

The human visual system preserves the hierarchy of 2-dimensional pattern regularity

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1 A century of vision research has demonstrated that symmetry contributes to numerous domains of visual perception (1–4). In a 2-D
2 image, the four fundamental symmetries, reflection, rotation, translation and glide reflection, can be combined in 17 distinct ways. These
3 17 “wallpaper” groups (5–7) obey a hierarchy, determined by mathematical group theory, in which simpler groups are subgroups of more
4 complex ones(8). Here we probe representations of symmetries in
5 wallpaper groups using two methods: (1) Steady-State Visual Evoked
6 Potentials (SSVEPs) recorded using EEG and (2) symmetry detection
7 thresholds measured psychophysically. We find that hierarchical re-
8 lationships between the wallpaper groups are almost perfectly pre-
9 served in both behavior and response amplitudes in visual cortex.
10 This remarkable consistency between the structure of symmetry rep-
11 resentations and mathematical group theory, is likely generated over
12 visual development, through implicit learning of regularities in the
13 environment.

Keyword 1 | Keyword 2 | Keyword 3 | ...

1 **S**ymmetries are present at many scales in images of natu-
2 ral scenes, due to a complex interplay of physical forces
3 that govern pattern formation in nature. The importance of
4 symmetry for visual perception has been known at least since
5 the gestalt movement of the early 20th century. Since then,
6 symmetry has been shown to contribute to the perception of
7 shapes (1, 3), scenes (4) and surface properties (2), as well as
8 the social process of mate selection (9). Most of this work has
9 focused on mirror symmetry or *reflection*, with much less attention
10 being paid to the other fundamental symmetries: *rotation*,
11 *translation* and *glide reflection*. In the two spatial dimensions
12 relevant for images, these four fundamental symmetries can be
13 combined in 17 distinct ways, the “wallpaper” groups (5–7).
14 Previous work has focused on four of the wallpaper groups,
15 and used functional MRI to show that rotation symmetries
16 within wallpapers are represented parametrically in several
17 areas in occipital cortex, beginning with visual area V3 (10).
18 This effect is also robust in electroencephalography (EEG),
19 whether measured using Steady-State Visual Evoked Poten-
20 tials (SSVEPs)(10) or event-related paradigms (11). Here we
21 extend on this work by collecting SSVEPs and psychophysical data
22 from human participants viewing the complete set of
23 wallpaper groups. We measure responses in visual cortex to
24 16 out of the 17 wallpaper groups, with the 17th serving as a
25 control stimulus, with the goal of providing a more complete
26 picture of how wallpaper groups are represented in the human
27 visual system.

28 A wallpaper group is a topologically discrete group of isome-
29 tries of the Euclidean plane, i.e. transformations that preserve
30 distance (7). Wallpaper groups differ in the number and
31 kind of these transformations. In mathematical group theory,
32 when the elements of one group is completely contained in

another, the inner group is called a subgroup of the outer group (7). Subgroup relationships between wallpaper groups can be distinguished by their indices. The index of a subgroup relationship is the number of cosets, i.e. the number of times the subgroup is found in the outer group (7). As an example, let us consider groups P6 and P2. If we ignore the translations in two directions that both groups share, group P6 consists of the set of rotations $0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ$, in which P2 $0^\circ, 180^\circ$ is contained. P2 is thus a subgroup of P6, and the full P6 set can be generated by every combination of P2 and rotations $0^\circ, 120^\circ, 240^\circ$. Because P2 is repeated three times in P6, P2 is a subgroup of P6 with index 3 (7). The 17 wallpaper groups thus obey a hierarchy of complexity where simpler groups are sub-groups of more complex ones (8). The full set of subgroup relationships is listed in Section 1.4.2 of the Supplementary Material.

49 The two datasets presented here puts on in the position
50 of being able to assess the extent to which both behavior
51 and brain responses follow that hierarchy. The results show
52 that activity in human visual cortex is remarkably consist-
53 ent with the hierarchical relationships between the wallpaper
54 groups, with SSVEP amplitudes and psychophysical thresh-
55 olds preserving these relationships at a level that is far beyond
56 chance. Visual cortex thus appears to encode all of the funda-
57 mental symmetries using a representational structure that
58 closely approximates the subgroup relationships from group
59 theory. Given that most participants had no knowledge of
60 group theory, the ordered structure of visual responses to
61 wallpaper groups is likely learned implicitly from regularities
62 in the visual environment.

