

# Advanced Analytics for a Better World

## Assignment 3 – Housing Allocation Optimisation (Report)

Hubert Radosław Perliński (13985612)  
Jorge Matamala Yepes (13955764)

November 23, 2024

### 1. Case Background

The report commissioner has tasked our group with developing a model, which would find a design for the planned residential tower that would maximize the profits from apartment sales, given a list of design requirements. We were provided with three possible tower heights (23, 40, 56 floors) and data about the profits of constructing an apartment of a given size, owner and sector. The data also included a list of possible floor designs and the number of apartments of a given type in each design.

To find the most profitable full tower design, we have implemented a linear optimization model using the Pyomo Python library. This model is guaranteed to find a solution that yields the highest profit possible given the data. This report presents the outcome of the model and considers how the profits would change if certain design restrictions were altered (sensitivity analysis). The full design specifications for towers of the proposed floor counts can be found in the Appendix.

### 2. Optimal Tower Design

The output of the model was implemented for three different building heights with 23, 40 and 56 floors. In this section, we will provide an overview of the optimal allocations.

Table 1 shows the maximum profit and average profit per floor for the proposed tower heights. Intuitively, the maximum profit is increasing with the number of floors. The profit per floor is decreasing, however, remains highly positive for all considered heights. Hence, our recommendation would be to construct a tower of 56 floors, leading to a maximum profit of 39 million €.

	Number of Floors	Maximum Profit	Profit per Floor
<b>Scenario 1</b>	23	17,483,910 €	760,170 €
<b>Scenario 2</b>	40	29,622,229 €	740,555 €
<b>Scenario 3</b>	56	39,187,428 €	699,775 €

Table 1: Profit and Profit per Floor for Different Scenarios

One of the requirements was that each floor should have just one owner type. The number of floors allocated to each owner can be seen in Figure 1. In all three scenarios, the number of floors assigned to investors is the greatest, followed by private owners and corporations. This aligns with their profitability – investors being the most profitable and corporations the least.

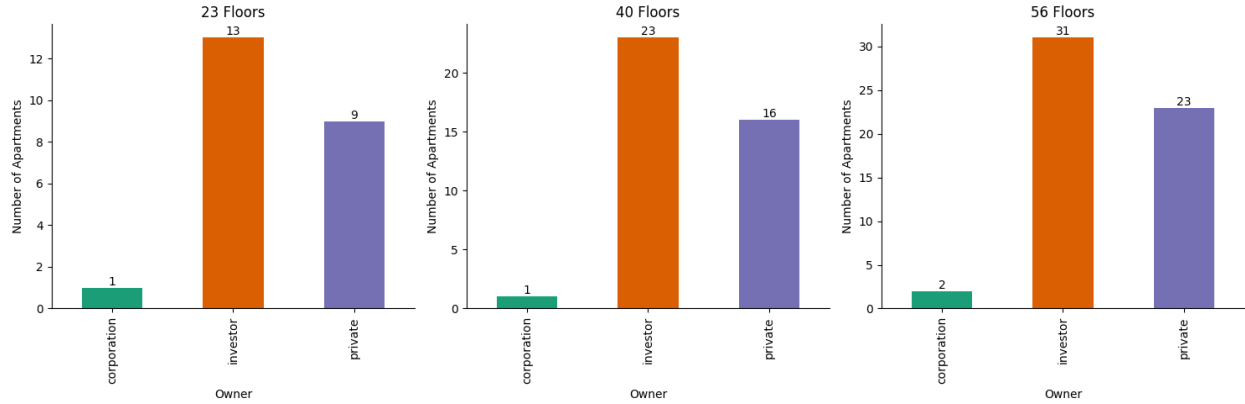


Figure 1: Owner Distribution for the Different Floor Scenarios

Similarly, Figure 2 shows the number of floors that have a specific design. Again, we see a similar distribution for the three scenarios, where the 'aa' and 'ee' designs are the most common, which are the designs with the smallest and biggest apartments, respectively.

The former ('aa') is only assigned to investors, and has a combination of social (58%) and middle sector (42%) apartments, while the latter ('ee') is fully assigned to private owners in the free sector. This is the case for all three scenarios.

