

# Bayesian multi-level modelling for predicting single and double feature visual search

Anna E. Hughes<sup>a</sup>, Anna Nowakowska<sup>b</sup>, Alasdair D. F. Clarke<sup>1</sup>

<sup>a</sup>*Department of Psychology, University of Essex, Colchester, CO4 3SQ, UK*

<sup>b</sup>*School of Psychology, University of Aberdeen, Aberdeen, AB24 3FX, UK*

---

## Abstract

Performance in visual search tasks is frequently summarised by “search slopes” - the additional cost in reaction time for each additional distractor. While search tasks with a shallow search slopes are termed efficient (pop-out, parallel, feature), there is no clear dichotomy between efficient and inefficient (serial, conjunction) search. Indeed, a range of search slopes are observed in empirical data. The Target Contrast Signal (TCS) Theory is a rare example of quantitative model that attempts to predict search slopes for efficient visual search. One study using the TCS framework has shown that the search slope in a double-feature search (where the target differs in both colour and shape from the distractors) can be estimated from the slopes of the associated single-feature searches. This estimation is done using a contrast combination model, and a collinear contrast integration model was shown to outperform other options. In our work, we extend TCS to a Bayesian multi-level framework. We investigate modelling using normal and shifted-lognormal distributions, and show that the latter allows for a better fit to previously published data. We run a new fully within-subjects experiment to attempt to replicate the key original findings, and show that overall, TCS does a good job of predicting the data. However, we do not replicate the finding that the collinear combination model outperforms the other contrast combination models, instead finding that it may be difficult to conclusively distinguish between them.

**Keywords:** Visual search, Efficient search, Parallel processing

---

## 1. Introduction

Visual search, where participants are asked to find a target within a cluttered scene, has been extensively studied within psychology. Several models have been

4 developed that can generate testable predictions about how different types of dis-  
5 tractors and targets affect search efficiency. One of the key distinctions in the field  
6 has been between efficient (also referred to as parallel or pop-out) and inefficient  
7 (serial) search. These are often studied in the context of the regression slope be-  
8 tween the number of distractors and mean reaction time, which has been termed  
9 the *search slope*. When the search slope is shallow (usually positive, but occasion-  
10 ally negative e.g. (Rangelov et al., 2017)), the search is called efficient or parallel,  
11 and the addition of more non-target distractors has little impact on an observers  
12 difficulty in finding a target. When the slope is steeper, each additional distrac-  
13 tor has a noticeable impact on increasing difficulty, and the search is described  
14 as inefficient or serial. However, the distinction between these types of search is  
15 often less clear in real experimental data, with a range of different search slopes  
16 being seen for different types of targets and distractors (Duncan and Humphreys,  
17 1989; Cave and Wolfe, 1990; Wolfe, 1998; Liesefeld et al., 2016). Recent work  
18 has also attempted to model the variation in search slopes at the boundary between  
19 inefficient and efficient search (Liesefeld et al., 2016).

20 In the current study, we are interested in what has traditionally been termed  
21 efficient or parallel search, and the factors that affect search slope in these condi-  
22 tions. Recent work has suggested that for efficient search, there is a logarithmic  
23 relationship between distractor set size and reaction time, and that this relation-  
24 ship can be modified by target-distractor similarity (Buetti et al., 2016), providing  
25 evidence that search behaviour in parallel search is more complex than has pre-  
26 viously been assumed. This observation has formed the basis of the ‘Target Con-  
27 trast Signal (TCS) Theory’ (Lleras et al., 2020), which aims to provide a means  
28 of predicting observer search slopes for new search arrays by quantifying target-  
29 distractor differences. For example, by measuring search slopes for conditions in  
30 which the distractors differ from the target along a *single feature* (e.g. colour *or*  
31 shape), it has been shown that you can predict search times for arrays in which  
32 the target differs from the distractors along two features (e.g., colour *and* shape)  
33 which we refer to here as *double feature* search (Buetti et al., 2019) (but simi-  
34 lar paradigms have been known by other names e.g. ‘redundant feature search’  
35 (Krummenacher and Müller, 2012; Mordkoff and Yantis, 1991)). Here, we aim  
36 to replicate and extend this work both theoretically and empirically, to test the  
37 generalisability of the TCS model, and to suggest ways in which the TCS model  
38 could be modified to generate better predictions.

### 1.1. Previous Work

Many different forms of visual search models have been proposed. One well developed class of models are the saliency models, which aim to predict eye movements during scene viewing, including visual search. They rest on the assumption that fixations are directed to objects or locations that are most dissimilar to the background or other objects in the visual display (Itti and Koch, 2000; Itti et al., 1998; Koch and Ullman, 1987). While the original saliency model was able to predict fixation allocation in a visual search task above chance (Parkhurst et al., 2002), further research demonstrated that a comparable level of performance could be achieved using a simple central fixation bias heuristic (Tatler, 2007). The saliency models have since been extended and improved (see for example Zhang et al. (2008)): however, the main issue with this family of models remains their limited usability in complex real-life search arrays (Tatler et al., 2011; Koehler et al., 2014), and even in abstract laboratory search arrays (Kotseruba et al., 2020). In addition, in most instances of visual search, the target is clearly defined (i.e. the goal is to find a specific object) and inspecting the most salient areas of the display may in these cases be inefficient. Finally, by focusing on eye movements, these models do not necessarily provide a theoretical framework for the cognitive processes underlying visual search.

Perhaps the most established class of models of visual search are based around Feature Integration Theory (Treisman and Gelade, 1980), which has been modified and extended by Wolfe and colleagues in the Guided Search Model (Wolfe et al., 1989; Wolfe, 2014). These theories have been developed using data from visual search tasks with discrete sets of abstract items. These models combine top-down influences (how closely an item resembles the observer’s goal) with bottom-up image properties. For example, if one’s goal (top-down processing) is to find a red horizontal bar, all the red and horizontal items in a visual search display will be given greater weight than distractors (e.g. vertical and blue items) in the model. The salience of a given object in the display (how distinctive it is from the surrounding objects) also activates bottom-up processing. For instance, a blue item among red items is ranked higher than red among orange items. In such cases, a salient item can capture attention even without resembling the target. Combining bottom-up and top-down sources of activation generates an activation map which generates a prediction of the order in which stimuli are processed in visual search. Other extensions to these models have been proposed, such as the Dimension Weighting Account, in which saliency weightings are assigned to different target ‘dimensions’ (e.g. colour or shape), helping to explain results where varying the target dimension within blocks of trials leads to longer reaction times

77 than where the dimension remains consistent within a block (Krummenacher and  
78 Müller, 2012). Thus, these models aim to produce a representation of the visual  
79 properties of the distractors at each location in the visual field. However, these  
80 are predominantly qualitative models, and thus it is difficult to use them to make  
81 specific quantitative predictions.

82 TCS falls under a class of models that take a different approach, in that they  
83 focus solely on representing the difference between targets and distractors. For  
84 example, in work on eye movement patterns, it has been proposed that perfor-  
85 mance in inefficient (serial) visual search is mostly determined by the size of the  
86 ‘functional viewing field’, whose size varies as a function of target-distractor sim-  
87 ilarity (Hulleman and Olivers, 2017). Similarly, work on attention has proposed  
88 the notion of ‘relative features’, where attention is tuned to feature relationships  
89 i.e. the appearance of the target relative to distractors in the environment (Becker  
90 et al., 2014; Becker, 2010). TCS also has features in common with other models  
91 that propose parallel identification of all items in a scene, with diffusion based  
92 mechanisms for identifying targets from distractors (Moran et al., 2013, 2016).  
93 However, TCS (Lleras et al., 2020) aims to provide a unifying framework that can  
94 make quantitative behavioural predictions for visual search based on this general  
95 assumption. As such, it is an attractive candidate model for a formal registered  
96 replication.

97 A key assumption of the TCS model is that behaviour is determined by com-  
98 paring the target template (held in memory) with every element present in the  
99 scene in parallel. This allows the visual system to reject peripheral non-targets  
100 quickly; the speed at which items are evaluated is determined by how different the  
101 item is from the template through an evidence accumulation process (formally,  
102 the slope of the logarithmic function is assumed to be inversely proportional to the  
103 overall magnitude of the contrast signal between the target and distractor). The  
104 model thus focuses on an initial, efficient processing stage of search; if sufficient  
105 evidence is not accumulated during this process, the model posits that a second  
106 stage is entered, requiring a sequence of eye movements to search for the target  
107 in a serial manner. TCS has been successful in predicting a number of empirical  
108 results, including search performance in heterogeneous scenes based on param-  
109 eters estimated in homogeneous scenes, both with artificial stimuli (Buetti et al.,  
110 2016; Lleras et al., 2019) and with real-world objects visualised on a computer  
111 display (Wang et al., 2017). Table 1 provides an overview of studies investigating  
112 the TCS framework to date.

113 The original version of the TCS model is essentially a (natural) log-linear  
114 model in the number of distractors. The full model contains a variable  $L$ , which

115 represents the number of different types of distractors present in the display. How-  
 116 ever, in our paper, we will follow Buetti et al. (2019) and only consider the specific  
 117 case of  $L = 1$ , of a target among a homogeneous set of distractors. In this case,  
 118 the TCS model can be represented in the following way:

$$\hat{RT} = a + D \log(N_T + 1) \quad (1)$$

119 The intercept,  $a$ , corresponds to search arrays in which only the target is  
 120 present and there are no distractors.  $N_T$  is the total number of distractors.

