

Addition:

for implementation we always!

Carries

Ex 1

$$\begin{array}{r} 375 \\ + 896 \\ \hline 1271 \end{array}$$

Iteration

it 1:

$$0 + 5 + 6 = 11, \quad 11 \text{ div } 10 = 1$$

$$11 \bmod 10 = 1$$

Ex 2

$$C = , \quad \begin{array}{r} 111 \\ 10111 \end{array}_{(2)} +$$

$$1_{(2)} + 1_{(2)} = 10_{(2)}$$

$$\begin{array}{r} 01011 \\ + 00010 \end{array}_{(2)}$$

$$it 1: 0_{(2)} + 1_{(2)} + 1_{(2)} = 0 + 1 + 1 = 2, \quad 2 \bmod 2 = 0$$

$$2 \text{ div } 2 = 1$$

$$9+1 = 10$$

$$7_{(8)} + 1_{(8)} = 10_{(8)}$$

$$F_{(16)} + 1_{(16)} = 10_{(16)}$$

Ex 3

$$1 \begin{array}{r} 1 \\ 5 \\ 6 \\ 7 \\ + \\ 4 \\ \hline \end{array} \begin{array}{l} ^7 \\ ^0 \\ (8) \\ + \\ (8) \\ \hline \end{array}$$

$$1 \begin{array}{r} 2 \\ 6 \\ 3 \\ \hline \end{array} \begin{array}{l} (8) \\ \hline \end{array}$$

it 1:  $0_{(8)} + 7_{(8)} + 4_{(8)} = 0 + 7 + 4 = 11$

$$11 \text{ div } 8 = 1$$

$$11 \bmod 8 = 3$$

it 2:  $1_{(8)} + 7_{(8)} + 2_{(8)} = 1 + 7 + 2 = 14$

$$14 \text{ div } 8 = 1$$

$$14 \bmod 8 = 6$$

it 3:  $1_{(8)} + 5_{(8)} + 4_{(8)} = 1 + 5 + 4 = 10$

$$10 \text{ div } 8 = 1$$

$$10 \bmod 8 = 2$$

ex 5

$$\begin{array}{r}
 C_{(16)} + \\
 C_{(16)} S_{(16)} + \\
 B_{(16)} F_{(16)} \\
 \hline
 184C
 \end{array}$$

at 1:  $0 + D_{(16)} + F_{(16)} = 13 + 15 = 28$

$28 \text{ div } 16 = 1$

$28 \bmod 16 = 12 \rightarrow C_{(16)}$

at 2:  $1_{(16)} + 9_{(16)} + A_{(16)} = 1 + 8 + 10 = 20$

$20 \text{ div } 16 = 1$

$20 \bmod 16 = 4$

at 3:  $1_{(16)} + C_{(16)} + B_{(16)} = 1 + 12 + 11 = 24$

$24 \text{ div } 16 = 1$

$24 \bmod 16 = 8$

# Multiplication by one digit

(ex5)

$$\begin{array}{r} \text{C: } 2 \ 5 \ 6 \ 7 \\ \times \quad \quad \quad 4 \\ \hline 2 \ 2 \ 6 \ 8 \end{array}$$

it1:  $0 + 7 \cdot 4 = 28$

$$28 \text{ div } 10 = 2$$

$$28 \bmod 10 = 8$$

Obs: if we have more digits, we will have more sums  
(nested loops)

(ex6)

$$\begin{array}{r} \text{C: } 2 \ 2 \ 1 \ 0 \\ \times \quad \quad \quad 5 \ 6 \ 4 \\ \hline 1 \ 2 \ 3 \ 5 \ 5 \end{array}$$

it1:  $0_{(2)} + 4_{(2)} \cdot 3_{(2)} = 0 + 4 \cdot 3 = 12$

$$12 \text{ div } 2 = 1$$

$$12 \bmod 2 = 0$$

It 2:  $6_{(7)} \cdot 3_{(7)} = 1+6 \cdot 3 = 19$

19 like 2 = 2

19 mod 7 = 5

It 3:  $2_{(7)} + 5_{(7)} \cdot 3_{(7)} = 2 + 5 \cdot 3 = 2 + 5 - 17$

17 like 2 = 2

17 mod 7 = 3

(ex 7)

c:

$$\begin{array}{r} 9AB \\ \times C_{(16)} \\ \hline E_{(16)} \end{array}$$

→ de lait aleré

# Division by one digit

Ex 1:

$$\begin{array}{r}
 37 \div 5 \\
 \hline
 3 \quad \text{quotient} \\
 \hline
 2 \quad \text{remainder} \\
 \hline
 4
 \end{array}$$

divisor

Ex 1:  $03_{(10)} = 0 \cdot 10 + 3 = 3$

$$3 \text{ div } 5 = 0$$

$$3 \bmod 5 = 3$$

Ex 2:

$$37_{(10)} = 3 \cdot 10 + 7 = 37$$

$$37 \text{ div } 5 = 7$$

$$37 \bmod 5 = 2$$

ex 5

$$\begin{array}{r} \textcircled{0} \\ 2736_{(8)} \\ \hline 24 \\ \hline 33 \\ \hline 26 \\ \hline 2 \end{array} \quad \left| \begin{array}{r} 5(6) \\ \hline 0454_{(8)} \end{array} \right.$$

it 1:  $02_{(8)} = 0 \cdot 8 + 2 = 2$

$$2 \text{ div } 5 = 0 = 0_{(8)}$$

$$2 \text{ mod } 5 = 2 = 2_{(8)}$$

it 2:  $27_{(8)} = 2 \cdot 8 + 7 = 16 + 7 = 23$

$$23 \text{ div } 5 = 4 = 4_{(8)}$$

$$23 \text{ mod } 5 = 3 = 3_{(8)}$$

it 3:  $33_{(8)} = 3 \cdot 8 + 3 = 27$

$$27 \text{ div } 5 = 5 = 5_{(8)}$$

$$27 \text{ mod } 5 = 2 = 2_{(8)}$$

$$\text{of } \{ : 26_{(8)} = 2 \cdot 8 + 6 = 16 + 6 = 22$$

$$22 \text{ div } 5 = 4_{(8)}$$

$$22 \bmod 5 = 2_{(8)}$$

(ex 10)

$$\begin{array}{r} 0 \\ \text{---} \\ 2B \quad 7F_{(16)} \end{array} \left| \begin{array}{r} A_{(16)} \\ \text{---} \\ 0459 \end{array} \right.$$

$\begin{array}{r} 2B \\ \text{---} \\ 37 \\ \text{---} \\ 5F \\ \text{---} \\ 5 \end{array}$

$$\text{of } \{ : 02_{(16)} = 0 \cdot 16 + 2 = 2$$

$$2 / 16 = 0 = 0_{(16)}$$

$$2 \% 16 = 2 = 2_{(16)}$$

$$\underline{t62}: \quad 2B_{(16)} = 2 \cdot 16 + 11 = 32 + 11 = 43$$

$$43 / 10 = 4 = 4_{(16)}$$

$$43 \% 10 = 3 = 3_{(16)}$$

$$\underline{t51} \quad 37_{(16)} = 3 \cdot 16 + 7 = 48 + 7 = 55$$

$$55 / 10 = 5 = 5_{(16)}$$

$$55 \% 10 = 5 = 5_{(16)}$$

$$\underline{t52}: \quad 57_{(16)} = 5 \cdot 16 + 15 = 95$$

$$95 / 10 = 9 = 9_{(16)}$$

$$95 \% 10 = 5 = 5_{(16)}$$

# Subtraction

(ex 1)

$$\begin{array}{r}
 & 1 & 9 & 3 & 1 \\
 2 & 8 & 0 & 5 & 4 \\
 - & 0 & 3 & 6 & 7 & 9 \\
 \hline
 & 2 & 8 & 3 & 1 & 7 & 5
 \end{array}$$

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 → substrahend =  
 = rezipitor

known:

$$\begin{array}{r}
 & 1 & -1 & -1 & -1 & 0 \\
 3 & 2 & 0 & 0 & 5 & 4 \\
 - & 0 & 3 & 6 & 7 & 9 \\
 \hline
 & 2 & 8 & 3 & 1 & 7 & 5
 \end{array}$$

it 1:  $0 + 4 - 9 = -5 < 0 \Rightarrow$

$$\Rightarrow b = -1$$

$$-5 + 10 = 5$$

**! base!**

it 2:  $-1 + 5 - 7 = -3 < 0 \Rightarrow$

$$b = -1$$

$$-3 + 10 = 7$$

it 3:  $-1 + 0 - 8 = -9 < 0 \Rightarrow$

$$b = -1$$

$$-9 + 10 = 1$$

it 4:  $-1 + 0 - 6 = -7 < 0 \Rightarrow$

$$b = -1$$

$$-7 + 10 = 3$$

it 5:  $-1 + 2 - 3 = -2 < 0 \Rightarrow$

$$b = -1$$

$$-2 + 10 = 8$$

$$\underline{\text{Ex 6}}: -1 + 3 - 0 = 2 \rightarrow b=0$$

(ex 12)

