Ch 7: 9, ~~(10 a) b)~~

Ch 8: 3, 4, 5, 10, 14

$Q: a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_{10}x + 2a_{01}y + a_{00} = 0$

$$M_Q = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}$$

$$\hat{M}_Q = \begin{pmatrix} a_{11} & a_{12} & a_{10} \\ a_{12} & a_{22} & a_{01} \\ a_{10} & a_{01} & a_{00} \end{pmatrix}$$

$$Q: \begin{pmatrix} x & y & 1 \end{pmatrix} \cdot \hat{M}_Q \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0$$

isometric invariants:

$\hat{\Delta} = \det(\hat{M}_Q)$

$\Delta = \det(M_Q)$

$T = \mathrm{Tr}(M_Q)$

$\hat{\Delta}$	Δ	T	Curve Q	
0	>0		a point	
	$=0$		two lines or the empty set	
	<0		two lines	
$\neq 0$	>0	<0	ellipse	(positivity)
	>0	>0	empty set	
	$=0$		parabola	(equals 0)
	<0		hyperbola	(negativity)

7.9. Discuss the type of the curve $x^2 + \lambda xy + y^2 - 6x - 16 = 0$

hw: solve this using the techniques from last time

$$\hat{\Delta} = \begin{vmatrix} 1 & \frac{\lambda}{2} & -6 \\ \frac{\lambda}{2} & 1 & 0 \\ -6 & 0 & -16 \end{vmatrix} = -16 + 0 + 0 - 36 - 0 + \cancel{16} \cdot \frac{\lambda^2}{4}$$

$$= 4\lambda^2 - 48.$$

I $\hat{\Delta} = 0 \Rightarrow 4\lambda^2 = 48 \Rightarrow \lambda^2 = 12 \Rightarrow \lambda = \pm 2\sqrt{3}$

$\delta = 1 - \frac{x^2}{4} < 0$ two lines.

II $\hat{\delta} > 0 \Rightarrow 4x^2 > 16 \Rightarrow x^2 > 4 \Rightarrow x \in (-\infty, -2) \cup (2, \infty)$

$\Rightarrow \Delta < 0$

7.16. Decide what surfaces are described by the following eq:

a) $x^2 + 2y^2 + z^2 + xy + yz + zx = 1$

b) $xy + yz + zx = 1$

$= x^2 + y^2 + z^2 + 2xy + 2yz + 2zx + y^2 - xy - yz - zx = 1$

$= (x+y+z)^2 + y^2 - xy - yz - zx = 1.$

$= (x+y+z)^2 + y^2 - x \cdot \frac{1}{2} \cdot y + \frac{x^2}{4} - \left(\left(\frac{x}{2}\right)^2 + 2 \cdot \frac{x}{2} \cdot z + z^2\right) + z^2 - 2 \cdot z \cdot \frac{y}{2} + \frac{y^2}{4} - \frac{y^2}{4}$

$= (x+y+z)^2 + \left(y - \frac{x}{2}\right)^2 - \left(\frac{x}{2} + z\right)^2 + \left(z - \frac{y}{2}\right)^2 - \frac{y^2}{4} = 1$

$= \underbrace{\left(x + \frac{y}{2} + \frac{z}{2}\right)}_{x'}^2 - \frac{y^2}{4} - \frac{z^2}{4} + \frac{yz}{2} = 1$

$= x'^2 - \frac{y^2}{4} - \frac{z^2}{4} + \frac{yz}{2} = 1$

$= x'^2 - \left(\left(\frac{y}{2}\right)^2 - 2 \cdot \frac{y}{2} \cdot \frac{z}{2} + \frac{z^2}{4}\right) = 1$