## 121 1.2. Rationale for proposed work

122 While many aspects of the TCS framework have been tested, with extremely  
 123 promising results, there remains a great deal of scope for verification of some of  
 124 the key findings to date, and extensions of aspects of the model. In all implementa-  
 125 tions of TCS so far, predictions of search efficiency (e.g. in heterogeneous scenes)  
 126 have been made on the average of a group of participants, using data from a dif-  
 127 ferent group performing a different task (e.g. searching in homogeneous scenes).  
 128 Thus, we know that TCS can replicate group-level averages between subjects in  
 129 search well, but we do not know to what extent it is also able to make predictions  
 130 at the individual level. This is particularly important given that conclusions based  
 131 on aggregate data can be different from those that take individual differences into  
 132 account; in one study where participants searched for a target in an array of ran-  
 133 domly oriented line segments, aggregating the data suggested that participants  
 134 were using a stochastic search model (Nowakowska et al., 2017). However, when  
 135 considering each participant individually, it became clear that there was a high  
 136 level of heterogeneity in responses, with some participants performing close to  
 137 optimally, and others actually performing worse than chance (Nowakowska et al.,  
 138 2017). Similarly striking variability has also been reported in other search studies  
 139 (Irons and Leber, 2016, 2018; Clarke et al., 2022a).

140 Taking search time distributions into account is also important for constrain-  
 141 ing theories of visual search (Wolfe et al., 2010; Liesefeld and Müller, 2020): for  
 142 example, they have been used to help distinguish between models that make sim-  
 143 ilar predictions at the level of average reaction times (Moran et al., 2016, 2017).  
 144 Including subject and trial level data into our implementation of the TCS will  
 145 therefore further aid model development and assumption testing.

146 We also extend the TCS model into a Bayesian framework, where we begin  
 147 with existing 'prior' beliefs that are updated with data to give 'posterior' beliefs  
 148 that can be used for inference (McElreath, 2020). We think this has a number

Reference	Overview
Buetti et al. (2016)	For efficient search with a specific target, there is a logarithmic relationship between distractor set size and reaction time. The steepness of this relationship is modulated by distractor-target similarity, with steeper slopes for more similar distractors.
Wang et al. (2017)	Data from homogeneous search arrays can be used to predict search reaction times in heterogeneous displays containing images of real-world objects, using an equation assuming parallel, unlimited capacity, exhaustive processing, and independence of inter-item processing.
Madison et al. (2018)	Logarithmic efficiency in efficient search cannot be explained by crowding in peripheral vision.
Ng et al. (2018)	Logarithmic efficiency in efficient search cannot be explained by eye movements.
Lleras et al. (2019)	Validation of previous results showing data from homogeneous search arrays can be used to predict reaction times in heterogeneous displays. Distractor-distractor interactions can also facilitate processing when nearby items are similar to each other.
<b>Buetti et al. (2019)</b>	Data from search arrays where the distractors are distinguished from the target by one feature can be used to predict search reaction times in displays with compound stimuli, defined by two features. Reaction times can be predicted using a collinear contrast integration model, which assumes that the overall target-distractor contrast is the sum of the contrasts from the two feature vectors separately.
Lleras et al. (2020)	Full proposal of the Target Contrast Signal Theory, proposing that the initial stage of processing computes a difference signal between each item in the scene and the target template, using this to determine which items in the scene are unlikely to be the target.
Ng et al. (2020)	Attention works in a two stage process, first discarding target-dissimilar distractors in a distributed, parallel way. Focused spatial attention then visits target-similar items at random.
Xu et al. (2021)	Extension of Buetti et al. (2019) to new features (shape and texture), which combine according to a Euclidean metric (orthogonal contrast integration model).

Table 1: An overview of work on the Target Contrast Signal Theory. The key paper for our replication is highlighted.

149 of advantages over frequentist approaches. Perhaps most importantly, Bayesian  
150 models are highly flexible. We demonstrate how we are able to specify a model  
151 that is able to more accurately represent the distribution of responses (for exam-  
152 ple, by specifying a response distribution that avoids predicting negative reaction  
153 times) with a relatively complex model structure, that can be fit to a relatively  
154 small amount of pilot data: something that would be challenging within a fre-  
155 quentist framework. We also believe that Bayesian models offer very intuitive  
156 methods for model testing and comparison and straightforward interpretation of  
157 results, and we hope that this manuscript can act as a demonstration of these ben-  
158 efits, showing how they can be applied to real scientific questions beyond the  
159 simplified examples often found in textbooks or tutorials.

160 In the current manuscript, we focus on replicating and extending findings from  
161 Buetti et al. (2019). In their study, participants searched for a target in a scene of  
162 homogeneous distractors (see Figure 1). First, parallel search efficiency (mea-  
163 sured by the logarithmic search slope) was estimated for cases where the distrac-  
164 tors varied from the target in one dimension: either colour (e.g. a cyan target being  
165 searched for in either yellow, blue or orange distractors) or shape (e.g. a semicir-  
166 cle target in either circle, diamond or triangle distractors). New participants then  
167 searched for the same targets in displays where the distractors were compounds,  
168 differing from the target in both colour and shape (e.g. searching for a cyan semi-  
169 circle in either blue circles, orange diamonds or yellow triangles). The logarithmic  
170 search slopes in the initial experiments were then used to predict the logarithmic  
171 slopes and reaction times using a number of models. The authors found that the  
172 best model was a ‘collinear contrast integration model’ where the distinctiveness  
173 scores were summed along each attribute in the unidimensional experiments, cre-  
174 ating an overall contrast score that was used for compound stimuli predictions.  
175 In our registered replication, we will attempt to verify the conclusions of Buetti  
176 et al. (2019), that the collinear contrast integration model does indeed offer the  
177 best characterisation of contrast signal combinations in visual search within the  
178 TCS framework.

179 We begin by verifying the analysis of Buetti et al. (2019). We then describe  
180 our proposed replication study, showing with pilot data how we are able to extend  
181 their model of how multi-dimensional contrasts are calculated, both by incorpo-  
182 rating a multi-level design to predict within-subjects effects and by utilising a  
183 Bayesian generalised linear model framework to better represent the distribution  
184 of responses (e.g. avoiding predicting negative reaction times, accounting for un-  
185 certainty in model predictions).

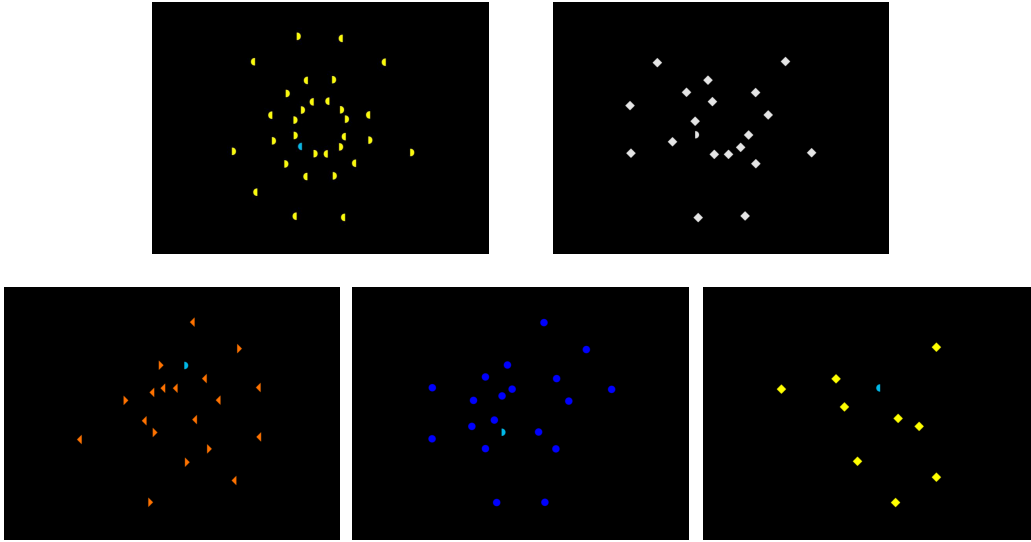


Figure 1: Example stimuli from Buetti et al. (2019) Top left: Expt 1A. Here, the target is a blue semicircle within a set of homogeneous (yellow semicircle) distractors. Top right: Expt 1B. The target is a grey semicircle in circular grey distractors. Bottom left: Expt 2A. The target is a blue semicircle in orange diamond distractors. Bottom middle: Expt 2B. The target is a blue semicircle in dark blue triangle distractors. Bottom right: Expt 2C. The target is a blue semicircle in yellow circular distractors.

## 2. The Target Contrast Model

We first describe the original Target Contrast Model, as presented in Buetti et al. (2019) and verify that we can successfully replicate the original analysis (both using frequentist modelling and Bayesian modelling; see *Supplementary Materials - Computational Verification*).

### 2.1. TCS modelling overview

In Experiment 1a of Buetti et al. (2019), participants searched for a cyan semicircle target among blue, yellow or orange semicircular distractors i.e. they searched for a target that differed from the distractors by a *single feature* (colour). The experiment was then repeated (1b) using a different single feature (shape, with participants searching for the semicircular target within triangle, circle or diamond distractors). In Experiments 2a, 2b and 2c, participants again searched for a cyan semicircle, but this time, the distractors differed in both shape and colour.



199 We will refer to these conditions as *double features*. Note, unlike in standard con-  
 200 junction searches, in this paradigm, the distractors are all identical with respect to  
 201 these features (i.e, orange triangles). Examples of all these stimuli are shown in  
 202 Figure 1. Buetti et al. (2019) also carried out a replication of their basic results  
 203 using slightly different target and distractor stimuli (Experiments 3 and 4).

204 The *Target Signal Contrast* theory is built around a linear model for predicting  
 205 mean reaction times from the logarithm of the number of distractors (see Equation  
 206 1). In particular, the TCS theory allows us to predict the value of the logarithmic  
 207 slope,  $D_{c,s}$ , in this condition based on the corresponding  $D_i$  in the single feature  
 208 search experiments.

### 209 2.1.1. Calculating the intercept, $a$ , and the logarithmic slope parameter, $D_i$

210 Experiments 1a and 1b and 3a and 3b were used to calculate the logarithmic  
 211 slope parameter  $D_i$ . In all experiments, the number of distractors varied, allowing  
 212 the data to be used to fit a log-linear model for reaction times, where reaction  
 213 times increase logarithmically with  $N_T$ , the number of distractors (see Equation  
 214 1). In the original model the error distribution was assumed to be normal. Thus  
 215 the results of Experiments 1 and 3 were used to calculate  $D_i$ , for each type of  
 216 distractor. When colour varied, we will refer to  $D_c$ , for  $c = 1, 2, 3$ . Similarly for  
 217 shape we will denote this ( $D_s$ ), and the compound features are denoted as ( $D_{c,s}$ ).