Significance Statement

Wallpaper groups were discovered in the mid-19th century, and the 17 groups constitute the complete set of possible ways of regularly tiling the 2D-plane. In recent years wallpaper groups have found use in the vision science community, as an ideal stimulus set for studying the perception of symmetries in textures. Here we present brain imaging and psychophysical data on the complete set of wallpaper groups and show the hierarchical organization among wallpaper groups in reflected in both representations in visual cortex and performance on a symmetry detection task. This shows that the visual system is highly sensitive to regularities in textures, and suggest that symmetries may play an important role in texture perception.

PJK and ADFC designed the study, PJK collected EEG data, ADFC collected psychophysical data, PJK and ADFC wrote the paper.

The authors have no conflicts of interests to declare

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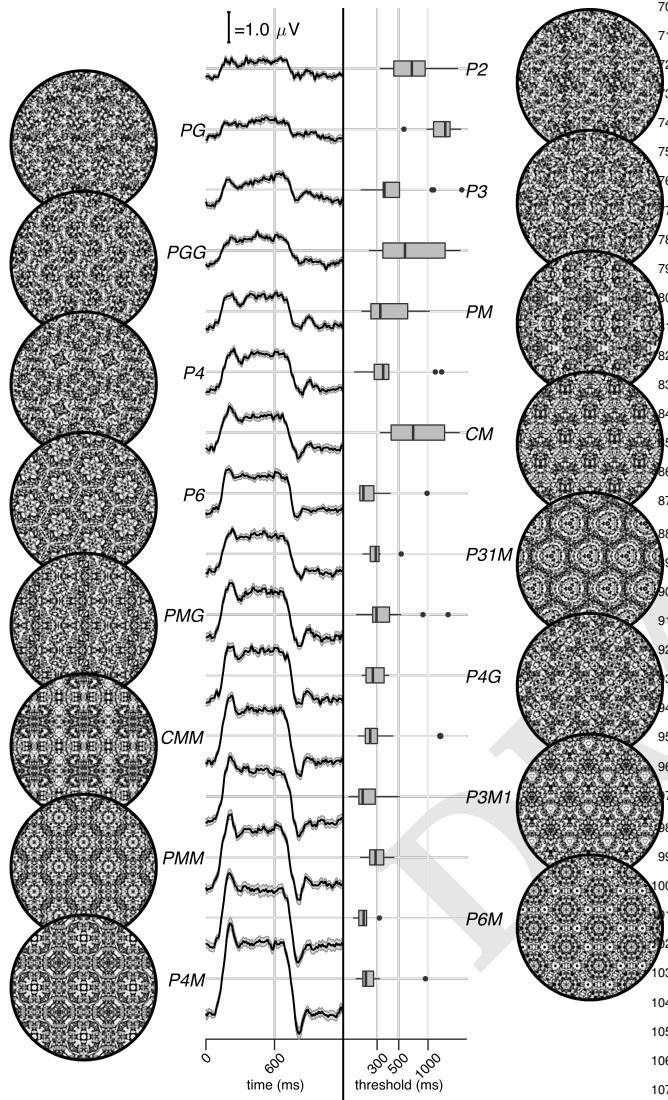


Fig. 1. Examples of each of the 16 wallpaper groups are shown in the left-⁹ right-most column of the figures, next to the corresponding SSVEP (center-left) and ¹⁰ psychological (center-right) data from each group. The SSVEP data are odd-harmonic-¹¹ filtered cycle-average waveforms. In each cycle, a P1 exemplar was shown for the ¹² first 600 ms, followed by the original exemplar for the last 600 ms. Errorbars are ¹³ standard error of the mean. Psychophysical data are presented as boxplots reflecting ¹⁴ the distribution of display duration thresholds. The 16 groups are ordered by the ¹⁵ strength of the SSVEP response, to highlight the range of response amplitudes. ¹⁶

