

March 10th, 2021

Dear Editors of Proceedings of the Royal Society B,

Symmetries are present at many scales in images of natural scenes, due to a complex interplay of physical forces that govern pattern formation in nature. Symmetrical patterns have been created and appreciated by human cultures throughout history and since the gestalt movement of the early 20th century, symmetry has been recognized as important for visual perception. Here we present a new approach to studying the role of symmetry in vision, using *wallpaper groups*.

The wallpaper groups are regular textures that form 17 distinct combinations of symmetries in the 2D plane, and thus represent the complete set of symmetries in images. We expand on prior work on symmetry in vision and neuroscience by investigating representations of the entire set of wallpaper groups in human visual cortex, using brain imaging (Steady-State Visual Evoked Potentials measured using high-density EEG) and behavioral measurements (symmetry detection thresholds). In accordance with best practices of Open Science, the Supplementary Materials include all of the data and code required to run our analyses, as well as additional helpful figures and tables (see: <https://osf.io/f3ex8/>).

We frame our findings in terms of a series of hierarchical relationships among the wallpaper groups that are based on mathematical group theory. Our main finding is that both behavioral and brain imaging measures are remarkably consistent with group theory, such that groups lower in the hierarchy produce smaller response amplitudes and require longer presentation time to be accurately detected. This is true for the overwhelming majority of the ~60 subgroup relationships and across every symmetry type. These results show that visual cortex have comprehensive representations of symmetries in regular textures and that individual symmetry types embedded in textures are coded with a high degree of precision. Subsequent analyses reveal that differences among groups depend not only on position in the hierarchy, but also on distance between the groups, suggesting that representations of symmetries in textures may be compositional. We also find that our behavioral and brain data are robustly correlated, indicating that the two measurements are likely reflecting the same underlying representations in visual cortex.

The physical forces underlying the structure of objects and their surface properties are subject to fundamental constraints that generate symmetries in the visual environment. Our findings represent an important step towards a more complete understanding of the role of symmetry in natural vision and opens the door to further investigations of the symmetry network. This should include a detailed analysis of the network's response properties and a detailed mapping of the differences and similarities there might be in the mechanisms for encoding symmetry over objects and symmetry in textures. The correspondence between brain responses and group theory that we observe here is likely learned implicitly from regularities in the natural world, suggesting that the well-known principle of *efficient coding* extends to a much higher level of structural redundancy – that of symmetries in visual images. It is interesting to consider whether the aesthetic appeal of symmetry in human cultures is perhaps a direct result of the coding strategy implemented by the visual system.

We look forward to hopefully embarking on a rigorous peer review process under your editorial guidance.

Sincerely,

Peter J. Kohler & Alasdair Clarke

