15.01.2024

5) Boolean functions using Quine's method

$f(0,1,0) = f(0,1,1) = f(1,0,1) = 0$

	x_1	x_2	x_3	$f(x_1,x_2,x_3)$
1.	0	0	0	1
2.	0	0	1	1
3.	0	1	0	0
4.	0	1	1	0
5.	1	0	0	1
6.	1	0	1	0
7.	1	1	0	1
8.	1	1	1	1

$DCF(f) = m_0 \vee m_1 \vee m_4 \vee m_6 \vee m_7 =$

$= \bar{x}_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_1 \bar{x}_2 x_3 \vee x_1 \bar{x}_2 \bar{x}_3 \vee x_1 x_2 \bar{x}_3 \vee$

$\vee\ x_1 x_2 x_3$

$S(f) = \{(0,0,0), (1,0,0), (0,0,1), (1,1,0), (1,1,1)\}$

↳ support of function

order with respect to no. of "1" values in the tupples.

Factorization process

		X_1	X_2	X_3	
representation of function	I	0	0	0	m_0 ✓
	II	1	0	0	m_4 ✓
		0	0	1	m_1 ✓
	III	1	1	0	m_6 ✓
	IV	1	1	1	m_7 ✓
simple factorisations	V = I + II	–	0	0	$m_0 \vee m_4 = \overline{X}_2 \overline{X}_3 = max_1$
		0	0	–	$m_0 \vee m_1 = \overline{X}_1 \overline{X}_2 = max_2$
	VI = II + III	1	–	0	$m_4 \vee m_6 = \overline{X}_1 \overline{X}_3 = max_3$
	VII = III + IV	1	1	–	$m_6 \vee m_7 = X_1 X_2 = max_4$

m_1 and m_6 are not neighbours bcs they have more than one variable (this applies for SIMPLE factorisation)

We cannot apply double factorisations

$M(f) = \{ max_1, max_2, max_3, max_4 \}$

min terms \ max monoms	max_1	max_2	max_3	max_4
m_0	*	*		
m_4	*		*	
m_1		*		
m_6			*	*
m_7				*

$C(f) = \{ max_2, max_4 \}$

- $g = max_2 \vee max_4$

$M(f) \neq C(f) \neq \emptyset \Rightarrow 2^{nd}$ simplif. case

m_4 is covered by max_1 or max_3 $\Rightarrow$

$\Rightarrow$ There are two simplified forms:

$$f_s^1 = g \vee max_1 = max_2 \vee max_4 \vee max_1 =$$
$$= \bar{x}_1 \bar{x}_2 \vee x_1 x_2 \vee \bar{x}_2 x_3$$

$$f_s^2 = g \vee max_3 = max_2 \vee max_4 \vee max_3 =$$

$$= \bar{x}_1\bar{x}_2 \vee x_1x_2 \vee x_1\bar{x}_3$$

6.1) $f(x_1,x_2,x_3) = \underset{\bar{x}_1\bar{x}_2\bar{x}_3}{m_0} \vee \underset{\bar{x}_1x_2x_3}{m_3} \vee \underset{x_1\bar{x}_2\bar{x}_3}{m_4} \vee \underset{x_1\bar{x}_2x_3}{m_5} \vee \underset{x_1x_2\bar{x}_3}{m_6} \vee \underset{x_1x_2x_3}{m_7}$

$S_f = \{ \underbrace{(0,0,0)}_{m_0}, \underbrace{(1,0,0)}_{m_4}, \underbrace{(0,1,1)}_{m_3}, \underbrace{(1,0,1)}_{m_5}, \underbrace{(1,1,1)}_{m_7} \}$

	x_1	x_2	x_3	
I	0	0	0	m_0
II	1	0	0	m_4
III	0	1	1	m_3
	1	0	1	m_5
	1	1	0	m_6
IV	1	1	1	m_7
V = I + II	–	0	0	$m_0 \vee m_4 = \bar{x}_2\bar{x}_3 = max_1$
VI = II + III	1	0	–	$m_4 \vee m_5 = x_1\bar{x}_2$ ✓
	1	–	0	$m_4 \vee m_6 = x_1\bar{x}_3$ ✓
VII = III + IV	–	1	1	$m_3 \vee m_7 = x_2x_3 = max_2$
	1	–	1	$m_5 \vee m_7 = x_1x_3$ ✓
	1	1	–	$m_6 \vee m_7 = x_1x_2$ ✓

Simple fact.

$\overline{VIII} = \overline{VI} + \overline{VII}$	1	–	–	$m_4 \vee m_5 \vee m_6 \vee m_7 = x_1 = max_3$

$M(f) = \{ max_1, max_2, max_3 \}$

	max_1	max_2	max_3
m_0	*		
m_4	*		*
m_3		*	
m_5			*
m_6			*
m_7		*	*

$C(f) = \{ max_1, max_2, max_3 \}$

$M(f) = C(f) \Rightarrow$ 1st simplif case $\Rightarrow$ unique minimal simplif form

$f_S = max_1 \vee max_2 \vee max_3 =$

$= \bar{x}_2 \bar{x}_3 \vee x_2 x_3 \vee x_1$

7.1) Simplify the following boolean function of 4 var.
(Quine)

$$f(1,1,1,1) = f(1,1,0,1) = f(0,1,1,1) = f(1,1,0,0) =$$
$$= f(0,1,0,0) = f(0,0,0,0) = f(0,0,0,1) = f(0,0,1,1) = 1$$

$$f = m_{15} \vee m_7 \vee m_{13} \vee m_3 \vee m_{12} \vee m_1 \vee m_4 \vee m_0$$

$$S = \{(1,1,1,1), (0,1,1,1), (1,1,0,1), (0,0,1,1), (1,1,0,0),$$
$$(0,1,0,0), (0,0,0,1), (0,0,0,0)\}$$

OBS: S needs to be written either in ascending or descending order based on the no. of "1"s

	x_1	x_2	x_3	x_4		
I	1	1	1	1	m_{15}	✓
II	0	1	1	1	m_7	✓
	1	1	0	1	m_{13}	✓
III	0	0	1	1	m_3	✓
	1	1	0	0	m_{12}	✓

