

OBS: rapid conv. are used when both bases are powers of 2 | 16.10.2023

## ! Substitution method

$N(b) = N(h)$ , when  $b \in h$

Ex ①  $4321, 23_{(5)} = ?_{(8)}$  with 2 digits  
at the fractional part in base 8

$\hookrightarrow 2^{10} - 1 - 2$

$$4321, 23_{(5)} = 4_{(5)} \cdot 5^3 + 3_{(5)} \cdot 5^2 + 2_{(5)} \cdot 5^1 + 1_{(5)} \cdot 5^0 +$$

$$+ 2_{(5)} \cdot 5^{-1} + 3_{(5)} \cdot 5^{-2} \quad \begin{array}{l} \text{rewrite sum} \\ \text{in base 8} \end{array}$$

$$\left| \begin{array}{l} 4_{(5)} = 4_{(8)} \\ 3_{(5)} = 3_{(8)} \\ 2_{(5)} = 2_{(8)} \\ 1_{(5)} = 1_{(8)} \\ 5_{(10)} = 5_{(8)} \end{array} \right| \begin{array}{l} = \underbrace{4_{(8)} \cdot 5_{(8)}^3}_{76_{(8)}} + \underbrace{3_{(8)} \cdot 5_{(8)}^2}_{113_{(8)}} + \underbrace{2_{(8)} \cdot 5_{(8)}^1}_{2_{(8)}} + \underbrace{1_{(8)} \cdot 5_{(8)}^0}_{1_{(8)}} \\ + \underbrace{2_{(8)} \cdot 5_{(8)}^{-1}}_{0,32_{(8)}} + \underbrace{3_{(8)} \cdot 5_{(8)}^{-2}}_{0,075_{(8)}} \\ \approx 764_{(8)} + 113_{(8)} + 2_{(8)} + 0,32_{(8)} + 0,075_{(8)} \\ \approx 1112,40_{(8)} \end{array}$$

## Calculations

a)

$$\frac{2_{(8)}}{5_{(8)}} \cdot \underline{12_{(8)}}$$

$$2_{(8)} \cdot 5_{(8)} = 2 \cdot 5 = 10_{(10)}$$

10 div 8 = 1

10 mod 8 = 2

b)

$$5_{(8)}$$

$$5_{(8)} \cdot 5_{(8)} = 5 \cdot 5 = 25$$

$$\overline{5_{(8)}}.$$

$$25 \text{ div } 8 = 3$$

$$\overline{31}_{(8)}.$$

$$\overline{3}_{(8)}$$

$$\overline{11}_{(8)}.$$

c)

$$31_{(8)}.$$

$$\overline{5}_{(8)}$$

$$\overline{17}_{(8)}$$

$$\begin{array}{r} 3^2 \\ 175_{(8)} \\ \hline \end{array}$$

$$\begin{array}{r} 175_{(8)} \\ \hline \end{array}$$

$$\begin{array}{r} 764_{(8)} \\ \hline \end{array}$$

d)

$$2_{(8)} \cdot 5^{-1}_{(8)} = 2_{(8)} / 5_{(8)}$$

$$2, 00_{(8)} \left| \begin{array}{r} 5_{(8)} \\ \hline \end{array} \right.$$

$$20_{(8)} = 2 \cdot 8 + 0 = 16_{(10)}$$

$$20 \left| \begin{array}{r} 0,31_{(8)} \\ \hline \end{array} \right.$$

$$16 / 5 = 3$$

$$\overline{=10}$$

$$16 \% 5 = 1$$

$$\overline{\overline{=3}}$$

$$10_{(8)} = 1 \cdot 8 + 0 = 8_{(10)}$$

$$8 / 5 = 1$$

$$8 \% 5 = 3$$

e)  $(3_{(8)} / 5_{(8)}) / 5_{(8)}$

$$\begin{array}{r}
 3,00_{(8)} \\
 \times 8 \\
 \hline
 24 \\
 -20 \\
 \hline
 40 \\
 -40 \\
 \hline
 0
 \end{array}$$

