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Random Variables

param $\rightarrow \boxed{X} \rightarrow$ random number / always a number $\in$ countable set of possible values

e.g. $X = \begin{cases} 1, & \text{if "heads"} \\ 0, & \text{otherwise} \end{cases}$ $\quad X\begin{pmatrix} 1 & 0 \\ 0.5 & 0.5 \end{pmatrix}$

e.g. X = nr. on a die after rolling

$X \begin{pmatrix} x_i \\ p_i \end{pmatrix}_{i \in I}$ $\quad X\begin{pmatrix} i \\ \frac{1}{6} \end{pmatrix}_{i \in \overline{1,6}}$

values

corresp. prob.

distribution

function

pdf

prob. distrib function

$X\begin{pmatrix} 1 & 0 \\ p & 1-p \end{pmatrix} \rightarrow$ Bernoulli(p)

1. A computer virus is trying to corrupt two files. The first file will be corrupted with probability 0.4. Independently of it, the second file will be corrupted with probability 0.3. Find the probability distribution function (pdf) of X, the number of corrupted files.

!

$$X \begin{pmatrix} 0 & 1 & 2 \\ 0.42 & 0.46 & 0.12 \end{pmatrix}$$ multiplication

X = no. of corrupted files out of 2

$$\underbrace{q_1 \cdot q_2}_{(P_1 \cap P_2) \overset{indep}{=} product} = (1-0.4)\cdot(1-0.3) = 0.42$$

$$q_1 \cdot p_2 + p_1 \cdot q_2 = (0.6)\cdot(0.3) + (0.4)\cdot(0.7)$$

$$= 0.18 + 0.28 = 0.46$$

$$p_1 \cdot p_2 = (0.4)\cdot(0.3) = 0.12$$

Careful, use more thorough explanations when solving

5. A number is picked randomly out of 1, 2, 3, 4 and 5. Let X denote the number picked. Let Y be 1 if the number picked was 1, 2 if the number was prime and 3, otherwise.

a) Find the pdf's of X, Y;

b) Find the pdf's of $X+Y$, XY.

X = random number from the set $\{1,2,3,4,5\}$

$$X \begin{pmatrix} i \\ 0.2 \end{pmatrix}_{i \in \overline{1,5}}$$

$$Y = \begin{cases} 1, & X=\{1\} \\ 2, & X=\{2,3,5\} \\ 3, & X=\{4\} \end{cases}$$

$$Y \begin{pmatrix} 1 & 2 & 3 \\ p_1 & p_2 & p_3 \end{pmatrix}$$

0.2 0.6 0.2

$$p_1 = 0.2 \text{ from } X = \underbrace{P(Y=1 \mid X=1)}_{1} \cdot \underbrace{P(X=1)}_{0.2}$$

$$p_2 = P(X=2) + P(X=3) + P(X=5) = 3 \cdot (0.2) = 0.6$$

$$p_3 = P(X=4) = 0.2$$

Uniform Discrete Distribution with $n=5$

$$X = \begin{pmatrix} i \\ 0.2 \end{pmatrix}_{i \in \overline{1,5}} \qquad Y \begin{pmatrix} 1 & 2 & 3 \\ 0.2 & 0.6 & 0.2 \end{pmatrix}$$

$$X+Y=\begin{pmatrix} 2 & 4 & 5 & 7 \\ 0.2 & 0.2 & 0.2 & 0.4 \end{pmatrix}$$

$$P(X+Y=2)=P(X=1,Y=1)=P(X=1)=0.2$$

$$P(X+Y=4)=P(X=2,Y=2)=P(X)=2$$

$$P(X+Y=5)=P(X=3,Y=2)=0.2$$

$$P(X+Y=7)=\underbrace{P(X=5,Y=2)}_{P(X=5)}+\underbrace{P(X=4,Y=3)}_{P(X=4)}=0.4$$

$$X\cdot Y=\begin{pmatrix} 1 & 4 & 6 & 10 & 12 \\ \_ & \_ & \_ & \_ & \_ \end{pmatrix}$$

2. A coin is flipped 3 times. Let X denote the number of heads that appear.
a) Find the pdf of X. What type of distribution does X have?
b) Find $P(X \leq 2)$ and $P(X < 2)$.