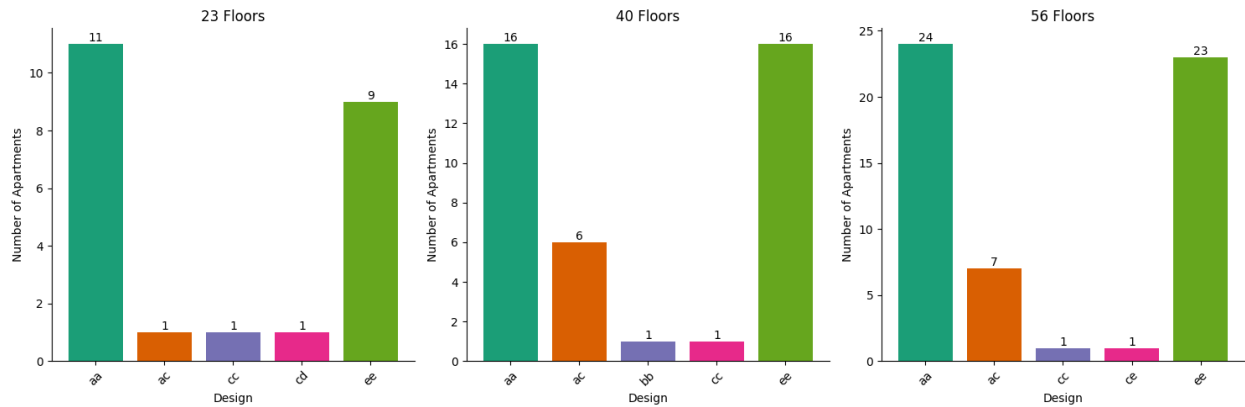


Figure 2: Design Distribution for the Different Floor Scenarios

Tables 5.1, 5.2 and 5.3 in the Appendix specify the exact design, owner and sector structure of each floor, including the implementation of restriction 12 ("If the smallest apartment of a floor is smaller than the smallest apartment of another floor, then the floor number should be lower."). It is noteworthy that the best designs are not unique - there are other combinations which would yield the same maximum profit. For example, the number of floors with the 'aa' design could be either 24, 25, 26 or 27. Such examples can be found in the Python code attached to the report. If the commissioner has any preference regarding the floor designs, a secondary objective could be declared to find a floor combination that yields the maximum profit, but also satisfies those additional preferences.

### 3. Sensitivity Analysis

This part of the report focuses on the sensitivity analysis, which checks how the profits would change if the design restriction parameters were to change. It can inform the real estate developer which guidelines given by the municipality could be renegotiated to increase the profit.

The sensitivity analysis will focus on Scenario 3 (56 floors), since it has the highest total profit. Given the similarity between the three scenarios, the key points of the analysis will also be applicable to the other two as well.

#### 3.1. Minimum percentage of apartments in the given sector requirements (restrictions 1 and 2)

This requirement states that the apartments in the social and middle sectors should each represent at least 40% of all apartments. Figure 3 shows that this has a substantial impact on the profits. If any of those requirements were lowered to 25%, the maximum attainable profit could rise up to 52,080,658 €. On the other hand, if the municipality decided to increase any of those requirements to for example 45%, the profits would decrease to even 30,284,755€. Hence, this is a crucial parameter for the negotiations with the municipality.

