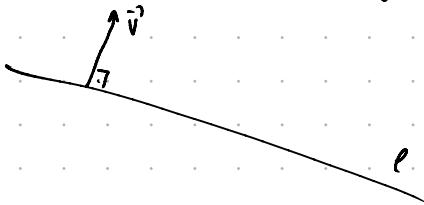


26, 29, 30, 31, 32, 33, 35, 37, 38, 39, 42

- coords in orthogonal system

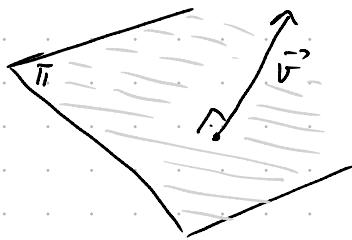
$l: ax + by + c = 0$
Line in \mathbb{E}^2

$\vec{v}(a, b)$ is a normal vector of l

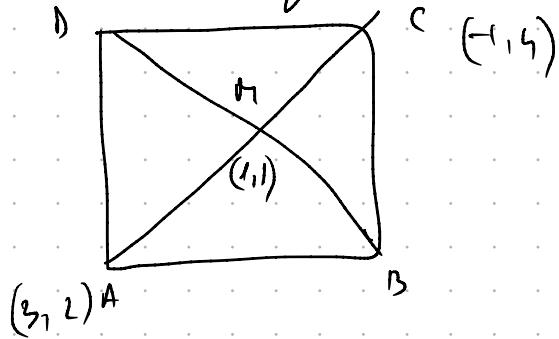


ii: $ax + by + cz + d = 0$ plane in \mathbb{E}^3

$\Rightarrow \vec{v}(a, b, c)$ - normal vector of π



3.26. $A(3, -2)$ is the vertex of a square ABCD, M - int point of the diag.
find the cartesian eq for the sides of the square



$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{-2 - (-2)}{-1 - 3} = \frac{0}{-4} = 0$$

$$x_M = \frac{x_A + x_C}{2} = 1 = \frac{3 + x_C}{2} \Rightarrow x_C = -1$$

$$y_M = \frac{y_A + y_C}{2} = 1 \Rightarrow -2 + y_C = 2 \Rightarrow y_C = 4$$

$$AC \perp BD \Rightarrow m_{AC} \cdot m_{BD} = -1 \Rightarrow m_{BD} = \frac{2}{3}$$

$$y - y_M = \frac{2}{3}(x - 1)$$

$$\Rightarrow BD: y = \frac{2}{3}x + \frac{1}{3}$$

$$AM = \sqrt{(1-3)^2 + (1+2)^2} = \sqrt{4+9} = \sqrt{13}$$

$$\Rightarrow AC = 2\sqrt{13}$$

$$B, D : \begin{cases} y = \frac{2}{3}x + \frac{1}{3} \\ (y - y_M)^2 + (x - x_M)^2 = AM^2 \end{cases}$$

$$\Rightarrow (y-1)^2 + (x-1)^2 = 13 \Rightarrow y^2 - 2y + x^2 - 2x = 11$$

$$\frac{4}{9}x^2 + \frac{4}{9}x + \frac{1}{9} - \frac{4}{3}x - \frac{2}{3} + x^2 - 2x = 11 \quad | \cdot 9$$

$$\underline{\underline{4x^2 + 4x + 1}} - \underline{\underline{12x}} - \underline{\underline{6}} + \underline{\underline{9x^2 - 18x}} = 99$$