$= x'^2 - \left(\frac{y}{2} - \frac{z}{2}\right)^2$

$= x'^2 + y'^2 + \frac{5}{7} z^2 = 1.$ – Ellipsoid

b) $xy + yz + zx = 1$

$x = y + x'$

$(y + x') \cdot y + yz + z \cdot (y + x') = 1$

$$y^2 + yx' + 2yz + zx' = 1$$

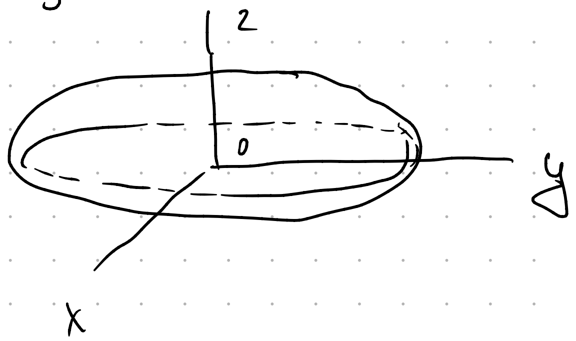
$$y^2 + yx' + 2yz + \left(\tfrac{1}{2}x'\right)^2 + z^2 + 2\cdot\tfrac{1}{2}x'z\big) - \tfrac{1}{2}x^2 - z^2 = 1$$

$$\underbrace{\left(y + \tfrac{1}{2}x' + z\right)^2}_{y'} - \left(\tfrac{1}{2}x'\right)^2 - z^2 = 1$$

$$\Rightarrow -\frac{x'^2}{4} + y'^2 - z^2 = 1 \Rightarrow \text{2 sheet hyperboloid}$$

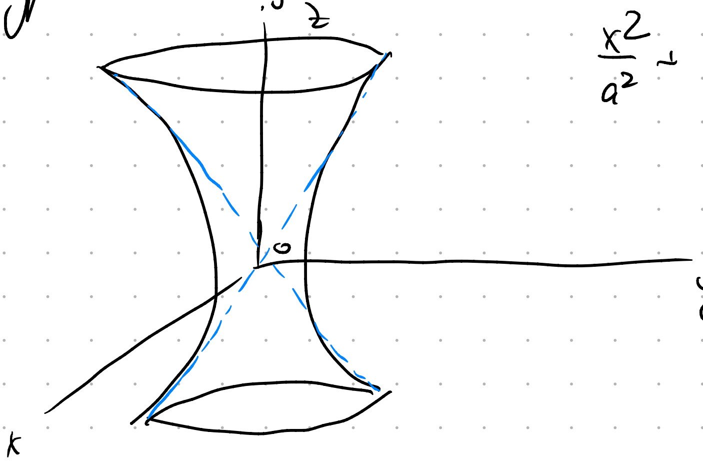
ellipsoid:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



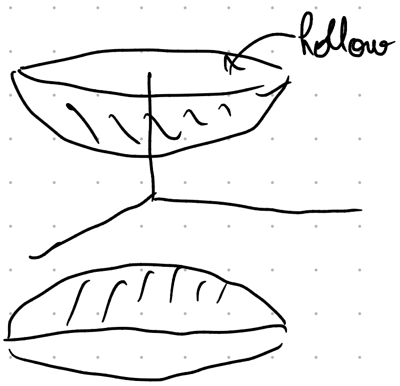
hyperboloid of one sheet.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



hyperboloid of two sheets

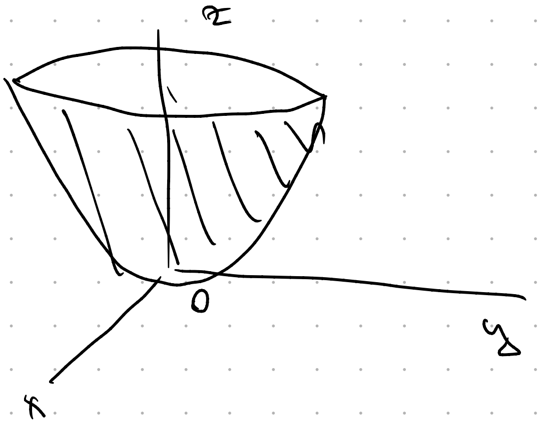
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