218 Fitting the model specified in Equation 1 to the data, we obtain the values for  
 219  $D_c$  and  $D_s$  given in Table 2. As can be seen, the more similar the distractors are to  
 220 the target, the steeper the slope parameter is.

feature	$D_c$	feature	$D_s$
blue	76.8	triangle	141.1
yellow	16.0	diamond	77.2
orange	9.8	circle	62.1

Table 2: A table of  $D_i$  values for Experiment 1a and 1b. See *Supplementary Materials - Computational Verification* for full values for all experiments.

### 221 2.1.2. Estimating $D_{c,s}$ , the logarithmic slope parameter for compound features

222 In the context of the current experiments, the core idea of TCS theory is that  
 223 we can estimate the (natural) logarithmic slope parameter for a double feature  
 224 visual search from the slopes parameters in the two independent single feature  
 225 searches i.e.,  $D_{c,s} = f(D_c, D_s)$ . Buetti et al. (2019) tested three different models  
 226 for predicting  $D$  for compound colour-shape stimuli. The best feature guidance

227 model (Equation 2) suggests that when the target and lures differ in two dimen-  
 228 sions, participants will choose to attend to whichever feature dimension is the  
 229 most discriminable (i.e. has the smallest  $D$  value):

$$D_{c,s} = \min(D_c, D_s) \quad (2)$$

230 The orthogonal contrast combination model instead suggests that independent  
 231 feature dimensions comprise a multidimensional space, where an object can be  
 232 described by the overall vector in this space, and thus  $D_{c,s}$  can be represented as:

$$D_{c,s} = \frac{1}{\sqrt{(\frac{1}{D_c})^2 + (\frac{1}{D_s})^2}} \quad (3)$$

233 Finally, the collinear contrast integration model also assumes independence of  
 234 feature dimensions, but assumes that while the visual features create a multidimensional space, the contrast between them is unidimensional. As  $D$  is assumed  
 235 to be inversely proportional to contrast, the equation can be written as follows:  
 236

$$\frac{1}{D_{c,s}} = \frac{1}{D_c} + \frac{1}{D_s} \quad (4)$$

237 Buetti et al. (2019) found that with their dataset, the collinear contrast inte-  
 238 gration model was best able to predict  $D_{c,s}$  from  $D_c$  and  $D_s$ , with  $R^2 = 0.915$ .  
 239 We verified we were able to replicate this result using the dataset available on  
 240 OSF (<https://osf.io/f3m24/>)<sup>1</sup> and using the exclusion criteria originally applied;  
 241 see Figure 2 (left panel) and *Supplementary Materials - Computational Verification*  
 242 for details. We show that we are able to do this using both the frequentist  
 243 modelling approaches used in the original paper, and using Bayesian modelling.

### 244 2.1.3. Estimating $a$ , the intercept parameter for compound features

245 As  $a$  is the intercept of the model, it represents how long observers take to find  
 246 a target when  $N_T = 0$ , i.e., there are no distractors. As such, it should be inde-  
 247 pendent of both shape and colour, and can be thought of as the role of non-search  
 248 processes (such as motivation, motor preparation etc.) that influence reaction time.  
 249 In Buetti et al. (2019),  $a$  was calculated for each sub-experiment. Here, we follow  
 250 that method in order to replicate their results exactly.

---

<sup>1</sup>downloaded on 28th August 2020



Figure 2: (left) The collinear method for calculating  $D$  offers a good prediction. (centre) Using the TCS to predict reaction times. (right) Each dot now represents a randomly sampled reaction time from an observer. Note that there is greater spread in the data points here, due to the fact that there will be trial-to-trial variability due to target position, inter-item distances, observer differences and so on.

#### 2.1.4. Estimating mean reaction times

Finally, we can use Equation 1 to predict mean reaction times. As can be seen in Figure 2 (centre panel), these predictions are essentially identical to the empirical RT results:  $R^2 = 0.93\%$ .

#### 2.1.5. Discussion

While TCS theory offers a good prediction of search slopes and corresponding mean reaction times for double feature search, there are two related limitations. Firstly, it is unable to account for individual differences between observers, only the changes to the sample average. Secondly, it cannot account for the distribution of reaction times over multiple trials. Figure 2 (right panel) shows clearly that these factors generate high levels of variability within the individual trial-level data. To address these issues, we propose adapting TCS to make use of multi-level modelling techniques. Multi-level models allow us to take into account the hierarchical structure of the data (i.e. that each participant completes multiple trials) in a way that does not require averaging, meaning that we are able to model participant variability as well as group-level effects (Gelman and Hill, 2006).

#### 2.2. A multi-level TCS

Switching from a linear regression model to a multi-level model will allow us to compute  $D$  for each participant, while simultaneously estimating the trial-to-trial variance. We also switch from a frequentist to Bayesian framework, as

271 this allows us to naturally account for the uncertainty in the model’s predictions.  
272 However, switching from linear regression to a multi-level model raises the prob-  
273 lem of which distribution to use for modelling reaction times. Using a normal  
274 distribution is unlikely to be satisfactory, as it is unable to account for the skew  
275 frequently seen in reaction time distributions, and also allows the possibility of  
276 negative reaction times. We can account for both of these problems by using a  
277 log-normal distribution. We will also test whether a slightly more complex ex-  
278 tension of this model, the shifted lognormal model (which allows the distribution  
279 to be offset to the right i.e. mimicking the patterns seen in reaction time data,  
280 where valid responses begin at around 100ms) offers any improvement in model  
281 fit. Note that a Wald, or inverse Gaussian distribution, would also be a reasonable  
282 distribution choice for this data given that TCS is based on a diffusion process e.g.  
283 (Moran et al., 2013), and this distribution has been argued to be psychologically  
284 more plausible (e.g. Kieffaber et al. (2006), though see Matzke and Wagenmakers  
285 (2009)): we chose not to use this distribution as it often leads to computational is-  
286 sues, which would make it harder for others to reproduce or build on our approach  
287 later.

### 288 3. Hypotheses

289 We plan an experiment to test the extent to which the original results in Buetti  
290 et al. (2019) replicate and generalise, using our new modelling approach.

#### 291 3.1. *Proposed Modifications to Experimental Design*

292 In order to better test the above, and increase sensitivity, we propose to make  
293 the following changes to the experiment described in Buetti et al. (2019):

- 294 1. **Within-subjects design.** This modification should give us greater power to  
295 detect differences between different models, as well as allowing us to in-  
296 vestigate how individual differences in the single-feature task might explain  
297 differences in the double-feature task.  
298
- 299 2. **Increase target-distractor similarity.** If the distractors are a very differ-  
300 ent colour from the target, they may not distinguish well between different  
301 contrast models. We will therefore run a version of the experiment where  
302 the target is a red semicircle, with distractors being either orange, purple or  
303 pink.  
304

### 3.2. Registered Hypotheses

1. **Shifted lognormal model.** We hypothesise that a shifted lognormal model will give the best fit to our single-feature data, when compared to a lognormal and a normal model.
2. **Log-linear effect of  $N_T$ .** We will test the TCS model assumption that  $N_T$  has a log-linear effect by testing models with and without the log of this term. We expect that this will confirm the results previously seen in papers testing TCS i.e. that the log-linear approach will be best.
3. **Contrast model comparisons.** We will test the hypothesis proposed by (Buetti et al., 2019): specifically, that the *collinear contrast integration model* outperforms the *best feature guidance*, and *orthogonal contrast combination models* for the calculation of  $D$ , by calculating and comparing the mean absolute prediction error for each model.
4. **Reaction time predictions.** We will further test the hypothesis proposed by (Buetti et al., 2019) by testing which model gives the best prediction at the trial-by-trial RT level.

We will test each of these hypotheses by calculating the marginal likelihood of the relevant models, and then calculating the posterior probabilities. This will give us a probability for each model that represents the likelihood that the model gives the best prediction. We will consider there to be evidence for one model over the others if a given model has a probability above 90%. We will consider there to be strong evidence for one model over the others if that model has a posterior probability above 99%. This approach is most appropriate for our model: other measures of model fit, such as AIC, require an assumption of flat priors (which is not valid for multi-level models) and are based on point estimates (which is not valid for Bayesian models) (McElreath, 2020).

### 3.3. Planned Explorations

We plan to investigate the effect of individual differences in this paradigm: to what extent performance in the single-feature task can predict performance in the double-feature task for a given individual (Buetti et al. (2019) were not able to investigate this due to the between-subjects design of their study). We plan to do this by specifying a more complex random effects structure for the model, that allows for individual differences across different slopes for different features. This

allows us to then study the random effect correlation structure. However, given these models can be challenging to fit, we will do this in an exploratory manner after carrying out our formally registered analysis.

One of the benefits of using a multi-level modelling approach is that it is relatively easy to extend to incorporate other factors that may contribute to reaction times, such as eccentricity and inter-item distance, which may help to explain behaviour further. To demonstrate this, we will also run exploratory analyses including a factor for which ring the target is in to assess whether this improves model fit or affects any of the conclusions that can be drawn from the model.

### 3.4. Pilot Experiment

Full details of a pilot experiment with  $n = 4$  participants (960 trials each) using our proposed analyses can be found in *Supplementary Materials - Pilot Analysis*. This suggests that even with a small sample, we can convincingly demonstrate H1 and H2. However, more data will be required to discriminate between the models, particularly for H4. Given that our methods are within-subject, we have reduced the number of trials per condition compared to Buetti et al. (2019) (12 in our pilot study, 20 in our proposed, compared to 40 in theirs). It is therefore possible that the increased noise in our estimated  $D$  single-feature parameters will make it more difficult to predict double-feature  $D$ s accurately. However, we think this is unlikely to be the case as we can see that even in a small amount of pilot data, we can verify H3, with the collinear model having the lowest mean absolute prediction error.

