

The human visual system preserves the hierarchy of 2-dimensional pattern regularity

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Symmetries are present at many scales in images of natural scenes. A large body of literature has demonstrated contributions of symmetry to numerous domains of visual perception. The four fundamental symmetries, reflection, rotation, translation and glide reflection, can be combined in exactly 17 distinct ways. These *wallpaper groups* represent the complete set of symmetries in 2D images. The goal of the current study is to provide a complete description of responses to symmetry in the human visual system, by collecting both brain and behavioral data using all 17 groups. This allows us to probe the hierarchy of complexity among wallpaper groups, in which simpler groups are subgroups of more complex ones. We find that this hierarchy is preserved almost perfectly in both behavior and brain activity: A multi-level Bayesian GLM indicates that for most of the 63 subgroup relationships, subgroups produce lower amplitude responses in visual cortex (posterior probability: >0.95 for 56 of 63) and require longer presentation durations to be reliably detected (posterior probability: >0.95 for 49 of 63). This systematic pattern is seen only in visual cortex and only in components of the brain response known to be symmetric-specific. Our results show that representations of symmetries in the human brain are precise and rich in detail, and that this precision is reflected in behavior. These findings expand our understanding of symmetry perception, and open up new avenues for research on how fine-grained representations of regular textures contribute to natural vision.

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Symmetries are present at many scales in images of natural scenes, due to a complex interplay of physical forces that govern pattern formation in nature. The importance of symmetry for visual perception has been known at least since the gestalt movement of the early 20th century. Since then, symmetry has been shown to contribute to the perception of shapes (1, 3), scenes (4) and surface properties (2), as well as the social process of mate selection (9). Most of this work has focused on mirror symmetry or *reflection*, with much less attention being paid to the other fundamental symmetries: *rotation*, *translation* and *glide reflection*. In the two spatial dimensions relevant for images, these four fundamental symmetries can be combined in 17 distinct ways, the *wallpaper groups* (5–7). Previous work on a subset of the wallpaper groups used functional MRI to demonstrate that rotation symmetries in wallpapers elicit parametric responses in several areas in occipital cortex, beginning with visual area V3 (10). This effect was also robust with electroencephalography (EEG), whether measured using Steady-State Visual Evoked Potentials (SSVEPs) (10) or event-related paradigms (11). Here we extend this work by collecting SSVEPs and psychophysical data from human participants viewing the full set of wallpaper groups. We measure responses in visual cortex to 16 out of the 17 wallpaper groups, with the 17th serving as a control

stimulus. Our goal is to provide a more complete picture of how wallpaper groups are represented in the human visual system.

A wallpaper group is a topologically discrete group of isometries of the Euclidean plane, i.e. transformations that preserve distance (7). Wallpaper groups differ in the number and kind of these transformations. In mathematical group theory, when the elements of one group are completely contained in another, the inner group is called a subgroup of the outer group (7). The full list of subgroup relationships is listed in Section 1.4.2 of the Supplementary Material. Subgroup relationships between wallpaper groups can be distinguished by their indices. The index of a subgroup relationship is the number of cosets, i.e. the number of times the subgroup is found in the supergroup (7). As an example, let us consider groups P_2 and P_6 . If we ignore the translations in two directions that both groups share, group P_6 consists of the set of rotations $\{0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ\}$, in which $P_2 \{0^\circ, 180^\circ\}$ is contained. P_2 is thus a subgroup of P_6 , and P_6 can be generated by combining P_2 with rotations $\{0^\circ, 120^\circ, 240^\circ\}$. Because P_2 is repeated three times in P_6 , P_2 is a subgroup of P_6 with index 3 (7). The 17 wallpaper groups thus obey a hierarchy of complexity where simpler groups are subgroups of more complex ones (8).

The two datasets we present here make it possible to assess the extent to which both behavior and brain responses follow the hierarchy of complexity expressed by the subgroup relationships. Based on previous brain imaging work showing that patterns with more axes of symmetry produce greater activity in visual cortex (10–13), we hypothesized that more complex groups would produce larger SSVEPs. For the psychophysical

Significance Statement

Wallpaper groups were discovered in the mid-19th century, and the 17 groups constitute the complete set of possible ways of regularly tiling the 2D-plane. In recent years wallpaper groups have found use in the vision science community, as an ideal stimulus set for studying the perception of symmetries in textures. Here we present brain imaging and psychophysical data using the complete set of wallpaper groups and show that the hierarchical organization among wallpaper groups is reflected both in representations in visual cortex and performance on a symmetry detection task. This shows that the visual system is highly sensitive to regularities in textures, and suggest that symmetries may play an important role in texture perception.