Results

The stimuli used in our two experiments were multiple exemplar images belonging to each of the wallpaper groups, generated from random-noise textures, as described in detail elsewhere (10). Exemplar images from group P1 was used as control stimuli, and each exemplar from the other 16 groups had a power-spectrum matched P1 exemplar. The matched P1 exemplars were generated by phase-scrambling the exemplar images. Because all wallpapers are periodic due to their lattice tiling structure, the phase-scrambled images all belong to group P1 regardless of group membership of the original exemplar. P1 contains no symmetries other than translation, while all other groups contain translation in combination with one or more of the other three fundamental symmetries (reflection, rotation, glide reflection) (7). In our SSVEP experiment, this stimulus set allowed us to isolate brain activity specific to the symmetry structure in the exemplar images from activity associated with modulation of low-level features, by alternating exemplar images and control exemplars. In this design, responses to structural features beyond the shared power spectrum, including any symmetries other than translation, are isolated in the odd harmonics of the image update frequency (10, 12, 13). Thus, the combined magnitude of the odd harmonic response components can be used as a measure of the overall strength of the visual cortex response.

The psychophysical experiment took a distinct but related approach. In each trial an exemplar image was shown with its matched control, one image after the other, and the order varied pseudo-randomly such that in half the trials the original exemplar was shown first, and in the other half the control image was shown first. After each trial, participants were told to indicate whether the first or second image contained more structure, and the duration of both images was controlled by a staircase procedure so that a display duration threshold could be computed for each wallpaper group.

A summary of our brain imaging and psychophysical measurements is presented with examples of the wallpaper groups in Figure 1. For our primary SSVEP analysis, we only considered EEG data from a pre-determined region-of-interest (ROI) consisting of six electrodes over occipital cortex (see Supplementary Figure 1.1). SSVEP data from this ROI was filtered so that only the odd harmonics that capture the symmetry response contribute to the waveforms. While waveform amplitude is quite variable among the 16 groups, all groups have a sustained negative-going response that begins at about the same time for all groups, 180 ms after the transition from the P1 control exemplar to the original exemplar. To reduce the amplitude of the symmetry-specific response to a single number that could be used in further analyses and compared to the psychophysical data, we computed the root-mean-square (RMS) over the odd-harmonic-filtered waveforms. The data in Figure 1 are shown in descending order according to RMS. The psychophysical results, shown in box plots in Figure 1, were also quite variable between groups, and there seems to be a general pattern where wallpaper groups near the top of the figure, that have lower SSVEP amplitudes, also have longer psychophysical display duration thresholds.

We now wanted to quantify the degree to which our two measurements were consistent with the subgroup relationships. We hypothesized that more complex groups would (a) produce symmetric-specific SSVEPs with higher amplitudes and (b)

shorter display duration thresholds. **PERHAPS WE NEED TO JUSTIFY THESE ASSUMPTIONS?** We tested each of these hypotheses using the same approach. We first fitted a Bayesian model with wallpaper group as a factor and participant as a random effect. We fit the model separately for SSVEP RMS and psychophysical data, and then computed posterior distributions for the difference between supergroup and subgroup. These difference distributions could allow us to compute the conditional probability that the supergroup would produce (a) larger RMS and (b) a shorter threshold durations, when compared to the subgroup. The posterior distributions are shown in Figure 2 for the SSVEP data, and in Figure 3 for the psychophysical data, which distributions color-coded according to conditional probability. For both data sets our hypothesis is confirmed: For the overwhelming majority of the 64 subgroup relationships, supergroups are more likely to produce larger symmetry specific SSVEPs and shorter threshold durations, and in most cases the conditional probability of this happening is extremely high.

We also ran a control analysis using (1) odd-harmonic SSVEP data from a six-electrode ROI over parietal cortex (see Supplementary Figure 1.1) and (2) even-harmonic SSVEP data from the same occipital ROI that was used in our primary analysis. By comparing these two control analysis to our primary SSVEP analysis, we can address the specificity of our effects in terms of location (occipital cortex vs parietal cortex) and harmonic (odd vs even). For both control analyses (plotted in Supplementary Figures 3.3 and 3.4), the correspondence between data and subgroup relationships was weaker than in the primary analysis. We can quantify the strength of the association between the data and the subgroup relationships, by asking what proportion of subgroup relationships that reach or exceed a range of probability thresholds. This is plotted in Figure 4, for our psychophysical data, our primary SSVEP analysis and our two control SSVEP analyses. It is that odd-harmonic SSVEP data from an occipital ROI and display duration thresholds both have a strong association with the subgroup relations, that for a clear majority of the subgroups survive even at the highest threshold we consider ($p(\Delta > 0 | \text{data}) > 0.99$), and that the association is far weaker for the two control analyses.

