

11.12.2023

Resolution in propositional logic

$$1.1) U_1 = (A \rightarrow B \wedge C) \rightarrow (A \rightarrow B) \wedge (A \rightarrow C), \vdash U_1$$

$$\neg U_1 = \neg [(A \rightarrow B \wedge C) \rightarrow (A \rightarrow B) \wedge (A \rightarrow C)],$$

we apply the norm. algorithm.

:

$$\equiv \underbrace{(\neg A \vee B)}_{C_1} \wedge \underbrace{(\neg A \wedge C)}_{C_2} \wedge \underbrace{A}_{C_3} \wedge \underbrace{(\neg B \vee \neg C)}_{C_4}$$

$$S = \{C_1, C_2, C_3, C_4\}, S \vdash_{\text{Res}}^? \square$$

$$C_5 = \text{Res}_A(C_1, C_3) = B$$

$$C_6 = \text{Res}_A(C_2, C_3) =$$

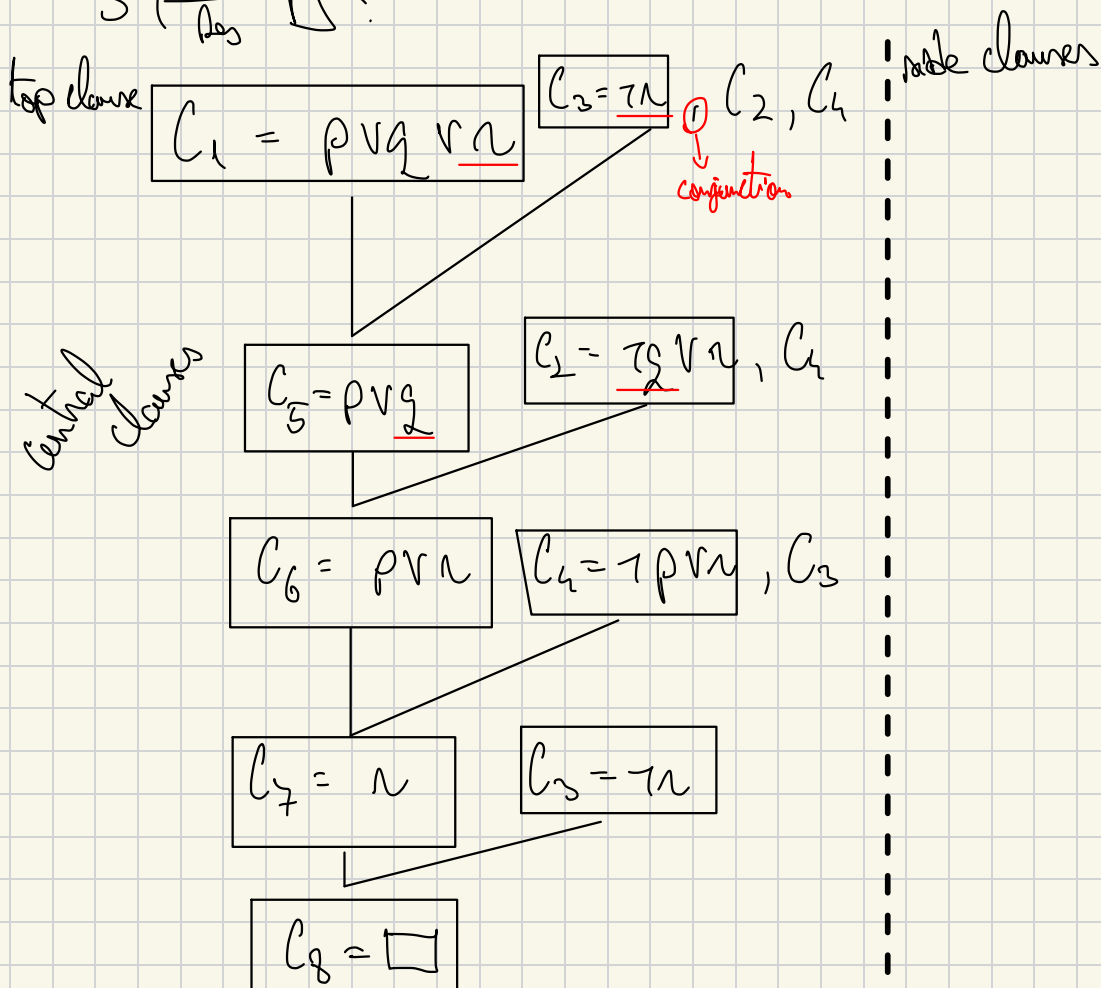
$$C_7 = \text{Res}_B(C_4, C_5) = \neg C$$

