Ch. 8.

11, 14, 17, 18, 21, 24 b

8.14. Det. the tangent plane of the hyperboloid:

$$H_{2,3,1}: \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{1} = 1$$

$M(2,3,1)$. Show that the tangent plane intersects the surface in 2 lines.

$$T_M H: \frac{x_0 x}{4} + \frac{y_0 y}{9} - \frac{z_0 z}{1} = 1$$

$$\Rightarrow T_M H: \frac{2x}{4} + \frac{3y}{9} - \frac{z}{1} = 1$$

$$T_M \cap H: \begin{cases} \frac{2x}{4} + \frac{3y}{9} - \frac{z}{1} = 1 \\ \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{1} = 1 \end{cases}$$

$$18x + 12y - 36z = 36$$

$$\Rightarrow 3x + 4y - 6z = 6 \Rightarrow z = -\frac{6 - 3x - 2y}{6}$$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{9} + \frac{36 + 9x^2 + 4y^2 - 36x + 12xy - 24y}{36} = 1$$

$$\cancel{\frac{x^2}{4}} + \cancel{\frac{y^2}{9}} + \cancel{1} + \frac{x^2}{4} + \frac{4y^2}{9} - x + \frac{2xy}{3} - \frac{2y}{3} = \cancel{1}$$

$$\cancel{\frac{x^2}{2} + \frac{y^2}{2} - x - \frac{2y}{3} + \frac{2xy}{3} = 0}$$

$$(3-y)(2-\cancel{x}) = 0$$

$$x = 2 \Rightarrow 2y - 6z = 0$$

$$y = 3 \Rightarrow 3x - 6z = 0$$

8.24. Which of the following surfaces is a hyperboloid?

a) $2xz + 2xy + 2yz = 1$

b) $5x^2 + 3y^2 + xz = 1$

c) $2xy + 2y^2 + y + z = 2$

Lagrange method to bring to a canonical form

a) $x = y + x'$

$$\Rightarrow 2zy + 2zx' + 2y^2 + 2yx' + 2yz = 1$$

b) $5x^2 + 3y^2 + xz = 1$

$$(\sqrt{5}x)^2 + 2 \cdot \sqrt{5}x \cdot \frac{z}{2\sqrt{5}} + \left(\frac{z}{2\sqrt{5}}\right)^2 - \frac{z^4}{20} + 3y^2 = 1$$

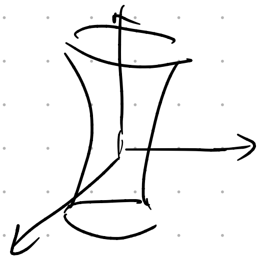
$$\underbrace{\left(\sqrt{5}x + \frac{z}{2\sqrt{5}}\right)}_{x'}^2 + 3y^2 - \frac{z^2}{20} = 1$$

$$x'^2 + 3y^2 - \frac{z^2}{20} = 1 \Rightarrow 1 \text{ sheet hyperboloid}$$

$$\begin{cases} z' = \frac{1}{\sqrt{20}} z \\ y' = \sqrt{3} y \end{cases}$$

$x'^2 + y'^2 - z'^2 = 1$

axis of symmetry for z.



$$M_\varphi : \begin{pmatrix} 5 & 0 & \frac{1}{2} \\ 0 & 3 & 0 \\ \frac{1}{2} & 0 & 0 \end{pmatrix}$$

$$P_{M_Q} = \begin{vmatrix} 5-x & 0 & \frac{1}{2} \\ 0 & 3-x & 0 \\ \frac{1}{2} & 0 & -x \end{vmatrix}$$

$$-x(5-x)(3-x) + 0 + 0 - \frac{1}{4}(3-x) - 0 - 0$$

$$(3-x)\left(-5x + x^2 - \frac{1}{4}\right) = 0$$

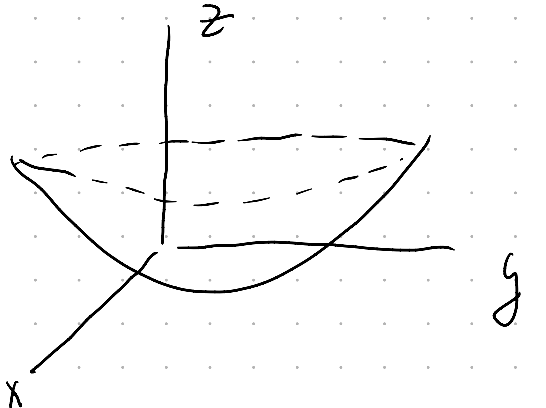
$\Rightarrow \lambda_1 = 3$

$x_{2,3} = \dfrac{5 \pm \sqrt{26}}{2}$

$\left.\begin{array}{l} 2 \text{ positive} \\ 1 \text{ negative} \end{array}\right\} \Rightarrow$ 1 sheet hyperboloid.

