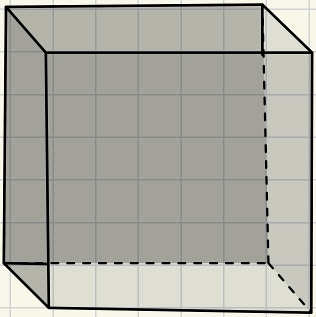
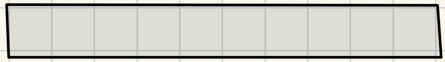
22.10.2024

1)





a) 8 – 3 faces $P(A) = \frac{8}{1000}$

b) 96 – 2 faces $P(B) = \frac{96}{1000}$

c) 384 – 1 face $P(C) = \frac{378}{1000}$

d) $\frac{1000-8-95-378}{1000} = \frac{519}{1000} = P(D) = 1 - P(A) - P(B) - P(C)$

E_1, E_2 events that are independent:

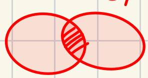
$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

$P(E_2) \neq 0 \iff P(E_1 | E_2) = P(E_1)$

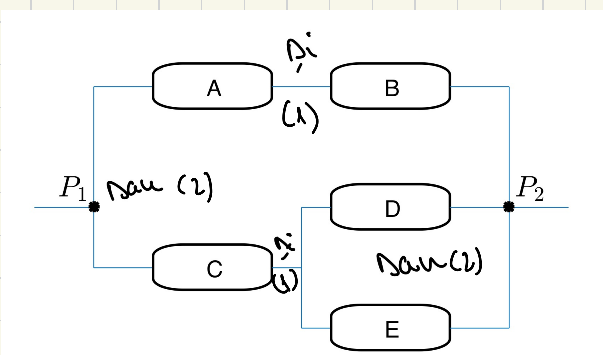
E_1, E_2 disjoint / incompatible

$E_1 \cap E_2 = \emptyset$

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$



2)

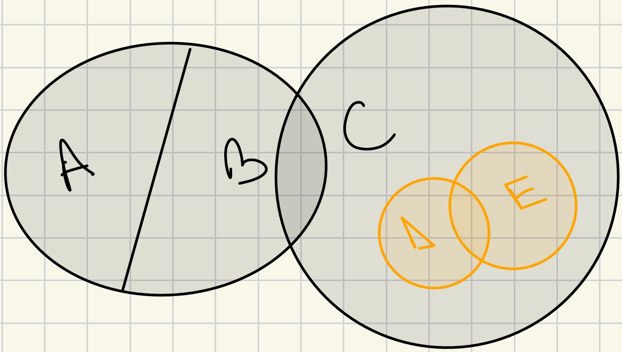


Let R = prob. of the system, then R =

$$= \underbrace{\underbrace{P(A)\cdot P(B)}_{0,8464} + \underbrace{\underbrace{P(C)}_{0,92}\cdot\left(\overbrace{P(D)+P(E)}^{1,84}-\overbrace{P(D)\cdot P(E)}^{0,8464}\right)}_{0,914112}}_{1,760512}$$

$$- \left(\underbrace{P(A)\cdot P(B)}_{0,8464}\right)\cdot\left(\underbrace{P(C)}_{0,92}\cdot\left(\underbrace{P(D)+P(E)}_{1,84}-\underbrace{P(D)\cdot P(E)}_{0,8464}\right)\right)$$

$$\cong 0,9868$$



5. Among employees of a certain firm, 70% know C/C++, 60% know Fortran and 50% know both. What portion of programmers
a) does not know Fortran?
b) does not know C/C++ and does not know Fortran?
c) knows C/C++, but not Fortran?
d) Are "knowing C/C++" and "knowing Fortran" independent of each other?
e) What is the probability that someone who knows Fortran, also knows C/C++?
f) What is the probability that someone who knows C/C++, does not also know Fortran?

C : knows C/C++
F : knows Fortran

$P(C) = 0.7$
$P(F) = 0.6$
$P(C \cap F) = 0.5$

a) $P(\overline{F}) = 1 - P(F) = 0.4$

b) $P(\overline{C} \cap \overline{F}) = P(\overline{C \cup F}) = 1 - P(C \cup F) =$

$= 1 - \big(P(C) + P(F) - P(C \cap F) \big) = 1 - 0.8 = 0.2$

c) $P(C \cap \overline{F}) = P(C) - P(C \cap F) = 0.2$

d) $P(C) \cdot P(F) = 0.7 \times 0.6 = 0.42 \neq P(C \cap F) = 0.5 \Rightarrow$ not indep.

e) $P(C | F) = \dfrac{P(F \cap C)}{P(F)} = \dfrac{0.5}{0.6} = 0.83$

f) $P(\overline{F} | C) = \dfrac{P(\overline{F} \cap C)}{P(C)} = \dfrac{0.2}{0.7} = 0.286$

2. (Pigeonhole Principle) A postman distributes n letters in N mailboxes. What is the probability of the event A: there are m letters in a given (fixed) mailbox ($0 \leq m \leq n$)?

$$P(A) = \frac{(N-1)^{n-m} \cdot C_n^m}{N^n}$$

C_n^m → number of ways in which we can choose m letters from the total n.

N^n → total possible distributions of n letters in N mailboxes (Product rule)

Out of the remaining mailboxes and remaining letters, we take the possible distributions (also product rule)

6. Three shooters aim at a target. The probabilities that they hit the target are 0.4, 0.5 and 0.7, respectively. Find the probability that the target is hit exactly once.

$$P(\mathrm{I}) = 0.4, \quad P(\mathrm{II}) = 0.5, \quad P(\mathrm{III}) = 0.7$$

$$P\left((\mathrm{I}, \overline{\mathrm{II}}, \overline{\mathrm{III}}) \cup (\overline{\mathrm{I}}, \mathrm{II}, \overline{\mathrm{III}}) \cup (\overline{\mathrm{I}}, \overline{\mathrm{II}}, \mathrm{III})\right)$$

$$(0{,}4 \cdot 0{,}5 \cdot 0{,}3) + (0{,}6 \cdot 0{,}5 \cdot 0{,}3) + (0{,}6 \cdot 0{,}5 \cdot 0{,}7) =$$

$$=$$