3.17, 3.30, 3.33, 3.34, 3.36, 3.40, 3.42, 4.11a, 4.13, 4.16, 4.18,

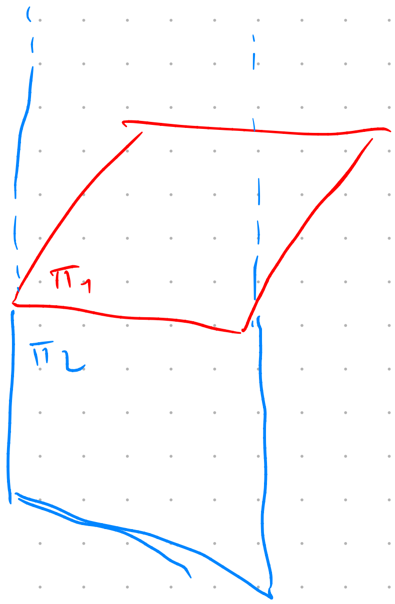
3.34. pairwise intersection of the planes

$\pi_1: \; 3x+y+z-5=0$

$\pi_2: \; 2x+y+3z+2=0$

$\pi_3: \; 5x+2y+4z+1=0$

are parallel lines.



$$\begin{cases} 3x+y+z-5=0 \\ 2x+y+3z+2=0 \end{cases}$$

$y=t \Rightarrow 3x+z=5-t \Rightarrow z=5-t-3x$

$2x+3z=-2-t$

$\Rightarrow \cancel{2x}+15-3t-\cancel{9x}=-2-t$

$-7x=-17+2t$

$x=\frac{1}{7}(17-2t)=\frac{17}{7}-\frac{2}{7}t.$

$$\vec{m}_{\pi_1} \times \vec{m}_{\pi_2} = D(\ell)$$

$$(3,1,1)\times(2,1,3)=\begin{vmatrix} i & j & k \\ 3 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 3i+3k+2j-2k-i-9j$$

$$=2i-7j+k=(2,-7,1)$$

$$\vec{m}_{\pi_2}\times\vec{m}_{\pi_3}=\begin{vmatrix} i & j & k \\ 2 & 1 & 3 \\ 5 & 2 & 4 \end{vmatrix} = -2\vec{i}-\vec{k}+7\vec{j}$$

$$=(-2,7,-1)$$

$$\vec{m}_{\pi_1}\times\vec{m}_{\pi_3}=\begin{vmatrix} i & j & k \\ 3 & 1 & 1 \\ 5 & 2 & 4 \end{vmatrix} = 2\vec{i}+\vec{k}-7\vec{j}$$

$$=(2,-7,1)$$

3.36. angles between planes

$\Pi_1 : x - \sqrt{2}y + z - 1 = 0$

$\Pi_2 : x + \sqrt{2}y - z + 3 = 0$

$\vec{n}_{\Pi_1} (1, -\sqrt{2}, 1)$, $\measuredangle(\Pi_1, \Pi_2) = \measuredangle(\vec{n}_{\Pi_1}, \vec{n}_{\Pi_2})$

$\vec{n}_{\Pi_2} (1, \sqrt{2}, -1)$

$$\cos \measuredangle(n_{\Pi_1}, n_{\Pi_2}) = \frac{(1, -\sqrt{2}, 1) \cdot (1, \sqrt{2}, -1)}{4}$$

$$= \frac{\cancel{1} - 2 \cancel{-1}}{4} = -\frac{1}{2} \Rightarrow \varphi = \frac{4\pi}{6} = \frac{2\pi}{3}$$

3.40. $A(1, 3, 5)$

$\ell : 2x + y + z - 1 = 0 \cap 2x + y + 2z - 3 = 0$

$$\begin{cases} 2x + y + z - 1 = 0 \\ 2x + y + 2z - 3 = 0 \end{cases}$$

$\boxed{t = y} \Rightarrow 2x + z = 1 - t$

$2x + 2z = 3 - t$ $\ominus$

$\boxed{z = 2}$

$\Rightarrow 2x = -1 - t$

$\boxed{x = -\frac{1}{2}(1 + t)}$

$\ell \perp \pi$

$\Rightarrow \{ A' \} = \pi \cap \ell$

$\vec{n}_1 = (2, 1, 1)$

$\vec{n}_2 = (3, 1, 1)$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 2\vec{i} \ldots$$

$\Rightarrow D(\ell) = \langle (1, -1, -1) \rangle$

$1(x - 1) + (-1)(y - 3) - 1(z - 5) = 0$

$x - 1 - y + 3 - z + 5 = 0$

$\pi: x - y - z = -7.$

$$\begin{cases} 2x + y + z - 1 = 0 \\ 3x + y + 2z - 3 = 0 \\ x - y - z + 7 = 0 \end{cases}$$

$y = t \Rightarrow x - z = t - 7 \Rightarrow -z = t - 5 \Rightarrow z = 5 - t.$

$\underline{2x + z = 1 - t}$ (+)