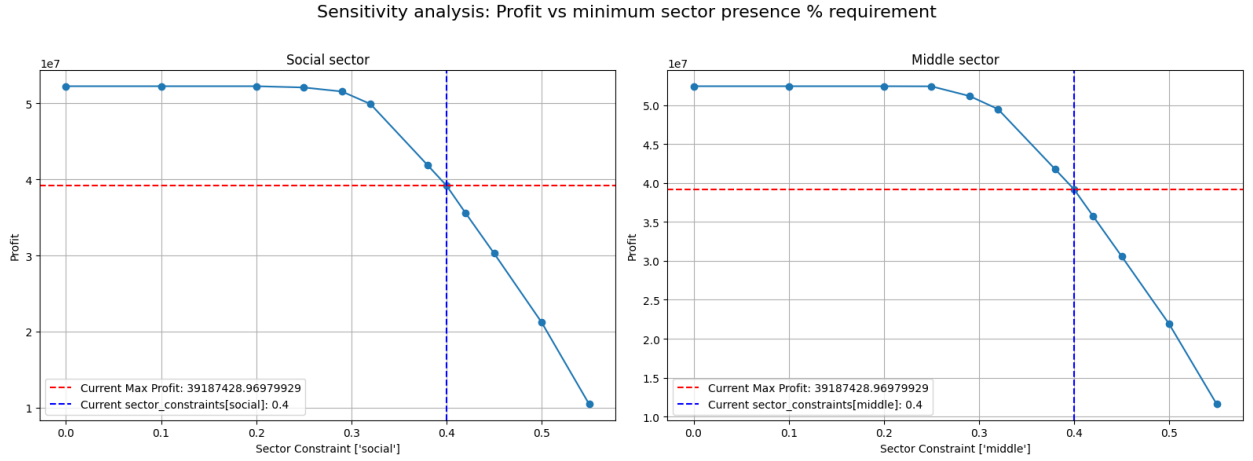


Figure 3: Impact of the minimum sector requirements on profit.

#### 3.2. Minimum average floor area of apartments in a sector (restrictions 3 and 4)

Currently, there are restrictions on the minimum average area of apartments in social and middle sectors, equal to  $40m^2$  and  $50m^2$  respectively. Lowering the requirement for the social sector to  $38m^2$  could increase the profit to 40,156,566€. Further decreases would not affect the profit. For the middle sector, lowering the requirement to  $44m^2$  could increase the profit to 44,307,388€. Further decreases would not affect the profit. If this requirement were to be also introduced in the free sector, the best profit would not be affected, which follows the fact that only the largest apartments were assigned to that sector. Impacts of other values can be seen in Figure 4.

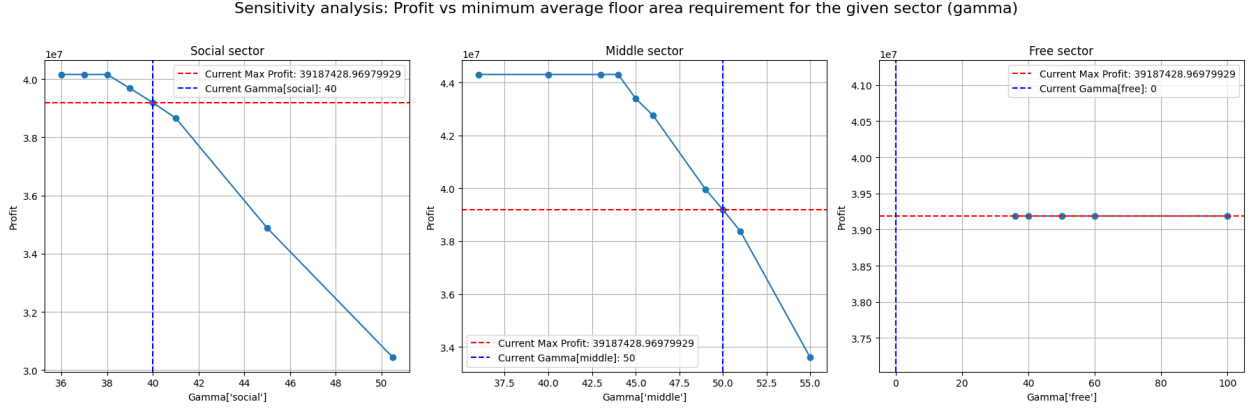


Figure 4: Impact of the minimum average floor area per sector on profit.

### 3.3. Remaining restrictions

The other design restrictions are not limiting factors in the maximum attainable profit. Removing requirements 5 and 6 (Minimal apartment area for “social” sector for corporation is 40 m<sup>2</sup> and Minimal apartment area for “middle” sector for corporation is 50 m<sup>2</sup>) has no impact. This is also true for requirement 8 (“The Corporation cannot buy apartments in the “free” sector.”) and 9 (Minimal 70% of apartments should go to “investors”). The effects of increasing the % of apartments reserved for investors, or introducing new requirements for reserving a % of apartments for corporations or private buyers, can be seen in Figure 5. Due to the impact of such requirements on profits, it is recommended they are not introduced.