PJK and ADFC designed the study, PJK collected EEG data, ADFC collected psychophysical data, PJK and ADFC wrote the paper.

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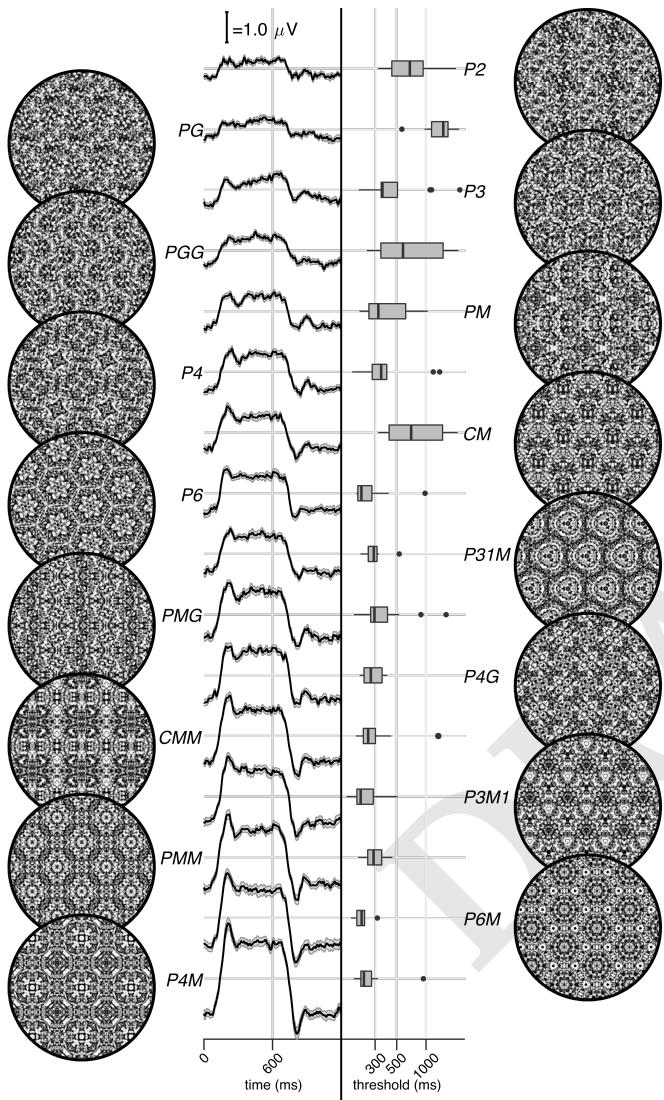


Fig. 1. Examples of each of the 16 wallpaper groups are shown in the left- and right-most column of the figures, next to the corresponding SSVEP (center-left) and psychological (center-right) data from each group. The SSVEP data are odd-harmonic-filtered cycle-average waveforms. In each cycle, a P_1 exemplar was shown for the first 600 ms, followed by the original exemplar for the last 600 ms. Errorbars are standard error of the mean. Psychophysical data are presented as boxplots reflecting the distribution of display duration thresholds. The 16 groups are ordered by the strength of the SSVEP response, to highlight the range of response amplitudes.

data, we hypothesized that more complex groups would lead to shorter symmetry detection thresholds, based on previous data showing that under a fixed presentation time, discriminability increases with the number of symmetry axes in the pattern (14). Our results confirm both hypotheses, and show that activity in human visual cortex is remarkably consistent with the hierarchical relationships between the wallpaper groups, with SSVEP amplitudes and psychophysical thresholds following these relationships at a level that is far beyond chance. The human visual system thus appears to encode all of the fundamental symmetries using a representational structure that closely approximates the subgroup relationships from group theory.

Results

The stimuli used in our two experiments were generated from random-noise textures, which made it possible to generate multiple exemplars from each of the wallpaper groups, as described in detail elsewhere (10). We generated control stimuli matched to each exemplar in the main stimulus set, by scrambling the phase but maintaining the power spectrum. All wallpaper groups are inherently periodic because of their repeating lattice structure. Phase scrambling maintains this periodicity so the phase-scrambled control images all belong to group P_1 regardless of group membership of the original exemplar. P_1 contains no symmetries other than translation, while all other groups contain translation in combination with one or more of the other three fundamental symmetries (reflection, rotation, glide reflection) (7). In our SSVEP experiment, this stimulus set allowed us to isolate brain activity specific to the symmetry structure in the exemplar images from activity associated with modulation of low-level features, by alternating exemplar images and control exemplars. In this design, responses to structural features beyond the shared power spectrum, including any symmetries other than translation, are isolated in the odd harmonics of the image update frequency (10, 15, 16). Thus, the combined magnitude of the odd harmonic response components can be used as a measure of the overall strength of the visual cortex response.

