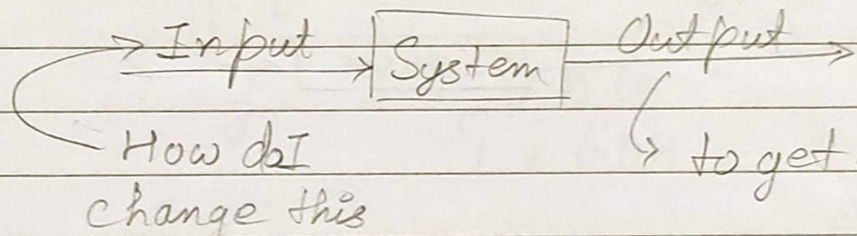
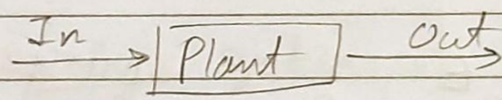
CONTROL SYSTEM

Control System:- Mechanism that alters the future state of a system.

Control Theory:- Strategy to select appropriate inputs.

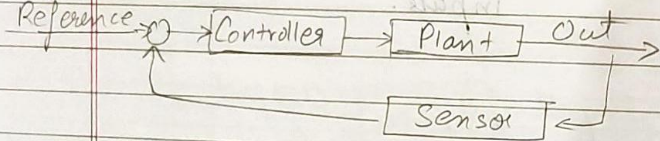


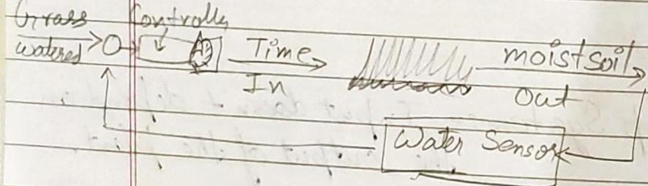
Open Loop System:- Input doesn't depend on the output of the plant.

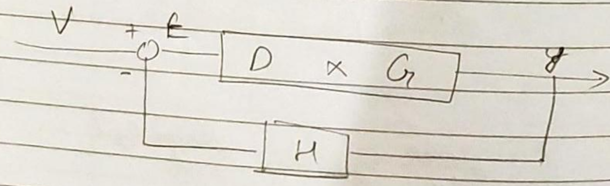


Thus a major drawback of open loop, the input to the system has no way of changing due to variations within the system

Closed loop Control –
Also called feedback control, negative feedback control or automatic control







$E = V - yH$

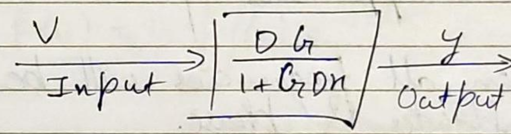
$y = E \cdot D \cdot G$

$\Rightarrow E = \dfrac{y}{D \cdot G}$

$\therefore \dfrac{y}{D \cdot G} = V - yH$

$\Rightarrow y = V \cdot D \cdot G - y\, G \cdot D \cdot H$

$\Rightarrow y = \dfrac{VDG}{1 + GDH}$

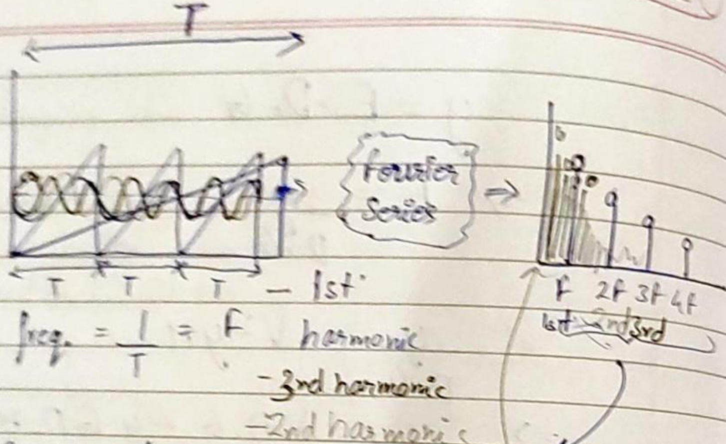


— x —

for a spring system

$m\ddot{x} = -kx$ or $m\ddot{x} + kx = 0$

General Solⁿ $x(t) = A \sin(\omega t + \phi)$



$$\text{freq.} = \frac{1}{T} = f$$

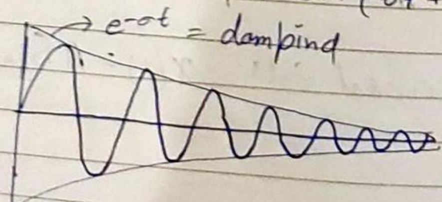
* Possible frequency are multiples of a particular freq. called first harmonic

