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5.1) $U_1 = (p \overset{1}{\rightarrow} \neg q) \wedge (q \vee r) \overset{2}{\rightarrow} (p \overset{3}{\rightarrow} r)$

write DNF and CNF and prove U_1 is a tautology.

DNF : disj. normal form $\rightarrow \bigvee(\bigwedge l_{ij})$ cubes

CNF : conj. normal form $\rightarrow \bigwedge(\bigvee l_{ij})$ clauses

$A \rightarrow B \equiv \neg A \vee B$

we apply the normalisation alg.

$U_1 \overset{\text{replace 1,3}}{=\!=} (\neg p \vee \neg q) \wedge (q \vee r) \overset{2}{\rightarrow} (\neg p \vee r)$ (higher priority)

$U_1 \overset{\text{replace 2}}{=\!=} \neg((\neg p \vee \neg q) \wedge (q \vee r)) \vee (\neg p \vee r)$ redundant parentheses

$U_1 \overset{\text{De Morgan}}{=\!=} \underbrace{(p \wedge q)}_{\text{cube I}} \vee \underbrace{(\neg q \wedge \neg r)}_{\text{cube II}} \vee \underbrace{\neg p}_{\text{cube III}} \vee \underbrace{r}_{\text{cube IV}}$ → DNF with 4 cubes

$U_1 \overset{\text{distrib. laws}}{=\!=} (p \vee q \vee \neg p \vee r) \wedge (p \vee r \vee \neg p \vee r) \wedge (q \vee q \vee r \vee \neg p \vee r) \wedge (q \vee \neg q \vee r \vee \neg p \vee r)$

($\equiv \top$ (always true)) → CNF with 4 clauses

$(3+4) \cdot (5+6) \cdot 7 = 3 \cdot 5 \cdot 7 + 3 \cdot 6 \cdot 7 + 4 \cdot 5 \cdot 7 + 4 \cdot 6 \cdot 7$

$U_1 \equiv \top \wedge \top \wedge \top \wedge \top$

$\Rightarrow$ U_1 always true $\Rightarrow$ tautology (valid formula)

> U is valid (tautology) if all the clauses of CNF(U) are valid. (ex 3)

A clause is valid if it contains a pair of opposite literals.

> U is inconsistent if all the cubes of DNF(U) are inconsistent

7.1) $U_1 = (p \overset{1}{\rightarrow} (q \overset{2}{\rightarrow} r)) \wedge \neg((p \overset{3}{\rightarrow} q) \overset{4}{\rightarrow} (p \overset{5}{\rightarrow} r))$

$U_1 \overset{\text{replace } 2,3,5}{=\!=\!=} (p \overset{1}{\rightarrow} (\neg q \vee r)) \wedge \neg((\neg p \vee q) \overset{4}{\rightarrow} (\neg p \vee r))$

$U_1 \overset{\text{replace } 1,4}{=\!=\!=} (\neg p \vee \neg q \vee r) \wedge \neg(\neg(\neg p \vee q) \vee (\neg p \vee r))$

$U_1 \overset{\text{De Morgan law}}{=\!=\!=} (\neg p \vee \neg q \vee r) \wedge (\neg p \vee q) \wedge p \wedge \neg r$

↳ CNF with 4 clauses.

$U_1 \overset{\text{distrib.}}{=\!=\!=} \underbrace{(\neg p \wedge \neg p \wedge p \wedge \neg r)}_{\equiv F} \vee \underbrace{(\neg p \wedge q \wedge p \wedge \neg r)}_{\equiv F} \vee \underbrace{(\neg q \wedge \neg p \wedge p \wedge \neg r)}_{\equiv F}$

$\vee \underbrace{(\neg q \wedge q \wedge p \wedge \neg r)}_{\equiv F} \vee \underbrace{(r \wedge \neg p \wedge p \wedge \neg r)}_{\equiv F} \vee \underbrace{(r \wedge q \wedge p \wedge \neg r)}_{\equiv F}$

$U_1 \equiv F \vee F \vee F \vee F \vee F \vee F$

$U_1 \equiv F \Rightarrow U_1$ is inconsistent. because all values from DNF are inconsistent