08.01.2024

1.8 get DCF and CCF and simplify both using

K-maps

x	y	z	f_8	m_0	M
0	0	0	$\boxed{1}$	1	1
0	0	1	0	0	0
0	1	0	$\boxed{1}$	0	1
0	1	1	0	0	1
1	0	0	0	0	1
1	0	1	$\boxed{1}$	0	1
1	1	0	$\boxed{1}$	0	1
1	1	1	0	0	1

DCF

CCF

minterm

$$m_0 = x^0 y^0 z^0 = \bar{x}\bar{y}\bar{z}$$

$$= 000_{(2)}$$

$$DCF(f_8) = m_0 \vee m_2 \vee m_5 \vee m_6$$

$$= x^0 y^0 z^0 \vee x^0 y^1 z^0 \vee x^1 y^0 z^1 \vee$$

$$\vee x^1 y^1 z^0 =$$

$$= \bar{x}\bar{y}\bar{z} \vee \bar{x} y \bar{z} \vee x \bar{y} z \vee x y \bar{z}$$

Karnaugh diagram

2^3
2^2
2^1
2^0

number of neighbors

$$m_2 = \bar{x} y \bar{z}$$

$$m_6 = x y \bar{z}$$

$$m_2 \vee m_6 = \bar{x} y \bar{z} \vee x y \bar{z}$$

$$= (\bar{x} \vee x) \vee y \bar{z} = y \bar{z}$$

$x \backslash yz$	00	01	11	10
0	m_0			m_2
1		m_5		m_6

max2

max3

max1

Factorizations

$max_1 = m_2 \vee m_6 = y\bar{z}$ (the common part)

$max_2 = m_0 \vee m_2 = \bar{x}\bar{z}$

$max_3 = m_5 = x\bar{y}z$

$M(f_8) = \{max_1, max_2, max_3\}$ the set of maximal monoms

$C(f_8) = \{max_1, max_2, max_3\}$

bcs. m_6 is circled once

bcs. m_5 is circled once

bcs. m_0 is circled once

$M(f_8) = C(f_8) \rightarrow 1^{st}$ simplification case

f_8^{DS} -> disjunction simplified

$f_8^{DS} = max_1 \vee max_2 \vee max_3 = y\bar{z} \vee \bar{x}\bar{z} \vee x\bar{y}z$

$M_1 = x^{\bar{0}} y^{\bar{0}} z^{\bar{1}} = x y \bar{z}$ maxterm

$1 = 001_{(2)}$

$$CCF(f_3) = M_1 \wedge M_3 \wedge M_4 \wedge M_7 =$$

$$= (x^{\bar{0}} \vee y^{\bar{0}} \vee z^{\bar{1}}) \wedge (x^{\bar{0}} \vee y^{\bar{1}} \vee z^{\bar{1}}) \wedge (x^{\bar{1}} \vee y^{\bar{0}} \vee z^{\bar{0}}) \wedge$$

$$\wedge (x^{\bar{1}} \vee y^{\bar{1}} \vee z^{\bar{1}})$$

$$= (x \vee y \vee \bar{z}) \wedge (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z) \wedge$$

$$\wedge (\bar{x} \vee \bar{y} \vee \bar{z})$$

x \ yz	00	01	11	10
0		M_1	M_3	
1	M_4		M_7	

we apply a dual simplification alg. bcs. we work with a CCF

Maximal disjunctions (dual factorisations)

$$\max d_1 = M_1 \wedge M_3 = x^{\bar{0}} \vee z^{\bar{1}} = x \vee \bar{z}$$

$$\max d_2 = M_3 \wedge M_7 = y^{\bar{1}} \vee z^{\bar{1}} = \bar{y} \vee \bar{z}$$

$$\max d_3 = M_4 = x^{\bar{1}} y^{\bar{0}} z^{\bar{0}} = \bar{x} \vee y \vee z$$

$$\mathrm{Max}\, d = \{ \max d_1, \max d_2, \max d_3 \}$$

$$Cd(f_3) = \{ \max d_1, \max d_2, \max d_3 \}$$

↳ same condition as before

$\mathrm{Max}\, d = Cd(f_3) =$ 1st case of the dual simplif. alg.

$$f_3^{CS \Rightarrow \text{conj. simplified}} = \max d_1 \wedge \max d_2 \wedge \max d_3 =$$

$$= (x \vee \bar{z}) \wedge (\bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z)$$