$$C_8 = \text{Res}_C(C_7, C_6) = \square$$

$$\text{Res}_2(f \vee l, g \vee r) = f \vee g$$

$$4.1) S = \{ \overbrace{p \vee q \vee r}^{C_1}, \overbrace{\neg q \vee r}^{C_2}, \overbrace{\neg r}^{C_3}, \overbrace{\neg p \vee r}^{C_4} \}$$

$S \stackrel{\text{lim}}{\underset{\text{Res}}{\sqsubset}} ?$



$S \stackrel{\text{lim}}{\underset{\text{Res}}{\sqsubset}},$ so S is inconsistent

5.1) $S = \{ \overset{C_1}{\tau \underline{g} V \underline{n}}, \overset{C_2}{P V \underline{n}}, \overset{C_3}{\underline{g} V \tau P} \}$, process consistency

using linear resolution.

(I) top down

$$C_1 = \tau \underline{g} V \underline{n}$$

$$C_2 = P V \underline{n} \quad , \quad C_3$$

$$C_4 = \tau \underline{g} V \underline{P}$$

$$C_3 = \underline{g} V \tau P, C_3$$

$$C_5 = P V \tau P \equiv T \Rightarrow \text{process is blocked} \Rightarrow \Rightarrow \text{backtracking}$$

(II) top down

$$C_1 = \tau \underline{g} V \underline{n}$$

$$C_2 = P V \underline{n} \quad , \quad C_3$$

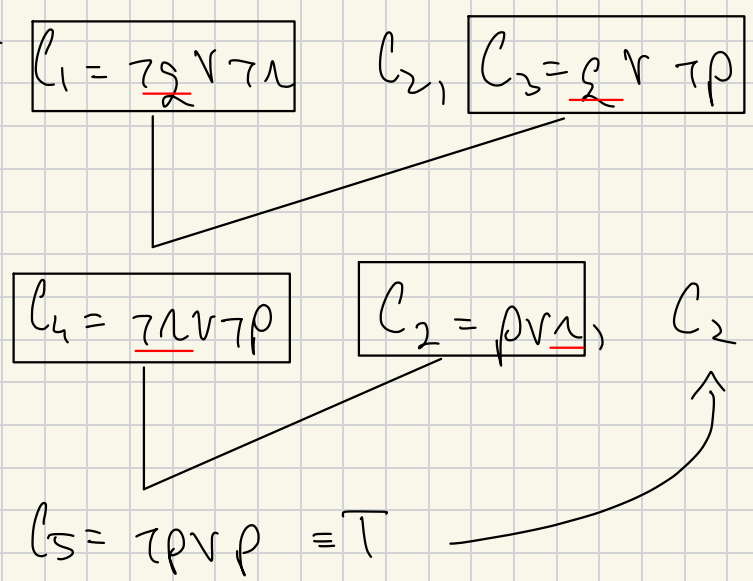
$$C_4 = \tau \underline{g} V \underline{P}$$