The psychophysical experiment took a distinct but related approach. In each trial an exemplar image was shown with its matched control, one image after the other, and the order varied pseudo-randomly such that in half the trials the original exemplar was shown first, and in the other half the control image was shown first. After each trial, participants were told to indicate whether the first or second image contained more structure, and the duration of both images was controlled by a staircase procedure so that a threshold duration for symmetry detection could be computed for each wallpaper group.

Examples of the wallpaper groups and a summary of our brain imaging and psychophysical measurements is in Figure 1. For our primary SSVEP analysis, we only considered EEG data from a pre-determined region-of-interest (ROI) consisting of six electrodes over occipital cortex (see Supplementary Figure 1.1). SSVEP data from this ROI was filtered so that only the odd harmonics that capture the symmetry response contribute to the waveforms. While waveform amplitude is quite variable among the 16 groups, all groups have a sustained negative-going response that begins at about the same time for all groups, 180 ms after the transition from the P_1 control exemplar. To reduce the amplitude of

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the symmetry-specific response to a single number that could be used in further analyses and compared to the psychophysical data, we computed the root-mean-square (RMS) over the odd-harmonic-filtered waveforms. The data in Figure 1 are shown in descending order according to RMS. The psychophysical results, shown in box plots in Figure 1, were also quite variable between groups, and there seems to be a general pattern where wallpaper groups near the top of the figure, that have lower SSVEP amplitudes, also have longer psychophysical threshold durations.

We now wanted to test our two hypotheses about how SSVEP amplitudes and threshold durations would follow subgroup relationships, and thereby quantify the degree to which our two measurements were consistent with the group theoretical hierarchy of complexity. We tested each hypothesis using the same approach. We first fitted a Bayesian model with wallpaper group as a factor and participant as a random effect. We fit the model separately for SSVEP RMS and psychophysical data, and then computed posterior distributions for the difference between supergroup and subgroup. These difference distributions allowed us to compute the conditional probability that the supergroup would produce (a) larger RMS and (b) a shorter threshold durations, when compared to the subgroup. The posterior distributions are shown in Figure 2 for the SSVEP data, and in Figure 3 for the psychophysical data, which distributions color-coded according to conditional probability. For both data sets our hypothesis is confirmed: For the overwhelming majority of the 63 subgroup relationships, supergroups are more likely to produce larger symmetry-specific SSVEPs and shorter symmetry detection threshold durations, and in most cases the conditional probability of this happening is extremely high.

We also ran a control analysis using (1) odd-harmonic SSVEP data from a six-electrode ROI over parietal cortex (see Supplementary Figure 1.1) and (2) even-harmonic SSVEP data from the same occipital ROI that was used in our primary analysis. By comparing these two control analysis to our primary SSVEP analysis, we can address the specify of our effects in terms of location (occipital cortex vs parietal cortex) and harmonic (odd vs even). For both control analyses (plotted in Supplementary Figures 3.3 and 3.4), the correspondence between data and subgroup relationships was substantially weaker than in the primary analysis. We can quantify the strength of the association between the data and the subgroup relationships, by asking what proportion of subgroup relationships that reach or exceed a range of probability thresholds. This is plotted in Figure 4, for our psychophysical data, our primary SSVEP analysis and our two control SSVEP analyses. It shows that odd-harmonic SSVEP data from the occipital ROI and symmetry detection threshold durations both have a strong association with the subgroup relationships such that a clear majority of the subgroups survive even at the highest threshold we consider ($p(\Delta > 0 | data) > 0.99$). The association is far weaker for the two control analyses.

SSVEP data from four of the wallpaper groups (P_2 , P_3 , P_4 and P_6) was previously published as part of our earlier demonstration of parametric responses to rotation symmetry in wallpaper groups(10). We replicate that result using our Bayesian approach, and find an analogous parametric effect in the psychophysical data (see Supplementary Figure 4.1). We also conducted an analysis testing for an effect of index in

our two datasets, and found that subgroup relationships with higher indices tended to produce greater pairwise differences between the subgroup and supergroup, for both SSVEP RMS and symmetry detection thresholds (see Supplementary Figure 4.2). The effect of index is relatively weak, but the fact that there is a measurable index effect can nonetheless be taken as preliminary evidence that representations of symmetries in wallpaper groups may be compositional.

