

$K = (O, B)$  - reference system (ref frame / coord. system)

$O \in \mathbb{R}^m$

$B$  - basis of  $\mathbb{V}^m$

Notations:  $V, V'$  - vec. spaces

$\varphi: V \rightarrow V'$  - lin. map

$B, B'$  - basis of  $V, V'$

Instead of  $[f]_{B'B}$ , we use  $M_{B'B}(f) = \left( [f(v_1)]_{B'}, [f(v_2)]_{B'}, \dots, [f(v_m)]_{B'} \right)$ , where  $D = (v_1, \dots, v_m)$

$V$  - vector space

$B, B'$  - basis of  $V$

Base change matrix from  $B$  to  $B'$  is  $M_{B'B} := M_{B'B}(Id) = [Id]_{B'B}$

We use it like this: If  $v \in V$ :

$$[v]_{B'} = M_{B'B} [v]_B$$

$K = (O, B)$

$K' = (O', B')$

$$\forall p \in \mathbb{R}^m: [p]_K = [\overrightarrow{op}]_B$$

We want  $[p]_{K'}$  in terms of  $[p]_K$

$$\Rightarrow [p]_{K'} = [\overrightarrow{op}]_{B'} = M_{B'B} [O'p]_B = M_{B'B} [\overrightarrow{op} - \overrightarrow{oo'}]_B = M_{B'B} ([\overrightarrow{op}]_B - [\overrightarrow{oo'}]_B) =$$

$$= M_{B'B} ([p]_K - [O']_K) = M_{B'B} [p]_K - M_{B'B} [\overrightarrow{oo'}]_B = M_{B'B} [p]_K - [\overrightarrow{oo'}]_B =$$

$$= M_{B'B} [p]_K + [\overrightarrow{oo'}]_{B'} = M_{B'B} [p]_K + [O]_{K'}$$

From now on, we will write  $[\vec{v}]_B = [\vec{v}]_K$  &

$$M_{B'K} = M_{KK}$$

1.16.  $K = (0, \vec{i}, \vec{j})$

$$K' = (0', \vec{i}', \vec{j}') \quad - \text{coord. sys. for } \mathbb{E}^2$$

$$[0']_K = \begin{pmatrix} ? \\ -1 \end{pmatrix}, \quad \vec{i}' = -2\vec{i} + \vec{j} \quad \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$

$$\vec{j}' = \vec{i} + 2\vec{j}$$

$A, B, C$  - points in  $\mathbb{E}^2$  w/  $[A]_K = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ,  $[B]_K = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ ,  $[C]_K = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

- det. the base change m. from  $K$  to  $K'$  & find coords.
- find the  $-11-11-$   $K'$  to  $K$  & check the results.

$$M_{KK'} = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow M_{K'K} = \frac{1}{5} \begin{bmatrix} 2 & -1 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

$$[A]_{K'} = M_{K'K} [\vec{o}']_K = M_{K'K} [A]_K - M_{K'K} [0']_K$$

$$= \frac{1}{5} \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \left[ \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 7 \\ -1 \end{bmatrix} \right]$$

$$= \begin{bmatrix} -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} 6 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$[B]_{K'} = M_{K'K} [\vec{o}']_K = M_{K'K} [B]_K - M_{K'K} [0']_K$$

$$= \begin{bmatrix} -\frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{bmatrix} \left[ \begin{bmatrix} 2 \\ 4 \end{bmatrix} - \begin{bmatrix} 7 \\ -1 \end{bmatrix} \right] =$$

$$= \frac{1}{5} \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$[e]_{,1} = M_{KK'} [\vec{o}c]_K = M_{KK'} [\vec{o}\vec{c} - \vec{o}\vec{o}]_K$$

$$= \frac{1}{5} \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \left[ \begin{bmatrix} 3 \\ 1 \end{bmatrix} - \begin{bmatrix} 7 \\ -1 \end{bmatrix} \right]$$

$$= \frac{1}{5} \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$= \frac{2}{5} \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$= \frac{2}{5} \begin{bmatrix} 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$M_{KK'} = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$[A]_K = [\vec{o}A]_K = M_{KK'} [\vec{o}A - \vec{o}\vec{o}]_{K'}$$

$$= M_{KK'} [A]_{K'} - M_{KK'} [o]_{K'}$$

$$= \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix} = \begin{bmatrix} -6 \\ 3 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} \vec{0} \\ \vec{0} \end{bmatrix}_K = M_{KK'} \cdot \begin{pmatrix} \vec{0} \\ \vec{0} \end{pmatrix}_{K'} = \frac{1}{2} \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ -1 \end{bmatrix} =$$

$$= \frac{1}{2} \begin{bmatrix} -15 \\ 5 \end{bmatrix} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

$$[B]_K = [\vec{OB}]_K = M_{KK'} \left( [\vec{OB}]_{K'} - [\vec{O}']_{K'} \right)$$

$$= M_{KK'} ([B]_{K'} - [O']_{K'})$$

$$= M_{KK'} ([B]_{K'} - [O']_{K'})$$

$$= M_{KK'} ([B]_{K'} + [O']_{K'})$$

$$= M_{KK'} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 \\ 5 \end{bmatrix} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$[C]_K = [OC]_K = M_{KK'} [O'C]_{K'}$$

$$= M_{KK'} ([O'C]_{K'} - [O']_{K'}) =$$

$$= M_{KK'} [C]_{K'} + [O']_{K'} =$$

$$= \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix} =$$

$$= \begin{bmatrix} -4 \\ 2 \end{bmatrix} + \begin{bmatrix} 7 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

1.17

$$ABCD - \text{delta hessian} \quad \& \quad K_A = (A, \bar{AB}, \bar{AC}, \bar{AD})$$

$$\text{H-mid. point of } [BC] \quad K_A' = (A, \bar{AB}, \bar{AD}, \bar{AC})$$

$$K_B = (B, \bar{BA}, \bar{BC}, \bar{BD})$$

a) Find the cords. of ABCD, H in the coord. sys

b) b-e. change matrix from  $K_A$  to  $K_A'$

c) -11-  $K_B$  to  $K_A$ .

$$a) A = r_A = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$B = r_B = \bar{AB} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = r_C = \bar{AC} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$D = r_D = \bar{AD} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$H - \text{mid of } BC \Rightarrow H = r_m = \frac{1}{2} (\bar{AB} + \bar{AC}) = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

 $K_A'$ 

$$A = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$H = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix}$$

 $K_B$ 

$$A = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{cases} \bar{AB} < \bar{AC} \\ \frac{1}{2} (-\cancel{\bar{BA}} + \bar{CB} + \cancel{\bar{BA}}) = \frac{1}{2} \bar{BC} \end{cases}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 \\ -\frac{1}{2} \\ 0 \end{bmatrix}$$