## 4. General Methods

### 4.1. Sample Size: Participants and Trials

We plan to test 40 participants during the experiment. Our pilot experiment shows that H1 and H2 are easily demonstrated with 10 times less data, and Buetti et al. (2019) used 20 participants per experiment. Our sample size will therefore be in line with previous work testing H3 and H4. Ethical approval for the study was granted by the University of Aberdeen (application number PEC/4677/2021/2).

Our pilot study above suggests that just 12 trials per condition may be sufficient to fit our models. To be conservative, we propose using 20 in our experiment. We have demonstrated that using just half the data (20/40 trials per condition) from Buetti et al. (2019) makes no difference to our computational verification (see *Supplementary Materials - Computational Verification*).



Figure 3: (left) The dark line shows the distribution we sampled from. The blue lines show distributions fitted to different samples of 20 data points. (right) Plot showing how the distribution of sample means vary with  $n$ . Shaded regions indicate the 50%, 80% and 95% confidence intervals.

375 Finally, we have carried out a simulation experiment to estimate the confi-  
 376 dence intervals on the mean when sampling from a log-normal distribution. We  
 377 defined our distribution to have a mean-log of 6.135 and a standard deviation of  
 378 0.32. These values were loosely based on the distributions of reaction times in  
 379 Buetti et al. (2019). The results are shown in Figure 3. Based on these simula-  
 380 tions, we find that a sample of  $n = 20$  leads to a 95% confidence interval that is  
 381 approximately 1.4 times larger than  $n = 40$ . We feel this is a suitable compromise  
 382 given we will be collecting our data within-subjects.

#### 383 4.2. Stimuli

384 The targets and distractors are randomly assigned to the display based on an  
 385 invisible grid. Within each quadrant of the screen, there are three 'spokes' each  
 386 with four possible target positions (starting from the centre of the screen and mov-  
 387 ing outwards), creating 36 different target positions in total, in three concentric  
 388 circles. A small amount of jitter is added to each possible position to make the  
 389 target locations less predictable.

390 **Distractor and target types:** we will replicate the distractor types used in  
 391 Buetti et al. (2019), apart from that we will change one distractor colour (from  
 392 blue to pink) to allow us to discriminate better between different models of the  
 393 data (see above). There are six single-feature conditions (purple, orange and pink  
 394 distractors and triangle, circle and diamond distractors) and nine double-feature  
 395 conditions (all possible pairings of the single-feature conditions). The target is al-  
 396 ways a red semicircle, except in the trials where the distractors are single-feature

397 shapes (triangles, circles and diamonds) in which case the target is a white semi-  
398 circle.

399 **Set sizes:** we will run all the distractor set sizes used in Buetti et al. (2019) (1,  
400 4, 9, 19 and 31). We will also run target-only 'zero distractor' trials (60 in total,  
401 with 12 being the white semicircle target and the remainder the red semicircle  
402 target).

403 The experiments were programmed in PsychoPy and Pavlovica (Peirce et al.,  
404 2019). Stimuli were pre-made to generate search array images with  $1920 \times 1080$   
405 resolution.

#### 406 4.3. Procedure

407 Participants will complete the experiment in the laboratory, sitting at a view-  
408 ing distance of 45cm from the screen (viewing distance will be fixed by using a  
409 chin rest). They will view a fixation cross before viewing a search array: they  
410 will press the space bar to continue to the trial. Participants will be told to search  
411 for the target among distractors (either a red semicircle or a white semicircle, de-  
412 pending on the block) and report if the semicircle target points to the left or right,  
413 by pressing either the left or right button on a button box (Cedrus RB-540). They  
414 will first complete 16 practice trials where they will receive feedback immediately  
415 after completing each trial. In the real experimental trials, participants will receive  
416 feedback on their average accuracy and reaction time after each block of 320 tri-  
417 als. Participants will complete 5 blocks of trials (1600 trials overall i.e. 320 trials  
418 in each of 5 experiments, consisting of 5 set sizes x 3 distractor conditions x 20 re-  
419 peats + 20 zero distractor trials). The trials where the distractors are single-feature  
420 shapes (i.e. the target is a white semicircle - Experiment 1b in Buetti et al. (2019))  
421 will all appear in one block (which will appear at a randomly selected position  
422 within the experiment). All other trials (where the target is red semicircle) will  
423 be fully randomised i.e. all different conditions will be completely intermixed.  
424 This approach will be taken as TCS requires the participant to have a well-defined  
425 target template in mind in order to compare this to the stimuli in the display. Thus,  
426 participants will be cued to search for the relevant target at the beginning of each  
427 block.

428 In both the practice and experimental trials, the search display will always  
429 remain on screen until a response is made, or until 5 seconds had passed.

#### 430 4.4. Data Pre-processing

431 Only participants who complete the full experiment will be considered candi-  
432 dates for inclusion in the data analysis. We will apply the same inclusion criteria



433 as the original paper: participants will only be included if their search accuracy  
434 is over 90% and their average response time is not smaller or larger than two  
435 standard deviations from the group average response time.

436 For participants included in the analysis, we will apply the data cleaning used  
437 in the pilot data analysis i.e. removing the top and bottom 1% of their data.<sup>2</sup>

#### 438 4.5. *Analysis Plan*

439 All analysis will be carried out using R (v4.2.0), brms (v2.17.0) and rStan  
440 (v2.26.11) As discussed above, we will use mixed-effect models with either nor-  
441 mal, lognormal or shifted lognormal distributions.

442 Please see the analysis of our pilot data for a full implementation of our anal-  
443 ysis pipeline, including all code (available on Github at [https://github.com/Riadsala/single\\_double\\_](https://github.com/Riadsala/single_double_feature_search)  
444 [feature\\_search](https://github.com/Riadsala/single_double_feature_search)).

#### 445 4.6. *Registered Report*

446 The original Stage 1 registered report for this manuscript is available at <https://osf.io/f9sua/>. All study data, materials and analysis code for both Stage 1  
447 and Stage 2 are available at [https://github.com/Riadsala/single\\_double\\_](https://github.com/Riadsala/single_double_feature_search)  
448 [feature\\_search](https://github.com/Riadsala/single_double_feature_search).

449 We report how we determined our sample size (see Section 4.1), all data exclu-  
450 sions (if any), all inclusion/exclusion criteria, whether inclusion/exclusion criteria  
451 were established prior to data analysis (see Section 4.4) all manipulations, and all  
452 measures in the study (see Section 4.2).

### 454 5. Results

455 All 40 participants had accuracy over 90% (minimum 93.1%). One participant  
456 had an average response time (1100ms) over two standard deviations from the  
457 group average response time (781ms) and was removed. Incorrect trials were  
458 then removed, and the data was trimmed (only including response times between  
459 the 1% and 99% quantiles) leaving us with 39 participants completing a total of  
460 59,587 trials.

---

<sup>2</sup>See *Supplementary Materials - Pilot Analysis* and *Supplementary Materials - Registered Anal-  
ysis* for full details of data cleaning.

461 All Bayesian models were fit to the new data using exactly the same proce-  
 462 dure<sup>3</sup> as the pilot data presented in the Stage One review process. We checked  
 463 for convergence of our models by visually inspecting the chains as well as ver-  
 464 ifying that the  $\hat{R}$  was close to 1 for all parameters of all the fitted models (see  
 465 *Supplementary Material - Main Analysis* for full model fit information).

### 466 5.1. Hypothesis 1: Shifted-lognormal model

467 Our first hypothesis concerns which distribution best fits the single feature  
 468 response time data. We fit multi-level models with a i) normal, ii) lognormal, and  
 469 iii) shifted-lognormal distribution. The models all used the same model formula  
 470 that estimated search slopes in terms of  $\log N_T$  for each feature. Maximal random  
 471 effect structures were used.

472 After each of these models had been fit to the data, leave-one-out (LOO) model  
 473 comparison was used to calculate posterior probabilities for each. The results of  
 474 this procedure allocated  $\sim 100\%$  of the weight to the shifted-lognormal model, so  
 475 we can conclude that it is the best distribution (out of the three we tested<sup>4</sup>) to use  
 476 for modelling response times in this paradigm. This model is shown in Figure 2.2  
 477 of the *Supplementary Materials - Registered Analysis*.

### 478 5.2. Hypothesis 2: log-linear effect of $N_T$

479 We then used the same methods to verify that using  $\log N_T$  for the search slope  
 480 does indeed give a better fit to the data than simply using  $N_T$ . The results are again  
 481 conclusive with  $\sim 100\%$  of the model weight being assigned to the model that is  
 482 log-linear in  $N_T$ .

### 483 5.3. Hypothesis 3: Contrast Model Comparison

484 Now that we have confirmed that the shifted-lognormal multilevel model (with  
 485 a log-linear effect of  $N_T$ ) is indeed the best fit to the data we will extract the search  
 486 slopes for each feature. These are summarised in Table 3. We can see that we have  
 487 successfully obtained a range of values for both  $D_c$  and  $D_s$ . As with Buetti et al.  
 488 (2019) we find that the values for  $D_s$  are larger than  $D_c$  (see Table 2), meaning  
 489 that search slopes for colour features are shallower than shape.

---

<sup>3</sup>The only departure was an increase in iterations from 5000 to 80000 for the model predicting reaction times, based on advice given in the Stan forums, to enable the bridge sampling process to work properly.

<sup>4</sup>See discussion for Wald, Weibull, etc.

feature	$D_c$	95%HDCI	feature	$D_s$	95%HDCI
orange	0.156	[0.139 , 0.173]	triangle	0.253	[0.230 , 0.275]
pink	0.042	[0.028 , 0.057]	diamond	0.187	[0.171 , 0.205]
purple	0.015	[0.002 , 0.030]	circle	0.191	[0.175 , 0.204]

Table 3: A summary of the posterior estimates of  $D_c$  and  $D_s$  values from our Experiment. Note that our values are reported in seconds, in contrast to Table 2, which follows (Buetti et al., 2019) and reports the slopes in milliseconds.