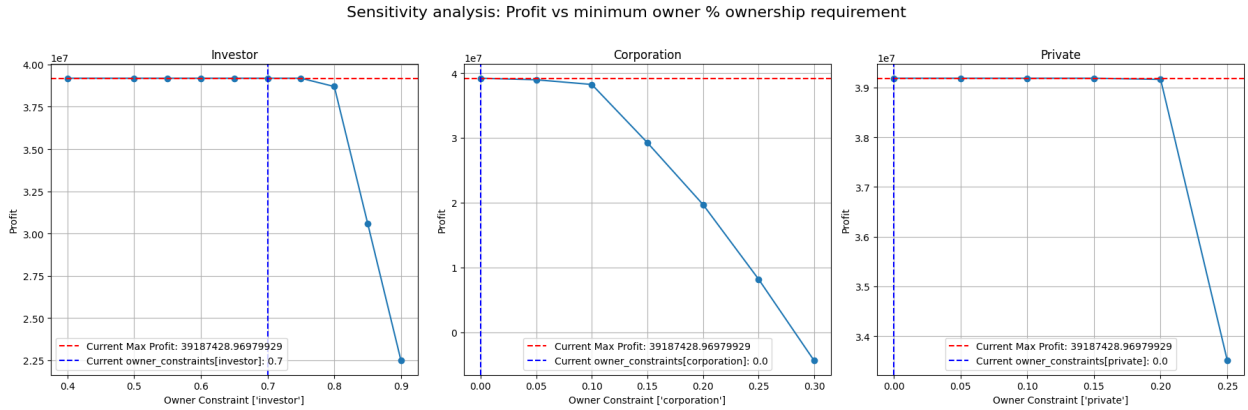


Figure 5: Impact of minimum owner % ownership requirement on profits

## 4. Summary

Three tower height scenarios (23, 40, and 56 floors) were analyzed, with the 56-floor option offering the highest profit of 39.19 million €. The full tower design specifications can be found in the Appendix. Sensitivity analysis showed how adjusting design restrictions, such as sector % requirements and apartment size requirements, could further impact profits. The analysis suggests renegotiating certain restrictions could enhance profitability.

## 5. Appendix

### 5.1 Optimal Apartment Configuration – 23 Floors

Floor	Design	Owner	Apartments (size, sector)
1	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
2	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
3	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
4	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
5	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
6	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
7	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:3, middle:4], 48m <sup>2</sup> : [middle:4]
8	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
9	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
10	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
11	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:3], 48m <sup>2</sup> : [middle:4]
12	ac	investor	36m <sup>2</sup> : [social:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
13	cc	investor	60m <sup>2</sup> : [middle:4], 71m <sup>2</sup> : [middle:3, free:1]
14	cd	corporation	60m <sup>2</sup> : [social:2], 70m <sup>2</sup> : [social:1], 71m <sup>2</sup> : [social:2], 96m <sup>2</sup> : [middle:2]
15	ee	private	131m <sup>2</sup> : [free:4]
16	ee	private	131m <sup>2</sup> : [free:4]
17	ee	private	131m <sup>2</sup> : [free:4]
18	ee	private	131m <sup>2</sup> : [free:4]
19	ee	private	131m <sup>2</sup> : [free:4]
20	ee	private	131m <sup>2</sup> : [free:4]
21	ee	private	131m <sup>2</sup> : [free:4]
22	ee	private	131m <sup>2</sup> : [free:4]
23	ee	private	131m <sup>2</sup> : [free:4]