$$11x^2 - 26x - 94 = 0$$

$$\frac{4}{9}(x-1)^2 + (y-1)^2 = 13$$

$$\Rightarrow \frac{13}{9}(x-1)^2 = 13 \Rightarrow (x-1)^2 = 9$$

$$\Rightarrow x-1 = \pm 3$$

$$\Rightarrow x_1 = 4 \Rightarrow y = \frac{8}{3} + \frac{1}{3} = 3$$

$$x_2 = -2 \Rightarrow y = -\frac{4}{3} + \frac{1}{3} = -1$$

$$\Rightarrow (4, 3), (-2, -1)$$

$$\text{If } B(-2, -1) \Rightarrow D(4, 3)$$

$$\vec{AB} = (-2, -1) - (3, -2)$$

$$= (-5, 1)$$

$$\vec{DC} = (4, 3) - (-1, 4)$$

$$= (5, -1)$$

$$AD: \frac{y - y_D}{y_D - y_A} = \frac{x - x_H}{x_D - x_A}$$

$$\therefore y - 3 = \frac{3 + 2}{1} (x - 3)$$

$$y - 3 = 5(x - 3)$$

$$y = 5x - 15 + 3$$

$$y = 5x - 12.$$

$$BC: \frac{y - y_C}{y_C - y_B} = \frac{x - x_C}{x_C - x_B}$$

$$y - 4 = \frac{4 + 2}{-1 - 1} (x + 1)$$

$$y - 4 = 3x + 3$$

$$y = 3x + 7$$

$$3.30. A(2, 1, 0)$$

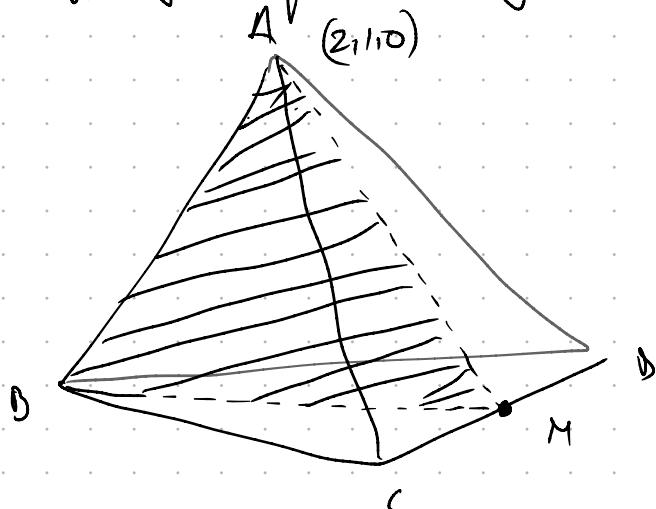
$$B(1, 3, 5)$$

$$C(6, 3, 4)$$

$$D(0, -7, 8)$$

Achsenabschnitt

Cent. eq. of the plane containing $[AB]$ and the midpoint M of $[CD]$



- 3 points

- 1 point & 2 vectors

$Ax + by + cz + d = 0$ implicit form.

$$\begin{aligned}\vec{BH} &= \frac{1}{2} (\vec{BC} + \vec{BD}) \\ &= \frac{1}{2} (\vec{BA} + \vec{AC} + \vec{BA} + \vec{AD}) \\ &= \vec{BA} + \frac{1}{2} (\vec{AC} + \vec{AD}) \\ \vec{AB} + \vec{BH} &= \vec{AM} = \frac{1}{2} (\vec{AC} + \vec{AD}) \\ &= \frac{1}{2} ((4, 2, 4) + (-2, -8, 8)) \\ &= \frac{1}{2} (2, -6, 12) \\ &= (1, -3, 6) \rightarrow \vec{AM}\end{aligned}$$

$$\text{II: } \begin{cases} x = x_A + 2x_J + \mu x_W \\ y = y_A + 2y_J + \mu y_W \\ z = z_A + 2z_J + \mu z_W \end{cases}$$

The cartesian (symmetric form):

$$\begin{vmatrix} x - x_A & y - y_A & z - z_A \\ x_J & y_J & z_J \\ x_W & y_W & z_W \end{vmatrix} = 0$$

$$\vec{AB} = (-1, 2, 5)$$

$$\vec{A} = (2, 1, 0)$$

$$x = 2 + 2 - 3$$

$$\rightarrow \begin{vmatrix} x - 2 & y - 1 & z \\ 1 & -3 & 6 \\ -1 & 2 & 5 \end{vmatrix}$$