SSVEP data from four of the wallpaper groups (P2, P3, P4 and P6) was previously published as part of our earlier demonstration of parametric responses to rotation symmetry in wallpaper groups(10). We replicate that result using our Bayesian approach, and find the same parametric effect in the psychophysical data (Supplementary Figure 4.1). We also conducted an analysis **DOES THIS NEED TO BE SPELLED OUT MORE** looking for effects of index and normality in our two datasets, and found that subgroup relationships with higher indices tended to produce stronger effects for both SSVEP RMS and symmetry detection thresholds, as shown by the fact that a leave-one-out test ... **MORE DETAILS NEEDED HERE**. The effect of index did not (???) reach significance.

Finally, we conducted a correlation analysis comparing SSVEP and psychophysical data, and found a small ($R^2 = 0.44$) but above-zero correlation, as indicated by our confidence intervals. There are several factors that might explain the relatively weak correlation, most prominently the fact that the same individuals did not participate in each of the two

experiments. Nevertheless, we find the relationship between the two datasets interesting, because it suggests that our psychophysical and SSVEP measurements are tapping into the same underlying mechanisms.

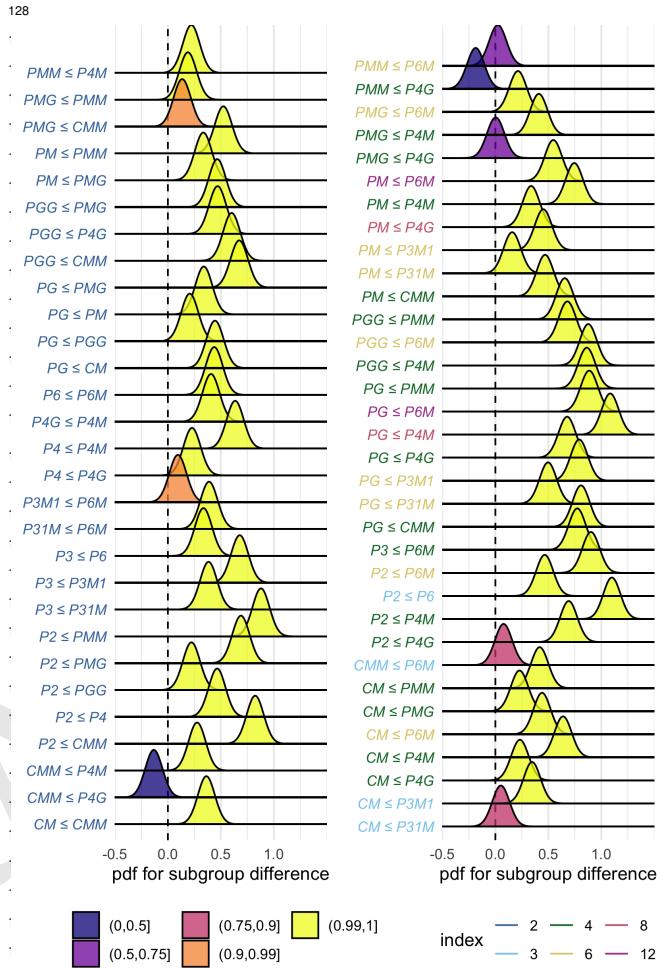


Fig. 2. Posterior distributions for the difference in mean RMS SSVEP response. Colour coding of the text indicates the index of the subgroup, while the colour of the filled distribution relates to the conditional probability that the difference in means is greater than zero. We can see that xx/64 subgroup relationships have $p(\Delta | \text{data}) > 0.99$.