* $\lim_{T \to \infty}$ then f approaches $\phi \rightarrow$

then all frequencies will be possible all amplitudes & phase.

* If we consider damping then.

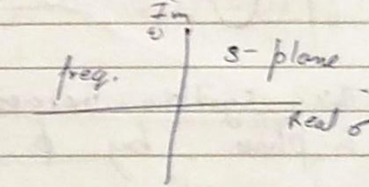
$$x(t) = e^{-\sigma t}(A \sin(\omega t + \phi))$$



Laplace Transformation

$$= \int^{\infty} f(t)\, e^{-\sigma t} \cdot e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} f(t) e^{-st} dt \qquad (s \triangleq \sigma + j\omega)$$



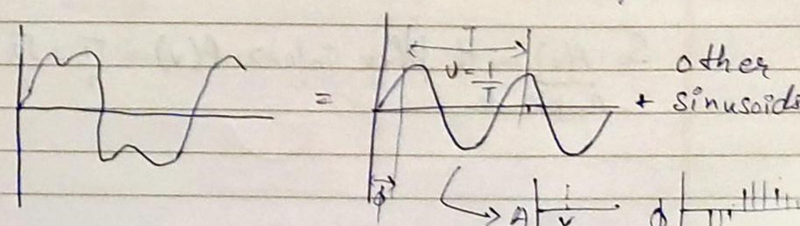
— x —

$$F(v) = \int_{-\infty}^{\infty} f(t)\, e^{-2\pi i v t}\, dv \quad \{\text{change f}^n \text{ from time to freq}\}$$

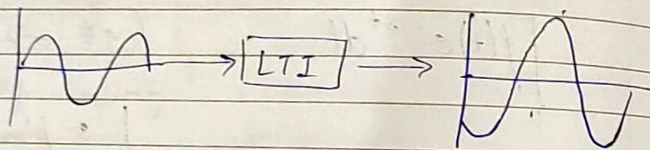
Fourier transform^n

$$f(t) = \int_{-\infty}^{\infty} F(v)\, e^{2\pi i v t}\, dv \quad \{\text{change from freq to time}\}$$

Inverse Fourier Transform^n.

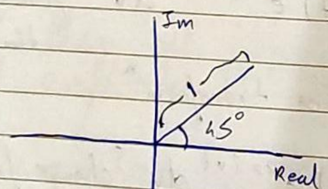


* we choose sinusoids b'coz they ~~behave~~ don't change shape when subjected to linear time invariance system (LTI system)



* This system increased gain by X & shifted in phase by ϕ at frequency ν.

* ~~fori~~ Fourier Transform is used to move from time domain to frequency domain



$A = \sqrt{real^2 + Im^2}$

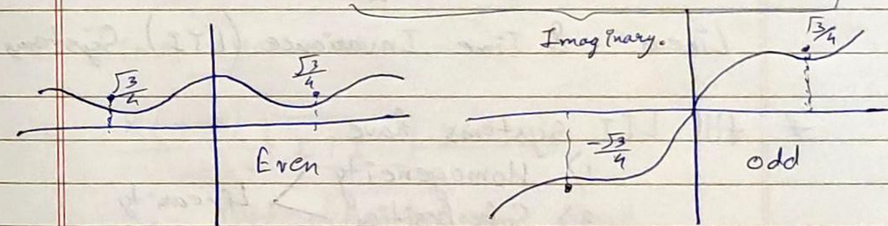
$\tan\phi = \frac{Im}{real}$

real $= \frac{\sqrt{2}}{2}$, Im $= \frac{\sqrt{2}}{2}\hat{j}$.