$$\begin{array}{r} \overset{-1}{6} \overset{-1}{0} \overset{-1}{3} \overset{-1}{2} \overset{0}{1}_{(7)} \\ \overset{-1}{0} \overset{-1}{2} \overset{-1}{6} \overset{-1}{4} \overset{-1}{5}_{(7)} \\ \hline \overset{-1}{5} \overset{-1}{4} \overset{-1}{3} \overset{-1}{4} \overset{-1}{3}_{(7)} \end{array}$$

$$\underline{\text{Ex 11}}: 0_{(7)} + 1_{(7)} - 5_{(7)} = 0 + 1 - 5 = -4 < 0$$

$$b = -1$$

$$-4 + 7 = 3 = 3_{(7)}$$

$$\underline{\text{Ex 12}}: -1_{(7)} + 2_{(7)} - 4_{(7)} = -1 + 2 - 4 = -3 < 0$$

$$b = -1$$

$$-3 + 7 = 4 = 4_{(7)}$$

$$\underline{\text{Ex 13}}: -1_{(7)} + 3_{(7)} - 6_{(7)} = -1 + 3 - 6 = -4 < 0$$

$$b = -1$$

$$-4 + 7 = 3 = 3_{(7)}$$

$$\underline{\text{Ex 14}}: -1_{(7)} + 0_{(7)} - 2_{(7)} = -1 + 0 - 2 = -3 < 0$$

$$b = -1$$

$$-3 + 7 = 4 = E_{(7)}$$

ots:  $- \frac{1}{(7)} + 6_{(7)} = 5 \rightarrow$

N:  $\begin{array}{r} 5 \{ 3 \{ 3 \\ 0 2 6 \{ 5_{(7)} \\ \hline \end{array}$

(ex15)  $\begin{array}{r} -1 \quad 0 \quad -1 \quad -1 \quad 0 \\ B \quad O \quad T \quad 3 \quad A \\ \hline 0 \quad 6 \quad E \quad 1 \quad C \\ \hline A \quad C \quad 0 \quad 5 \quad E \end{array}$

st1:  $Q_{(6)} + A_{(16)} - C_{(16)} = 0 + 10 - 12 = -2 < 0$

$$b = -1$$

$$-2 + 16 = 14 = E_{(16)}$$

st2:  $-1_{(16)} + 3_{(16)} - A_{(16)} = -1 + 3 - 13 = -11 < 0$

$$b = -1$$

$$-11 + 16 = 5 = S_{(16)}$$

$$\text{Stz: } -k_{(6)} + f_{(6)} - f_{(6)} = -1 + 15 - 14 = 0$$

$$b=0$$

$$\text{Stz: } 0_{(6)} + 0_{(6)} - k_{(6)} = 0 + 0 - 4 = -4 < 0$$

$$b=-1$$

$$-4 + 16 = 12 = C_{(6)}$$

$$\text{Stz: } -k_{(6)} + b_{(6)} - 0_{(6)} = -1 + 11 - 0 = 10 \neq 0$$

$$b=0$$

## Rapid conversions

Ex 14

$$370241_{(8)} = ?_{(16)}$$

$$(370241, 65)_{(8)} = 0111100001010001, 11010101_{(2)}$$

$\equiv \text{FOAL}, \Delta^4_{(16)}$

0133002101, 510

$$7 = 4 + 1 + 1 = 2^2 + 2 + 2^0 = 111_{(2)}$$

$$\Delta_{(6)} = 13 = 8 + 4 + 1 = 2^3 + 2^2 + 2^0 = 1101_{(2)}$$

$$A_{(6)} = 10 = 2 \cdot 4^1 + 2 \cdot 4^0 = 2d(4)$$

$$F_{(6)} = 15 = 3 \cdot 1 + 3 \cdot 4^0 = 33 \cdot 4^0$$

$$f(6) = 15 = 3 \cdot 4^1 + (-1)^0 = 3f(4)$$

$$h_{(16)} = 10(s)$$