Discussion

Here we show that beyond merely responding to the elementary symmetry operations of reflection (14) and rotation (10), the visual system explicitly represents hierarchical structure of the 17 wallpaper groups, and thus the compositions of all four of the fundamental symmetry transformations (rotation, reflection, translation, glide reflection) which comprise regular textures. The SSVEP amplitude and the symmetry detection threshold both preserve the complex hierarchy of subgroup relationships among the wallpaper groups (8). For the SSVEP, this remarkable consistency was specific to the odd harmonics of the stimulus frequency, that capture the symmetry-specific response (10) and to electrodes in an ROI over occipital cortex. When the same analysis was done using the odd harmonics from electrodes over parietal cortex (Supplementary Figure 3.3) or even harmonics from electrodes over occipital cortex

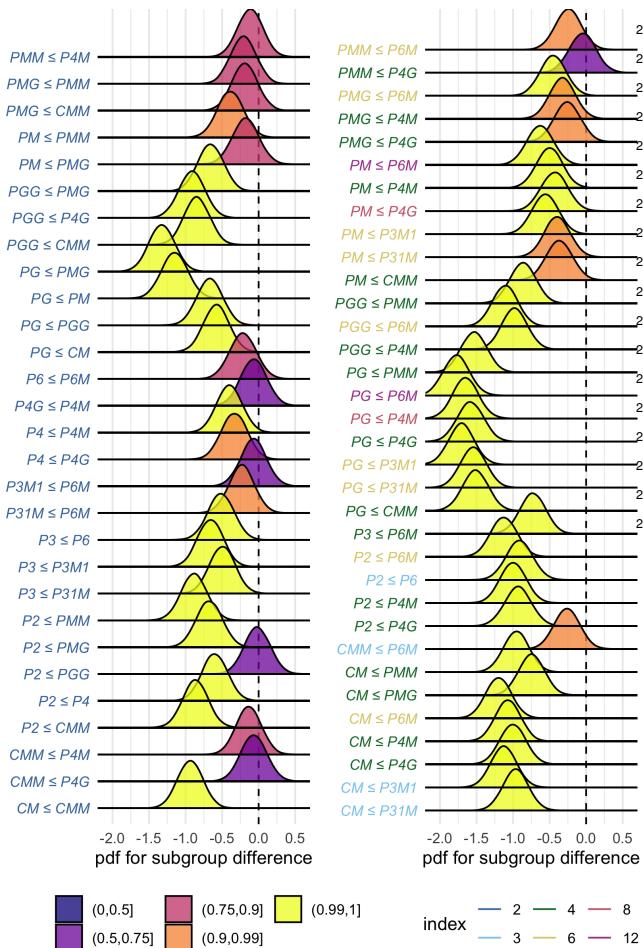


Fig. 3. Posterior distributions for the difference in mean display duration threshold. Colour coding of the text indicates the index of the subgroup, while the colour of the filled distribution relates to the conditional probability that the difference in means is greater than zero. We can see that $xx/64$ subgroup relationships have $p(\Delta|data) > 0.9$.

(Supplementary Figure 3.4), the data was much less consistent with the subgroup relationships (yellow and green lines, Figure 4).

The current data provide a complete description of the visual system's response to symmetries in the 2-D plane. Our design precludes us from independently measure the response to P1, but because each of the 16 other groups produce non-zero odd harmonic amplitudes (see Figure 1), we can conclude that the relationships between P1 and all other groups, where P1 is the subgroup, are also preserved by the visual system. The subgroup relationships are in many cases not obvious perceptually, and most participants had no knowledge of group theory. Thus, the visual system's ability to preserve the subgroup hierarchy does not depend on explicit knowledge of the relationships. Furthermore, previous behavioral experiments have shown that although naïve observers can distinguish many of the wallpaper groups (15), they are generally error-prone when asked to assign exemplar images to the appropriate group (16) **ALASDAIR PERHAPS YOU WANT TO SAY MORE OR DIFFERENT HERE.**

The correspondence between responses in the visual system and group theory that we demonstrate here, may reflect a

form of implicit learning that depends on the structure of the natural world. The environment is itself constrained by physical forces underlying pattern formation and these forces are subject to multiple symmetry constraints (17). The ordered structure of responses to wallpaper groups could be driven by a central tenet of neural coding, that of efficiency. If coding is to be efficient, neural resources should be distributed in such a way that the structure of the environment is captured with minimum redundancy considering the visual geometric optics, the capabilities of the subsequent neural coding stages and the behavioral goals of the organism (18–21). Early work within the efficient coding framework suggested that natural images had a $1/f$ spectrum and that the corresponding redundancy between pixels in natural images could be coded efficiently with a sparse set of oriented filter responses, such as those present in the early visual pathway (22, 23). Our results suggest that the principle of efficient coding extends to a much higher level of structural redundancy – that of symmetries in visual images.

