13.05.2024

test exercise

2) $\begin{cases} x' = 3x \\ y' = -2y \end{cases}$, $A = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix} \Rightarrow \lambda_1 = 3, \lambda_2 = -2$ $\Rightarrow$

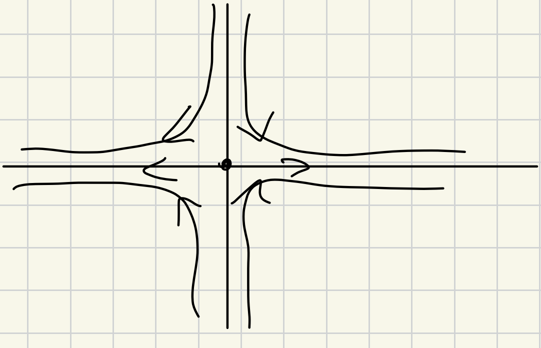
$\begin{cases} 3x = 0 \\ -2y = 0 \end{cases} \Rightarrow (0,0)$ is eq. point

$\Rightarrow$ saddle point, unstable

$$\frac{dx}{dy} = \frac{3x}{-2y}$$

$$\frac{3\,dx}{x} = \frac{dy}{-2y} \overset{\int}{\Leftrightarrow} 3\int \frac{1}{x}\,dx = -\frac{1}{2}\int \frac{1}{y}\,dy \Leftrightarrow$$

$$\Leftrightarrow 3\ln|x| + c = -\frac{1}{2}\cdot \ln|x| \quad \ldots$$



S6 ex 1 Find the general solution of each of the following equations looking first for some sol. of the form $x = t^r$, $r \in \mathbb{R}$.

a) $t^2 x'' + 8 t x' + 20 x = 0$, $t \in (0, +\infty)$

2^{nd} order linear hom. diff. equation with non-const. coef.

Hint: has sol. $x = t^r$

$$x' = (t^r)' = r\, t^{r-1}$$

$$x'' = (r\, t^{r-1})' = r \cdot (r-1) \cdot t^{r-2}$$

$$r(r-1) \cdot t^r - 8 \cdot r \cdot t^r + 20 t^r = 0$$

$$t^r \underbrace{\left(r(r-1) - 8r + 20 \right)}_{\text{must be } 0} = 0$$

$$r^2 - r - 8r + 20 = 0$$

$$r^2 - 9r + 20 = 0$$

$$(r-4)(r-5) = 20 \rightarrow \begin{cases} r_1 = 4 \mapsto t^4 \\ r_2 = 5 \mapsto t^5 \end{cases}$$

$$x = c_1 \cdot t^3 + c_2 \cdot t^5 \quad , c_1, c_2 \in \mathbb{R}.$$

2) Find the general solution of the following linear planar system using the reduction method.

$$\begin{cases} x' = 2x - 5y \\ y' = x - 2y \end{cases}$$

$$x' = 2x - 5y \Rightarrow 5y = 2x - x' \Rightarrow y = \frac{2x - x'}{5}$$

$$y' = x - 2y = x - 2 \cdot \frac{2x - x'}{5} =$$

$$= x - \frac{4x}{5} - \frac{2x'}{5} = \frac{x}{5} \cdot \frac{2x'}{5}$$

$$x'' = \cancel{2x'} - x - \cancel{2x'}$$

$$x'' = -x$$

$$x'' + x = 0$$

$$r^2 + 1 = 0 \Rightarrow r_{1,2} = \pm i \xrightarrow[m=1]{\alpha = 0,\ \beta = 1} e^{0} \cdot \cos t,\ e^{0} \cdot \sin t$$

$$\boxed{x = c_1 \cdot \cos t + c_2 \cdot \sin t \quad , c_1, c_2 \in \mathbb{R}.}$$

$$x' = -c_1 \sin t + c_2 \cos t$$

$$y = \frac{2x - x'}{5} = \frac{2c_1 \cdot \cos t + 2c_2 \sin t + c_1 \sin t - c_2 \cos t}{5} =$$

$$= \boxed{\frac{\cos t \cdot (2c_1 - c_2) + \sin t \cdot (2c_2 + c_1)}{5}}, \quad c_1, c_2 \in \mathbb{R}.$$