QML-Mod2-Quantum Mechanics

Riccardo Marega

March 2025

Indice

1 Introduction to Quantum Mechanics -4/04/2025

Newton's laws explained all known macroscopic problems of dynamics of solids. Hamilton had developed a very elegant method to write and solve general mechanical problems. Maxwell's equations could solve almost all known problems related to electricity and magnetism, and of light propagation. However there were a list of thing that couldn't be explained:

- the blackbody radiation
- the photoelectric effect
- the light spectra if gases
- the specific heat of solids
- the stability of matter

1.1 Historical introduction

1.1.1 Blackbody radiation

The experimental evidence was that the emitted light depends only on the temperature of the body. The problem of explaining this phenomenon with classical physics leads to the so called "ultraviolet catastrophe". The solution to this problem arrives when the assumption that the energy is quantized is formulated ($E = h\nu$, where h is the Planck constant).

1.1.2 The photoelectric effect

The experimental evidence was that a metal exposed to light shows a measurable current if the light to witch the metal is exposed is in the UV range (even if it's very weak). Classical physics however predicts that current is proportional to intensity and not to frequency. To explain this phenomenon, Einstein proposed that not only energy was quantized but also light, in this way the photoelectric effect can be explained in the following way: electrons are trapped in the bodies, if a minimum energy is given, the electron is freed form the atom.

1.1.3 Bohr's atomic model

Planetary model of atoms: a nucleus is surrounded by electrons on different orbits. Transitions from two orbits releases energy in form of light.

1.1.4 De Broglie

By combining the Planck's equation and the Einsteins's equation ($E = mc^2$), De Broglie derives:

$$p = \frac{h}{\lambda} = \frac{2\pi h}{\lambda}.$$

The relationship by De Broglie introduce the concept of wave into the atomic problem. The orbits from the Bohr's model are now seen as standing waves, like those formed in drum when you hit it.

1.2 Mathematical fundamentals

We start by formulating the Hamiltonian:

$$H = T + V; T = \frac{p^2}{2m}, V = V(q).$$

We can write the equations of motion as:

$$\frac{dq}{dt} = \frac{\partial H}{\partial p}, \frac{dp}{dt} = -\frac{\partial H}{\partial q}.$$

The Shrodinger's equation is formulated as:

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi.$$

The probabilistic interpretation asserts that

$$\int_a^b |\Psi(x,t)|^2 dt = \text{ \{probability of finding particle between a and b at time t\}}$$

Note that: (1) Ψ is complex and (2) measurements can only return squares of amplitudes: if I have Ψ, I need to multiply it by its complex conjugate Ψ^*.
If I have a particle, the probability of measuring the presence of the particle somewhere in space is 100 %.

$$\int |\Psi(x,t)|^2 dt = 1.$$

This sets the condition for a natural interpretation of the probability. Is also sets another requirement for acceptable wavefunctions: their modulus square need to be integrable (i.e. no diverging wavefunctions).