$3x \quad = -6 \Rightarrow \boxed{x = -2}$

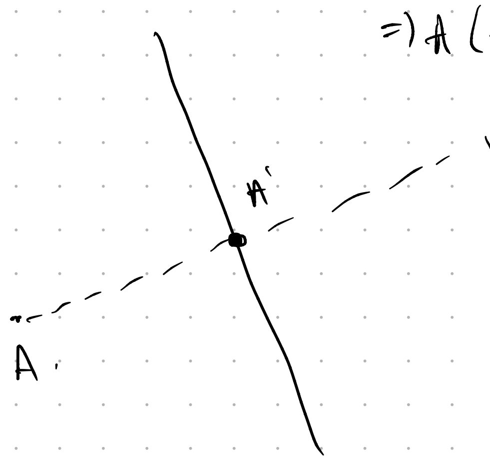
$+1$

$\cancel{-6} + t \cancel{+10} - 2t \cancel{-3} = 0$

$-t = -1 \Rightarrow t = 1 \Rightarrow \boxed{y = 1}$

$\Rightarrow \boxed{z = 4}$

$\Rightarrow A(-2, 1, 4)$



$-2 = \frac{1 + x_{A'}}{2} \Rightarrow x_{A'} = -5$

$1 = \frac{3 + y_{A'}}{2} \Rightarrow y_{A'} = -1$

$z_{A'} = 3$

3.42,

Determine the orthogonal proj of

$\ell: 2x - y - 1 = 0 \cap x + y - z + 1 = 0$

w/ $\pi: x + 2y - z = 0$

$\vec{n}_\pi = (1, 2, -1)$

$\ell: \begin{array}{l} 2x - y - 1 = 0 \\ x + y - z + 1 = 0. \end{array}$

$3x - z = 0$

$x = \alpha \Rightarrow z = 3\alpha$

$y = 2\alpha - 1$

$$\ell: \begin{cases} x = \alpha + 1 \cdot t \\ y = 2\alpha - 1 + 2t \\ z = 3\alpha - 1t \end{cases} =$$

$$P_{\pi_A} = \begin{cases} x = \alpha + t \\ y = 2\alpha + 2t - 1 \\ z = 3\alpha - t \end{cases}$$

$$x + 2y - z = 0$$

$$\cancel{\alpha} + \cancel{t} + \cancel{4\alpha} + \cancel{4t} - 2 - \cancel{3\alpha} + \cancel{t} =$$

$$2\alpha + 6t - 2 = 0$$

$$\alpha + 3t - 1 = 0$$

$$t = \frac{1-\alpha}{3}$$

$$\Rightarrow \pi_A : \begin{cases} x = \frac{2\alpha + 1}{3} \\ y = \frac{4\alpha - 1}{3} \\ z = \frac{10\alpha - 1}{3} \end{cases}$$

4.11. Prove Grassmann identity.

$$\vec{v_1} \times (\vec{v_2} \times \vec{v_3}) = \begin{vmatrix} \vec{v_3} & \vec{v_2} \\ \vec{v_1} \cdot \vec{v_3} & \vec{v_1} \cdot \vec{v_2} \end{vmatrix}$$

$$= (\vec{v_1} \cdot \vec{v_2}) \cdot \vec{v_3} - (\vec{v_1} \cdot \vec{v_3}) \cdot \vec{v_2}$$

Proof:

$\vec{v_i}(x_i, y_i, z_i)$

$i = 1, 2, 3.$

$$\vec{v_2} \times \vec{v_3} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$\begin{vmatrix} y_2 & z_2 \\ y_3 & z_3 \end{vmatrix} \vec{i} + \begin{vmatrix} z_2 & x_2 \\ z_3 & x_3 \end{vmatrix} \vec{j} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} \vec{k}$$

$$-\vec{v_1} \times (\vec{v_2} \times \vec{v_3}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ \begin{vmatrix} y_2 & z_2 \\ y_3 & z_3 \end{vmatrix} & \begin{vmatrix} z_2 & x_2 \\ z_3 & x_3 \end{vmatrix} & \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} \end{vmatrix}$$

$$x \vec{v_1} \times (\vec{v_2} \times \vec{v_3}) = \begin{vmatrix} y_1 & z_1 \\ \begin{vmatrix} z_2 & x_2 \\ z_3 & x_3 \end{vmatrix} & \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} \end{vmatrix}$$

$$= y_1(x_2y_3 - x_3y_2) - z_1(z_2x_3 - z_3x_2)$$

$$= x_2(y_1y_3 + z_1z_3) - x_3(y_1y_2 + z_1z_2)$$

$$= x_2(x_1y_3 + y_1y_3 + z_1z_3)$$
$$- x_3(x_1x_2 + y_1y_2 + z_1z_2)$$

$$= (\vec{v}_1 \cdot \vec{v}_3)x_2 - (\vec{v}_1 \cdot \vec{v}_2) \cdot x_3$$

= sare,