X = number of heads in 3 coin flips

$$X=\begin{pmatrix} 0 & 1 & 2 & 3 \\ 0{,}125 & 0{,}375 & 0.375 & 0.125 \end{pmatrix}$$ Binomial Distrib

$$P(X=0) = C_3^0 \cdot (0.5)^0 \cdot (0.5)^3 = 0,125$$

$$P(X=1) = C_3^1 \cdot (0.5)^1 \cdot (0.5)^{3-1} = 3 \cdot (0.5) \cdot (0.25)$$

$$= 0.375$$

$$P(x=2) = C_3^2 \cdot (0.5)^2 \cdot (0,5)^{3-2} = 0.375$$

$$P(x=3) = P(x=0)$$

b) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) =$

$$= 1 - P(x=3) = 1 - 0,125 = 0,875$$

$$P(X < 2) = P(x=0) + P(x=1) = 0.5$$

$$X = \left(\begin{array}{c} k \\ C_3^k \cdot \left(\frac{1}{2}\right)^3 \end{array} \right)_{k} = \overline{0,3}$$

$P(X=k)$ prefix distrib pdf

binopdf(k, params)

$P(X \le k)$ prefix distrib cdf

binocdf(k, params)

3. (New Accounts) Customers of an internet service provider initiate new accounts at the average rate of 10 accounts per day.
a) Find the probability that more than 8 new accounts will be initiated today;
b) Find the probability that at most 16 new accounts will be initiated within 2 days.

Solve octave online

a) poisscdf(8, 10) $= P(X \le 8) = 0.3328$

$P(X > 8) = 1 - P(X \le 8) = 1 - 0.3328 = 0,6672$

b) poisscdf(16, 20) $= P(X \geq 16) = 0.2211$

4. It was found that the probability to log on to a computer from a remote terminal is 0.7. Let X denote the number of attempts that must be made to gain access to the computer:
a) Find the pdf of X;
b) Find the probability (express it in terms of the cdf F_X) that at most 4 attempts must be made to gain access to the computer;
c) Find the probability that at least 3 attempts must be made to gain access to the computer.

X = no. of attempts

shifted geometric distrib

a) $\text{pdf } X = \begin{pmatrix} k \\ p \cdot q^{k-1} \end{pmatrix}_{k=\{1,\dots\}}$

Geometric Model: The probability of the 1^{st} success occurring after k failures in a sequence of Bernoulli trials with probability of success p ($q = 1 - p$), is

$p_k = pq^k$.

b) $P(X \leq 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4) = 0{,}9919$

$P(X=1) = \text{geopdf}(0, 0.7) = 0.7$
$P(X=2) = \text{geopdf}(1, 0.7) = 0.2100$
$P(X=3) = \text{geopdf}(2, 0.7) = 0{,}063$
$P(X=4) = \text{geopdf}(3, 0.7) = 0{,}0189$

just compute geocdf$(3, 0.7)$ = $P(X \leq 4)$

c) $P(X \geq 3) = 1 - \underbrace{P(X \leq 2)}_{P(X=1)+P(X=2)} = 1 - 0.7 - 0{,}21 = 0.09$

$X-1 \in Geo(p) \quad X-1 = \begin{pmatrix} k \\ p\cdot q^{k} \end{pmatrix}_{k=0,1,\dots} \Rightarrow$

$$\Rightarrow X = \begin{pmatrix} k+1 \\ p\cdot q^{k} \end{pmatrix}_{k=0,1,2,\dots}$$

↳ don't change the prob!

this can be written as $\begin{pmatrix} k \\ p\cdot q^{k-1} \end{pmatrix}_{k=1,2\dots}$