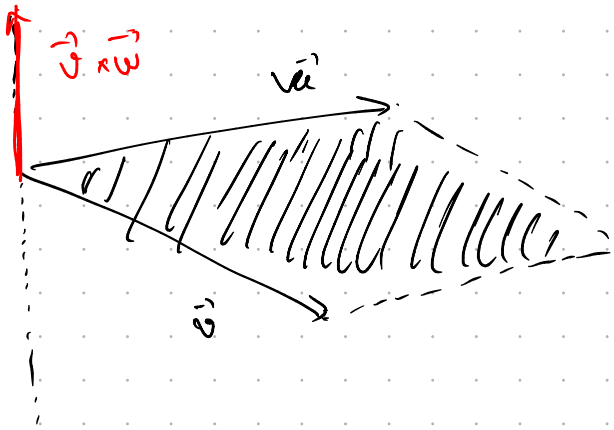
Ch 4:

2, 3, 4, 10 a), 11 a), 13, 16, 17

Cross product (vector product)



$|\vec{v} \cdot \vec{w}| = |\vec{v}| \cdot |\vec{w}| \cdot \sin(v, w)$

$\vec{v} \cdot \vec{w} \perp \vec{v}$ and $\vec{v} \cdot \vec{w} \perp \vec{w}$

Orientation of $\vec{v} \times \vec{w}$ given by the right hand rule

If we work over a right orthonormal system and $\vec{v_1}(x_1, y_1, z_1)$
$\vec{v_2}(x_2, y_2, z_2)$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

Properties:

- bilinearity:

$\forall \alpha_1, \alpha_2 \in \mathbb{R}, \forall \vec{v_1}, \vec{v_2}, \vec{w} \in V^3$

$(\alpha_1 \vec{v_1} + \alpha_2 \vec{v_2}) \times \vec{w} =$

$\alpha_1(\vec{v}_1 \times \vec{w}) + \alpha_2(\vec{v}_2 \times \vec{w})$

- skew-symmetry:

$\forall \vec{v}, \vec{w} \in V^3$:

$\vec{v} \times \vec{w} = -\vec{w} \times \vec{v}$

- if $\vec{v}, \vec{w}$, lin. indep $\Rightarrow \vec{v} \times \vec{w} = \vec{0}$

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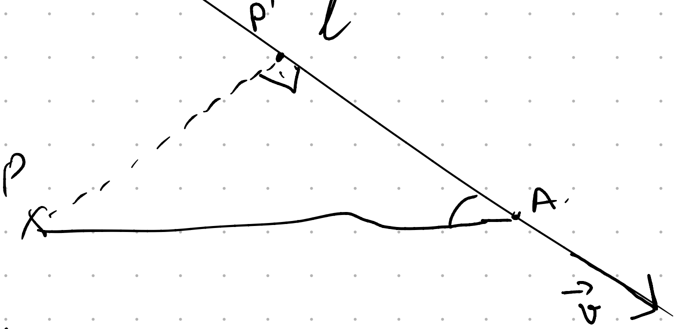
w. respect to a right oriented sys.
oriented orthogonal basis of V^3 consider the vectors
$\vec{a}(3,-1,2)$, $\vec{b}(1,2,-1)$
Calculate
$\vec{a} \times \vec{b}$, $(2\vec{a}+\vec{b}) \times \vec{b}$
$(2\vec{a}+\vec{b}) \times (2\vec{a}-\vec{b})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{vmatrix} = \vec{i} + 6\vec{k} + 2\vec{j} + \vec{k} - 4\vec{i} + 3\vec{j}$$
$$= -3\vec{i} + 7\vec{k} + 5\vec{j} = (-3, 5, 7)$$

$$(2\vec{a}+\vec{b}) \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & 0 & 3 \\ 1 & 2 & -1 \end{vmatrix} = 14\vec{k} + 3\vec{j} - 6\vec{i} + 7\vec{j} = 14\vec{k} + 10\vec{j} - 6\vec{i}$$

$$(2\vec{a}+\vec{b}) \times (2\vec{a}-\vec{b}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 7 & 0 & 3 \\ 5 & -4 & 5 \end{vmatrix} = \vec{0} \ldots$$