So $\underset{\text{Amplitude}}{\underline{A(\nu)}}$ with phase info $\Rightarrow f(\nu) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\hat{i}$

Also, sinusoids can also described by Euler's form. Therefore with phase $A(\nu)\cos 2\pi\nu t$ becomes $\underbrace{f(\nu)\, e^{2\pi i\nu t}}_{\text{Complex}}$

$$f(\nu)\, e^{2\pi i\nu t} = \overbrace{\frac{\sqrt{2}}{2}\cos(2\pi\nu t) - \frac{\sqrt{2}}{2}\sin(2\pi\nu t)}^{\text{Real}}$$
$$\underbrace{+ \frac{\sqrt{2}}{2}\cos(2\pi\nu t)\hat{i} - \frac{\sqrt{2}}{2}\sin(2\pi\nu t)\hat{i}}_{\text{Imaginary.}}$$

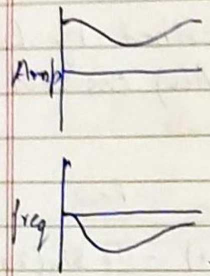


Real part of $f(\nu)$ Imaginary part of $f(\nu)$

Solve $f(\nu)\, e^{2\pi i\nu t}$ for $-$ve freq. & sum with positive, thus imaginary part will cancel itself.

$= \frac{\sqrt{2}}{2}\cos(2\pi\nu t) - \frac{\sqrt{2}}{2}\sin(2\pi\nu t)$

$= \cos\left(2\pi\nu t + \frac{\pi}{4}\right)$



Convert to complex f(v) b/w $-\infty$ to ∞ => Then multiply by all sinusoids

=> Then sum it all to get the time domain.

—x—

Linear & Time Invariance (LTI) Systems

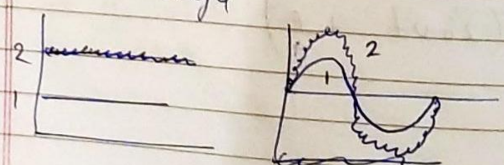
* All LTI systems have
  1> Homogeneity
  2> Superposition
  (Homogeneity and Superposition) > Linearity
  3> Time ~~Insuran~~ Invariance

Homogenity

$x(t) \rightarrow \boxed{LTI} \rightarrow y(t)$

$a(x(t)) \rightarrow \boxed{LTI} \rightarrow a\,y(t)$

scale of a



Super position

$x_1(t) \rightarrow \boxed{LTI} \longrightarrow y_1(t)$

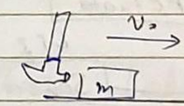
$x_2(t) \rightarrow \boxed{LTI} \longrightarrow y_2(t)$

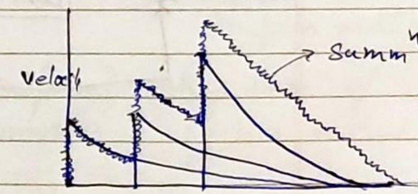
$x_1(t) + x_2(t) \longrightarrow \boxed{LTI} \longrightarrow y_1(t) + y_2(t)$

Time Invariance

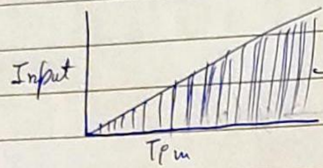
$x(t-a) \rightarrow \boxed{LTI} \rightarrow y(t-a)$

$\therefore$ Not dependent on time.

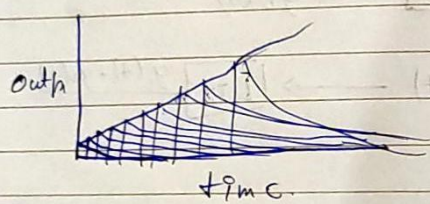




However the Impulses are more continuu



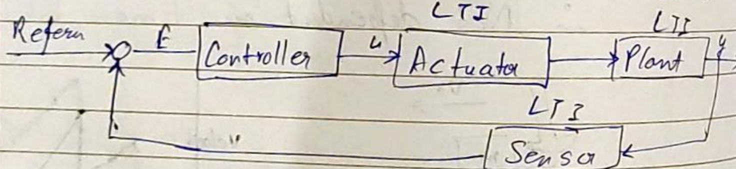
→ Impulses broken into small part



Input * Impulse Respon

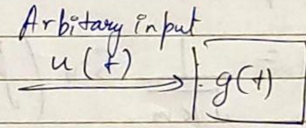
↓ Laplac into S domain | ↓ Laplace into S domain

↓ $f_1(s)$ * $f_2(s)$



Lamplace Transform Table.