21. Use a parametrization of a parabola and a rotation matrix to deduce a parametrization of an elliptic paraboloid of revolution

$$\mathcal{C}: \begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases} \Rightarrow \text{parametrization}$$



$$P: \begin{cases} x = 0 \\ \dfrac{y^2}{2p} = z \end{cases} \Longleftrightarrow \begin{cases} x = 0 \\ y = t \\ z = \dfrac{t^2}{2p} \end{cases}, \quad t \in \mathbb{R}$$

$\theta \in [0, 2\pi]$

$$[Rot_{Oz, \theta}] = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$[Rot_{Oy, \theta}] = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$$

$$[Rot_{Ox, \theta}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$P^e \cdot \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\mathcal{P}^{\rho} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \text{-11-} \quad \begin{matrix} 0 \\ t \\ t^2/2p \end{matrix}$$

$$= \begin{pmatrix} -t \sin\sigma \\ t\cos\sigma \\ t^2/2p \end{pmatrix}, \quad t \in \mathbb{R}, \ \sigma \in [0, 2\pi)$$

We see that the points of $\mathcal{P}^{\rho}$ satisfy the eq

$$x^2 + y^2 - 2\rho z = 0$$

Euler Rodriguez

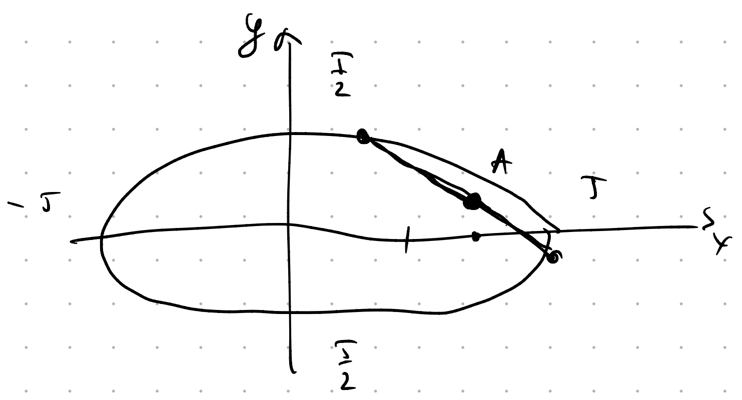
$$\mathrm{Rot}_{\vec{v},\theta}(p) = (\cos\theta)\cdot p + \sin\theta\,(\vec{v} \times p) + (1-\cos\sigma)\langle \vec{v}, p\rangle \cdot \vec{v}$$

6.13.

$\mathcal{E}: \ x^2 + 4y^2 = 25$

Find the chords that have $A(\frac{7}{2}, \frac{7}{4})$ midpoint.

$$\frac{x^2}{25} + \frac{4}{25}y^2 = 1$$



let l_m - line of slope m containing M

$l_m: y - \frac{7}{4} = m\left(x - \frac{7}{2}\right)$

$l_m \cap \mathcal{E} \begin{cases} x^2 + 4y^2 = 25 \\ y = \frac{7}{4} + mx - \frac{7m}{2} \end{cases}$

$\Rightarrow x^2 + \frac{49}{4} + 4m^2x^2 + 49m^2 + 14mx - 49m - 14m^2x = 25$

$(1+4m^2)x^2 + (14m - 14m^2)x + 49m^2 - 49m + \frac{49}{4} = 25$

$l_m \cap \mathcal{E} : \{A, B\}$

M - mid AB $\Rightarrow M\left(\underbrace{\frac{x_1+x_2}{2}}_{\frac{7}{2}}, \underbrace{\frac{y_1+y_2}{2}}_{\frac{7}{4}}\right)$

$\Rightarrow x_1 + x_2 = 7, \quad y_1 + y_2 = \frac{7}{2}$

$x_1 + x_2 = -\frac{b}{a} = \frac{14m^2 - 14m}{1+4m^2} = 7.$

$\cancel{14m^2} - 14m = 7 + \cancel{28m^2}$

$14m^2 + 14m + 7 = 0 \; | :7$

$2m^2 + 2m + 1 = 0$

$\Rightarrow m^2 - 2m - 1$

$\Rightarrow m = 1 \pm \sqrt{2}$

$$\Rightarrow \ \ell_m : \begin{cases} y - \frac{7}{4} = 1+\sqrt{2}\left(x - \frac{7}{2}\right) \\ y - \frac{7}{4} = 1-\sqrt{2}\left(x - \frac{7}{2}\right) \end{cases}$$