490 We now combine the *single-feature* search slopes,  $D_c$  and  $D_s$ , to predict the  
491 *double-feature* conditions ( $D_{c,s}$ ) using Equations 2, 3 and 4 and above. The results  
492 are summarised in Figure 4. We find that while the collinear contrast model has  
493 the highest  $R^2$  (0.922, compared to  $R^2 = 0.884$  for best feature, and  $R^2 = 0.916$  for  
494 orthogonal contrast), the orthogonal contrast model is the most accurate, both in  
495 terms of mean absolute error (0.165, compared to 0.185 for best feature and 0.271  
496 for collinear) and having a regression slope closest to 1 (1 compared to 0.753 and  
497 1.48). Therefore, Hypothesis 3 does not hold: orthogonal contrast rather than  
498 collinear contrast offers the best prediction of search slopes in the double-feature  
499 condition.

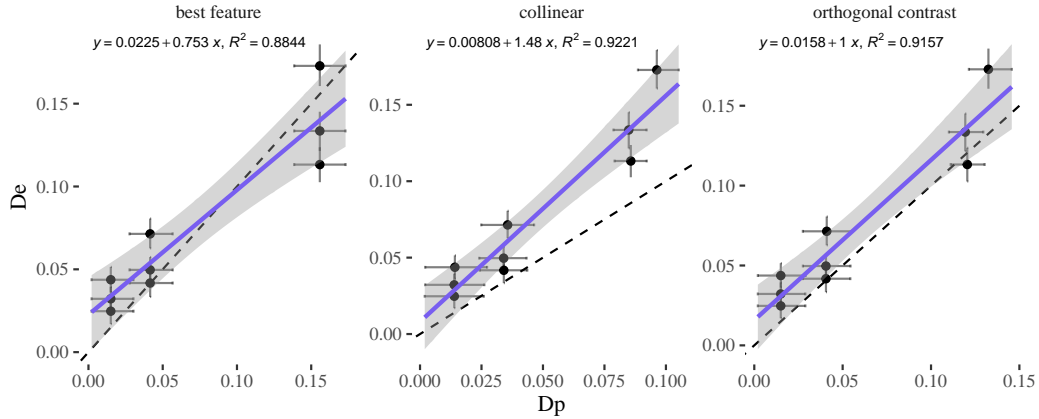


Figure 4: Predicting  $D_{c,s}$  from  $D_c$  and  $D_s$ . The x-axis shows our predictions,  $D_p$ , using the best feature, collinear contrast, and orthogonal contrast models.

#### 500 5.4. Hypothesis 4: Reaction Time Predictions

501 Upon reflection, the approach to model comparison we outlined in our reg-  
502 istered analysis was limited in a number of ways. Our original plan was to use

the posterior predictions from a model trained on the single-feature data to act as a prior for the double-feature data. While we initially thought this would be an elegant approach, there are a large number of parameters that are outside the main focus of this paper yet still require priors (intercepts, group level variance and residual variance). Furthermore, while the methods for estimating  $D_{c,s}$  presented above give good predictions in terms of the mean value, it is not clear that the standard deviation for these distributions will be accurate. As such, we have developed a new, simpler method for this final comparison. To maintain full transparency, we present both methods here.

#### 5.4.1. Registered Method

Our final hypothesis concerns how well the different feature combination models perform when predicting reaction times. We find very little difference between the three methods in terms of LOO model weights: 0.318 for best feature, 0.346 for collinear and 0.336 for orthogonal contrast.

#### 5.4.2. Updated Method

Our new method for exploring this hypothesis involves taking  $n = 100$  samples of the fixed effects from both the model fitted to the single-feature data and the model fitted to the double-feature data. Each of these samples includes an intercept ( $a$ ), slope ( $D$ ), non-decision time (ndt), and residual variance ( $\sigma$ ). We then take the parameters from the double-feature model, but replace the  $D$  values with our predicted  $D$  using the single-feature model. Finally the predicted mean  $\log(rt)$  is calculated for each feature and number of distractors. These are then compared to the empirical reaction times and we compute the absolute error.

We can also calculate an upper-bound by carrying out the above process, but without replacing the fitted  $D_{c,s}$  with the predicted. This allows us to report ‘relative absolute error’. As all of the methods under consideration make identical predictions for trials with no distractors, these are omitted from this calculation.

The results of this procedure are in-line with the registered analysis presented above: all three methods perform well relative to our baseline (see Table 4).

## 6. Planned Explorations

Our interpretation of the null/neutral results for Hypothesis 4 (the prediction of reaction times) is that the differences in predictions from the three contrast combination methods are small relative to the (i) individual differences between participants and (ii) trial-to-trial variability due to target eccentricity. Thus, in our

metric	abs error		
	lower	median	upper
orthogonal	0.994	1.00	1.02
collinear	0.990	1.01	1.05
best feature	0.999	1.01	1.02

Table 4: How well can we predict RTs using  $D_p$  (collinear, best feature or orthogonal contrast) compared to using  $D_e$ ? A value of 1 means that our estimates of  $D$  derived from the single-feature trials does an equally good job at predicting the double-feature trials as using the  $D$  fit to the data.

exploratory analysis, we investigate how incorporating these factors affects our conclusions.

### 6.1. Individual Differences

We start this exploratory analysis looking at how the  $D_c$  and  $D_s$  values vary from participant to participant. From Figure 5 (*left*) we can see that there is considerable variation between observers - in fact, the variation from one observer to the next is often larger than the variation across features. To investigate this further we calculated the correlations between each of the features, by calculating Pearson's  $r$  for each sample from our posterior, which gives us a full posterior distribution for the correlations. We can see in Figure 5 (*right*) that while both the  $D_c$  and  $D_s$  are correlated within feature classes ( $\sim 0.75$ ), there is no correlation of any of the colour features with any of the shape features. The individual differences for the *double-feature* conditions are much less pronounced - these conditions are easy and the search slopes are quite close to flat. Hence, the correlations are all much weaker, presumably due to range restriction.

Given these results, it is perhaps unsurprising that our analysis for Hypothesis 4 leads to an inconclusive result for distinguishing between the three contrast combination methods. Perhaps taking these individual differences into account when we predict reaction times will lead to improved power to discriminate between the models. However, before we do so, we will also investigate incorporating information about target eccentricity into the model.

### 6.2. Target Eccentricity

It is well known that there are eccentricity effects in visual search, with reaction times being longer for targets that are further away from fixation (Carrasco et al., 1995; Wang et al., 2017). To investigate this in our dataset, we will use the same methods as above (fitting a multi-level shifted-lognormal model) but now

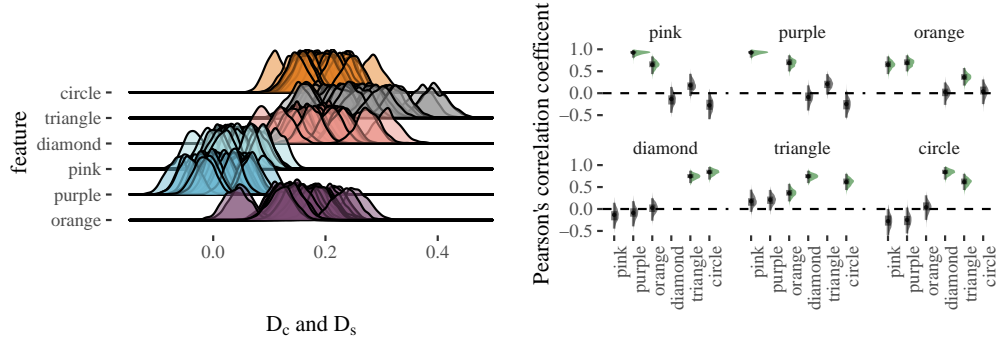


Figure 5: Individual differences in  $D_c$  and  $D_s$ . (left) Posterior probability distributions for  $D_c$  and  $D_s$  for each individual. (right) Estimated correlations between each of the  $D_c$  and  $D_s$ .

563 including an additional factor that represents how far the target was from the fix-  
 564 ation cross. This is coded as a three-level categorical factor representing which  
 565 ring contained the target (see stimulus details, above). Allowing for interactions  
 566 with the *feature* and  $\log N_T$  increases the number of fixed effect parameters in the  
 567 model from 8 to 22.

$$y \sim 0 + r + r : f : \log(N_T) + (1|id) \quad (5)$$

568 We experimented with including  $r$  in the random effect structure, but this  
 569 proved difficult to fit. We also had to revise the priors used in our registered analy-  
 570 sis, in order to lower the intercept. Full details can be found in the supplementary  
 571 materials.

572 After obtaining a model that passed all convergence checks, we examined the  
 573 posterior distribution for the effect of *ring*. Figure 6 paints an interesting and  
 574 complex picture in which some features (e.g. some colours, particularly those  
 575 that are more distinct from the target colour) are clearly leading to ‘pre-attentive  
 576 search’ in which response times are unaffected by either the number of distractors  
 577 or target eccentricity. However, shape features seem to be strongly affected by  
 578 eccentricity, particularly when there are multiple distractors in the stimulus.

579 We can now compute our predictions ( $D_p$ ) for  $D_{c,s}$  taking the *ring* into ac-  
 580 count. Doing so leads us to a similar result as before with orthogonal contrast  
 581 outperforming the best feature and collinear measures in terms of absolute error  
 582 (0.023 compared to 0.025 (best feature) and 0.034 (collinear)). However, the re-  
 583 gression slopes are all relatively similar (0.90 for best feature, 1.58 for collinear  
 584 and 1.15 for orthogonal contrast).



Figure 6: Fixed effects for predicting the effect of ring, feature and number of distractors on response times. Shaded regions represent the 53% and 97% HDICs. We can see that *ring* has an effect on search slopes, and that this effect is more pronounced for some features (i.e., triangles) than others.

