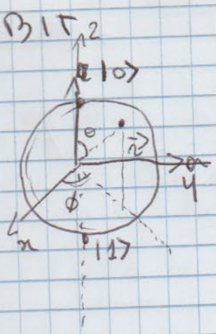
QUBIT



$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

$|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ dove $\alpha, \beta \in \mathbb{C}$

$\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix} =$

$= \left(\cos\left(\frac{\theta}{2}\right)\begin{pmatrix}1\\0\end{pmatrix} + \underbrace{e^{i\phi}}_{\text{local phase}} \sin\left(\frac{\theta}{2}\right)\begin{pmatrix}0\\1\end{pmatrix}\right) \underbrace{e^{i\gamma}}_{\text{global phase}}$

(θ, ϕ, n) coordinate sferiche

$|\vec{n}| = 1$

$e^{i\phi}$

$\langle 0| = (1 \quad 0)$

$\langle 1| = (0, 1)$

$\alpha^2 + \beta^2 = 1$

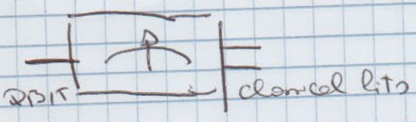
Qubit state: $\mathbb{C}^2$

———— $|1\rangle$

sistema a due livelli

———— $|0\rangle$

Misurazione



$\text{prob}(\overset{\text{lo classico}}{0}) = |\langle 0|\psi\rangle|^2 = \left|(1 \quad 0)\begin{pmatrix}\alpha\\\beta\end{pmatrix}\right|^2 = |\alpha|^2$

$\text{prob}(1) = |\langle 1|\psi\rangle|^2 = \left|(0 \quad 1)\begin{pmatrix}\alpha\\\beta\end{pmatrix}\right|^2 = |\beta|^2$.

$|\alpha|^2 + |\beta|^2 = \text{prob}(0) + \text{prob}(1) = 1$

Caso in cui $\alpha = \beta = \frac{1}{\sqrt{2}}$:

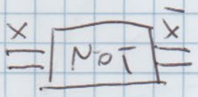
$|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$

Caso $\alpha = \cos\left(\frac{\theta}{2}\right)$ $\qquad |\alpha|^2 = \cos^2\left(\frac{\theta}{2}\right)$

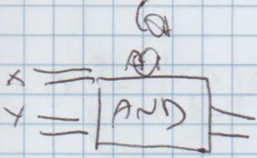
$\beta = \sin\left(\frac{\theta}{2}\right) e^{i\phi}$ $\qquad |\beta|^2 = \sin^2\left(\frac{\theta}{2}\right)$

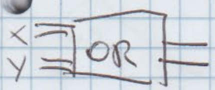
$|e^{2i\phi}| = 1$

GATES



X	$\bar{X}$
0	1
1	0





(porte classiche)

QUANTUM GATES

$M \in \mathbb{C}^{2\times 2}$ $\qquad \begin{pmatrix} a_0 & a_1 \\ a_2 & a_3 \end{pmatrix}$

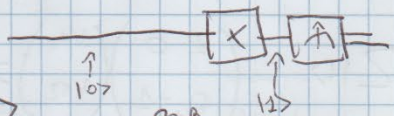
Matrice complessa unitaria

$M^\dagger$ = trasposta coniugata

Matrice unitaria: M : $MM^\dagger = M^\dagger M = I$

Un quantum gate è una matrice complessa unitaria

X GATE



$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $X|1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$

prob(0) $= |\langle 0|1\rangle|^2 = 0$

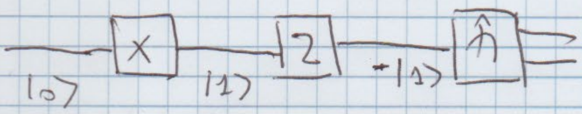
prob(1) $|\langle 1|1\rangle|^2 = 1$

(equivalente del NOT classico)

Z - GATE

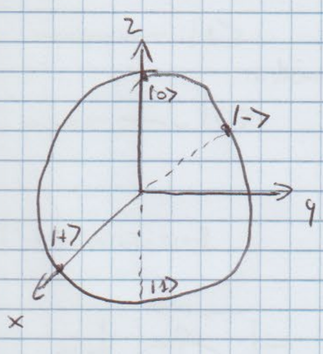
$Z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ $\qquad Z|0\rangle = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix} = \begin{pmatrix}1\\ 0\end{pmatrix} = |0\rangle$

$Z|1\rangle \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = \begin{pmatrix}0\\ -1\end{pmatrix} = -|1\rangle$