Finally, we conducted a correlation analysis comparing SSVEP and psychophysical data, and found a modest ($R^2 = 0.44$) but above-zero correlation. The correlation between the two datasets is interesting because it suggests that our psychophysical and SSVEP measurements are at least to some extent tapping into the same underlying mechanisms.

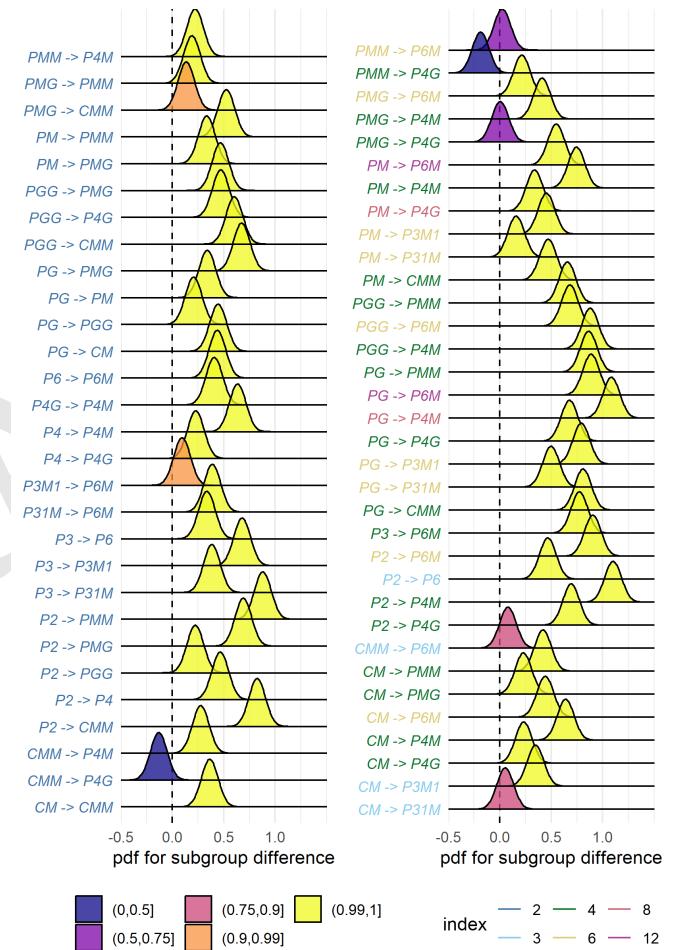


Fig. 2. Posterior distributions for the difference in mean SSVEP RMS amplitude. Colour coding of the text indicates the index of the subgroup, while the colour of the filled distribution relates to the conditional probability that the difference in means is greater than zero. We can see that 56/63 subgroup relationships have $p(\Delta | data) > 0.95$.

Discussion

Here we show that beyond merely responding to the elementary symmetry operations of reflection (12) and rotation (10), the visual system explicitly represents hierarchical structure of the 17 wallpaper groups, and thus every composition of the four fundamental symmetries (rotation, reflection, translation,

