8. Consider family of ellipses $\mathcal{E}_a: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
$a = ?$ s.t. $\mathcal{E}_a$ is tangent to the line $l: x - y + 5 = 0$

$\mathcal{E}: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$l: y = kx + m$

$l$ is tangent to $\mathcal{E}$ if $m^2 = a^2k^2 + b^2$

$\Rightarrow 25 = a^2 + 4^2$

$25 = a^2 + 16 \Rightarrow a^2 = 9 \Rightarrow a = \pm 3$

$\sqrt{5}x + 6 = y.$

9. Consider a family of lines $l_c: \sqrt{5}x - y + 6 = 0$
for what values of $c$ is $l_c$ tangent to $\mathcal{E}: x^2 + \frac{y^2}{4} = 1$?

From the quadratic $\Rightarrow$

$c^2 = 5 + 4 \Rightarrow c^2 = 9 \Rightarrow c = \pm 3$

10. Det. the common tangents of the ellipses.

$\frac{x^2}{45} + \frac{y^2}{9} = 1$ assume $y = kx + m.$

$\frac{x^2}{9} + \frac{y^2}{18} = 1$ $\Rightarrow m^2 = 45k^2 + 9.$

$m^2 = 9k^2 + 18.$

$\Rightarrow 45k^2 + 9 = 9k^2 + 18.$

$36k^2 = 27$

$k^2 = \frac{1}{36}$

$k = \pm\frac{1}{2}$

$\Rightarrow m^2 = \frac{81}{4} \Rightarrow m = \pm\frac{9}{2}$

$\Rightarrow l: \pm\frac{1}{2}x \pm \frac{9}{2}$

18. tangents to the hyperbola: $H: x^2 - y^2 = 16$ which contain $M(-1, 7)$

$x^2 - y^2 = 16 \quad |:16$

$\frac{x^2}{16} - \frac{y^2}{16} = 1$

$H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\ell_k: y = kx \pm \sqrt{a^2k^2 - b^2}$

$7 = -k \pm \sqrt{k^2 16 - 16}$

$7 + k = \pm\sqrt{k^2 16 - 16} \quad ()^2$

$\cancel{49} + 14k + k^2 = k^2 \cdot 16 - \cancel{16}$

$65 + 14k - 15k^2 = 0$

$15k^2 - 14k - 65 = 0$

$\Delta = 196 + 60 \cdot 65$

$= 196 + 3900$

$= 4096 = 2^{12}$

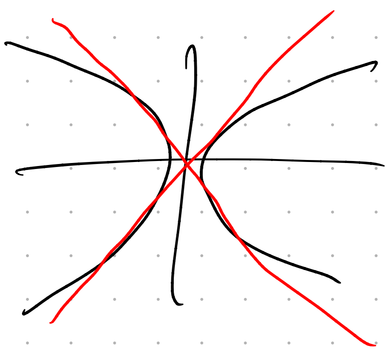
$k_{1,2} = \frac{14 \pm 64}{30} = \frac{78}{30} = \frac{39}{15} = \frac{13}{5}$

$= \frac{-50}{30} = -\frac{5}{3}$

Exercise 20.

Find the area of the triangle determined by the asymptotes of the hyperbola $H: \frac{x^2}{4} - \frac{y^2}{9} = 1$ and the line $\ell: 9x + 2y - 24 = 0$.

$\Rightarrow y = -\frac{9}{2}x + 12$



asymptotes:

$y = \pm \frac{b}{a} x$

$\Rightarrow y = \pm \frac{3}{2} x$

$\frac{3}{2}x = -\frac{9}{2}x + 12$

$\Rightarrow 6x = 12 \Rightarrow \boxed{x=2} \Rightarrow y = -9+12 = 3$

$\Rightarrow A(2,3)$

$-\frac{3}{2}x = -\frac{5}{2}x + 12$

$3x = 12 \Rightarrow \boxed{x=4} \Rightarrow y = -18+12 = -6 \Rightarrow B(4,-6)$

$A_\Delta = \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ 4 & -6 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}\left| -12+0+0-0-0-12 \right|$

$= \boxed{12}$

26 a)

Find an eq for the tangent lines to $H: \frac{x^2}{20} - \frac{y^2}{5} = 1$

which are orthogonal to the line $l: 4x+3y-7=0$

$y = \pm\frac{b}{a}x = \pm\frac{\cancel{\sqrt{5}}}{2\cancel{\sqrt{5}}}x = \pm\frac{1}{2}x \Rightarrow$

$y = -\frac{4}{3}x + \frac{7}{3} \Rightarrow k_l = -\frac{4}{3} \Rightarrow k = \frac{3}{4}$

$l: y = kx \pm \sqrt{20k^2 - 5}$

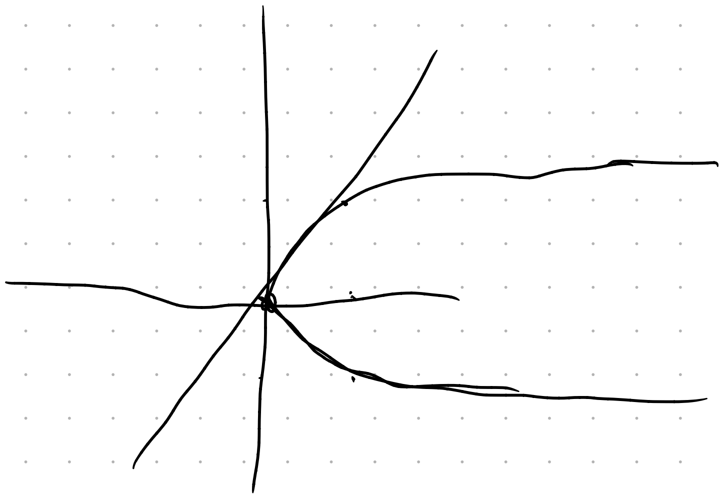
$y = \frac{3}{4}x \pm \sqrt{\frac{20 \cdot 9}{\cancel{16}} - 5}$

$y = \frac{3}{4}x \pm \frac{5}{2}$

26. For which value of $k$ is the line $y = kx + 2$ tangent to the parabola $P: y^2 = 4x$?

$P_p = y^2 = 2px$

tangent lines: $y = kx + \frac{p}{2k}$



$y^2 = 4x = 2px \Rightarrow p = 2.$

$y = kx + \frac{1}{k}$

$\Rightarrow k = \frac{1}{2}$

27: $P: y^2 = 16x$ . Determine the tangents to P.
$\rightarrow p = 8$
s.t. a) parallel to $\ell: 3x - 2y + 30 = 0$
b) $\perp$ to $4x + 2y + 7 = 0$

$-3x + 2y - 30 = 0$

$y = \frac{3}{2}x + 15 \Rightarrow k_\ell = \frac{3}{2} = k$

$$\boxed{y = \frac{3}{2}x + \frac{8}{3}}$$

$y = -2x - \frac{7}{2} \Rightarrow k_\ell = -2 \Rightarrow k = \frac{1}{2}$

$$\boxed{y = \frac{1}{2}x + 8}$$

28. Determine the tangents to the parabola P: $y^2 = 16x$, which contain the point $P(-2,2)$

$\Downarrow$

$p = 8$

$y = kx + \frac{p}{2k}$

$2 = -2k + \frac{8}{2k} \quad / \cdot 2k$

$4k = -4k^2 + 8 \quad / : 4$

$k = -k^2 + 2 \Rightarrow k^2 + k - 2 = 0$

$(k-1)(k+2) \Rightarrow k = 1$, $k = -2$