

Math-Problems Sharing Program

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Contents

1	Introduction	2
2	p-limit	2
2.1	no.1	3
2.2	no.2	3

1 Introduction

2 p-limit

2.1 no.1

[14/4/2023 LiYongHuan sharing]

P:

$$\lim_{n \rightarrow \infty} \frac{\sqrt[n]{n!}}{n}$$

S:

$$\begin{aligned} & \lim_{n \rightarrow \infty} e^{\frac{1}{n} \ln(1*2*3*\dots*n)} \\ &= e^{\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \ln(k)} \\ &= e^{\int_0^1 \ln(x) dx} \\ &= e^{x \ln(x) \Big|_0^1 - \int_0^1 x \cdot \frac{1}{x}} \\ &= e^{-1} \end{aligned}$$

2.2 no.2

[14/4/2023 LiuWeiXiong sharing]

P:

$$\lim_{x \rightarrow 0} \frac{\ln(1+\arcsin(x)) - \ln(1+\tan(x))}{x^3}$$

S:

$$\begin{aligned} & \lim_{x \rightarrow 0} \frac{\ln(1+\arcsin(x)) - \ln(1+\tan(x))}{x^3} \\ &= \lim_{x \rightarrow 0} \frac{1}{1+\xi} \frac{\arcsin(x) - \tan(x)}{x^3} \\ &= \lim_{x \rightarrow 0} \frac{\arcsin(x) - x - (\tan(x) - x)}{x^3} \\ &= \lim_{x \rightarrow 0} \frac{[(1-x^2)^{-\frac{1}{2}} - 1] - [\sec(x)^2 - 1]}{3x^2} \\ &= \lim_{x \rightarrow 0} \frac{-\frac{1}{2}(-x^2) - \tan(x)^2}{3x^2} \\ &= -\frac{1}{6} \end{aligned}$$