### 6.3. Predicting Response Times

We will now test to see if we can discriminate between the three contrast combination methods when we take target eccentricity (ring) and individual-level slopes into account. We use the same model comparison as before (see *Supplementary Materials - Planned Explorations* for full code) and find orthogonal contrast performs best, closely followed by best feature.

metric	abs error		
	lower	median	upper
orthogonal	1.01	1.00	1.04
collinear	1.03	1.00	5.15
best feature	1.03	1.00	1.07

Table 5: How well can we predict RTs using  $D_p$  (collinear, best feature or orthogonal contrast) compared to using  $D_e$  when using a model containing the ring of the target? A value of 1 means that our estimates of  $D$  derived from the single-feature trials does an equally good job at predicting the double-feature trials as using the  $D$  fit to the data.

#### 6.3.1. Issues with the Collinear Contrast method

The upper bound on the error in the collinear contrast method is high (see Table 5). To explain this, we can look back at Equation 4 and realise that empirical

data is not required to reject this hypothesis as a suitable method for predicting search slopes in feature search. This is because when search slopes are close to 0, it is possible that we will observe negative values in the empirical data. Breaking down our data to compute search slopes for each person and each target eccentricity increases the chances of this being observed. Looking at Equation 4 we can see that  $D_1 \sim -D_2 \Rightarrow 1/D_1 + 1/D_2 \sim 0$ . This leads to our estimated  $D = \frac{1}{1/D_1 + 1/D_2} \gg D_1, D_2$  which is clearly incorrect.

## 7. General Discussion

In this paper, we aimed to test the extent to which the results of Buetti et al. (2019) replicate and generalise, using a new modelling approach. Our results allow us to confirm our pre-registered hypotheses 1 and 2. Firstly, a shifted-lognormal distribution of response times outperforms normal and lognormal distributions, demonstrating that reaction time data are best modelled by a skewed distribution with an offset. Similarly, we confirmed that the number of distractors has a log-linear effect in this model, in line with the predictions of TCS theory. We also replicated other aspects of the original Buetti et al. (2019) paper with a different experimental set up, such as to observing shallower search slopes for colour features compared to shape features.

We do not find support for our pre-registered hypotheses 3 and 4. For predicting  $D$  in the double-feature conditions, our analyses found that the orthogonal contrast model was favoured over collinear, which is not in line with the registered hypothesis, which predicted that the collinear contrast model would be best (in line with Buetti et al. (2019)). Similarly, for hypothesis 4, we found that there was relatively little difference between the three combination methods for prediction of trial-by-trial reaction times. Our exploratory analyses suggest that incorporating additional factors (e.g. individual differences in participant  $D_c$  and  $D_s$  values, and the eccentricity of the target) allows better discrimination between models, but again suggests that the orthogonal contrast combination method gives the best predictions.

### 7.1. Modelling of reaction times

In much of the literature on visual search, mean reaction times are modelled using a simple linear model  $\bar{y} = bN_T + a$  (e.g. Treisman and Gormican (1988); Rosenholtz et al. (2012); Hughes et al. (2016)). The  $b$  coefficients are often referred to as “search slopes” and are often treated as measurements of theoretical importance. Our results indicate that a shifted-lognormal model that is loglinear



629 in  $N_T$  offers a much better fit to the data ( $\log(y) - ndt = b \log(N_T) + a$ ), which  
630 is perhaps not surprising, given the properties of reaction time data, where valid  
631 responses normally begin at around 100ms, and the distribution often has a long  
632 “tail” of slower responses.

633 However, there have been concerted efforts within the literature to model re-  
634 action time distributions more effectively: indeed, Buetti et al. (2019) use  $\log N_T$   
635 when computing their search slopes. In terms of reaction time distributions, log  
636 transformations are frequently taught as a way to normalise reaction time data  
637 (although often with caveats regarding how this can change the interpretation of  
638 the results (Osborne, 2002)) and are frequently used in analysing reaction time  
639 data e.g. (Clarke et al., 2022b). Researchers have also looked at other distribu-  
640 tions to assess which offer the best fit to empirical response times in visual search.  
641 For example, Palmer et al. (2011) compared ex-Gaussian, ex-Wald, Gamma, and  
642 Weibull distributions and found that the distributions with exponential compo-  
643 nents offer a better fit to the data. Our results are in line with this. However,  
644 we opted to use a shifted-lognormal distribution in our analysis above for mostly  
645 pragmatic reasons, as these more complex distributions are often computationally  
646 difficult to fit <sup>5</sup>. It has also been argued that trying to select a “correct” distribu-  
647 tion is likely to be problematic for empirical data, which is probably a mixture  
648 of multiple components (Wolfe et al., 2010). Similarly, some recent approaches  
649 make use of drift-diffusion methods (e.g. Wolfe and Van Wert (2010); Yu et al.  
650 (2022); Corbett and Smith (2020)), though again these models can be challenging,  
651 particularly when considering how to interpret the parameters (Evans and Wagen-  
652 makers, 2019; Bompas et al., 2023). While important, these debates are outside  
653 the scope of the present Registered Report.

654 Despite these previous findings, the use of linear search slopes is still prevalent  
655 in the visual search literature. Our work shows that these choices of distribution  
656 can influence results and conclusions (see section 7.2 below), and therefore we  
657 recommend that other researchers consider carefully how they want to model their  
658 data. Even in the case where the search slopes are the primary outcome measure  
659 of interest (as opposed to the potentially more ‘cognitive’ parameters of e.g. Wald  
660 distributions, or drift diffusion models), we demonstrate that approaches that bet-  
661 ter account for the data distribution can be taken with relative ease.

---

<sup>5</sup>See: <https://discourse.mc-stan.org/t/model-fails-to-converge-when-using-brms/9062>

## 662 7.2. *Discriminating between combination methods*

663 In Buetti et al. (2019), the collinear contrast integration model was found to  
664 provide the best fit for their data, providing a more precise prediction than the  
665 orthogonal contrast combination model (as measured by both the closeness of  
666 the slope of the regression line to one, and the mean average prediction error).  
667 Accepting this model of how the combination process works has theoretical im-  
668 plications e.g. it implies that colour and shape contrasts independently contribute  
669 to attentional guidance. However, we did not find strong support for this model,  
670 instead finding that the orthogonal contrast combination model provides the best  
671 fit with the data.

672 One possibility is that our small modifications to the experimental stimuli  
673 changed the strategy that participants used. However, this seems unlikely given  
674 that we only made changes to the colour of the stimuli, a manipulation that Buetti  
675 et al. (2019) also used, with no changes to their overall conclusions. In addition,  
676 our reanalysis of the original (Buetti et al., 2019) data using our new methods  
677 also suggested that the orthogonal contrast model was best supported. Thus, we  
678 suggest that the choice of modelling distribution (e.g. shifted-lognormal v.s. log-  
679 normal) affects the conclusions drawn, and thus we should aim to use the models  
680 that best align with the data in order to better understand the theoretical implica-  
681 tions of our findings.

682 We also modified the way stimuli were presented in our experiment com-  
683 pared to Buetti et al. (2019): rather than running each experiment separately, we  
684 (mostly) intermixed conditions. Models of attention presume that we hold a target  
685 template in our memory (Duncan and Humphreys, 1989), and thus we ensured that  
686 the trials where the target was the white semicircle were blocked separately from  
687 the trials where the target was the red semicircle, to try to avoid conflict between  
688 maintaining multiple target templates in memory. However, the distractors could  
689 change from trial to trial in all other blocks, unlike in Buetti et al. (2019). In their  
690 experiments, it is possible participants used strategies such as shifting the target  
691 representation away from the distractors, or generally using relational strategies  
692 (Navalpakkam and Itti, 2007; Becker, 2010; Yu et al., 2023), which would be  
693 more challenging in our experimental set up. In relation to the models, this type  
694 of target representation shift could occur more strongly for one feature dimension  
695 (e.g. colour) than the other, perhaps changing the relationship between the con-  
696 trasts for different feature dimensions and therefore the preferred model. If future  
697 work were to confirm this hypothesis, it would suggest that observers are able to  
698 cognitively shift their strategy based on the information available in the task.

699 Another possibility is that because some participants had negative search slopes,  
700 the collinear contrast model predicts implausibly large reaction times, due to the  
701 mathematical formulation of this model, leading to worse predictions. Given that  
702 negative search slopes do occur in some situations (Utochkin, 2013), we suggest  
703 that a future improvement for the collinear contrast integration model would be to  
704 modify it to be able to give sensible predictions in these situations.

705 Finally, we would argue that it is difficult from these results to definitively  
706 make a decision about which model is best: all three models give very similar  
707 predictive weights during our model evaluation process. One challenge is that  
708 in general, the double feature searches are easy, and therefore the search slopes  
709 are fairly flat and there is not much variability to allow different models to make  
710 different predictions. For the current paradigm, a fruitful approach for future re-  
711 search could be to consider trials in which there are a large number of distractors,  
712 while still using many different set sizes: the small differences in predicted search  
713 slope diverge as  $D_T$  increases, leading to larger differences in predicted reaction  
714 times.

### 715 7.2.1. Individual differences

716 Our (planned) exploratory analysis of the individual differences in search slopes  
717 suggests that there are large differences from one observer to the next. Indeed  
718 in some cases, these are larger than the differences from one feature to another.  
719 The difference between the steepest and shallowest search slopes (fixed effects) is  
720 0.238 ( $D_{triangle} = 0.253$  3 while  $D_{purple} = 0.015$ ). If we compare this to the range  
721 of observer search slopes within a feature, we find this varies from 0.242 ( $D_{triangle}$   
722 per-observer ranges from 0.395 to 0.152) to 0.149 ( $D_{pink}$  ranges from -0.065 to  
723 0.092). This suggests a challenge for modelling based on average performance:  
724 can we be sure that averages represent a meaningful summary of the data, given  
725 that we see very clear individual differences? It could certainly be argued that ob-  
726 servers might be using different strategies, and thus some members of the sample  
727 population might use (for example) a collinear combination strategy, while others  
728 use an orthogonal contrast strategy. Variable strategies have been found for other  
729 search behaviour (Clarke et al., 2022a; Kristjánsson et al., 2014; Proulx, 2011;  
730 Li et al., 2022), highlighting the importance of considering individual differences  
731 when understanding behaviour.