The 17 wallpaper groups are completely regular, and relatively rare in the visual environment, especially when considering distortions due to perspective and occlusion. Near-regular textures, however abound in the visual world, and can be approximated as deformed versions of the wallpaper groups (24). The correspondence between visual cortex responses and group theory demonstrated here may indicate that the visual system represents visual textures using a similar scheme, with the wallpaper groups serving as anchor points in representational space. This framework resembles norm-based encoding strategies that have been proposed for other stimulus classes, most notably faces (25), and leads to the prediction that adaptation to wallpaper patterns should distort perception of near-regular textures, similar to the aftereffects found for faces (26). Field biologists have demonstrated that animals respond more strongly to exaggerated versions of a learned stimulus, referred to as “supernormal” stimuli (27). In the norm-based encoding framework, wallpaper groups can be considered super-textures, exaggerated examples of the near-regular textures that surround us. Artists may consciously or unconsciously create supernormal stimuli, to capture the essence of the subject and evoke strong responses in the audience (28). Wallpaper groups are visually compelling and have been widely used in human artistic expression going back to the Neolithic age (29). If wallpapers are in fact super-textures, it would imply that this prevalence may be a direct consequence of the strategy the human visual system uses for encoding textures.

Participants. Twenty-five participants (11 females, mean age 28.7 ± 13.3) took part in the EEG experiment. Their informed consent was obtained before the experiment under a protocol that was approved by the Institutional Review Board of Stanford University. 11 participants (8 females, mean age 20.73 ± 1.21) took part in the psychophysics experiment. All participants had normal or corrected-to-normal vision. Their informed consent was obtained before the experiment under a protocol that was approved by the University of Essex's Ethics Committee.

Stimulus Generation. Exemplars from the different wallpaper groups were generated using a modified version of the methodology developed by Clarke and colleagues (16) that we have

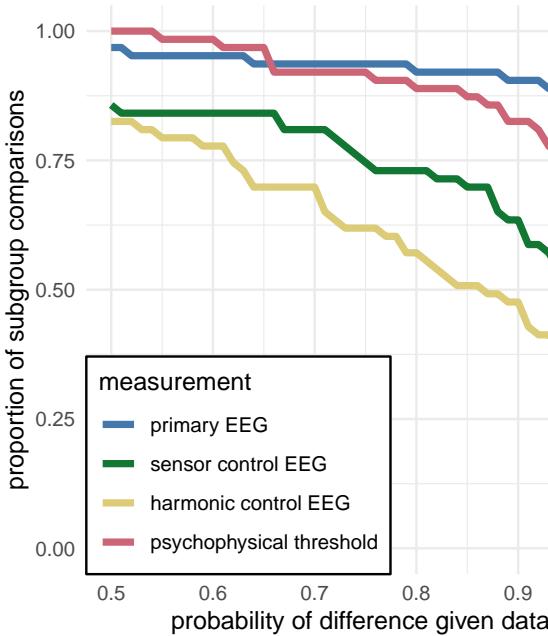


Fig. 4. This plot shows the proportion of subgroup relations that satisfy $p(\Delta \nmid \text{data}) > x$. We can see that if we take $x = 0.95$ as our threshold, the subgroup relations are preserved in $56/63 = 89\%$ and $49/64 = 78\%$ of the comparisons for the primary EEG and display durations respectively. This compares to the $32/64 = 50\%$ and $22/64 = 35\%$ for the control EEG conditions.

319 was controlled using in-house software.
320

321 For the psychophysics experiment, the stimuli were shown
322 on a $48 \times 27\text{cm}$ VIEWPixed/3D LCD Display monitor, model
323 VPX-VPX-2005C, resolution 1920×1080 pixels, with a viewing
324 distance of approximately 40cm and linear gamma. Stimulus
325 presentation was controlled using MatLab and Psychtoolbox-3
326 ([30, 31](#)). The diameter of the circular aperture for the stimuli
327 was 21.5° .