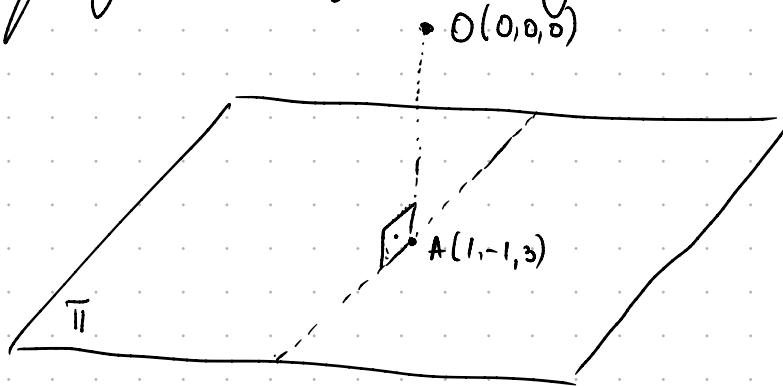
$$\begin{aligned}&-15(x-2) + 2z - 6y + 6 - 3z - 12x \\&+ 2y - 5y + 5 \\&= -\underline{15x} + 30 + \underline{2z} - \underline{6y} + 6 - \underline{3z} - \underline{12x} \\&+ \underline{2y} - \underline{5y} + \underline{5} \\&= -27x - 11y - 2 + 65 = 0\end{aligned}$$

SNU

(exam)

$$\begin{vmatrix} x & y & z & 1 \\ 2 & 1 & 0 & 1 \\ 1 & 3 & 5 & 1 \\ 3 & -2 & 6 & 1 \end{vmatrix} = 0$$

32. Det. a cartesian eq. of the plane π if $A(1, -1, 3)$ is the orthogonal projection of the origin on π



\vec{OA} - normal vector of π

$$\Rightarrow \vec{OA} (1, -1, 3)$$

$$\Rightarrow \pi: x - y + 3z + d = 0$$

$$1 + 1 + 9 = -d$$

$$\Rightarrow d = -11$$

$$\Rightarrow \pi: x - y + 3z - 11 = 0$$

33. Det. the dist. bet. the planes.

$$\pi_1: x - 2y - 2z + 7 = 0$$

$$\pi_2: 2x - 4y - 4z + 17 = 0$$

1. Check if they are \parallel or not using the normal vector.

$$\begin{aligned} \vec{v}_1 &= (1, -2, -2) \\ \vec{v}_2 &= (2, -4, -4) = 2\vec{v}_1 \end{aligned} \quad \left. \begin{array}{l} \Rightarrow \text{they are } \parallel \\ \Rightarrow \text{dist. } (\pi_1, \pi_2) = \text{dist. } (P, \pi_2) \text{ if } P \in \pi_1 \end{array} \right.$$

$P \in \pi_1 \Rightarrow \text{let } P(-7, 0, 0)$

$$\Rightarrow d(P, \pi_2) = \frac{|-14 - 14|}{\sqrt{4+16+16}} = \frac{3}{\sqrt{36}} = \frac{1}{2}$$

37. Det. values a and c for which the line l given by the eq. $\begin{cases} 3x - 2y + z + 3 = 0 \\ 4x - 3y + 4z + 1 = 0 \end{cases}$ is perpendicular.

To the plane π given by $ax + 8y + cz + 2 = 0$

$$3x - 2y + z - 3 = 4x - 3y + 4z + 1$$

$$\Rightarrow x - y + 3z - 2 = 0$$

$$\left| \begin{array}{ccc|c} 3 & -2 & 1 & -3 \\ 4 & -3 & 4 & -1 \\ 1 & -1 & 3 & 2 \end{array} \right|$$

$$\cancel{-24} - 4 \cancel{-8} + \cancel{3} + 12 + \cancel{24} \cancel{3} \\ \cancel{6} \neq 0.$$

$$\Rightarrow \begin{aligned} 3x - 2y &= -3 - 2 & | \cdot 4 \\ 4x - 3y &= -1 - 4 \cancel{2} & | \cdot 3 \end{aligned}$$

