MIP 4 Water Flow

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1 Introduction

E: the set of edges
T: the set of time periods

$X_{e,t}$: the amount of water flow from at edge e at time t
$Y_{A,t}$: the amount of released water at node A at time t
Z_{AB}: 1 if pipe from A to B is used, 0 otherwise

$f(t) = max(-t^3 + 4t^2 - t + 4, 0)$, for t $\geq$ 0, 0 otherwise: the amount of water entering the system at node A at time t
c_e: the capacity of edge e
k: the maximum number of active pipes allowed

Please note that 1. $X_{e,0}$ refers to the initial state of edge e which is always equal to 0 for all edges.

$$\text{Maximize} \sum_{t \in T} X_{EG,t} + X_{FG,t}$$

$$\text{Subject to:}$$

$$
\begin{aligned}
X_{AB,t} + X_{AC,t} + Y_{A,t} &= f(t-1), & \forall t \in T \\
X_{BC,t} + X_{BD,t} X_{BE,t} + Y_{B,t} &= X_{AB,t-1} + X_{FB,t-1}, & \forall t \in T \\
X_{CE,t} + X_{CF,t} + Y_{C,t} &= X_{AC,t-1} + X_{BC,t-1}, & \forall t \in T \\
X_{DE,t} + Y_{D,t} &= X_{BD,t-1}, & \forall t \in T \\
X_{EG,t} + Y_{E,t} &= X_{DE,t-1} + X_{CE.t-1}, & \forall t \in T \\
X_{FG,t} + Y_{F,t} &= X_{CE,t-1}, & \forall t \in T \\
X_{e,-1} &= 0, & \forall e \in E \\
X_{e,t} &\leq Z_e c_e, & \forall e \in E \\
\sum_{e \in E} Z_e &\leq k \\
X_{e,t} &\in R^+, & \forall e \in E, t \in T \\
Y_{n,t} &\in R^+, & \forall n \in N, t \in T \\
Z_e &\in \{0,1\}, & \forall e \in E
\end{aligned}
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