$f(t)$	$F(s)$
$\delta(t)$	1
$x(t)$	$X(s)$
$x'(t)$	$sX(s) - x(0)$
$x''(t)$	$s^2X(s) - sx(0) - x'(0)$



$y(t) = \int_0^t u(t)\, g(t-\tau)\, d\tau$

or

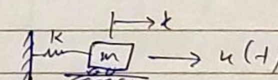
$u(t) * g(t)$

$u(s)$ Impulse Respons $G(s)$ $\qquad Y(s) = u(s) \cdot G(s)$

Simpler multiply.

Thus Laplace transform takes care of Convulation & provides a simpler method.

Eg.



Impulse.

$m\ddot{x}(t) + kx(t) = u(t) = \delta(t)$

$M\left(s^2X(s) - \cancel{s(x(0))}^{0} - \cancel{x'(0)}^{0}\right) + KX(s) = 1$

$$M s^2 X(s) + K X(s) = 1$$

$$X(s)\left[ms^2 + K\right] = 1$$

$$X(s) = \frac{1}{ms^2 + K}$$

To find in time domain take the inverse laplace transform".

$$\Rightarrow L^{-1}(X(s)) = \frac{1}{\sqrt{Km}} \sin\left(\sqrt{\frac{K}{m}}\,t\right)$$

* what if ramp input.

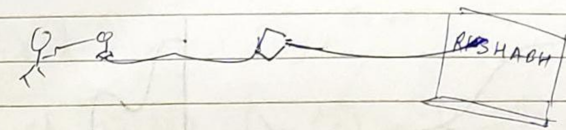
$u(t) = t$

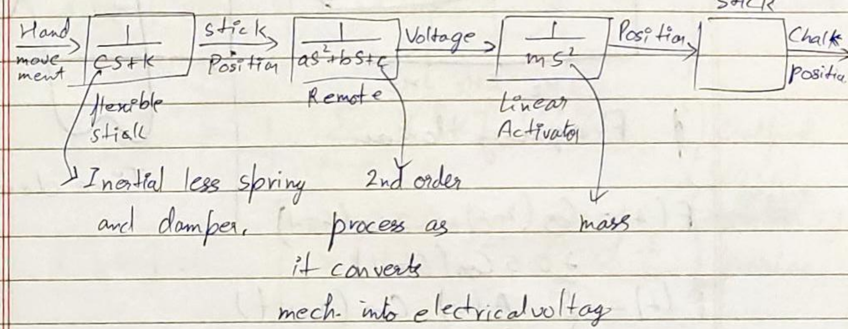
$L(u(t)) = \frac{1}{s^2}$

S Domain

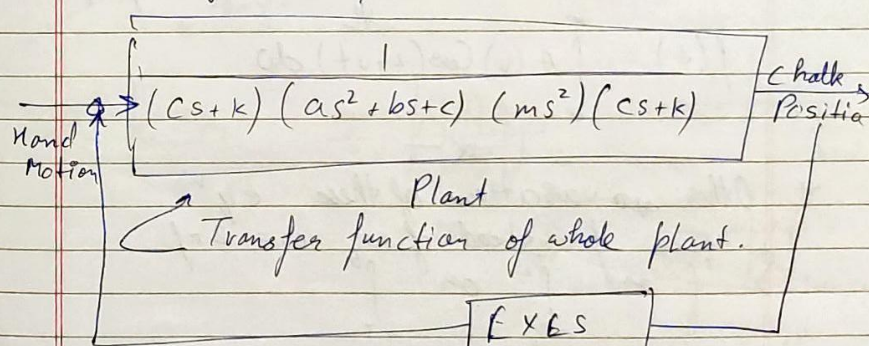
$$\frac{1}{s^2}\left(\underbrace{\frac{1}{ms^2+K}}\right)$$

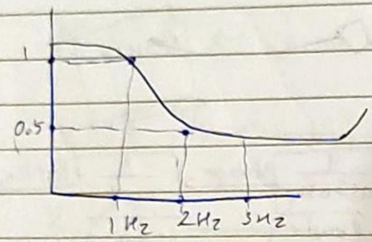
Impulse respone in transfer function.