$$12x - 8y = -12 - 4 \cancel{2}$$

$$12x - 9y = -3 - 12 \cancel{2} \quad \text{---} \quad (\ominus)$$

$$-y = 9 - 8 \cancel{2} \Rightarrow y = 8 \cancel{2} - 9$$

$$3x - 16 \cancel{2} - 18 = -3 - 2$$

$$3x = -3 - 2 + 16 \cancel{2} + 18$$

$$x = \frac{1}{3}(15 \cancel{2} + 5) \Rightarrow x = 5 \cancel{2} + 5$$

$$\Rightarrow S = \{(5\alpha + 5, 8\alpha - 9, \alpha)\}$$

Normal vector $\rightarrow (9, 8, c)$

$$\Rightarrow 5\alpha + 5 = 9 \Rightarrow 5 \cdot \frac{14}{8} + 5 = 9$$

$$8\alpha - 9 = 8 \Rightarrow 8\alpha = 17 \Rightarrow \alpha = \frac{17}{8}$$

$$\alpha = c \Rightarrow c = \frac{17}{8}$$

$$\frac{85 + 40}{8} = c \quad \boxed{\frac{125}{8} = c}$$

$$-8x + 5y - 11 = 0$$

$$y = \frac{11 + 8x}{5}$$

$$\Rightarrow z = -3x + \frac{22 + 16x}{5} - 3$$

$$z = \frac{-4x - 14}{5}$$

$$\Rightarrow l = \left\{ \alpha, \frac{11 + 8\alpha}{5}, \frac{\alpha - 7}{5} \right\}$$

$$D(l) = \langle (1, \frac{8}{5}, \frac{1}{5}) \rangle = \langle (5, 8, 1) \rangle$$

$$l \perp \pi \Rightarrow (5, 8, 1) \parallel (a, b, c) \Rightarrow a=5 \\ c=1$$

5
7

$$\vec{p}^2 \propto (8, -7, 5)$$

$$\alpha(8, -7, 5)(2, -3, 4)$$

$$\alpha(16 + 8(-20)) = 51$$

$$\alpha \cdot 17 = 51 \Rightarrow \alpha = 3$$

$$16. a. (\vec{i}, \vec{j}, \vec{k})$$

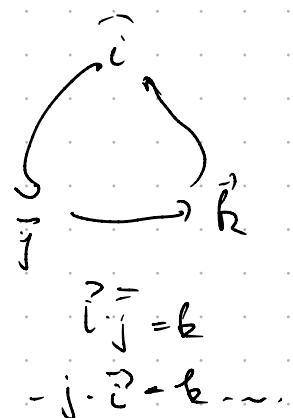
Determine $M(f)$

$$f: V^3 \rightarrow V^3$$

$$\vec{v} \mapsto \vec{w} \times \vec{v} \Rightarrow \vec{w} \cdot \vec{v} = -\vec{v} \cdot \vec{w}$$

$$\text{where } \vec{w} = -\vec{i} + 3\vec{j} + \vec{k}$$

$$\Rightarrow M_b(f) = -(\vec{i}, \vec{j}, \vec{k}) \times (-i, 3j, k)$$



$$f(i) = -i \cdot \vec{w} = -i \cdot (-i + 3j + k) = -i^2 - 3ij - ik = -3k + j = (0, 1, -3)$$

$$f(j) = \vec{w} \cdot \vec{j} = -ij + kj = -k - i = (-1, 0, -1)$$

$$f(k) = \vec{w} \cdot \vec{k} = -ik + 3jk = j - i = (1, 1, 0)$$

$$M_b(f) = \begin{vmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ -3 & -1 & 0 \end{vmatrix}$$

9.16. ABCD-festr. hydro.

$$\text{Vol}(ABCD) = 5$$

$$A(2, 1, -1), B(3, 0, 1), C(2, -1, 3)$$

Find coords of P

$$\text{Vol}(\text{Achtzylinder}) = \frac{1}{6} \text{Vol}(\text{Parallelepiped}) = \frac{1}{6} [\vec{v_1}, \vec{v_2}, \vec{v_3}]$$

$$[\vec{v_1}, \vec{v_2}, \vec{v_3}] = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} \quad D \in O_g$$

$$\text{Vol}(ABCD) = \frac{1}{6} [AB, AC, AD]$$

$$\pm 30 = \begin{vmatrix} 1 & -1 & 2 \\ 0 & -2 & 9 \\ 0 & y & 0 \end{vmatrix} \Rightarrow y = \pm \frac{15}{2}$$