elliptic paraboloid

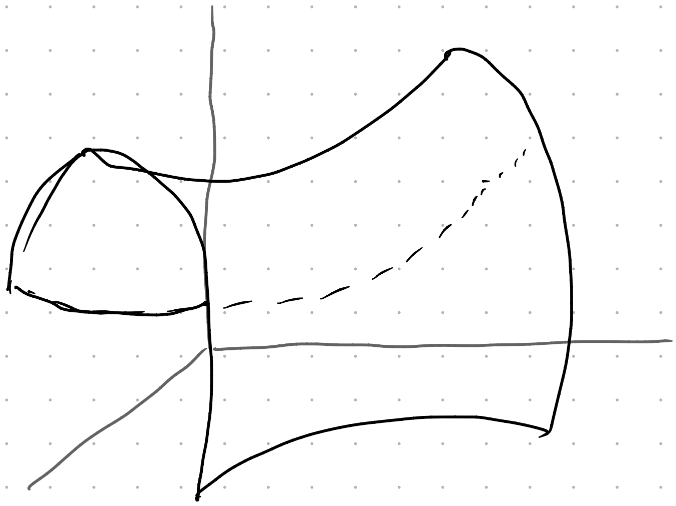
$P_{p,q}$ $\frac{x^2}{p} + \frac{y^2}{q} = 2z$

$p, q > 0$



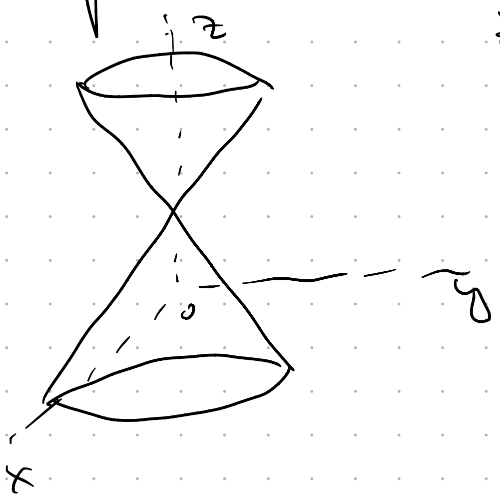
hyperbolic paraboloid.

$P^h_{p,q}$ $\frac{x^2}{p} - \frac{y^2}{q} = 2z$



Elliptic cone

$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$



Tangent lia:

$$\frac{\partial f}{\partial x}(x_0,y_0,z_0)\underbrace{(x-x_0)}+\frac{\partial f}{\partial y}(x_0,y_0,z_0)\underbrace{(y-y_0)}+\frac{\partial f}{\partial z}(x_0,y_0,z_0)\underbrace{(z-z_0)}$$

8.4. Det. the tagent planes to the ellipsoid

$\mathcal{E}_{2,3,2\sqrt{2}}: \frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{8}=1$

which are parallel to the plane $\pi: 3x-2y+5z+1=0$

$$\frac{xx_0}{4}+\frac{yy_0}{9}+\frac{zz_0}{8}=1$$

$\vec{m}_{T\mathcal{E}}=\left(\frac{x_0}{4},\frac{y_0}{9},\frac{z_0}{8}\right)$

$\vec{m}_{\pi}=(3,-2,5)$

$T\mathcal{E}\parallel\pi \Rightarrow \vec{m}_{\pi}\parallel\vec{m}_{T\mathcal{E}} \Rightarrow \frac{x_0}{12}=\frac{y_0}{-18}=\frac{z_0}{40}=t$

$$\frac{x_0}{6}=\frac{y_0}{-9}=\frac{z_0}{20}=t^*$$

$$\Rightarrow x_0=6t$$
$$y_0=-9t$$
$$z_0=40t$$

$$\Rightarrow \frac{(12t)^2}{4}+\frac{(-18t)^2}{9}+\frac{(40t)^2}{8}=1$$

$$36t^2+36t^2+200t^2=1$$

$$272t^2=1 \Rightarrow t=\pm\frac{1}{\sqrt{272}}=\pm\frac{1}{2\sqrt{68}}=\pm\frac{1}{4\sqrt{17}}$$

$$\Rightarrow P_1\left(\frac{3}{\sqrt{17}},-\frac{9}{2\sqrt{17}},\frac{10}{\sqrt{17}}\right),\ P_2\left(-\frac{3}{\sqrt{17}},\frac{9}{2\sqrt{17}},-\frac{10}{\sqrt{17}}\right)$$