$P \notin l$



Pick $A \in L$

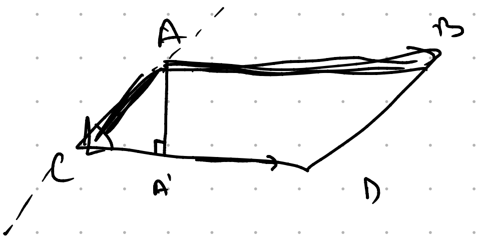
$PP' = PA \cdot \sin(\widehat{PAP'}) =$

$$= \frac{|\vec{PA} \times \vec{P'A}|}{|\vec{P'A}|}$$

Let $\vec{v} \in \delta(\ell) \Rightarrow PA' = \alpha \cdot \vec{v}, \alpha \in \mathbb{R}$

$$dist(P,\ell) = PP' = \frac{|\vec{PA} \times (\alpha\vec{v})|}{|\alpha\vec{v}|} = |\cancel{\alpha}| \cdot \frac{|\vec{PA} \times \vec{v}|}{\cancel{|\alpha|} \cdot |\vec{v}|} = \frac{|\vec{PA} \times \vec{v}|}{|\vec{v}|}$$

4.3. Det. the distances between opposite sides of a paralelog. spanned by ABDC. $\vec{AB}(6,0,1)$, $\vec{AC}(\frac{3}{2},2,1)$



$$\vec{AA'} = dist.(AB,CD) = \frac{|\vec{AC} \cdot \vec{v}|}{\vec{v}} =$$

$$S_{ABCD} = |\vec{AB} \times \vec{AC}| = |\vec{AB} \times \vec{CC'}|$$

$$\Rightarrow \vec{CC'} = \frac{|\vec{AB} \cdot \vec{AC}|}{\vec{AB}} = \begin{vmatrix} i & j & k \\ 6 & 0 & 1 \\ \frac{3}{2} & 2 & 1 \end{vmatrix} = \left(-2, -\frac{9}{2}, 12\right)$$

$$\Rightarrow CC' = \frac{(-2, -\frac{9}{2}, 12)}{(6, 0, 1)}$$

$$|\vec{AB} \cdot \vec{AC}| = \sqrt{4 + \frac{81}{4} + 144} = \sqrt{\frac{16 + 81 + 576}{4}}$$

$$= \sqrt{\frac{592 + 81}{4}} = \sqrt{\frac{673}{4}}$$

$$\Rightarrow \vec{CC'} = \frac{1}{2}\sqrt{673} \cdot \frac{1}{\sqrt{37}} = \frac{\sqrt{673}}{2\sqrt{37}}$$

$|\vec{AB}| = \sqrt{37}$

$$dist(AC,BD) = \frac{|\vec{CD} \cdot \vec{BD}|}{|\vec{BD}|} = \frac{\sqrt{673}}{\sqrt{29}}$$

$$\vec{CD} \cdot \vec{BD} = \begin{vmatrix} i & j & k \\ 6 & 0 & 1 \\ \frac{3}{2} & 2 & 1 \end{vmatrix} = \left(-2, -\frac{9}{2}, 12\right)$$

$$|\vec{BD}| = \sqrt{\frac{9}{4} + 4 + 1} = \sqrt{\frac{9}{4} + 16 + 4} = \frac{\sqrt{29}}{2}$$

4.4. $\vec{a}\,(2, 3, -1)$

$\vec{b}\,(1, -1, 3)$

a) Det. the vector subspace $\langle \vec{a}, \vec{b} \rangle^{\perp}$

b) find $\vec{p}$ which is orthogonal to $\vec{a}$ & $\vec{b}$ for which $p \cdot (2\vec{i} - 3\vec{j} + 4\vec{k}) = 51$

$$S \leq_{\mathbb{R}} V_3, \; S^{\perp} = \{ \vec{v} \in V_3 \mid \vec{v} \cdot \vec{w} = \vec{0}, \forall\, \vec{w} \in S \}$$

$$\Rightarrow \langle \vec{a}, \vec{b} \rangle^{\perp} = \langle \vec{a} \times \vec{b} \rangle$$

c) $$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{vmatrix} = 9i - 2k - j - 3k - i - 6j$$

$$= 8i - 7j - 5k = (5, -7, -5)$$

$$\Rightarrow \langle (8, -7, -5) \rangle$$

b) $\vec{p} \cdot (2\vec{i} - 3\vec{j} + 4\vec{k}) = 51$

$$2\vec{i}\,\vec{p} - 3\vec{j} \cdot \vec{p} + 4\vec{k}\,\vec{p} = 51$$