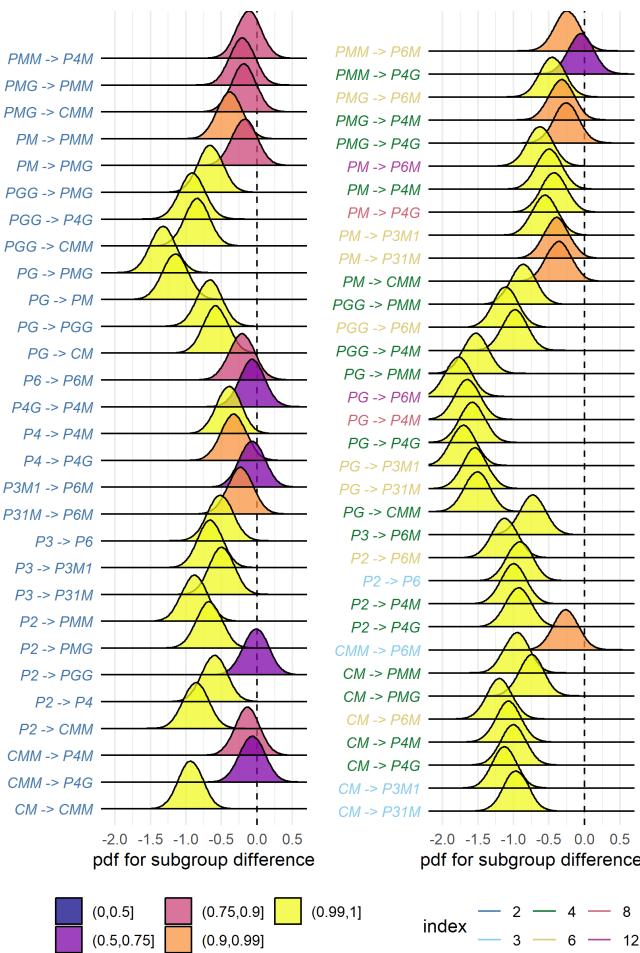


Fig. 3. Posterior distributions for the difference in mean symmetry detection threshold durations. Colour coding of the text indicates the index of the subgroup, while the colour of the filled distribution relates to the conditional probability that the difference in means is greater than zero. We can see that 49/63 subgroup relationships have $p(\Delta|data) > 0.95$.

ceptually, and most participants had no knowledge of group theory. Thus, the visual system's ability to preserve the subgroup hierarchy does not depend on explicit knowledge of the relationships. Previous behavioral experiments have shown that although naïve observers can distinguish many of the wallpaper groups (17), they tend to sort them into fewer groups than there actually are (4–12 groups) and it is common for exemplars from different wallpaper groups to be sorted in the same group (18). The more controlled two-interval forced choice approach used in the current behavioral experiment allows us to show that more granular representations of wallpaper groups are measurable in behavior. The relatively modest correlation we observe between behavior and brain imaging data may suggest that the relationship between the two measurements is more complex than what can be captured in our summary statistics. Future work in which the same individuals participate in both behavioral and brain imaging experiments will help tease apart these complexities and enhance our understanding of the brain mechanisms that drive both measurements.

We also find an index of our brain imaging measurements and our symmetry detection thresholds. This means that the visual system not only represents the hierarchical relationship captured by individual subgroups, but also distinguishes between subgroups depending on how many times the subgroup is repeated in the supergroup, with more repetitions leading to larger pairwise differences. Our measured effect of index is relatively weak. This is perhaps because the index analysis does not take into account the type of isometries that differentiate the subgroup and supergroup. The effect of symmetry type can be observed by contrasting the measured SSVEP amplitudes and detection thresholds for groups *PM* and *PG* in Figure 1. The two groups are comparable except *PM* contains reflection and *PG* contains glide reflection, and the former clearly elicits higher amplitudes and lower thresholds. The charting of how different symmetry types contribute to the representational hierarchy will be an important goal for future work.

The correspondence between responses in the visual system and group theory that we demonstrate here, may reflect a form of implicit learning that depends on the structure of the natural world. The environment is itself constrained by physical forces underlying pattern formation and these forces are subject to multiple symmetry constraints (19). The ordered structure of responses to wallpaper groups could be driven by a central tenet of neural coding, that of efficiency. If coding is to be efficient, neural resources should be distributed to capture the structure of the environment with minimum redundancy considering the visual geometric optics, the capabilities of the subsequent neural coding stages and the behavioral goals of the organism (20–23). Early work within the efficient coding framework suggested that natural images had a $1/f$ spectrum and that the corresponding redundancy between pixels in natural images could be coded efficiently with a sparse set of oriented filter responses, such as those present in the early visual pathway (24, 25). Our results suggest that the principle of efficient coding extends to a much higher level of structural redundancy – that of symmetries in visual images.

The 17 wallpaper groups are completely regular, and relatively rare in the visual environment, especially when considering distortions due to perspective and occlusion. Near-regular

glide reflection) which comprise the set of regular textures. Both SSVEP amplitudes and symmetry detection thresholds preserve the hierarchy of complexity among the wallpaper groups that is captured by the subgroup relationships (8). For the SSVEP, this remarkable consistency was specific to the odd harmonics of the stimulus frequency that are known to capture the symmetry-specific response (10) and to electrodes in a region-of-interest (ROI) over occipital cortex. When the same analysis was done using the odd harmonics from electrodes over parietal cortex (Supplementary Figure 3.3) or even harmonics from electrodes over occipital cortex (Supplementary Figure 3.4), the data was substantially less consistent with the subgroup relationships (yellow and green lines, Figure 4).