$prob(0) = |\langle 0|\psi\rangle|^2 = |-\langle 0|1\rangle|^2 = |\langle 0|1\rangle|^2 = \left|(1\ 0)\begin{pmatrix}0\\ -1\end{pmatrix}\right|^2 = 0$

$prob(1) \left|(0\ 1)\begin{pmatrix}0\\ -1\end{pmatrix}\right|^2 = 1$



$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$prob_{|-\rangle}(0) = |\langle 0|-\rangle|^2 = \left|(1\ 0)\begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix}\right|^2 = \frac{1}{2}$

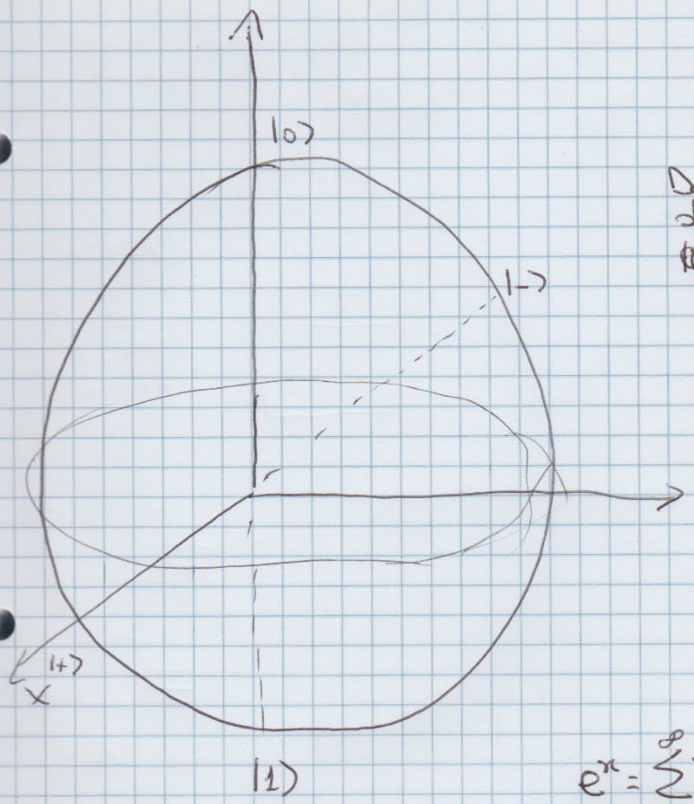
$prob_{|-\rangle}(1) = |\langle 1|-\rangle|^2 = \left|(0\ 1)\begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix}\right|^2 = \frac{1}{2}$

$X|+\rangle = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix} = |+\rangle$

$X|-\rangle = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix} = \begin{pmatrix}-1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix} = -|-\rangle$

$Z|+\rangle = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix} = |-\rangle$

$Z|-\rangle = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\ -1/\sqrt{2}\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix} = |+\rangle$



Da un punto di vista della sfera di Bloch,
$|0\rangle$ e $-|0\rangle$,
$|1\rangle$ e $-|1\rangle$
$|+\rangle$ e $-|+\rangle$
$|-\rangle$ e $-|-\rangle$,
ecc.
sono la stessa cosa, in quanto il "−" fa riferimento alla fase globale, che è ininfluente relativamente alla sfera di Bloch

$$e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!} \quad \text{(taylor)}$$

$$e^{i\gamma A} = \sum_{j=0}^{\infty} \frac{(iA)^j}{j!}$$ se $A^2 = I$ (idempotente), allora

$$e^{i\gamma A} = \cos(\gamma) I + i \sin(\gamma) A$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \mathbb{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Porte di Pauli
- hermitiane ($M = M^\dagger$)
- unitarie ($MM^\dagger = \mathbb{I}$)
- idempotenti: ($M^2 = I$)
- autovalori: ± 1
- $\det(M) = \pm 1$

Rotaz. sull'asse x, y o z

$-\boxed{R_x(\theta)}- \quad e^{-i\frac{\theta}{2}X} = \cos(\theta/2) I - i\,\mathrm{sen}(\theta/2) X$

$-\boxed{R_y(\theta)}- \quad e^{-i\frac{\theta}{2}Y} = \cos(\theta/2) I - i\,\mathrm{sen}(\theta/2) Y$

$-\boxed{R_z(\theta)}- \quad e^{-i\frac{\theta}{2}Z} = \cos(\theta/2) I - i\,\mathrm{sen}(\theta/2) Z$

$-\boxed{R_n(\theta)}- \quad e^{-i\frac{\theta}{2}(\vec{n}\cdot\vec{\sigma})}$ SCALAR PRODUCT

$\vec{\sigma} = (X, Y, Z)$