EEG Procedure. Visual Evoked Potentials were measured using
328 a steady-state design, in which P1 control images alternated
329 with test images from each of the 16 other wallpaper groups[[2](#)].
330 Exemplar images were always preceded by their matched P1
331 control image. A single 0.83 Hz stimulus cycle consisted of a
332 control P1 image followed by an exemplar image, each shown
333 for 600 ms. A trial consisted of 10 such cycles (12 sec) over
334 which 10 different exemplar images and matched controls from
335 the same rotation group were presented. For each group type,
336 the individual exemplar images were always shown in the same
337 order within the trials. Participants initiated each trial with
338 a button-press, which allowed them to take breaks between
339 trials. Trials from a single wallpaper group were presented
340 in blocks of four repetitions, which were themselves repeated
341 twice per session, and shown in random order within each
342 session. To control fixation, the participants were instructed
343 to fixate a small white cross in the center of display. To control
344 vigilance, a contrast dimming task was employed. Two times
345 per trial, an image pair was shown at reduced contrast, and the
346 participants were instructed to press a button on a response
347 pad. We adjusted the contrast reduction such that average
348 accuracy for each participant was kept at 85% correct, so that
349 the difficulty of the vigilance task was kept constant.

Psychophysics Procedure. The experiment consisted of 16
350 blocks, one for each of the wallpaper groups (excluding P1).
351 In each trial, participants were presented with two stimuli
352 (one of which was the wallpaper group for the current block of
353 trials, the other being P1), one after the other (inter-stimuli
354 interval of 700ms). After each stimuli had been presented, it
355 was masked with white noise for 300ms. After both stimuli had
356 been presented, participants made a response on the keyboard
357 to indicate whether they thought the first or second contained
358 the most symmetry. Each block started with 10 practise trials,
359 (stimulus display duration of 500ms) to allow participants
360 to familiarise themselves with the current block's wallpaper
361 pattern. If they achieved an accuracy of 9/10 in these trials
362 they progressed to the rest of the block, otherwise they carried
363 out another set of 10 practise trials. This process was repeated
364 until the required accuracy of 9/10 was obtained. The rest of
365 the block consisted of four interleaved staircases (using the
366 QUEST algorithm ([32](#)), full details given in the SI) of 30 trials
367 each. On average, a block of trials took around 10 minutes to
368 complete.

EEG Acquisition and Preprocessing. Electroencephalogram
369 Acquisition and Preprocessing. The time-locked Steady-State
370 Visual Evoked Potentials were collected with 128-sensor HydroCell
371 Sensor Nets (Electrical Geodesics, Eugene, OR) and
372 were band-pass filtered from 0.3 to 50 Hz. Raw data were evaluated
373 off line according to a sample-by-sample thresholding
374 procedure to remove noisy sensors that were replaced by the
375 average of the six nearest spatial neighbors. On average, less
376

described in detail elsewhere([10](#)). Briefly, exemplar patterns
377 for each group were generated from random-noise textures
378 which were then repeated and transformed to cover the plane
379 according to the symmetry axes and geometric lattice spe-
380 specific to each group. The use of noise textures as the starting
381 point for stimulus generation allowed the creation of an at-
382 most infinite number of distinct exemplars of each wallpaper
383 group. For each exemplar image, phase-randomized control
384 exemplars were generated that had the same power spectrum
385 as the exemplar images for each group. The phase scrambling
386 eliminates rotation, reflection and glide-reflection symmetries
387 within each exemplar, but the phase-scrambled images inher-
388 ent the spectral periodicity arising from the periodic tilings.
389 This means that all control exemplars, regardless of which
390 wallpaper group they are derived from, degenerate to another
391 symmetry group, namely P1. P1 is the simplest of the wallpa-
392 per groups, and contains only translations of a region whose
393 shape derives from the lattice. Because the different wallpaper
394 groups have different lattices, P1 controls matched to different
395 groups have different power spectra. Our experimental design
396 takes these differences into account by comparing the neural
397 responses evoked by each wallpaper group to responses evoked
398 by the matched control exemplars.