2.1. Simplify the following Boolean functions of 4 variables using

$$f_1(x_1,x_2,x_3,x_4) = \underbrace{x_1 x_2 \bar{x}_3 x_4}_{m_{13}} \vee \underbrace{x_1 x_2 x_3 \bar{x}_4}_{m_{14}} \vee \underbrace{x_1 x_2 \bar{x}_3 \bar{x}_4}_{m_{12}}$$

$$\underbrace{x_1 \bar{x}_2 x_3 \bar{x}_4}_{m_{10}} \vee \underbrace{\bar{x}_1 \bar{x}_2 \bar{x}_3 \bar{x}_4}_{m_0} \vee \underbrace{\bar{x}_1 \bar{x}_2 x_3 \bar{x}_4}_{m_2} \vee \underbrace{x_1 \bar{x}_2 x_3 x_4}_{m_{11}} \vee$$

$$\vee \underbrace{\bar{x}_1 \bar{x}_2 \bar{x}_3 x_4}_{m_1} \vee \underbrace{\bar{x}_1 \bar{x}_2 x_3 x_4}_{m_3}$$

x_1x_2 \ x_3x_4	00	01	11	10
00	m_0	m_1	m_3	m_2
01				
11	m_{12}	m_{13}		m_{14}
10			m_{11}	m_{10}

We apply factorisation

double fact: 2^2

$\boxed{max_1} = m_0 \vee m_1 \vee m_2 \vee m_3 = \bar{x}_1 \bar{x}_2$

$\boxed{max_2} = m_2 \vee m_3 \vee m_{10} \vee m_{11} = \bar{x}_2 x_3$

simple fact: 2^1

$\boxed{max_3} = m_{12} \vee m_{13} = x_1 x_2 \bar{x}_3$

$\boxed{max_4} = m_{12} \vee m_{14} = x_1 x_2 \bar{x}_4$

$\boxed{max_5} = m_{10} \vee m_{14} = x_1 x_3 \bar{x}_4$

$M(f) = \{ max_1, max_2, max_3, max_4, max_5 \}$

$C(f) = \{ \underset{(m_0, m_1 \text{ 0-ed once})}{max_1}, \underset{(m_{11} \text{ 0-ed once})}{max_2}, \underset{(m_{13} \text{ 0-ed once})}{max_3} \}$

central monoms

$M(f) \neq C(f) \neq \emptyset \Rightarrow 2^{nd}$ simplif case

$g = max_1 \vee max_2 \vee max_3$

$f^{S_1} = g \vee max_4 = \bar{x}_1 \bar{x}_2 \vee \bar{x}_2 x_3 \vee x_1 x_2 \bar{x}_3 \vee x_1 x_2 x_4$

$f^{S_2} = g \vee max_5 = \bar{x}_1\bar{x}_2 \vee \bar{x}_2 x_3 \vee x_1 x_2 \bar{x}_3 \vee x_1 x_3 \bar{x}_1$

A central monome BELONGS to all the simplified forms! (elements from $\underset{\sim}{C}(f)$)

↳ all central monomes.

3.1 Using Veitch diagram simplify the functions.

$f_1(x_1, x_2, x_3) = \bar{x}_1(x_2 \downarrow x_3) \vee \bar{x}_1\bar{x}_2 x_3 \vee$

$\vee (x_1 \vee (x_2 \uparrow x_3)) \vee x_1 x_2 \bar{x}_3$

$= \bar{x}_1(\overline{x_2 \vee x_3}) \vee \bar{x}_1 \bar{x}_2 x_3 \vee$

$\vee \overline{(x_1 \vee (\overline{x_2 \wedge x_3})} \vee x_1 x_2 \bar{x}_3$

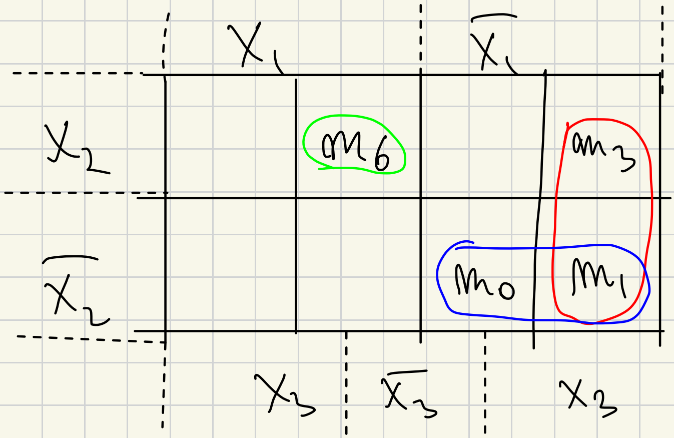
$= \bar{x}_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_1 \bar{x}_2 x_3 \vee (\bar{x}_1 \wedge x_2 \wedge x_3) \vee$