The current data provide a complete description of the visual system's response to symmetries in the 2-D plane. Our design precludes us from independently measure the response to *P1*, but because each of the 16 other groups produce non-zero odd harmonic amplitudes (see Figure 1), we can conclude that the relationships between *P1* and all other groups, where *P1* is the subgroup, are also preserved by the visual system. The subgroup relationships are in many cases not obvious per-

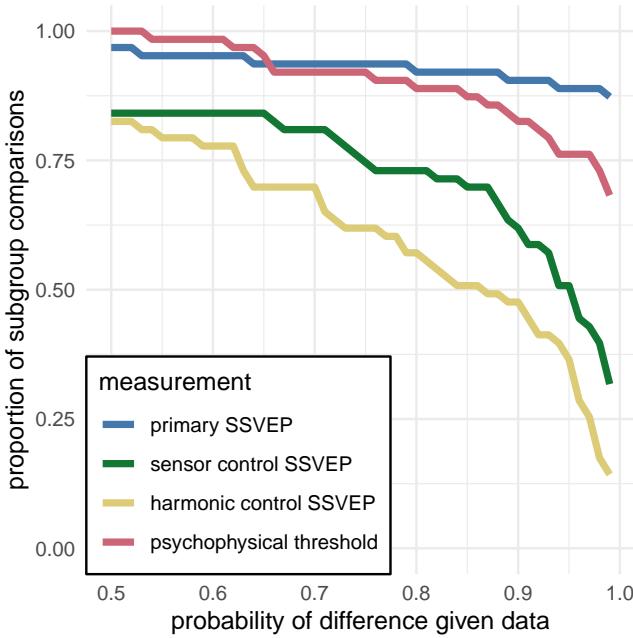


Fig. 4. This plot shows the proportion of subgroup relationships that satisfy $p(\Delta > 0 | \text{data}) > x$. We can see that if we take $x = 0.95$ as our threshold, the subgroup relationships are preserved in $56/63 = 89\%$ and $49/64 = 78\%$ of the comparisons for the primary SSVEP and threshold duration datasets, respectively. This compares to the $32/64 = 50\%$ and $22/64 = 35\%$ for the SSVEP control datasets.

protocol that was approved by the University of Essex's Ethics Committee.

Stimulus Generation. Exemplars from the different wallpaper groups were generated using a modified version of the methodology developed by Clarke and colleagues(18) that we have described in detail elsewhere(10). Briefly, exemplar patterns for each group were generated from random-noise textures, which were then repeated and transformed to cover the plane, according to the symmetry axes and geometric lattice specific to each group. The use of noise textures as the starting point for stimulus generation allowed the creation of an almost infinite number of distinct exemplars of each wallpaper group. To make individual exemplars as similar as possible we replaced the power spectrum of each exemplar with the median across exemplars within a group. We then generated control exemplars that had the same power spectrum as the exemplar images by randomizing the phase of each exemplar image. The phase scrambling eliminates rotation, reflection and glide-reflection symmetries within each exemplar, but the phase-scrambled images inherit the spectral periodicity arising from the periodic tiling. This means that all control exemplars, regardless of which wallpaper group they are derived from, are transformed into another symmetry group, namely $P1$. $P1$ is the simplest of the wallpaper groups, and contains only translations of a region whose shape derives from the lattice. Because the different wallpaper groups have different lattices, $P1$ controls matched to different groups have different power spectra. Our experimental design takes these differences into account by comparing the neural responses evoked by each wallpaper group to responses evoked by the matched control exemplars.

Stimulus Presentation. Stimulus Presentation. For the EEG experiment, the stimuli were shown on a 24.5" Sony Trimaster EL PVM-2541 organic light emitting diode (OLED) display at a screen resolution of 1920×1080 pixels, 8-bit color depth and a refresh rate of 60 Hz, viewed at a distance of 70 cm. The mean luminance was 69.93 cd/m^2 and contrast was 95%. The diameter of the circular aperture in which the wallpaper pattern appeared was 13.8° of visual angle presented against a mean luminance gray background. Stimulus presentation was controlled using in-house software. For the psychophysics experiment, the stimuli were shown on a 48 × 27cm VIEWPixx/3D LCD Display monitor, model VPX-VPX-2005C, resolution 1920×1080 pixels, with a viewing distance of approximately 40cm and linear gamma. Stimulus presentation was controlled using MatLab and Psychtoolbox-3 (32, 33). The diameter of the circular aperture for the stimuli was 21.5° .