$$30_{(8)} = 3 \cdot 8 + 0 = 24$$

$$24 / 5 = 4$$

$$24 \% 5 = 4$$

$$40_{(8)} = 4 \cdot 8 + 0 = 32$$

$$32 / 5 = 6$$

$$32 \% 5 = 2$$

$$\begin{array}{r}
 0,46_{(8)} \\
 \times 8 \\
 \hline
 3 \\
 -4 \\
 \hline
 46 \\
 -40 \\
 \hline
 6
 \end{array}$$

$$46_{(8)} = 4 \cdot 8 + 6 = 38$$

$$38 / 5 = 7$$

$$38 \% 5 = 3$$

$$\begin{array}{r}
 764,00_{(8)} + \\
 113,00_{(8)} \\
 12,00_{(8)} \\
 1,00_{(8)} \\
 0,31_{(8)} \\
 0,07_{(8)} \\
 \hline
 1112,40_{(8)}
 \end{array}$$

$$\textcircled{2} \quad 2345,1\overset{?}{\underset{(6)}{}} = ?_{(16)} \quad \text{with 2 digits}$$

$$2345,1\overset{?}{\underset{(6)}{}} = 2_{(6)} \cdot 6^3 + \underbrace{3_{(6)} \cdot 6^2}_{18_{(6)}} + \underbrace{4_{(6)} \cdot 6^1}_{4_{(16)}} + \underbrace{5_{(6)} \cdot 6^0}_{5_{(16)}} + \\ + 1_{(6)} \cdot 6^{-1} + 1\overset{-1}{\underset{(6)}{}} \cdot 6^{-2} =$$

$2_{(6)} = 2_{(16)}$ $3_{(6)} = 3_{(16)}$ $4_{(6)} = 4_{(16)}$ $5_{(6)} = 5_{(16)}$ $1_{(6)} = 1_{(16)}$	$= \underbrace{2_{(16)} \cdot 6_{(16)}^3}_{18_{(16)}} + \underbrace{3_{(16)} \cdot 6_{(16)}^2}_{6_{(16)}} + \underbrace{4_{(16)} \cdot 6_{(16)}^1}_{4_{(16)}} + \underbrace{5_{(16)} \cdot 6_{(16)}^0}_{5_{(16)}} +$ $+ \underbrace{1_{(16)} \cdot 6_{(16)}^{-1}}_{0,2A_{(16)}} + \underbrace{1\overset{-1}{\underset{(6)}{}} \cdot 6_{(16)}^{-2}}_{0,1C_{(16)}}$ $\approx$
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$180,00$   
 $6C,00$   
 $18,00$   
 $5,00$   
 $0,2A$   
 $0,1C$   


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 $239$

$$\begin{array}{r} a) \quad 4_{(16)} \cdot \\ \hline 6_{(16)} \\ \hline 18_{(16)} \end{array}$$

$$4_{(16)} \cdot 6_{(16)} = 4 \cdot 6 = 24$$

$$24 / 16 = 1$$

$$24 \% 16 = 8$$

$$\begin{array}{r} b) \quad 6_{(16)} \cdot \\ \hline 6_{(16)} \\ \hline 12_{(16)} \end{array}$$

$$6_{(16)} \cdot 6_{(16)} = 6 \cdot 6 = 36$$

$$36 / 16 = 2$$

$$36 \% 16 = 4$$

$$\begin{array}{r} 2 \cdot \ell_{(16)} \\ \underline{- 3 \cdot 16} \\ 6 \cdot C_{(16)} \end{array}$$

c)  $\begin{array}{r} 2 \cdot \ell_{(16)} \\ \underline{- 6 \cdot 16} \\ 08 \end{array}$

$$\ell_{(16)} \cdot 6_{(16)} = \ell \cdot 6 = 2\ell$$

$$2\ell / 16 = 1$$

$$2\ell \% 16 = 8$$

d)  $\begin{array}{r} 1 + 2 \cdot \ell_{(16)} \\ \underline{- 1 + 13 \cdot 2} \\ 150 \cdot 16 \end{array}$

