30.10.2023

$X = 0{,}13$, single precision, mantissa < 1

1. Convert into binary. (first into base 8, intermediate base)

$0{,}13 * 8 = 1{,}52$

$0{,}52 * 8 = 4{,}16$

$X = 0{,}14\ldots_{(8)}$

$= 0{,}\underbrace{001}_{1_{(8)}}\underbrace{100}_{4_{(8)}}\ldots_{(2)} \Rightarrow$

⇒ non-normalized mantissa

to normalize, we move the decimal point

$\underbrace{0{,}1100}_{\text{normalized mantissa}} \cdot 2^{-2}$, $-2 = e$

⇒ in order to not change the value

Single precision

S	$c = \xi + e$	mantissa
31	8 bits	23 bits

! we have 4 bits, we need 19 more to complete the mantisa !!

7 multiplications

$7 \cdot 3(\text{digits}) = 21$

1. $0,16 * 8 = 1,28$
2. $0,28 * 8 = 2,24$
3. $0,24 * 8 = 1,92$
4. $0,92 * 8 = 7,36$
5. $0,36 * 8 = 2,88$
6. $0,88 * 8 = 7,04$

7. $0,04 * 8 = 0,32$

not needed

$$0,\boxed{1100\ \underbrace{001}_{1}\ \underbrace{010}_{2}\ \underbrace{001}_{1}\ \underbrace{111}_{7}\ \underbrace{010}_{2}\ \underbrace{111}_{7}\ \underbrace{0}_{0}}\boxed{00}$$

mantisa

mantisa (23 bits)

2^7 constant

e

$c = 127 + (-2) = 125$

$125 = 175_{(8)} = 001111101_{(2)}$

S	C	mantissa
0	01111101	

2) Find the real number $x = C1DE0000_{(16)}$ as its fixed-point representation on 32 bits $I=12, F=15$

C	1	D	E	0	0	0	0
1100	0001	1101	1110	0000	0000	0000	0000

$1 \,|\, 100\,0001\,1101\,1 \,|\, 110\;0000\;0000\;0000\;0000$

S

I = 12 bits integer part

F = 15 bits fractional part.

$$X = -\,\overset{11\;10\;9\;8\;7\;6\;5\;4\;3\;2\;1\;0}{100000111011},\overset{-1\,-2}{11}\underbrace{0\;0000\;0000\;0000\;0000}_{\text{not needed!!}}\,{}_{(2)}$$

$$X = -(2048 + 32 + 16 + 8 + 2 + 1 + \tfrac{1}{2} + \tfrac{1}{4}) = -2107{,}75$$

③ Find the real number x having $C1DE0000_{(16)}$ as its floating point repr. SP, $m \geq 1$

single precision => 32 bits.

C	1	D	E	0	0	0	0
1100	0001	1101	1110	0000	0000	0000	0000

$\underbrace{1}_{s}\ \underbrace{1000\,0011}_{c = 8 \text{ bits}}\ \underbrace{101\,1110\,0000\ldots0}_{\text{mantisa} = 23 \text{ bits}}$

$c = \varsigma + e$

$$c = 1000\,0011_{(2)} = 1 + 2 + 128 = 131$$

$$c = \underbrace{\varsigma}_{\text{constant}} + e = 127 + e = 131 \Rightarrow e = 131 - 127 = 4$$

$$x = -1,\underbrace{0101\,111}_{1\ 2\ 3\ 4}{}_{(2)} \cdot 2^{4} = -1\,1011,11_{(2)} =$$

hidden bit because mantisa ≥ 1 (given in problem statement)

$$= -\Big(\underbrace{1 + 2 + 8 + 16}_{27} + \underbrace{\tfrac{1}{2} + \tfrac{1}{4}}_{0,75}\Big) = -27,75_{(10)}$$

$\boxed{\text{ex } 1.1}$

$$\underbrace{p \uparrow (q \uparrow r)}_{U} \stackrel{?}{\equiv} \underbrace{(p \uparrow q) \uparrow r}_{V}$$

Theor (step 1) $U \equiv V$ iff they have identical truth tables

$$p \uparrow q := \neg(p \wedge q)$$

Truth table (step 2)

	p	q	r	$q \uparrow r$	$\overbrace{p \uparrow (q \uparrow r)}^{U}$	$p \uparrow q$	$\overbrace{(p \uparrow q) \uparrow r}^{V}$
i_1	T	T	T	F	T	F	T
i_2	T	T	F	T	F	F	T
i_3	T	F	T	T	F	T	F
i_4	T	F	F	T	F	T	T
i_5	F	T	T	F	T	T	F
i_6	F	T	F	T	T	T	T
i_7	F	F	T	T	T	T	F
i_8	F	F	F	T	T	T	T

Conclusion (step 3)

U and V don't have identical truth tables, so "$\uparrow$" is NOT associative.