$$C_3, C_3 = \underline{g} V \tau \underline{P}$$

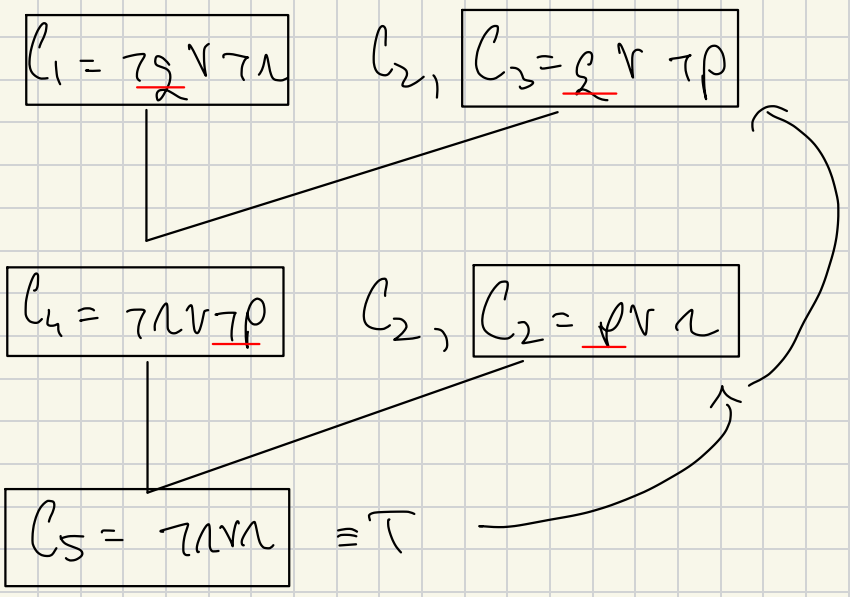
$$C_5 = \tau \underline{g} V \underline{g} \equiv T \Rightarrow \text{process is blocked} \Rightarrow$$

\Rightarrow backtracking

III top clause



IV top clause



After a complete linear search using the backtracking strategy, without the detection of the empty clause, we conclude that S is consistent.

6.1) Using block resolution, prove inconsistency

$$(S \stackrel{\text{block}}{\vdash} \square)$$

$$S^0 = S = \{ \overbrace{p \vee r}^{C_1}, \overbrace{\neg p \vee \neg q \vee r}^{C_2}, \overbrace{\neg p \vee q \vee r}^{C_3}, \overbrace{\neg r}^{C_4} \}$$

$$C_1 = (1) p \vee r$$

$$C_2 = (2) \neg p \vee_{(4)} \neg q \vee_{(5)} r$$

$$C_3 = (6) \neg p \vee_{(7)} q \vee_{(8)} r$$

$$C_4 = (9) \neg r$$

In this method we cannot resolve C_3 with C_4 as r does not have the lowest index in its literal.

$$a) C_5 = \text{Res}_p^{\text{block}}(C_1, C_2) = (2) \neg q \vee_{(5)} r$$

$$C_6 = \text{Res}_p^{\text{block}}(C_1, C_3) = (2) \neg q \vee_{(7)} r$$

$$C_7 = \text{Res}_r^{\text{block}}(C_4, C_5) = (5) \neg q$$

$$C_8 = \text{Res}_r^{\text{block}}(C_4, C_6) = (7) \neg q$$

$$C_5 = \text{Res}_\Sigma^{\text{lock}}(C_7, C_8) = \square \Rightarrow$$

$\Rightarrow S$ is inconsistent.

b) lock no + level saturation strategy (and diff index)

$$C_1 = {}_{(2)}P \vee {}_{(1)}\neg$$

$$C_2 = {}_{(5)}\neg P \vee {}_{(5)}\neg Q \vee {}_{(3)}\neg$$

$$C_3 = {}_{(8)}\neg P \vee {}_{(7)}\neg V_{(6)}\neg$$

$$C_4 = {}_{(5)}\neg \neg$$

$$S^0 = \{C_1, C_2, C_3, C_4\}$$

$$S^1 = \{ \text{Res}_\Sigma^{\text{lock}}(C_i, C_j) \mid C_i, C_j \in S^0 \}$$

$$C_5 = \text{Res}_\Sigma^{\text{lock}}(C_1, C_4) = {}_{(2)}P$$

$$C_6 = \text{Res}_\Sigma^{\text{lock}}(C_2, C_4) = {}_{(5)}\neg P \vee {}_{(4)}\neg Q$$

$$C_7 = \text{Res}_\Sigma^{\text{lock}}(C_3, C_4) = {}_{(8)}\neg P \vee {}_{(7)}\neg$$

