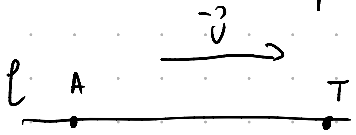
– affine variety:

$\mathcal{A} = a + U = \{ a + \vec{v} \mid \vec{v} \in U \}$, $a \in \mathcal{E}^n$, $U \subseteq V^n$, $d = \dim U$, $D(\mathcal{A}) = U$

if $d = 1 \Rightarrow a + U$ – line
$d = 2 \Rightarrow a + U$ – plane



Fix an origin O

$$\left.\begin{array}{l} \vec{r}_T = \vec{r}_A + \vec{AT} \\ \exists \lambda \in \mathbb{R} : \vec{AT} = \lambda \cdot \vec{v} \end{array}\right\} \Rightarrow \boxed{\vec{r}_T = \vec{r}_A + \lambda \cdot \vec{v}}$$ Vector eq. of ℓ.

Fix the reference system $K = (O, \mathcal{B})$

$[T]_K = \begin{pmatrix} x \\ y \end{pmatrix}$, $[A]_K = \begin{bmatrix} x_A \\ y_A \end{bmatrix}$

$[\vec{v}]_K = \begin{pmatrix} x_{\vec{v}} \\ y_{\vec{v}} \end{pmatrix}$

$$\ell : \begin{cases} x = x_A + \lambda x_{\vec{v}} \\ y = y_A + \lambda y_{\vec{v}} \end{cases}, \quad \lambda \in \mathbb{R}$$

(parametric eq. of ℓ)

If $x_{\vec{v}}$ & $y_{\vec{v}} \neq 0$: $\ell : \dfrac{x - x_A}{x_{\vec{v}}} = \dfrac{y - y_A}{y_{\vec{v}}}$

(symmetric form of the cartesian eq)

If $x_{\vec{v}} = 0 \Rightarrow x = x_A$
$y_{\vec{v}} = 0$ –"–

Implicit form: $y_{\vec{v}}(x - x_A) - x_{\vec{v}}(y - y_A) = 0$

$Ax + By + C = 0$

Explicit form:

If $A = 0$, $B \neq 0$: $y = -\dfrac{C}{B}$

If $A \neq 0$, $y = 0$: $x = -\frac{C}{A}$

If $A, B \neq 0$: $y = -\frac{A}{B}x - \frac{C}{B}$

$$\boxed{y = mx + n}$$

$\ell: Ax + By + C = 0$

$\ell: \begin{cases} x = \lambda \\ y = -\frac{A}{B}\lambda - \frac{C}{B} \end{cases}$

$D(\ell) = \langle \vec{v} \rangle = \langle (x_{\vec{v}}, y_{\vec{v}}) \rangle$

Det. parametric & cart. eq. along w/ their dir. vectors for line ℓ.

a) $\ell \ni A(1,2)$, $\ell \parallel \vec{a}(3,-1)$

b) $\ell \ni O(0,0)$, $\ell \parallel \vec{b}(4,5)$

c) $\ell \ni M(1,7)$, $\ell \parallel Oy$

d) $\ell \ni M(2,4)$, $N(2,-5)$

a) $\ell \parallel \vec{a} \Rightarrow \ell = \lambda \vec{a}$

$\ell: \begin{cases} x = x_A + \lambda x_{\vec{a}} \\ y = y_A + \lambda y_{\vec{a}} \end{cases} - \begin{cases} x = 1 + 3\lambda \\ y = 2 - \lambda \end{cases}$ param. way. $\ell = \frac{x-1}{3}, 2-y$

$\vec{a} \neq \vec{0} \Rightarrow$ (cartesian way) $\ell: \frac{x-1}{3} = 2 - y$

$\frac{-x+7}{3} = y \Rightarrow y = -\frac{x}{3} + \frac{7}{3}$

$D(\ell) = \langle \vec{a} \rangle = \langle (3,-1) \rangle$

b) $\ell: \begin{cases} x = 4\lambda \\ y = 5\lambda \end{cases} \Rightarrow \ell: \frac{x}{4} = \frac{y}{5}$

$\Rightarrow 5x - 4y = 0 \Rightarrow y = \frac{5}{4}x$

$D(\ell) = \langle \vec{\ell} \rangle$

c) $\vec{c} = (0,1)$

$$\ell : \begin{cases} x = 1 \\ y = 7 + \lambda \end{cases}$$

$\Rightarrow x = 1 \Rightarrow x - 1 = 0 \quad \ell : x = 1$

$D(\ell) = \langle \vec{c} \rangle = \langle (0,1) \rangle$

d) $\vec{MN} \ (0,-9)$

$$\ell : \begin{cases} x = 2 \\ y = 4 - 9\lambda \end{cases}$$

$\ell : x - 2 = 0, \ x = 2$

$D(\ell) = D\langle \vec{MN} \rangle = D\langle (0,-9) \rangle = D\langle (0,1) \rangle$