## 5.2 Optimal Apartment Configuration – 40 Floors

Floor	Design	Owner	Apartments (size, sector)
1	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
2	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
3	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
4	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
5	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
6	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
7	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
8	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
9	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
10	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
11	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
12	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
13	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:2, middle:4], 48m <sup>2</sup> : [middle:4]
14	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
15	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
16	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
17	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
18	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
19	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
20	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
21	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
22	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [free:2], 71m <sup>2</sup> : [middle:2]
23	bb	investor	42m <sup>2</sup> : [middle:2], 52m <sup>2</sup> : [middle:4], 68m <sup>2</sup> : [middle:2]
24	cc	corporation	60m <sup>2</sup> : [social:4], 71m <sup>2</sup> : [social:4]
25	ee	private	131m <sup>2</sup> : [free:4]
26	ee	private	131m <sup>2</sup> : [free:4]
27	ee	private	131m <sup>2</sup> : [free:4]
28	ee	private	131m <sup>2</sup> : [free:4]
29	ee	private	131m <sup>2</sup> : [free:4]
30	ee	private	131m <sup>2</sup> : [free:4]
31	ee	private	131m <sup>2</sup> : [free:4]
32	ee	private	131m <sup>2</sup> : [free:4]
33	ee	private	131m <sup>2</sup> : [free:4]
34	ee	private	131m <sup>2</sup> : [free:4]
35	ee	private	131m <sup>2</sup> : [free:4]
36	ee	private	131m <sup>2</sup> : [free:4]
37	ee	private	131m <sup>2</sup> : [free:4]
38	ee	private	131m <sup>2</sup> : [free:4]
39	ee	private	131m <sup>2</sup> : [free:4]
40	ee	private	131m <sup>2</sup> : [free:4]

### 5.3 Optimal Apartment Configuration – 56 Floors

Floor	Design	Owner	Apartments (size, sector)
1	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [social:1, middle:4]
2	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
3	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
4	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
5	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
6	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
7	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
8	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
9	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
10	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
11	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
12	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
13	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
14	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
15	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:4], 48m <sup>2</sup> : [middle:4]
16	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [social:3, middle:4], 48m <sup>2</sup> : [middle:4]
17	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
18	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
19	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
20	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
21	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
22	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
23	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
24	aa	investor	36m <sup>2</sup> : [social:4], 42m <sup>2</sup> : [middle:4], 48m <sup>2</sup> : [middle:4]
25	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
26	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
27	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
28	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
29	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:2], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
30	ac	investor	36m <sup>2</sup> : [social:2], 42m <sup>2</sup> : [middle:1], 48m <sup>2</sup> : [middle:2], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
31	ac	investor	36m <sup>2</sup> : [social:2], 48m <sup>2</sup> : [middle:1], 60m <sup>2</sup> : [middle:2], 71m <sup>2</sup> : [middle:2]
32	cc	corporation	60m <sup>2</sup> : [social:4], 71m <sup>2</sup> : [social:4]
33	ce	corporation	60m <sup>2</sup> : [social:2], 71m <sup>2</sup> : [social:2], 131m <sup>2</sup> : [middle:2]
34	ee	private	131m <sup>2</sup> : [free:4]
35	ee	private	131m <sup>2</sup> : [free:4]
36	ee	private	131m <sup>2</sup> : [free:4]
37	ee	private	131m <sup>2</sup> : [free:4]
38	ee	private	131m <sup>2</sup> : [free:4]
39	ee	private	131m <sup>2</sup> : [free:4]
40	ee	private	131m <sup>2</sup> : [free:4]
41	ee	private	131m <sup>2</sup> : [free:4]
42	ee	private	131m <sup>2</sup> : [free:4]
43	ee	private	131m <sup>2</sup> : [free:4]
44	ee	private	131m <sup>2</sup> : [free:4]
45	ee	private	131m <sup>2</sup> : [free:4]
46	ee	private	131m <sup>2</sup> : [free:4]
47	ee	private	131m <sup>2</sup> : [free:4]
48	ee	private	131m <sup>2</sup> : [free:4]
49	ee	private	131m <sup>2</sup> : [free:4]
50	ee	private	131m <sup>2</sup> : [free:4]
51	ee	private	131m <sup>2</sup> : [free:4]
52	ee	private	131m <sup>2</sup> : [free:4]
53	ee	private	131m <sup>2</sup> : [free:4]
54	ee	private	131m <sup>2</sup> : [free:4]
55	ee	private	131m <sup>2</sup> : [free:4]
56	ee	private	131m <sup>2</sup> : [free:4]