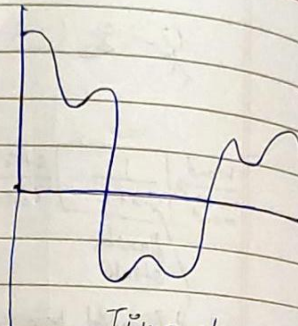


Combining all functions.





Frequency domain



Time domain

$$F(t) = \cos(2\pi t) + 0.5\cos(4\pi t) + 0.5\cos(6\pi t)$$

$$f(t) = \sum_{v=-\infty}^{\infty} \underbrace{A(v)}_{} \cos(2\pi v t)$$

→ Amplitude at each freq. v

⇓

$$f(t) = \int_{-\infty}^{\infty} A(v) \cos(2\pi v t)\, dv$$

— x —

* Other variations of these eq^n

i) using f instead of ω, $\omega = 2\pi f$

ii) $\int_0^T$ or $\int_{-T/2}^{T/2}$ or $\int_{-T}^{0}$

iii) Replacing T with f_0, $f_0 = 1/T$

iv) Moving the scaling term to other equation.

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{i2\pi k t/T}$$

$$X[k] = \frac{1}{T}\int_0^T x(t) e^{-i2\pi k t/T}\, dt$$

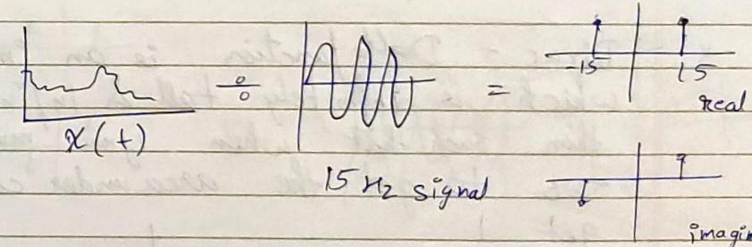
} Output Signal is unscaled

$$x(t) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X[k] e^{i2\pi t/T}$$

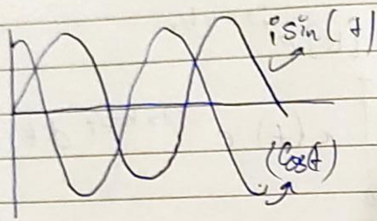
$$X[k] = \int_0^T x(t) e^{-2\pi k t/T}\, dt$$

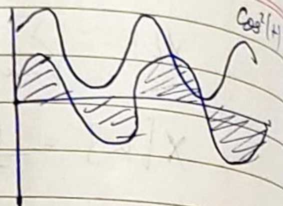
} Output Signal ~~scaled~~ scaled by period

$$\frac{x(t)}{e^{i2\pi v t}} = x(t) e^{-i2\pi v t}$$



frequency content



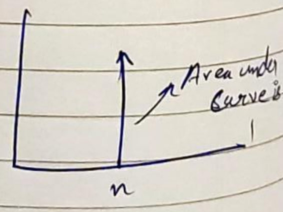


Sum of upper part $= \infty$

Sum of lower shaded part $= 0$;

* Output of fourier transport is scaled by the period, which is integrating time in this case which is ∞

* Dirac - Delta function is an impulse, which is infinitely tall & infinitesimily thin such that when you multiply the two to get the area under curve you get 1

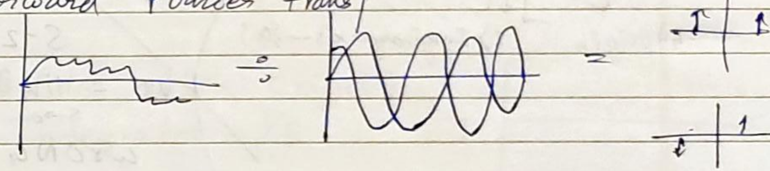


* So Amplitude $= \int_{-\infty}^{\infty} \text{Amplitude} \cdot \delta(\nu - \nu_0)\, d\nu$

