1. Five percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 16 parts contains
a) exactly 3 defective parts (ev. A)?
b) more than 3 defective parts? (ev. B)?
c) at least one defective part (ev. C)?
d) less than 3 defective parts (ev. D)?

X = no. of defective comp. in the sample

Binomial

a) $P(A) = P(X=3) = C_{16}^{3} \cdot \left(\frac{1}{20}\right)^{3} \cdot \left(\frac{19}{20}\right)^{13}$

$\sum$ of Binomial

b) $P(B) = P(X>3) = P(X \geq 4)$

$= C_{16}^{4} \cdot \left(\frac{1}{20}\right)^{4} + C_{16}^{5} \cdot \left(\frac{1}{20}\right)^{5} + \dots + C_{16}^{16} \cdot \left(\frac{1}{20}\right)^{16} =$

$= \sum_{k=4}^{16} C_{16}^{k} \left(\frac{1}{20}\right)^{k} \cdot \left(\frac{19}{20}\right)^{16-k} = 1 - \sum_{k=0}^{3} C_{16}^{k} (0.05)^{k} \cdot (0.95)^{16-k}$

c) $P(C) = P(X \geq 1) = 1 - (0.95)^{16}$

d) $P(D) = P(X \leq 2) = \sum_{k=0}^{2} C_{16}^{k} (0.05)^{k} \cdot (0.95)^{16-k}$

4. A computer program is tested by 5 independent tests. If there is an error, these tests will detect it with probabilities 0.1, 0.2, 0.3, 0.4 and 0.5, respectively. Suppose that the program contains an error. What is the probability that it will be found by

a) at least one test (ev. A)?

b) more than two tests (ev. B)?

c) all five tests (ev. C)?

$T_1, \ldots, \overline{T_5}$

$P(T_i) = \frac{i}{10}, \quad i = \overline{1,5}$

X = no. of succeses

↳ the no. of succeses detected

a) $P(X \geq 1) = 1 - P(X=0) = 1 - \prod_{i=1}^{5} P(\overline{T_i})$

b) $P(X > 2) = P(X \in \{3,4,5\}) = P_{(X=3)} + P_{(X=4)} + P_{(X=5)}$

c) $P(X=5)$

$(0.1 \cdot x + 0.9)(0.2 \cdot x + 0.8)(0.3 \cdot x + 0.7)(0.4 \cdot x + 0.6)(0.5x + 0.5)$

$= P_{(X=5)} x^5 + P_{(X=4)} x^4 + P_{(X=3)} x^3 + P_{(X=2)} x^2 + P_{(X=1)} + x^1 + P_{(X=0)} x^0$

3. Among 10 laptop computers, seven are good, the rest have defects. Unaware of this, a customer buys 5 laptops.

a) What is the probability of exactly 2 defective ones among them (ev. A)?

b) Knowing that *at least* 2 purchased laptops are defective, what is the probability that *exactly* 2 are defective (ev. B)?

3

success = defective $\rightarrow P(5,2)+P(5,3)$

a) $P(A)$ $\xrightarrow[\text{bcs. we don't put back}]{\text{Hypergeometric}}$ $P(5,2) = \dfrac{C_3^2 \cdot C_7^3}{C_{10}^5}$

$k=2,\ M_1=3,\ N=10,\ n=5$

defective laptops

b) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)}{P(B)} =$

$P(B) = P(X \geq 2) = P(5,2) + P(5,3)$

$P(A) = P(X=2)$

$= \dfrac{P(X=2)}{P(X=2)+P(X=3)}$

5. In a public library, 1 out of 10 people using the computers do not close Windows properly. What is the probability that Windows is closed properly only by the 3$^{\text{rd}}$ user (event A)?

success $\Rightarrow p = 0.9$, $q = 0.1$.

independent

X = no. of users before the first user to close Windows properly

Geometric model

$P(A) = p \cdot q^2 = (0.9)\cdot(0.1)^2$

prob $= 0 \Leftrightarrow$ impossible

$\Leftarrow$

$\not\Rightarrow$

6. An engineer tests the quality of produced computers. Suppose that 5% of computers have defects and defects occur independently of each other. Find the probability

a) of exactly 3 defective computers in a shipment of 20 (ev. A);

b) that the engineer has to test at least 5 computers in order to find 2 defective ones (ev. B).

5% defective computers

a) $P(\text{3 out of 20 are defective}) = C_{20}^{3} \cdot (0.05)^3 \cdot (0.95)^{17}$

b)

Pascal (Negative Binomial) Model: The probability of the n^{th} success occurring after k failures in a sequence of Bernoulli trials with probability of success p ($q = 1 - p$), is $P(n; k) = C^{n-1}_{n+k-1} p^n q^k = C^{k}_{n+k-1} p^n q^k$.

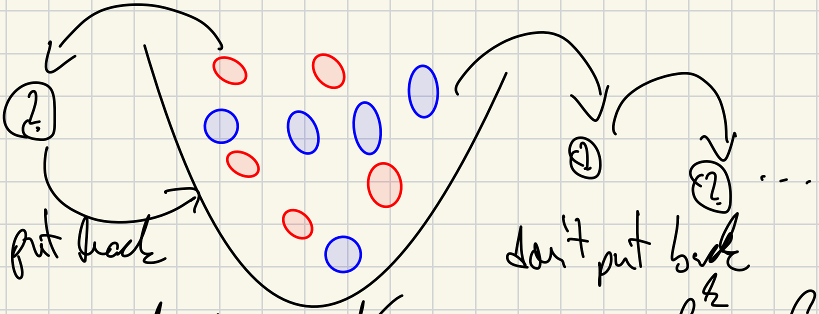
$$\underbrace{\underline{f/s}\ \underline{f/s}\ \underline{f/s}\ \_\ \_\ \_\ \_\ \_\ \_}_{x = \text{no of "f"}}\ \underset{2^{nd}}{S}$$

$P($"test at least 5 to find 2 def"$) =$

number of failures

$$= P(x \geq 3) = 1 - \sum_{k=0}^{n} C^{k}_{2-k+1} (0.95)^{k} \cdot (0.05)^{2}$$

Binomial vs Hypergeometric



$$P(X=k) = C_n^k \, p^k (1-p)^{n-k}$$

p = prob of ○

$$P(X=k) = \frac{C_{N_1}^{k} \cdot C_{N_2}^{n-k}}{C_{N}^{n}}$$

Newton Binomial expansion

Binomial vs Poisson

$n \to$ no. trails

$$(p \cdot x + q)^n = \sum_{k=0}^{n} \underbrace{C_n^k \, p^k q^{n-k}}_{P(X=k)} \cdot x^k$$

different prob. for each indep. trail.

$$(p_1 \cdot x + q_1)(p_2 \cdot x + q_2) \cdots (p_n \cdot x + q_n) =$$

$$= \dots x^n + \dots x^{n-1} + \dots + \dots x^1 + \dots x^0$$