$\overline{\underline{IV}}$	0	1	0	0	m_4 ✓
	0	0	0	1	m_1 ✓
$\overline{\underline{V}}$	0	0	0	0	m_0 ✓
$VI = I + II$	–	1	1	1	$m_{15} \vee m_7 = x_2x_3x_4 = max_1$
	1	1	–	1	$m_{15} \vee m_{13} = x_1x_2x_4 = max_2$
$VII = II + III$	0	–	1	1	$m_7 \vee m_3 = \bar{x}_1x_3x_4 = max_3$
	1	1	0	–	$m_{13} \vee m_{12} = x_1x_2\bar{x}_3 = max_4$
$VIII = III + IV$	0	0	–	1	$m_3 \vee m_1 = \bar{x}_1\bar{x}_2x_4 = max_5$
	–	1	0	0	$m_{12} \vee m_4 = x_2\bar{x}_3\bar{x}_4 = max_6$
$IX = IV + V$	0	0	0	–	$m_1 \vee m_0 = \bar{x}_1\bar{x}_2\bar{x}_3 = max_7$
	0	–	0	0	$m_4 \vee m_0 = \bar{x}_1\bar{x}_3\bar{x}_4 = max_8$

No double factorization. We have 8 simple factorizations.

	max_1	max_2	max_3	max_4	max_5	max_6	max_7	max_8
~~m_{15}~~	*	*						
~~m_7~~	*		*					
~~m_{13}~~		*		*				
~~m_3~~			*		*			
~~m_{12}~~				*		*		

$C(f) = \emptyset \Rightarrow 3^{rd}$ simplif case

$\exists$ more equivalent simplif forms!

$f_S^1 = max_1 \vee max_4 \vee max_5 \vee max_6$

$= \dots$

$f_S^2 = max_3 \vee max_4 \vee max_5 \vee max_8$

$= \dots$

8) <u>Moisil's method</u>

$f(x_1, x_2, x_3) = m_0 \vee m_3 \vee m_4 \vee m_6 \vee m_7$

$= \bar{x}_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_1 x_2 x_3 \vee x_1 \bar{x}_2 \bar{x}_3 \vee x_1 x_2 \bar{x}_3 \vee$

$\vee \; x_1 x_2 x_3$

Ascending order $= \{ \overset{m_0}{(0,0,0)}, \overset{m_4}{(1,0,0)}, \overset{m_3}{(0,1,1)}, \overset{m_6}{(1,1,0)},$

$\underset{m_7}{(1,1,1)} \}$

	x_1	x_2	x_3	
I	0	0	0	m_0 ✓
II	1	0	0	m_4 ✓
III	0	1	1	m_3 ✓
	1	1	0	m_6 ✓
IV	1	1	1	m_7 ✓
V = I + II	–	0	0	$m_0 \vee m_4 = \overline{x_2}\,\overline{x_3} = max_1$
VI = II + III	1	–	0	$m_4 \vee m_6 = x_1\overline{x_3} = max_2$
VII = III + IV	–	1	1	$m_3 \vee m_7 = x_2 x_3 = max_3$
	1	1	–	$m_6 \vee m_7 = x_1 x_2 = max_4$

$M(f) = \{ max_1, max_2, max_3, max_4 \}$

$p_i =$ "max_i belongs to a simplified form", $i = \overline{1,4}$

m_0 is covered by $max_1 \Rightarrow p_1 \equiv T$

m_4 is covered by max_1 or $max_2 \Rightarrow p_1 \vee p_2 \equiv T$

m_6 is covered by max_2 or $max_4 \Rightarrow p_2 \vee p_4 \equiv T$

m_3 is covered by $max_3 \Rightarrow p_3 \equiv T$

m_7 is covered by max_3 or max_4 $\Rightarrow$ $p_3 \vee p_4 \equiv T$

$$\underbrace{p_1}_{=} \wedge \underbrace{(p_1 \vee p_2)}_{=} \wedge (p_2 \vee p_4) \wedge p_3 \wedge (p_3 \vee p_4) \equiv T$$

$T \wedge T \wedge T \wedge T \wedge T$

$a \wedge (a \vee b) = a$ (Absorbtion law)

$$\underbrace{p_1 \wedge (p_2 \vee p_4) \wedge p_3}_{CNF} \equiv T$$

We need a DNF $\Rightarrow$ we apply distributive laws

$$\underbrace{(p_1 \wedge p_2 \wedge p_3) \vee (p_1 \wedge p_4 \wedge p_3)}_{DNF} \equiv T$$

$p_1 \wedge p_2 \wedge p_3 \equiv T$ provides the simplified form:

$$f_s^1 = max_1 \vee max_2 \vee max_3 =$$

$$= \bar{x}_2 \bar{x}_3 \vee x_1 \bar{x}_3 \vee x_2 x_3$$

$p_1 \wedge p_4 \wedge p_3 \equiv T$ provides the simplified form:

$$f_s^2 = max_1 \vee max_4 \vee max_3$$

$$= \bar{x}_2\bar{x}_3 \vee x_1x_2 \vee x_2x_3$$

<u>OBS</u>: In Moisil's method we DO NOT have simplif. cases. But, we can notice that, $C(f) = \{mcx_1, mcx_3\}$