Stimulus Presentation. Stimulus Presentation. For the EEG
399 experiment, the stimuli were shown on a 24.5" Sony Trimaster
400 EL PVM-2541 organic light emitting diode (OLED) display
401 at a screen resolution of 1920×1080 pixels, 8-bit color depth
402 and a refresh rate of 60 Hz, viewed at a distance of 70 cm.
403 The mean luminance was 69.93 cd/m^2 and contrast was 95%.
404 The diameter of the circular aperture in which the wallpaper
405 pattern appeared was 13.8° of visual angle presented against
406 a mean luminance gray background. Stimulus presentation

than 5% of the electrodes were substituted; these electrode³⁸⁸ were mainly located near the forehead or the ears. The substitutions can be expected to have a negligible impact on our³⁹⁰ results, as the majority of our signal can be expected to come³⁹¹ from electrodes over occipital, temporal and parietal cortices³⁹². After this operation, the waveforms were re-referenced to the³⁹³ common average of all the sensors. The data from each 12s³⁹⁴ trial were segmented into five 2.4 s long epochs (i.e., each³⁹⁵ of these epochs was exactly 2 cycles of image modulation³⁹⁶). Epochs for which a large percentage of data samples exceeding³⁹⁷ a noise threshold (depending on the participant and ranging³⁹⁸ between 25 and 50 μ V) were excluded from the analysis on a³⁹⁹ sensor-by-sensor basis. This was typically the case for epochs⁴⁰⁰ containing artifacts, such as blinks or eye movements. The use⁴⁰¹ of steady-state stimulation drives cortical responses at specific⁴⁰² frequencies directly tied to the stimulus frequency. It is thus⁴⁰³ appropriate to quantify these responses in terms of both phase⁴⁰⁴ and amplitude. Therefore, a Fourier analysis was applied on⁴⁰⁵ every remaining epoch using a discrete Fourier transform with⁴⁰⁶ a rectangular window. The use of epochs two-cycles (i.e., 2.4⁴⁰⁷ s) long, was motivated by the need to have a relatively high⁴⁰⁸ resolution in the frequency domain, $\delta f = 0.42$ Hz. For each⁴⁰⁹ frequency bin, the complex-valued Fourier coefficients were⁴¹⁰ then averaged across all epochs within each trial. Each partici⁴¹¹pant did two sessions of 8 trials per condition, which resulted⁴¹² in a total of 16 trials per condition.

SSVEP Analysis. Response waveforms were generated for each group by selective filtering in the frequency domain. For each participant, the average Fourier coefficients from the two sessions were averaged over trials and sessions. The Steady-State Visual Evoked Potentials (SSVEP) paradigm we used allowed us to separate symmetry-related responses from non-specific contrast transient responses. Previous work has demonstrated that symmetry-related responses are predominantly found in the odd harmonics of the stimulus frequency, whereas the even harmonics consist mainly of responses unrelated to symmetry⁴⁷⁰. Try, that arise from the contrast change associated with the appearance of the second image[2-4]. This functional distinction of the harmonics allowed us to generate a single-cycle waveform containing the response specific to symmetry, by filtering out the even harmonics in the spectral domain, and then back-transforming the remaining signal, consisting only of odd harmonics, into the time-domain. For our main analysis⁴⁷¹ we averaged the odd harmonic single-cycle waveforms within a six-electrode region of interest (ROI) over occipital cortex⁴⁷² (electrodes 70, 74, 75, 81, 82, 83). These waveforms, averaged⁴⁷³ over participants, are shown in Figure 2 in the main paper. The same analysis was done for the even harmonics (see Figure S1) and for the odd harmonics within a six electrode ROI over parietal cortex (electrodes 53, 54, 61, 78, 79, 86; see Figure S2). The root-mean square values of these waveforms, for each individual participant, were used to determine whether each of the wallpaper subgroup relationships were preserved in the brain data.

Bayesian Analysis of EEG and Psychophysical data. Bayesian analysis was carried out using R (v3.6.1) (33) with the brms package (v2.9.0) (34) and rStan (v2.19.2 (35)). The data from each experiment were modelled using a Bayesian generalised mixed effect model with wallpaper group being treated as a 16 level factor, and random effects for participant. The EEG

data and display thresholds were modelled using log-normal distributions with weakly informative, $\mathcal{N}(0, 2)$, priors. After fitting the model to the data, samples were drawn from the posterior distribution for each mean of the EEG response (display duration) for each wallpaper group. These samples were then combined to calculate the distribution of differences for each pair of subgroup and super-group. These distributions were then summarised by computing the conditional probability of obtaining a positive (negative) difference, $p(\Delta|data)$.

For further technical details, please see the supplementary materials where the full R code, model specification, prior and posterior predictive checks, and model diagnostics, can be found.

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