EEG Procedure. Visual Evoked Potentials were measured using a steady-state design, in which $P1$ control images alternated with exemplar images from each of the 16 other wallpaper groups. Exemplar images were always preceded by their matched $P1$ control image. A single 0.83 Hz stimulus cycle consisted of a control $P1$ image followed by an exemplar image, each shown for 600 ms. A trial consisted of 10 such cycles (12 sec) over which 10 different exemplar images and matched controls from the same rotation group were presented. For each group type, the individual exemplar images were always shown in the same order within the trials. Participants initiated each trial with a button-press, which allowed them

textures, however, abound in the visual world, and can be approximated as deformed versions of the wallpaper groups (26). The correspondence between visual cortex responses and group theory demonstrated here may indicate that the visual system represents visual textures using a similar scheme, with the wallpaper groups serving as anchor points in representational space. This framework resembles norm-based encoding strategies that have been proposed for other stimulus classes, most notably faces (27), and leads to the prediction that adaptation to wallpaper patterns should distort perception of near-regular textures, similar to the aftereffects found for faces (28). Field biologists have demonstrated that animals respond more strongly to exaggerated versions of a learned stimulus, referred to as "supernormal" stimuli (29). In the norm-based encoding framework, wallpaper groups can be considered *supertextures*, exaggerated examples of the near-regular textures that surround us. Artists may consciously or unconsciously create supernormal stimuli, to capture the essence of the subject and evoke strong responses in the audience (30). Wallpaper groups are visually compelling and have been widely used in human artistic expression going back to the Neolithic age (31). If wallpapers are in fact supertextures, this prevalence may be a direct result of the strategy the human visual system has adopted for texture encoding.

Participants. Twenty-five participants (11 females, mean age 28.7 ± 13.3) took part in the EEG experiment. Their informed consent was obtained before the experiment under a protocol that was approved by the Institutional Review Board of Stanford University. 11 participants (8 females, mean age 20.73 ± 1.21) took part in the psychophysics experiment. All participants had normal or corrected-to-normal vision. Their informed consent was obtained before the experiment under a

370 to take breaks between trials. Trials from a single wallpaper
371 group were presented in blocks of four repetitions, which were
372 themselves repeated twice per session, and shown in random
373 order within each session. To control fixation, the participants
374 were instructed to fixate a small white cross in the center of
375 display. To control vigilance, a contrast dimming task was
376 employed. Two times per trial, an image pair was shown
377 at reduced contrast, and the participants were instructed to
378 press a button on a response pad. We adjusted the contrast
379 reduction such that average accuracy for each participant was
380 kept at 85% correct, in order to keep the difficulty of the
381 vigilance at a constant level.

382 **Psychophysics Procedure.** The experiment consisted of 16
383 blocks, one for each of the wallpaper groups (excluding *P1*).
384 We used a two-interval forced choice approach. In each trial,
385 participants were presented with two stimuli (one of which
386 was the wallpaper group for the current block of trials, the
387 other being *P1*), one after the other (inter-stimulus interval
388 of 700ms). After each stimuli had been presented, it was
389 masked with white noise for 300ms. After both stimuli had
390 been presented, participants made a response on the key-
391 board to indicate whether they thought the first or second
392 image contained more symmetry. Each block started with 10
393 practice trials, (stimulus display duration of 500ms) to allow
394 participants to familiarise themselves with the current block's
395 wallpaper pattern. If they achieved an accuracy of 9/10 in
396 these trials they progressed to the rest of the block, otherwise
397 they carried out another set of 10 practise trials. This process
398 was repeated until the required accuracy of 9/10 was obtained.
399 The rest of the block consisted of four interleaved staircases
400 (using the QUEST algorithm (34), full details given in the SI)
401 of 30 trials each. On average, a block of trials took around 10
402 minutes to complete.

