

30.10.2023

$X = 0,13$, single precision, mantissa < 1

1. Convert into binary. (first into base 8, intermediate base)

$$0,13 * 8 = 1,04$$

$$0,04 * 8 = 0,32$$

$$X = 0,13 \dots (8)$$

$$= 0,001100 \dots (2)$$

\Rightarrow non-normalised mantissa

to normalise, we move the decimal point

$$0,1100 \cdot 2^{-2} = e$$

normalised mantissa

in order to not change the value

Single precision



! we have 4 bits, we need 13 more to complete the mantissa !!
7 multiplications
 $7 \cdot 3(\log_2 10) = 21$

1. $0,16 \cdot 8 = 1,28$

2. $0,28 * 8 = 2,24$

$$3) \quad 0,24 \cdot 8 = 1,92$$

$$4. \quad 0,92 * 8 = 7,36$$

5. $0,36 * 8 = 2,88$

6. $0,88 \cdot 8 = 7,04$

$$7. 0,04 * 8 = 0,32$$

not needed

0, 1100 001 010 001 111 010 111 000

mantissa (23 bits)

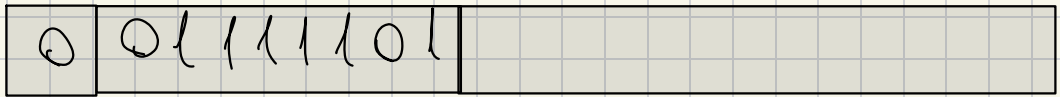
constant
 \hat{g}^2
 \hat{e}
 $(= 127 + (-2)) = 125$

$$125 = 125_{(8)} = 00111101_{(2)}$$

S

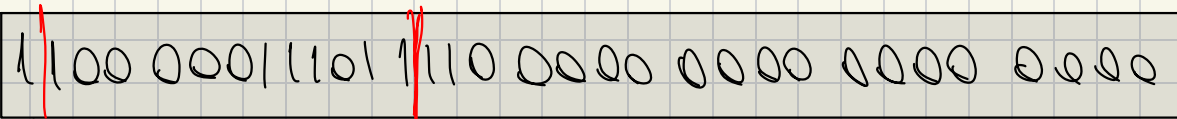
C

mantissa



② Find the real number $x = C1DE0000_{(16)}$
 or its fixed-point representation on 32 bits $I=12, F=13$

C 1 D E 0 0 0 0



S
 $I=12$ bits
 integer part

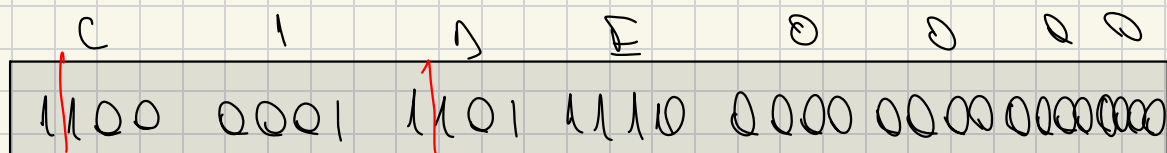
$F=13$ bits
 fractional part.

not needed!!

$$X = \ominus \overset{11}{1} \overset{10}{0} \overset{9}{0} \overset{8}{0} \overset{7}{0} \overset{6}{0} \overset{5}{1} \overset{4}{1} \overset{3}{1} \overset{2}{0} \overset{1}{1} \overset{0}{1} \overset{-1}{1} \overset{-2}{1} \overset{-3}{0} \overset{-4}{0} \overset{-5}{0} \overset{-6}{0} \overset{-7}{0} \overset{-8}{0} \overset{-9}{0} \overset{-10}{0} \overset{-11}{0} \overset{-12}{0} \overset{-13}{0} \overset{-14}{0} \overset{-15}{0} \overset{-16}{0} \overset{-17}{0} \overset{-18}{0} \overset{-19}{0} \overset{-20}{0} \overset{-21}{0} \overset{-22}{0} \overset{-23}{0} \overset{-24}{0} \overset{-25}{0} \overset{-26}{0} \overset{-27}{0} \overset{-28}{0} \overset{-29}{0} \overset{-30}{0} \overset{-31}{0}$$

$$X = \left(2^{12} + 32 + 16 + 8 + 2 + 1 + \frac{1}{2} + \frac{1}{4} \right) = -2107.75$$

② Find the real number x having C1SE0000₍₁₆₎ as its floating point repr. SP, $m \geq 1$ single precision $\rightarrow 32$ bits.



S C = 8 bits mantissa = 23 bits

$C = 9 + e$

$C = 10000011_{(2)} = 1 + 2 + 128 = 131$

$C = 9 + e = 127 + e = 131 \Rightarrow e = 131 - 127 = 4$

constant.