732 We also found that search slopes were correlated within feature, but not be-  
733 tween: i.e, knowing that an observer’s search slope for a colour condition allows  
734 us to predict their search slopes for the other colour conditions, but not any of  
735 the shape conditions. However, given the block design of our experiment, it is

736 possible that this reflects a type of priming effect: knowing the search slope for  
737 a feature in the first block tells allows us to predict the search slopes of the other  
738 features in that block, but tells us nothing of the observer’s behaviour in the second  
739 block. Understanding whether these correlations reflect something about an ob-  
740 server’s behaviour with different features, or instead how an observer’s behaviour  
741 changes over time, would be an interesting question for future research.

#### 742 7.2.2. *Eccentricity*

743 (Buetti et al., 2019) argues that the processing undertaken in this type of task  
744 can be done in parallel, with observers using peripheral vision to distinguish be-  
745 tween target and distractors, and that there is systematic variation in reaction times  
746 as a function of set size associated with parallel processing. Target Contrast Sig-  
747 nal Theory incorporates eccentricity effects in this type of parallel processing via  
748 a time-out parameter ( $T_0$ ) (Lleras et al., 2020; Wang et al., 2018; Ng et al., 2018).  
749 Here, we confirm in our exploratory analyses that we are able to detect relatively  
750 strong eccentricity effects, as a model with target ring number included was a  
751 better predictor of the data than one without. Following the original experimen-  
752 tal design, participants freely viewed the displays. It is therefore likely that in  
753 some cases, observers felt that peripheral information was insufficient to make  
754 judgements, and thus made eye movements, moving into a more serial, focused-  
755 attention processing stage. Future work could more exclusively investigate periph-  
756 eral effects in parallel processing by ensuring fixation when viewing the displays.

#### 757 7.3. *Conclusions and future directions*

758 In the current paper, we have independently reproduced the findings of Buetti  
759 et al. (2019) by extending their modelling to a multi-level framework. We have  
760 used a Bayesian approach, but note that this is in many ways entirely arbitrary: all  
761 of the modelling decisions we have taken would be possible within a frequentist  
762 framework as well. We also aimed to replicate the previous findings by running  
763 a within-subjects experiment, and broadly find that the Target Contrast Signal  
764 Theory does a good job of predicting the data. When using single-feature search  
765 slopes to predict double-feature search slopes, we do not replicate the previous  
766 finding that the collinear contrast integration method outperforms other options,  
767 but instead find that all combination methods do reasonably well, and in this par-  
768 ticular experimental design, it may be difficult to conclusively distinguish between  
769 them.

770 One of the clear benefits of Target Contrast Signal Theory (Lleras et al., 2020)  
771 is its quantitative nature, allowing it to be empirically tested in a straightforward

manner. Here, we demonstrate that we can independently replicate many aspects of TCS, while also offering extensions to the model that we hope will stimulate more research and refinement of this theory. Some suggestions for possible future directions and hypotheses that could be tested include:

1. It is relatively straightforward to make predictions about the mean reaction time per participant in the double-feature search condition: however, we have not attempted to predict an individual's trial-to-trial variance for different features, which could improve the model fit further.
2. We find correlations within feature classes (i.e.  $D_c$  and  $D_s$ ) but not between: however, these may be a side-effect of the block design of the experiment. A future experiment could randomise trial type in order to more fully understand the nature of these correlations.
3. To more fully explore which combination model best predicts the data, we suggest a) modifying the collinear contrast model to accommodate negative search slopes b) attempting to find experimental conditions that best differentiate between the models, perhaps by using feature dimensions other than colour and c) modifying the experimental design to enforce parallel processing e.g. by making the display gaze contingent.

Computational modelling approaches alongside detailed, quantitative theory building has been argued to be one way to improve the reliability of psychological research (Oberauer and Lewandowsky, 2019; Guest and Martin, 2021). By combining this approach with fully open datasets and analysis scripts, we can hopefully begin to take a more “distributed collaborative network” approach (Moshontz et al., 2018) to our scientific questions. As such, we would like to conclude by encouraging other researchers to critique, build on and improve the approach we have taken in this manuscript, in order to further improve our ability to model performance in visual search tasks.

### **Conflict of interest**

The authors declare that they have no conflict of interest.

### **Acknowledgements**

This work was supported by an Economic and Social Research Council grant (ES/S016120/1) to ADFC and employing AN. We thank the reviewers for their

806 helpful comments which greatly improved the manuscript.

## 807 **References**

- 808 Stefanie I Becker. The role of target–distractor relationships in guiding attention  
809 and the eyes in visual search. *Journal of Experimental Psychology: General*,  
810 139(2):247, 2010.
- 811 Stefanie I Becker, Christian Valuch, and Ulrich Ansorge. Color priming in pop-  
812 out search depends on the relative color of the target. *Frontiers in psychology*,  
813 5:289, 2014.
- 814 Aline Bompas, Petroc Sumner, and Craig Hedge. Non-decision time: the higg’s  
815 boson of decision. *bioRxiv*, pages 2023–02, 2023.
- 816 Simona Buetti, Deborah A Cronin, Anna M Madison, Zhiyuan Wang, and Ale-  
817 jandro Lleras. Towards a better understanding of parallel visual processing in  
818 human vision: Evidence for exhaustive analysis of visual information. *Journal*  
819 *of Experimental Psychology: General*, 145(6):672, 2016.
- 820 Simona Buetti, Jing Xu, and Alejandro Lleras. Predicting how color and shape  
821 combine in the human visual system to direct attention. *Scientific reports*, 9(1):  
822 1–11, 2019.
- 823 Marisa Carrasco, Denise L Evert, Irene Chang, and Svetlana M Katz. The eccen-  
824 tricity effect: Target eccentricity affects performance on conjunction searches.  
825 *Perception & psychophysics*, 57:1241–1261, 1995.
- 826 Kyle R Cave and Jeremy M Wolfe. Modeling the role of parallel processing in  
827 visual search. *Cognitive psychology*, 22(2):225–271, 1990.
- 828 Alasdair DF Clarke, Jessica L Irons, Warren James, Andrew B Leber, and  
829 Amelia R Hunt. Stable individual differences in strategies within, but not be-  
830 tween, visual search tasks. *Quarterly Journal of Experimental Psychology*, 75  
831 (2):289–296, 2022a.
- 832 Alasdair DF Clarke, Anna Nowakowska, and Amelia R Hunt. Visual search habits  
833 and the spatial structure of scenes. *Attention, Perception, & Psychophysics*, 84  
834 (6):1874–1885, 2022b.

- 835 Elaine A Corbett and Philip L Smith. A diffusion model analysis of target detec-  
836 tion in near-threshold visual search. *Cognitive Psychology*, 120:101289, 2020.
- 837 John Duncan and Glyn W Humphreys. Visual search and stimulus similarity.  
838 *Psychological review*, 96(3):433, 1989.
- 839 Nathan J Evans and Eric-Jan Wagenmakers. Evidence accumulation models: Cur-  
840 rent limitations and future directions. 2019.
- 841 Andrew Gelman and Jennifer Hill. *Data analysis using regression and multi-*  
842 *level/hierarchical models*. Cambridge university press, 2006.
- 843 Olivia Guest and Andrea E Martin. How computational modeling can force theory  
844 building in psychological science. *Perspectives on Psychological Science*, 16  
845 (4):789–802, 2021.
- 846 Anna E Hughes, Rosy V Southwell, Iain D Gilchrist, and David J Tolhurst. Quan-  
847 tifying peripheral and foveal perceived differences in natural image patches to  
848 predict visual search performance. *Journal of vision*, 16(10):18–18, 2016.
- 849 Johan Hulleman and Christian NL Olivers. On the brink: The demise of the item  
850 in visual search moves closer. *Behavioral and Brain Sciences*, 40, 2017.
- 851 Jessica L Irons and Andrew B Leber. Choosing attentional control settings in a  
852 dynamically changing environment. *Attention, Perception, & Psychophysics*,  
853 78(7):2031–2048, 2016.
- 854 Jessica L Irons and Andrew B Leber. Characterizing individual variation in the  
855 strategic use of attentional control. *Journal of Experimental Psychology: Hu-*  
856 *man Perception and Performance*, 44(10):1637, 2018.
- 857 Laurent Itti and Christof Koch. A saliency-based search mechanism for overt and  
858 covert shifts of visual attention. *Vision research*, 40(10-12):1489–1506, 2000.
- 859 Laurent Itti, Christof Koch, and Ernst Niebur. A model of saliency-based visual  
860 attention for rapid scene analysis. *IEEE Transactions on pattern analysis and*  
861 *machine intelligence*, 20(11):1254–1259, 1998.
- 862 Paul D Kieffaber, Emily S Kappenman, Misty Bodkins, Anantha Shekhar, Brian F  
863 O’Donnell, and William P Hetrick. Switch and maintenance of task set in  
864 schizophrenia. *Schizophrenia research*, 84(2-3):345–358, 2006.