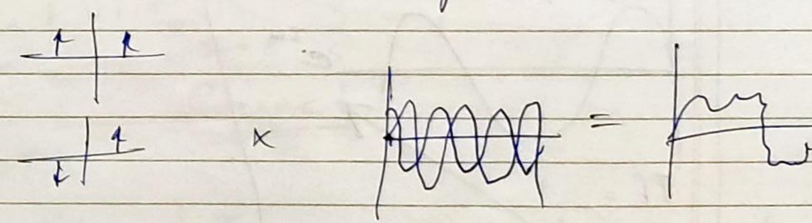
* Fourier Transform of $\cos(2\pi \nu_0 t)$

$$= \frac{1}{2}\left[\delta(\nu - \nu_0) + \delta(\nu + \nu_0)\right]$$

* Forward fourier transf^n

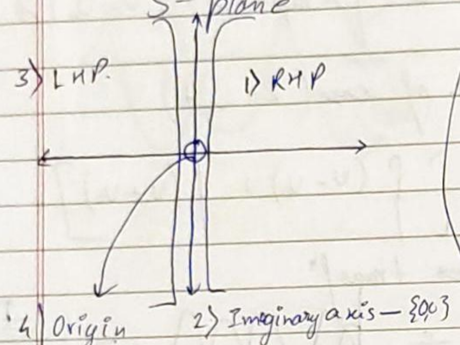


* Inverse Fourier transf^n

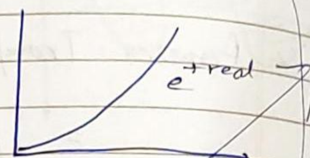


* CN

S-plane



RHP (Unstable)

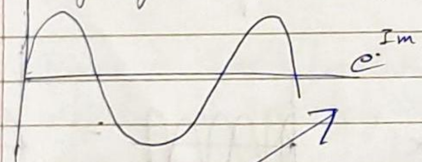


$$TF = \frac{1}{S-2}$$

$$FVT = \lim_{S \to 0} = 0$$

WRONG

Imaginary Axis (Oscillatory)

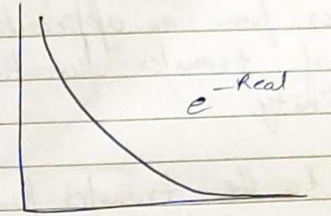


$$TF = \frac{1}{S^2+4}$$

$$FVT = \lim_{S \to 0} \frac{S}{S^2+4} = 0$$

WRONG

3) LHP (Stable)



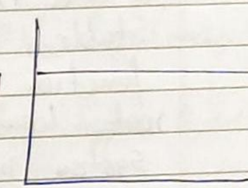
$$TF = \frac{1}{S+2}$$

$$FVT = \lim_{S \to 0} \frac{S}{S+2} = 0$$

Correct!

4) Pole at the origin

Integrator $1/S$



$$TF = \frac{1}{S}$$

$$FVT = \lim_{S \to 0} \frac{S}{S} = 1$$

Correct!

Poles & Zeroes are the frequencies for which the value of the denominator & numerator of transfer function becomes zero respectively. The values of the poles & the zeroes of a system determine whether the system is stable, & how well the system perform classmate

Date

Page

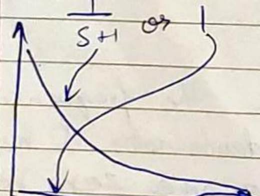
* As s approaches a pole, the denominator of the transfer function approaches 0, and the value of transfer function approches infinity.

* Add^n of poles to the transfer function has the effect of pulling the root locus to the right, making the system less Stable. Add^n of zeroes to the transfer function has the effect of pulling the root locus to the left, making the system more stable.