$$1 + \ell_{(16)} \cdot 2_{(16)} = 1 + 13 \cdot 2 = 27$$

$$27 / 16 = 1$$

$$27 \% 16 = 11 = R_{(16)}$$

e)  $\begin{array}{r} 1,00 \cdot \ell_{(16)} \\ \underline{- 10} \\ 1 \\ \underline{\underline{- 4}} \\ 6 \end{array}$

$$\begin{array}{r} 6_{(16)} \\ \hline 0,2A \end{array}$$

$$10_{(16)} = 16$$

$$16 / 16 = 1$$

$$16 \% 16 = 4$$

$$\begin{array}{r} 60_{(16)} = 6 \cdot 16 + 0 = 64 \\ 64 / 16 = 4 = A_{(16)} \\ 64 \% 16 = 4 \end{array}$$

$$\begin{array}{r}
 \text{e) } 4,00 \text{ ((6)} \\
 \overline{) 4} \quad \text{---} \\
 \text{---} \quad \text{---} \\
 \hline
 0, \text{AA} \text{ ((6)} \\
 \text{period}
 \end{array}$$

$$\begin{aligned}40_{(6)} &= 4 \cdot 16 + 0 = 64 \\64_{(6)} &= 10 = A_{(6)} \\64\%_6 &= 4\end{aligned}$$

$$\begin{array}{r}
 0, AA_{(16)} \\
 \hline
 0 \quad A \\
 \hline
 4 \quad A \\
 \hline
 1 \\
 \hline
 = 2
 \end{array}
 \quad \boxed{6(16)}$$

$$\begin{aligned} A_{(16)} &= 10 \\ 10 / 6 &= 1 \\ 10^{\circ} / 6 &= 4 \end{aligned}$$

$$4A_{(16)} = 4 \cdot (6+10=) 74$$

$$74^{\circ} / .6 = 2$$

$$74^{\circ} / .6 = 2$$

1 1  
 1 150,00 (16)  
 60,00 (16)  
 18 100 (16)  
 5,00 (16)  
 0,2A (16)  
 0,1C (16)  


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 239,46 (16)

$$\begin{array}{r} A + \\ C_{(16)} \\ \hline 16 \end{array}$$

$$\begin{array}{r} 1 \\ C \\ + \\ 8 \\ \hline 1 \\ 5 \\ \hline 1 \\ 9 \end{array}$$

$$C + H + S = 12 + 1 + 5 = 18$$

$$N_{(5)} = N'(h)$$

! Successive divisions and multiplications:

- $b > h$
- calculations in base  $b$

OBS: We always perform operations from in the higher base

$$\textcircled{3} \quad 643_{(7)} : 5_{(7)} = ?_{(5)} \text{ with 2 digits at the fractional part}$$

$$643_{(7)} : 5_{(7)} = 2300_{(5)}$$

$$\begin{array}{r} 643_{(7)} \Big| 5_{(7)} \\ \hline 14 \qquad \Big| 122_{(7)} \Big| 5_{(7)} \\ \hline 13 \qquad \Big| 0 \qquad \Big| 16_{(7)} \Big| 5_{(7)} \\ \hline 1 \qquad \qquad \qquad \Big| 3 \qquad \Big| 2_{(7)} \Big| 5_{(7)} \\ \hline 0 \end{array}$$

$$0,25_{(7)} = 0,1\textcolor{blue}{1}\textcolor{green}{4}$$

$$0,25_{(7)} * 5_{(7)} = \textcolor{blue}{1},64$$

$$0,64_{(7)} * 5_{(7)} = \textcolor{green}{4},46$$

$$14(7) = 7+4=11$$

$$11/5=2$$

$$11\%5=1$$

$$13(7) = 7+3=10$$

$$10/5=2$$

$$10\%5=0$$

$$12(7) = 7+2=9$$

$$9/5=1$$

$$9\%5=4$$

$$12(7)=4\cdot7+8=30$$

$$30/5=6$$

$$30\%5=0$$

$$\begin{array}{r} 6 \ 2 \\ 0,64(7)* \\ \hline 4,46 \end{array}$$

$$16(7) = 7+6=13$$

$$15/5=2$$

$$13\%5=3$$

$$\begin{array}{r} 1 \ 3 \\ 0,25(7) \\ \hline 1,64 \end{array}$$

$$5(7)\cdot 5(7)=5\cdot 5=25$$

$$25/7=3$$

$$25\%7=4$$

$$3(7)+2(7)\cdot 5(7)=3+10=13$$

$$13/7=1$$

$$13\%7=6$$

④  $ABC, gF_{(16)} = ?_{(7)}$  with 2 digits.