- 865 Christof Koch and Shimon Ullman. Shifts in selective visual attention: towards  
866 the underlying neural circuitry. In *Matters of intelligence*, pages 115–141.  
867 Springer, 1987.
- 868 Kathryn Koehler, Fei Guo, Sheng Zhang, and Miguel P Eckstein. What do  
869 saliency models predict? *Journal of vision*, 14(3):14–14, 2014.
- 870 Iuliia Kotseruba, Calden Wloka, Amir Rasouli, and John K Tsotsos. Do saliency  
871 models detect odd-one-out targets? new datasets and evaluations. *arXiv*  
872 *preprint arXiv:2005.06583*, 2020.
- 873 Árni Kristjánsson, Ómar I Jóhannesson, and Ian M Thornton. Common attentional  
874 constraints in visual foraging. *PloS one*, 9(6):e100752, 2014.
- 875 Joseph Krummenacher and Hermann J Müller. Dynamic weighting of feature di-  
876 mensions in visual search: behavioral and psychophysiological evidence. *Fron-*  
877 *tiers in psychology*, 3:221, 2012.
- 878 Walden Y Li, Molly R McKinney, Jessica L Irons, and Andrew B Leber. As-  
879 sessing the generality of strategy optimization across distinct attentional tasks.  
880 *Journal of Experimental Psychology: Human Perception and Performance*, 48  
881 (6):582, 2022.
- 882 Heinrich René Liesefeld and Hermann J Müller. A theoretical attempt to revive  
883 the serial/parallel-search dichotomy. *Attention, Perception, & Psychophysics*,  
884 82(1):228–245, 2020.
- 885 Heinrich René Liesefeld, Rani Moran, Marius Usher, Hermann J Müller, and  
886 Michael Zehetleitner. Search efficiency as a function of target saliency: The  
887 transition from inefficient to efficient search and beyond. *Journal of Experi-*  
888 *mental Psychology: Human Perception and Performance*, 42(6):821, 2016.
- 889 Alejandro Lleras, Zhiyuan Wang, Anna Madison, and Simona Buetti. Predicting  
890 search performance in heterogeneous scenes: Quantifying the impact of homo-  
891 geneity effects in efficient search. *Collabra: Psychology*, 5(1), 2019.
- 892 Alejandro Lleras, Zhiyuan Wang, Gavin Jun Peng Ng, Kirk Ballew, Jing Xu, and  
893 Simona Buetti. A target contrast signal theory of parallel processing in goal-  
894 directed search. *Attention, Perception, & Psychophysics*, pages 1–32, 2020.



- 895 Anna Madison, Alejandro Lleras, and Simona Buetti. The role of crowding in par-  
896 allel search: Peripheral pooling is not responsible for logarithmic efficiency in  
897 parallel search. *Attention, Perception, & Psychophysics*, 80(2):352–373, 2018.
- 898 Dora Matzke and Eric-Jan Wagenmakers. Psychological interpretation of the ex-  
899 gaussian and shifted wald parameters: A diffusion model analysis. *Psycho-*  
900 *nomics bulletin & review*, 16(5):798–817, 2009.
- 901 Richard McElreath. *Statistical rethinking: A Bayesian course with examples in R*  
902 *and Stan*. Chapman and Hall/CRC, 2020.
- 903 Rani Moran, Michael Zehetleitner, Hermann J Mueller, and Marius Usher. Com-  
904 petitive guided search: Meeting the challenge of benchmark rt distributions.  
905 *Journal of Vision*, 13(8):24–24, 2013.
- 906 Rani Moran, Michael Zehetleitner, Heinrich René Liesefeld, Hermann J Müller,  
907 and Marius Usher. Serial vs. parallel models of attention in visual search: ac-  
908 counting for benchmark rt-distributions. *Psychonomic bulletin & review*, 23(5):  
909 1300–1315, 2016.
- 910 Rani Moran, Heinrich René Liesefeld, Marius Usher, and Hermann J Muller. An  
911 appeal against the item’s death sentence: Accounting for diagnostic data pat-  
912 terns with an item-based model of visual search. *Behavioral and Brain Sci-*  
913 *ences*, 40:e148, 2017.
- 914 J Toby Mordkoff and Steven Yantis. An interactive race model of divided at-  
915 tention. *Journal of Experimental Psychology: Human Perception and Perfor-*  
916 *mance*, 17(2):520, 1991.
- 917 Hannah Moshontz, Lorne Campbell, Charles R Ebersole, Hans IJzerman,  
918 Heather L Urry, Patrick S Forscher, Jon E Grahe, Randy J McCarthy, Erica D  
919 Musser, Jan Antfolk, et al. The psychological science accelerator: Advancing  
920 psychology through a distributed collaborative network. *Advances in Methods*  
921 *and Practices in Psychological Science*, 1(4):501–515, 2018.
- 922 Vidhya Navalpakkam and Laurent Itti. Search goal tunes visual features optimally.  
923 *Neuron*, 53(4):605–617, 2007.
- 924 Gavin JP Ng, Simona Buetti, Trisha N Patel, and Alejandro Lleras. Prioritiza-  
925 tion in visual attention does not work the way you think it does. *Journal of*  
926 *Experimental Psychology: Human Perception and Performance*, 2020.

- 927 Gavin Jun Peng Ng, Alejandro Lleras, and Simona Buetti. Fixed-target efficient  
928 search has logarithmic efficiency with and without eye movements. *Attention,*  
929 *Perception, & Psychophysics*, 80(7):1752–1762, 2018.
- 930 Anna Nowakowska, Alasdair DF Clarke, and Amelia R Hunt. Human visual  
931 search behaviour is far from ideal. *Proceedings of the Royal Society B: Biolog-*  
932 *ical Sciences*, 284(1849):20162767, 2017.
- 933 Klaus Oberauer and Stephan Lewandowsky. Addressing the theory crisis in psy-  
934 chology. *Psychonomic bulletin & review*, 26:1596–1618, 2019.
- 935 Jason Osborne. Notes on the use of data transformations. *Practical assessment,*  
936 *research, and evaluation*, 8(1):6, 2002.
- 937 Evan M Palmer, Todd S Horowitz, Antonio Torralba, and Jeremy M Wolfe. What  
938 are the shapes of response time distributions in visual search? *Journal of ex-*  
939 *perimental psychology: human perception and performance*, 37(1):58, 2011.
- 940 Derrick Parkhurst, Klinto Law, and Ernst Niebur. Modeling the role of salience  
941 in the allocation of overt visual attention. *Vision research*, 42(1):107–123, 2002.
- 942 Jonathan Peirce, Jeremy R Gray, Sol Simpson, Michael MacAskill, Richard  
943 Höchenberger, Hiroyuki Sogo, Erik Kastman, and Jonas Kristoffer Lindeløv.  
944 Psychopy2: Experiments in behavior made easy. *Behavior research methods*,  
945 51(1):195–203, 2019.
- 946 Michael J Proulx. Individual differences and metacognitive knowledge of visual  
947 search strategy. *PLoS One*, 6(10):e27043, 2011.
- 948 Dragan Rangelov, Hermann J Müller, and Michael Zehetleitner. Failure to pop  
949 out: Feature singletons do not capture attention under low signal-to-noise ratio  
950 conditions. *Journal of Experimental Psychology: General*, 146(5):651, 2017.
- 951 Ruth Rosenholtz, Jie Huang, Alvin Raj, Benjamin J Balas, and Livia Ilie. A sum-  
952 mary statistic representation in peripheral vision explains visual search. *Journal*  
953 *of vision*, 12(4):14–14, 2012.
- 954 Benjamin W Tatler. The central fixation bias in scene viewing: Selecting an op-  
955 timal viewing position independently of motor biases and image feature distri-  
956 butions. *Journal of vision*, 7(14):4–4, 2007.

- 957 Benjamin W Tatler, Mary M Hayhoe, Michael F Land, and Dana H Ballard. Eye  
958 guidance in natural vision: Reinterpreting salience. *Journal of vision*, 11(5):  
959 5–5, 2011.
- 960 Anne Treisman and Stephen Gormican. Feature analysis in early vision: evidence  
961 from search asymmetries. *Psychological review*, 95(1):15, 1988.
- 962 Anne M Treisman and Garry Gelade. A feature-integration theory of attention.  
963 *Cognitive psychology*, 12(1):97–136, 1980.
- 964 Igor S Utochkin. Visual search with negative slopes: The statistical power of  
965 numerosity guides attention. *Journal of vision*, 13(3):18–18, 2013.
- 966 Zhiyuan Wang, Simona Buetti, and Alejandro Lleras. Predicting search perfor-  
967 mance in heterogeneous visual search scenes with real-world objects. *Collabra:*  
968 *Psychology*, 3(1), 2017.
- 969 Zhiyuan Wang, Alejandro Lleras, and Simona Buetti. Parallel, exhaustive process-  
970 ing underlies logarithmic search functions: Visual search with cortical magni-  
971 fication. *Psychonomic bulletin & review*, 25:1343–1350, 2018.
- 972 Jeremy M Wolfe. What can 1 million trials tell us about visual search? *Psycho-*  
973 *logical Science*, 9(1):33–39, 1998.
- 974 Jeremy M Wolfe. Approaches to visual search: Feature integration theory and  
975 guided search. *Oxford handbook of attention*, pages 11–55, 2014.
- 976 Jeremy M Wolfe and Michael J Van Wert. Varying target prevalence reveals two  
977 dissociable decision criteria in visual search. *Current biology*, 20(2):121–124,  
978 2010.
- 979 Jeremy M Wolfe, Kyle R Cave, and Susan L Franzel. Guided search: an alterna-  
980 tive to the feature integration model for visual search. *Journal of Experimental*  
981 *Psychology: Human perception and performance*, 15(3):419, 1989.
- 982 Jeremy M Wolfe, Evan M Palmer, and Todd S Horowitz. Reaction time distri-  
983 butions constrain models of visual search. *Vision research*, 50(14):1304–1311,  
984 2010.
- 985 Zoe Jing Xu, Alejandro Lleras, and Simona Buetti. Predicting how surface texture  
986 and shape combine in the human visual system to direct attention. *Scientific*  
987 *reports*, 11(1):1–13, 2021.

- 988 Xinger Yu, Timothy D Hanks, and Joy J Geng. Attentional guidance and match  
989 decisions rely on different template information during visual search. *Psycho-*  
990 *logical science*, 33(1):105–120, 2022.
- 991 Xinger Yu, Zhiheng Zhou, Stefanie I Becker, Sage EP Boettcher, and Joy J Geng.  
992 Good-enough attentional guidance. *Trends in Cognitive Sciences*, 2023.
- 993 Lingyun Zhang, Matthew H Tong, Tim K Marks, Honghao Shan, and Garrison W  
994 Cottrell. Sun: A bayesian framework for saliency using natural statistics. *Jour-*  
995 *nal of vision*, 8(7):32–32, 2008.