* No. of poles at the origin = System Type / Transfer function type

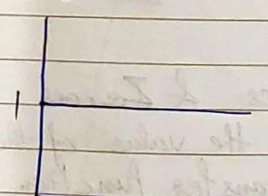
0 poles
Type 0

$\frac{1}{S+1}$ or 1



FV = 0

1 pole
Type 1



FV = 1

classmate

Date

Page

Type 2
2 pole

FV = ∞

Type 3 & up are same & FV = ∞

* Use Final Value Theorem only if all poles in the Left Hand Plane or origin

* If all poles in LHP then type 0, & FV = 0

* If system is type 1 then, FV = real value

* If system is type 2 or higher FV = ∞

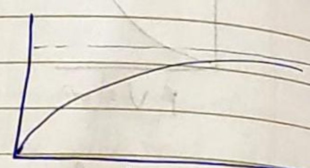
* $\boxed{\frac{1}{S^2+S}}$ $\quad u(s) = 1$
$Y(s)$

Now $\frac{1}{S+1} \cdot \frac{1}{S} \Rightarrow S = -1, 0$

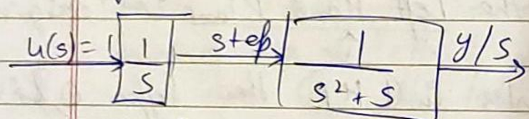
Thus this is a type 1 system

So we use final Value Theorem.

$$\lim_{s \to 0} s \cdot \frac{1}{s^2+s} = 1$$



* But if we wanted ~~to~~ the ~~find the~~ final value of a step input system? Impulse Response of $\frac{1}{s^2+s}$



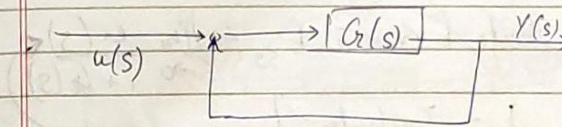
This is now a Type 2 system

$$FV = \lim_{s \to \infty} s \cdot \frac{1}{s} \cdot \frac{1}{s^2+s}$$

$$= \infty$$

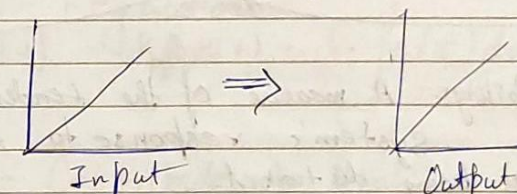
As expected by a type 2

* So for a step response to a transfer function we add a pole at origin ie multiply by $\frac{1}{s}$.



* Error = Difference b/w output & input

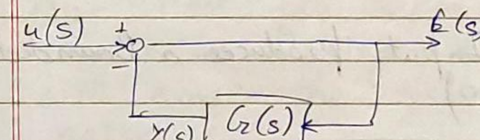
* A good system is designed to make the Error 0



* ~~Exp~~ If the input is a ramp then the Output is also supposed to be a ramp.

* Final Value of Error is called steady State error

*



$$E(s) = U(s) - Y(s)$$

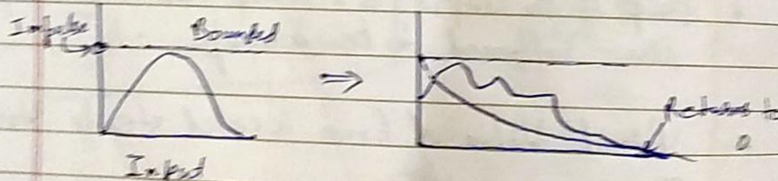
$$Y(s) = G(s) \cdot E(s)$$

$$E(s) = \frac{U(s)}{1 + G(s)}$$

Steady State Error = $E_{ss} = \lim_{s \to 0} \frac{u(s)}{1+G(s)}$

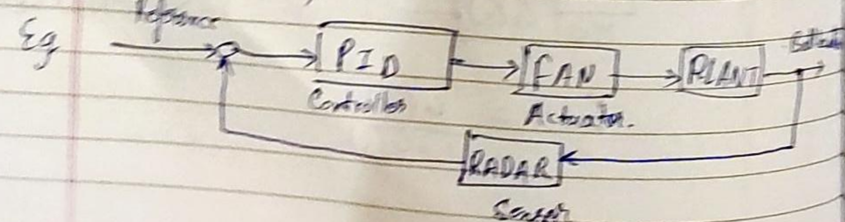
1 = Impulse
$1/s$ = Step
$1/s^2$ = Ramp
$1/s^3$ = Acceleration

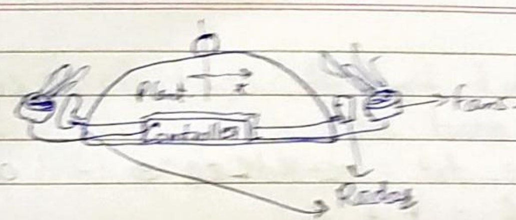
* Stability:- A measure of the tendency of a system's response to return to 0 after being disturbed.



