004) * (0.00004/134.00278)$$

The sensitivity of profit to parameter `k` is 0.0013356. This is quite small. Thus, the profit is not that sensitivity to parameter `k`.

d. Compare the results of part (b) to the optimal solution contained in the text. Comment on the robustness of our assumptions about price.

The resulting profit is robust even though the predicted sale date is very sensitive to the model.