403 **EEG Acquisition and Preprocessing.** Steady-State Visual
404 Evoked Potentials (SSVEPs) were collected with 128-sensor
405 HydroCell Sensor Nets (Electrical Geodesics, Eugene, OR) and
406 were band-pass filtered from 0.3 to 50 Hz. Raw data were eval-
407 uated off line according to a sample-by-sample thresholding
408 procedure to remove noisy sensors that were replaced by the
409 average of the six nearest spatial neighbors. On average, less
410 than 5% of the electrodes were substituted; these electrodes
411 were mainly located near the forehead or the ears. The sub-
412 stitutions can be expected to have a negligible impact on our
413 results, as the majority of our signal can be expected to come
414 from electrodes over occipital, temporal and parietal cortices.
415 After this operation, the waveforms were re-referenced to the
416 common average of all the sensors. The data from each 12s
417 trial were segmented into five 2.4 s long epochs (i.e., each
418 of these epochs was exactly 2 cycles of image modulation).
419 Epochs for which a large percentage of data samples exceeding
420 a noise threshold (depending on the participant and rang-
421 ing between 25 and 50 μ V) were excluded from the analysis
422 on a sensor-by-sensor basis. This was typically the case for
423 epochs containing artifacts, such as blinks or eye movements.
424 Steady-state stimulation will drive cortical responses at spe-
425 cific frequencies directly tied to the stimulus frequency. It
426 is thus appropriate to quantify these responses in terms of
427 both phase and amplitude. Therefore, a Fourier analysis was
428 applied on every remaining epoch using a discrete Fourier
429 transform with a rectangular window. The use of two-cycle

430 long epochs (i.e., 2.4 s) was motivated by the need to have a
431 relatively high resolution in the frequency domain, $\delta f = 0.42$
432 Hz. For each frequency bin, the complex-valued Fourier coeffi-
433 cients were then averaged across all epochs within each trial.
434 Each participant did two sessions of 8 trials per condition,
435 which resulted in a total of 16 trials per condition.

436 **SSVEP Analysis.** Response waveforms were generated for each
437 group by selective filtering in the frequency domain. For each
438 participant, the average Fourier coefficients from the two ses-
439 sions were averaged over trials and sessions. The SSVEP
440 paradigm we used allowed us to separate symmetry-related
441 responses from non-specific contrast transient responses. Pre-
442 vious work has demonstrated that symmetry-related responses
443 are predominantly found in the odd harmonics of the stimu-
444 lus frequency, whereas the even harmonics consist mainly of
445 responses unrelated to symmetry, that arise from the contrast
446 change associated with the appearance of the second image
447 (10, 16). This functional distinction of the harmonics allowed
448 us to generate a single-cycle waveform containing the response
449 specific to symmetry, by filtering out the even harmonics in the
450 spectral domain, and then back-transforming the remaining
451 signal, consisting only of odd harmonics, into the time-domain.
452 For our main analysis, we averaged the odd harmonic single-
453 cycle waveforms within a six-electrode region of interest (ROI)
454 over occipital cortex (electrodes 70, 74, 75, 81, 82, 83). These
455 waveforms, averaged over participants, are shown in Figure 1.
456 The same analysis was done for the even harmonics and for the
457 odd harmonics within a six electrode ROI over parietal cortex
458 (electrodes 53, 54, 61, 78, 79, 86; see Supplementary Figure
459 1.1). The root-mean square values of these waveforms, for
460 each individual participant, were used to determine whether
461 each of the wallpaper subgroup relationships were preserved
462 in the brain data.

463 **Defining the list of subgroup relationships.** In order to get the
464 complete list of subgroup relationships, we digitized Table 4
465 from Coxeter (8) (shown in Supplementary Table 1.2). After
466 removing identity relationships (i.e. each group is a subgroup
467 of itself) and the three pairs of wallpapers groups that are
468 subgroups of each other (e.g. *PM* is a subgroup of *CM*, and
469 *CM* is a subgroup of *PM*) we were left with a total of 63
470 unambiguous subgroups that were included in our analysis.

471 **Bayesian Analysis of SSVEP and Psychophysical data.** 472 Bayesian analysis was carried out using R (v3.6.1) (35) with
473 the *brms* package (v2.9.0) (36) and rStan (v2.19.2 (37)). The
474 data from each experiment were modelled using a Bayesian
475 generalised mixed effect model with wallpaper group being
476 treated as a 16 level factor, and random effects for participant.
477 The SSVEP data and symmetry detection threshold durations
478 were modelled using log-normal distributions with weakly in-
479 formative, $\mathcal{N}(0, 2)$, priors. After fitting the model to the data,
480 samples were drawn from the posterior distribution of the
481 two datasets, for each wallpaper group. These samples were
482 then recombined to calculate the distribution of differences
483 for each of the 63 pairs of subgroup and supergroup. These
484 distributions were then summarised by computing the condi-
485 tional probability of obtaining a positive (negative) difference,
486 $p(\Delta|data)$. For further technical details, please see the supple-
487 mentary materials where the full R code, model specification,

488 prior and posterior predictive checks, and model diagnoastics,
489 can be found.

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