Bounded Input produces a Bounded Output (BIBO)

Eg





~~Q- Why use Nyquist Stability Criterion?~~

PID- Regulators

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\,d\tau + K_D \frac{de(t)}{dt}$$

P:- Contributes to stability, medium-rate responsiveness

I:- Tracking & disturbance rejection, slow-rate responsiveness. May cause oscillations. If k_i is very high then oscillations will occur.

D:- Fast-rate responsiveness. Sensitive to noise. Making K_D too large will cause the system to overreact to noise.

LAPLACE TRANSFORM

* $f(t) \rightleftharpoons L\{f(t)\} = F(s)$

$$F(s) = \int_{-\infty}^{\infty} f(t).e^{-st}\,dt$$

where $s = \sigma + j\omega$

* Laplace transform of some standard Signals.

Signal	Laplace Transform
t	$1/s^2$
t^n	$\frac{n!}{s^{n+1}}$
$u(t)$	$1/s$
e^{-at}	$\frac{1}{s+a}$
$\delta(t)$	1
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$

* Inverse Laplace Transform.

$$f(t) = \frac{1}{2\pi j}\int_{\sigma-j\omega}^{\sigma+j\omega} F(s)\,e^{st}\,ds$$

* Properties of laplace Transform.

$\Rightarrow$ $f_1(t) \rightleftharpoons F_1(s)$ & $f_2(t) \rightleftharpoons F_2(s)$

$\Rightarrow a.f_1(t) + b.f_2(t) \rightleftharpoons a.F_1(s) + b.F_2(s)$

$\Rightarrow$ $f(t) \rightleftharpoons F(s)$

$f(at) \rightleftharpoons \frac{1}{|a|} F\left(\frac{s}{a}\right)$

$\Rightarrow$ $f(t) \rightleftharpoons F(s)$

$\Rightarrow f(t-t_0) \rightleftharpoons F(s).e^{-st_0}$

& $f(t+t_0) \rightleftharpoons F(s).e^{+st_0}$ } Time shift

$\Rightarrow$ $f(t) \rightleftharpoons F(s)$

$\Rightarrow e^{s_0 t} f(t) \rightleftharpoons F(s-s_0)$

& $e^{-s_0 t} f(t) \rightleftharpoons F(s+s_0)$ } Freq. shift

$\Rightarrow$ $f(t) \rightleftharpoons F(s)$

$\frac{d}{dt}\{f(t)\} \rightleftharpoons s.F(s) - \underbrace{f(0^-)}_{\text{Initial Value}}$ } Time differentiation

$\Rightarrow \frac{d^2}{dt^2}\{f(t)\} \rightleftharpoons s[s.F(s) - f(0^-)] - f'(0^-)$ } Time differentiation

⇒ Convolution Property

$x(t) \rightleftharpoons X(s)$

$y(t) \rightleftharpoons Y(s)$

$x(t) * y(t) \rightleftharpoons X(s).Y(s)$

∴ Convolution in time domain is the multiplication in frequency domain.

— X —

⇒ Linear - Time Invariant (LTI) Systems.

* These systems follow the properties of

1) Linear Systems, ie. (Superposition Principle)

2) Time Invariant Systems (Time Invariance)

* In an LTI System, all the initial conditions are 0.

1) Total Output = Z.I.R + Z.S.R

$$\underbrace{Z.I.R}_{\text{Zero Input Response}} + \underbrace{Z.S.R}_{\text{Zero State Response}}$$

2) In Linear Systems if I/P = 0, then O/P = 0

* The Impulse Response of a system is represented ~~by an impulse signal.~~ $h(t)$.

— X —

D.