$\vee x_1 x_2 \bar{x}_3 =$

$= \underbrace{\bar{x}_1 \bar{x}_2 \bar{x}_3}_{m_0} \vee \underbrace{\bar{x}_1 \bar{x}_2 x_3}_{m_1} \vee \underbrace{\bar{x}_1 x_2 x_3}_{m_2} \vee \underbrace{x_1 x_2 \bar{x}_3}_{m_6}$

$= m_0 \vee m_1 \vee m_2 \vee m_6$

Veitch Diagram



$$max_1 = m_0 \vee m_1 = \bar{x}_1 \bar{x}_2$$

$$max_2 = m_1 \vee m_3 = \bar{x}_1 x_3$$

$$max_3 = m_6 = x_1 x_2 \bar{x}_3$$

$$M(f_1) = \{ max_1, max_2, max_3 \}$$

$$C(f_1) = \{ max_1, max_2, max_3 \}$$

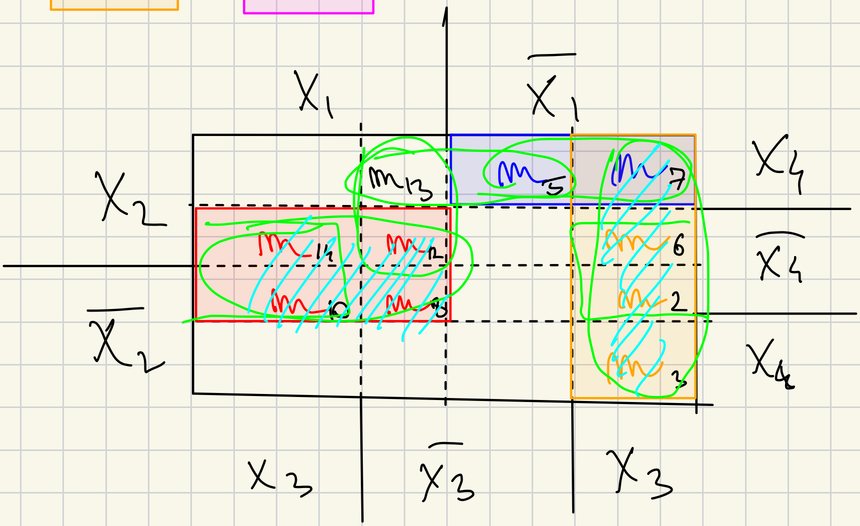
$M(f_1) = C(f_1) \Rightarrow$ 1st simplif. case $\Rightarrow$ unique simplif. form

$$f_1^s = max_1 \vee max_2 \vee max_3 = \ldots$$

4.1. $f(x_1, x_2, x_3, x_4) = \boxed{x_1 \bar{x}_4} \vee \underbrace{x_1 x_2 \bar{x}_3 x_4}_{m_{13}} \vee \boxed{\bar{x}_1 x_2 x_4}$

m m

$\vee \boxed{\bar{x}_1 x_3} \vee \boxed{x_3 \bar{x}_4}$ → contained in the others



$$f = m_{13} \vee m_5 \vee m_7 \vee m_{14} \vee m_{12} \vee m_6 \vee m_{10} \vee$$
$$\vee\, m_8 \vee m_2 \vee m_3$$

$$max_1 = m_7 \vee m_6 \vee m_2 \vee m_3 = \bar{x}_1 x_3$$

$$max_2 = m_{14} \vee m_{12} \vee m_{10} \vee m_8$$

$$max_3 = m_{14} \vee m_{10} \vee m_6 \vee m_2$$

$$max_4 = m_{13} \vee m_5$$

$$max_5 = m_{13} \vee m_{12}$$

$$max_6 = m_5 \vee m_7$$

$M(f) = \{ max_i \mid i = \overline{1,6} \}$

$C(f) = \{ max_1 , max_2 \}$

$M(f) \neq C(f) \neq \emptyset \Rightarrow 2^{nd}$ simplest case

$g = max_1 \vee max_2$

$f^S = g \vee max_4$ → unique $=$

$= max_1 \vee max_2 \vee max_4 =$

$= \ldots$