$\boxed{\text{ex } 2.1}$ $\to$ Decide the type of the formula:

$$U_1 = \underbrace{q \vee \neg p \vee r}_{U} \to \underbrace{\neg(p \vee r)}_{V}$$

(I) Theory.

Def: 1) U is consistent iff $\exists i$, $i(U) = T$

2) U is inconsistent iff $\forall i$, $i(U) = F$

3) U is valid (tautology) iff $\forall i$, $i(U) = T$

$$\models U$$

4) U is contingent iff $\exists i$, $i(U) = T$ and $\exists j$, $j(U) = F$

II Truth table

	p	q	r	$\mathcal{U}$	$\mathcal{V}$	$\mathcal{U}_1 = \mathcal{U} \to \mathcal{V}$
i_1	T	T	T	T	F	F
i_2	T	T	F	T	F	F
i_3	T	F	T	T	F	F
i_4	T	F	F	F	F	T
i_5	F	T	T	T	F	F
i_6	F	T	F	T	T	T
i_7	F	F	T	T	F	F
i_8	F	F	F	T	T	T

III Conclusion

$\mathcal{U}_1$ is a contingent formula having 3 models (i_4, i_6, i_8)

($i_4 : \{p, q, r\} \to \{T, F\}$, $i_4(p) = T$, $i_4(q) = F$, $i_4(r) = F$, $i_4(\mathcal{U}_1) = T$) and 5 anti-models (i_1, i_2, i_3, i_5, i_7)

($i_1(p) = T$, $i_1(q) = T$, $i_1(r) = T$, $i_1(\mathcal{U}_1) = F$)

$\boxed{\text{ex } 3.1}$ Using the truth table method, check whether the following logical consequences hold.

1. $\underbrace{p \to q}_{\mathcal{U}} \models \underbrace{(p \to r) \to (p \to q \wedge r)}_{= \mathcal{V}}$

I Def: $\mathcal{V}$ is a logical consequence of $\mathcal{U}$ ($\mathcal{U} \models \mathcal{V}$) iff all the models of $\mathcal{U}$ are also models of $\mathcal{V}$.

	p	q	r	$p \to r$	$\underbrace{p \to q}_{\mathcal{U}}$	$p \to q \wedge r$	$\mathcal{V}$
i_1	T	T	T	T	T	T	T
i_2	T	T	F	F	T	F	T
i_3	T	F	T	T	F	F	F
i_4	T	F	F	F	F	F	T
i_5	F	T	T	T	T	T	T
i_6	F	T	F	T	T	T	T
i_7	F	F	T	T	T	T	T
i_8	F	F	F	T	T	T	T

Conclusion: All the models of $\mathcal{U}$ are also models of $\mathcal{V}$

$i_{1,2,5,6,7,8}(\mathcal{U}) = T$ and $i_{1,2,5,6,7,8}(\mathcal{Y}) = T$

$\boxed{\text{ex } 4.1}$ $\mathcal{U}_1 = \underbrace{(p \to (q \wedge r))}_{A} \to \underbrace{((p \to q) \wedge (p \to r))}_{B}$

$\overset{?}{\models} \mathcal{U}_1$

distributivity of "$\to$" over "$\wedge$" is proven if $\mathcal{U}_1$ is or not a tautology.

	p	q	r	$p \to q$	$p \to r$	B	$q \wedge r$	A
i_1	T	T	T	T	T	T	T	T
i_2	T	T	F	T	F	F	F	F
i_3	T	F	T	F	T	F	F	F
i_4	T	F	F	F	F	F	F	F
i_5	F	T	T	T	T	T	T	T
i_6	F	T	F	T	T	T	F	T
i_7	F	F	T	T	T	T	F	T
i_8	F	F	F	T	T	T	F	T

Conclusion

$\forall i\ ,\ i(\mathcal{U}_1) = T \implies \models \mathcal{U}_1$

distrib: of „$\vee$" over „$\wedge$"