$$ABC, gF_{(16)} = \underline{11004} \rightarrow \underline{42}_{(7)}$$

$$\begin{array}{r} ABC_{(16)} \\ \swarrow \\ 3B \\ \swarrow \\ 3C \\ \swarrow \\ = h \end{array}$$

$$\begin{array}{r} 7_{(16)} \\ | \\ 188_{(16)} \\ | \\ 38 \\ | \\ 0 \end{array}$$

$$\begin{array}{r} 7_{(16)} \\ | \\ 8_{(16)} \\ | \\ 1 \end{array} \quad \begin{array}{r} 7_{(16)} \\ | \\ 1_{(16)} \\ | \\ 0_{(16)} \end{array}$$

$$\begin{array}{r} A_{(16)} = 10 \\ (0 \% 7 = 1) \\ 10 \% 7 = 3 \end{array}$$

$$\begin{array}{r} 3B_{(16)} = 3 \cdot 16 + 11 = 59 \\ 59 \% 7 = 8 \\ 59 \% 7 = 3 \end{array}$$

$$3C_{(16)} = 3 \cdot 16 + 12 = 60$$

$$60 \% 7 = 8$$

$$60 \% 7 = 1$$

$$\begin{array}{r} 18_{(16)} = 16 + 8 = 24 \\ 24 \% 7 = 3 \\ 24 \% 7 = 3 \\ 38_{(16)} = 3 \cdot 16 + 8 = 56 \\ 56 \% 7 = 8 \\ 56 \% 7 = 0 \\ \hline 38_{(16)} = 3 \cdot 16 + 8 = 56 \\ 56 \% 7 = 8 \\ 56 \% 7 = 0 \end{array}$$

$$0, gF_{(16)} \approx 0, \underline{4} \underline{2}_{(7)}$$

$$0, gF_{(16)} * 7_{(16)} = \underline{4}, 59$$

$$0, 59_{(16)} * 7_{(16)} = \underline{2}, 6F$$

$$Q_{(6)} + F_{(6)} \cdot r_{(16)} = 0 + 15 \cdot 2 = 105$$

$$0_{(16)} + 9_{(16)} \cdot 7_{(16)} = 0 + 63 = 63$$

$$63 / 16 = 3$$

$$16\% \cdot 16 = 15 = F_{16}$$

$$3_{(16)} + 5_{(16)} \cdot 7_{(16)} = 3 + 5 \cdot 7 = 38$$

$$38 / 16 = 2$$

$$33\% \text{ of } 16 = 6$$

$$6_{(16)} + 5_{(16)} \cdot 7_{(16)} = 6 + 5 \cdot 7 = 6 + 35 = 41$$

$$69 / 16 = 4$$

$$69\% \text{ of } 16 = 5$$

## Codes for integers

$$x = 53 = 32 + 16 + 4 + 1 = \underbrace{110101}_{(2)}$$

$$[S3]_{\text{dn}} = [S3]_{\text{inv}} = [S3]_{\text{compl}} \xrightarrow{\text{sign bit}} \begin{matrix} S \\ \hline 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ \hline 7 & & & & & & & 0 \end{matrix}$$

$$[-53]_{\text{div}} = \boxed{\begin{array}{r} S \\ 1011010 \\ \hline 7 \end{array}}$$

$$\text{two's complement } \neg [S]_{\text{comp}} = \begin{matrix} S \\ \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ \hline \end{array} \end{matrix}$$

reverse all the digits of the initial number

from right  
to left