$x = - \textcircled{1} \textcircled{2} \underbrace{101111}_{1\ 2\ 3\ 4} \textcircled{4}_{(2)} \cdot 2^4 = -11011,11_{(2)} =$

hidden bit because mantissa > 1 (given in problem statement)

$= - \left(\underbrace{1 + 2 + 8 + 16}_{27} + \underbrace{\frac{1}{2} + \frac{1}{5}}_{0,75} \right) = -27,75_{(10)}$

ex 1.1

$$\underbrace{p \uparrow (q \uparrow \neg)}_U \stackrel{?}{=} \underbrace{(p \uparrow q) \uparrow \neg}_V$$

Theorem (step 1) $U \equiv V$ iff they have identical truth tables

$$p \uparrow q := \neg(p \wedge q)$$

Truth table (step 2)

	p	q	\neg	$q \uparrow \neg$	$\underbrace{p \uparrow (q \uparrow \neg)}_U$	$p \uparrow q$	$\underbrace{(p \uparrow q) \uparrow \neg}_V$
i_1	T	T	T	F	T	F	T
i_2	T	T	F	T	F	F	T
i_3	T	F	T	T	F	T	F
i_4	T	F	F	T	F	T	T
i_5	F	T	T	F	T	T	F
i_6	F	T	F	T	T	T	T
i_7	F	F	T	T	T	T	F
i_8	F	F	F	T	T	T	T

Conclusion (step 3)

\mathcal{U} and \mathcal{V} don't have identical truth tables,
so " \rightarrow " is NOT associative.

ex 2.1 \rightarrow Decide the type of the formula:

$$\mathcal{U}_1 = \underbrace{p \vee \neg p \vee q}_{\mathcal{U}} \rightarrow \underbrace{\neg(p \vee q)}_{\mathcal{V}}$$

① Theory.

Def: 1) \mathcal{U} is consistent iff $\exists i, i(\mathcal{U}) = T$

2) \mathcal{U} is inconsistent iff $\forall i, i(\mathcal{U}) = F$

3) \mathcal{U} is valid (tautology) iff $\forall i, i(\mathcal{U}) = T$

$$\models \mathcal{U}$$

4) \mathcal{U} is contingent iff $\exists i, i(\mathcal{U}) = T$ and $\exists j, j(\mathcal{U}) = F$

II Truth table

	p	q	r	U	\neg	$U_1 = U \rightarrow \neg$
i_1	T	T	T	T	F	F
i_2	T	T	F	T	F	F
i_3	T	F	T	T	F	F
i_4	T	F	F	F	F	T
i_5	F	T	T	T	F	F
i_6	F	T	F	T	T	T
i_7	F	F	T	T	F	F
i_8	F	F	F	T	T	T

III Conclusion

U_1 is a contingent formula having 3 models (i_4, i_6, i_8)

($i_4: \{p, q, r\} \rightarrow \{T, F\}, i_4(p)=T, i_4(q)=F, i_4(r)=F, i_4(U_1)=T$) and 5 anti-models (i_1, i_2, i_3, i_5, i_7)

($i_1(p)=T, i_1(q)=T, i_1(r)=T, i_1(U_1)=F$)

ex 3.1 Using the truth table method, check whether the following logical consequences hold.

$$1. \quad p \rightarrow q \models (p \rightarrow r) \rightarrow (p \rightarrow q \wedge r)$$

U
= V

I Def: V is a logical consequence of U ($U \models V$) iff all the models of U are also models of V .

	p	q	r	$p \rightarrow r$	$p \rightarrow q$	$p \rightarrow q \wedge r$	V
\hat{u}_1	T	T	T	T	T	T	T
\hat{u}_2	T	T	F	F	T	F	T
\hat{u}_3	T	F	T	T	F	F	F
\hat{u}_4	T	F	F	F	F	F	T
\hat{u}_5	F	T	T	T	T	T	T
\hat{u}_6	F	T	F	T	T	T	T
\hat{u}_7	F	F	T	T	T	T	T
\hat{u}_8	F	F	F	T	T	T	T

Conclusion: All the models of U are also models of V .

$i_{1,2,5,6,7,8}(U) = T$ and $i_{1,2,5,6,7,8}(V) = T$

ex 4.1

$$U_1 = (p \rightarrow (q \wedge r)) \rightarrow ((p \rightarrow q) \wedge (p \rightarrow r))$$

$$\stackrel{②}{=} U_1$$

distributivity of \rightarrow over \wedge is proven if

U_1 is or not a tautology.

	p	q	r	p \rightarrow q	p \rightarrow r	B	q \wedge r	A
i_1	T	T	T	T	T	T	T	T
i_2	T	T	F	T	F	F	F	F
i_3	T	F	T	F	T	F	F	F
i_4	T	F	F	F	F	F	F	F
i_5	F	T	T	T	T	T	T	T
i_6	F	T	F	T	T	T	F	T
i_7	F	F	T	T	T	T	F	T
i_8	F	F	F	T	T	T	F	T

Conclusion

$$\forall i, i(U_i) = T \Rightarrow \models U_i$$

disturb: of " \sim " over " \wedge "