$$S^1 = \{ C_5, C_6, C_7 \}$$

$$S^2 = \{ \text{Res}^{\text{lock}}(C_i, C_j) \mid C_i \in S^1, C_j \in S^0 \cup S^1 \}$$

$$C_8 = \text{Res}_2^{\text{lock}}(C_6, C_7) \stackrel{(\exists)P}{=} \perp$$

$$S^2 = \{ C_8 \}$$

$$S^3 = \{ \text{Res}^{\text{lock}}(C_i, C_j) \mid C_i \in S^2 \mid C_j \in S^0 \cup S^1 \cup S^2 \}$$

$$C_9 = \text{Res}_P^{\text{lock}}(C_8, C_5) = \square \Rightarrow S \text{ is inconsistent}$$

Obs: a) and b) are two different strategies paired with two diff indexings. In strat b) we care about indexing since we cannot resolve two literals if the indexes of the resolvent is not the smallest one in the literal it's from.

! Indexing is only relevant for the RELATIVE order!

we stop if one clause is empty on a certain level or the last level is empty!

Theory: if $B \in S^k$, then S is inconsistent
if $S^k = \emptyset$, then S is consistent

7.1) Check the consistency using lock resolution.

$$a) S = \{ \underbrace{p v_2 v_1}_{C_1}, \underbrace{v_2 v_1}_{C_2}, \underbrace{v_1 v_1}_{C_3} \}$$

$$a) C_1 = \underbrace{(1) p}_{(1)} \underbrace{v_2}_{(2)} \underbrace{v_1}_{(3)}$$

$$C_2 = \underbrace{(6) v_2}_{(6)} \underbrace{v_1}_{(7)} \quad S^0 = S$$

$$C_3 = \underbrace{(3) v_1}_{(3)} \underbrace{v_1}_{(5)} p$$

$$S' = \{ \text{Res}^{\text{lock}}(C_i, C_j) \mid C_i, C_j \in S^0 \}$$

$$S' = \emptyset \Rightarrow S \text{ is consistent}$$

Obs: if we choose the appropriate indexing we can get to consistency in a faster way!

$$b) C_1 = \underbrace{(2) p}_{(2)} \underbrace{v_1}_{(1)} \underbrace{v_2}_{(6)}$$

$$C_2 = \underbrace{(4) v_2}_{(4)} \underbrace{v_1}_{(5)} \quad S^0 = S$$

$$C_3 = \underbrace{(7) v_1}_{(7)} \underbrace{v_1}_{(6)} p$$

$$S' = \{ \text{Res}^{\text{lock}}(C_i, C_j) \mid C_i, C_j \in S^0 \}$$

$$C_4 = \text{Res}_2^{\text{lock}}(C_1, C_2) = \underbrace{(2) p}_{(2)} \underbrace{v_1}_{(1)} \underbrace{v_2}_{(4)}$$

$$S^2 = \{ \text{Res}_S^{\text{lock}}(C_i, C_j) \mid C_i \in S^0, C_j \in S^0 \cup S^1 \}$$

$C_5 = \text{Res}_p^{\text{lock}}(C_1, C_3) = \underbrace{\neg}_{(2)} \underbrace{V}_{(7)} \neg \underbrace{L}_{(7)} = T$ in lock resolution we continue with tautologies.

$$C_5 = \underbrace{(1)P}_{\text{red}} \vee \underbrace{(2)L}_{\text{red}}$$

$$C_{10} = \underbrace{(3)T}_{\text{red}} \vee \underbrace{(4)P}_{\text{red}}$$

$$C_u = \text{Res}_p^{\text{lock}}(C_5, C_{10}) = \underbrace{\sum_{(2)} \underbrace{V}_{(7)} P}_{\text{red}}$$

$$\text{index}(P) < \text{index}(L)$$

$$\Rightarrow \text{index}(L) < \text{index}(P)$$

Here is an example that shows us that we can change the relative order using a tautology!

$$S^3 = \{ \text{Res}_S^{\text{lock}}(C_i, C_j) \mid C_i \in S^2, C_j \in S^0 \cup S^1 \cup S^2 \}$$

$$S^3 = \emptyset \Rightarrow S \text{ is constant.}$$