2.5. Det. the eq. for the line in A^2 parallel to $\vec{v}$ and passing through $S \cap T$:

$\vec{v} = (2,4)$

$S : \ 3x - 2y - 7 = 0$

$T : \ 2x + 3y = 0$

$$\Rightarrow \begin{cases} 3x - 2y = 7 \\ 2x + 3y = 0 \end{cases} \Rightarrow$$

$2x = -3y$

$x = -\frac{3}{2} y \quad \Big| \Rightarrow -\frac{9}{2} y - 2y = 7 \ | \cdot 2$

$-9y - 4y = 14$

$-13y = 14 \Rightarrow y = -\frac{14}{13}$

$x = \frac{21}{13}$

$\Rightarrow P\left(\frac{21}{13}, -\frac{14}{13}\right)$

$$\Rightarrow \ell : \begin{cases} x = \frac{21}{13} + 2\lambda \\ y = -\frac{14}{13} + 4\lambda \end{cases} \Rightarrow \frac{x - \frac{21}{13}}{2} = \frac{y + \frac{14}{13}}{4}$$

$$\Rightarrow 2x - \frac{42}{13} = y + \frac{14}{13}$$

$$2x - y - \frac{28}{13} = 0$$

$$\cancel{\frac{42}{13}} + 4\lambda - y - \cancel{\frac{28}{13}} = 0$$

$$-y = -\frac{28}{13} - 4\lambda$$

$$y = \frac{28}{13} + 4\lambda$$

$$y = 4\left(\frac{7}{3} + \lambda\right)$$

π plane, $A \in \pi$

$\vec{v}, \vec{w}$ lin. indep vec. in $D(\pi)$ (i.e they form a basis of $D(\pi)$)

$$\vec{r}_T = \vec{r}_A + \vec{AT}$$

$\vec{AT} \in D(\pi) \Rightarrow \exists\, \lambda, \mu \in \mathbb{R}$ s.t. $\vec{AT} = \lambda \vec{v} + \mu \cdot \vec{w}$

$\vec{r}_T = \vec{r}_A + \lambda \vec{v} + \mu \vec{w} \leftarrow$ vector eq. of the plane

Fix a reference system, $K = (O, B)$

$$\pi : \begin{cases} x = x_A + \lambda x_{\vec{v}} + \mu x_{\vec{w}} \\ y = y_A + \lambda y_{\vec{v}} + \mu y_{\vec{w}} \\ z = z_A + \lambda z_{\vec{v}} + \mu z_{\vec{w}} \end{cases}$$

The cartesian (symmetric form):

$$\begin{vmatrix} x - x_A & y - y_A & z - z_A \\ x_{\vec{v}} & y_{\vec{v}} & z_{\vec{v}} \\ x_{\vec{w}} & y_{\vec{w}} & z_{\vec{w}} \end{vmatrix} = 0$$

The implicit form:

$$Ax + By + Cz + D = 0$$

2.10. Determine Cartesian eq. for the plane π in the cases:

a) $\pi: \begin{cases} x = 2+3u-4v \\ y = 4-v \\ z = 2+3u \end{cases}$

b) $\pi: \begin{cases} x = u+v \\ y = u-v \\ z = 5+6u-4v. \end{cases}$

$x = z + 4y - 16$

$$\begin{vmatrix} x-2 & y-4 & z-2 \\ 3 & 0 & 3 \\ 4 & -1 & 0 \end{vmatrix}$$

$= 0 - 3z + \cancel{6} + 12y - 48 - 0 + 3x - \cancel{6} + 0$

$= 0 - 3z + 3x + 12y - 48 = 0 \quad /:3$

$-z + x + 4y - 16 = 0$

$x + 4y - z - 16 = 0 \Rightarrow D(\pi) = \langle (3,0,3), (-4,-1,0) \rangle$

b) $\begin{vmatrix} x & y & z-5 \\ 1 & 1 & 6 \\ 1 & -1 & -4 \end{vmatrix} \underline{\underline{L_2+L_3}} \begin{vmatrix} x & y & z-5 \\ 2 & 0 & 2 \\ 1 & -1 & -4 \end{vmatrix} = 0 - 2z + 10 + 2y - 0 + 2x + 8y$

$= -2z + 10y + 2x + 10$

$\Rightarrow x + 5y - z + 5 = 0$

$D(\pi) = \langle (2,0,2), (1,-1,-4) \rangle$

$A \in \pi$

$\vec{AB}, \vec{AC} \in D(\pi)$

$$\begin{vmatrix} x - x_A & y - y_A & z - z_A \\ x_B - x_A & y_B - y_A & z_B - z_A \\ x_C - x_A & y_C - y_A & z_C - z_A \end{vmatrix}$$

Det. param. eq. for π:

a) $3x - 6y + z = 0$

b) $2x - y - z - 3 = 0$

$z = 6y - 3x$

$$\begin{cases} x = u \\ y = v \\ z = -3u + 6v \end{cases}$$

$$\begin{pmatrix} x & y & z \\ 1 & 0 & -3 \\ 0 & 1 & 6 \end{pmatrix}$$

$D(\bar{u}) = \langle (1,0,-3),(0,1,6) \rangle$

$2x - y - z = 3$

$y = -3 + 2x - z$

$x = u$
$z = v$
$y = 2u - v - 3$

$$\begin{pmatrix} x & y+3 & z \\ 1 & 2 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$D(\bar{\pi}) = \langle (1,2,0),(0,-1,1) \rangle$