# Recipe 3: The newsvendor model

## Step 0: Write down all your inputs $(c,\ r,\ s,\ \mu,\ \sigma)$.

- unit (manufacturing) cost, $c$: unit wholesale price that the newsvendor pays per unit of good sold.
- unit revenue, $r$: selling price per unit charged by the newsvendor to its customers.
- unit salvage value, $s$: value that the newsvendor can recover for left-over goods.
- Demand $(D)$ follows $F(D)$ with mean $\mu$ and standard deviation $\sigma$.

*Example*: Demand is estimated to follow $\mathcal{N}(100,\ 25)$. Thus, $\mu = 100$ and $\sigma = 5$.

- unit underage cost, $C_u$: the opportunity cost of not ordering a unit that could have been sold.

$$C_u = r - c$$

  Intuitively, each unit of shortage represents missing a opportunity of the unit margin $r - c$.

- unit overage cost, $C_o$ : the loss incured when a unit is ordered but not sold.

$$C_o = c - s$$

  Intuitively, each unit of overage represents a salvage at loss of the difference of what was paid for it $(c)$ less the recovered salvage value $(s)$.

- Critical ratio: $\frac{C_u}{C_u + C_o}$.

## Step 1: Find the expected profit-maximizing order quantity, $Q$

- Case 1: If $D$ follows $\mathcal{N}(\mu, \sigma^2)$ then:
  - Find the $z$-value in the Standard Normal Distribution Function Table such that

$$\Phi(z) = \frac{C_u}{C_u + C_o}$$

*Round-up rule*: If the critical ratio value does not exist in the table, then find the two $z$-values it falls between and choose the larger of them. For example, the critical ratio 0.7778 falls between $z_1 = 0.76$ and $z_2 = 0.77$, so choose $z = 0.77$. On MS Excel: =normsinv(Cu/(Cu+Co)).

  - Convert $z$ into the order quantity that maximizes expected profit, $Q$:

$$Q = \mu + z \cdot \sigma$$

- Case 2: If $D$ follows a discrete distribution $F(\cdot)$, then find the quantity on the values given which equals the critical ratio, i.e. find the $Q$ such that

$$F(Q) = \frac{C_u}{C_u + C_o}$$

*Round-up rule*: If the critical ratio value falls between two entries on the table, choose the entry with the larger quantity.

## Step 2: Calculate several performance measures

1. Use the $z$-value you found before to look up in the Standard Normal Loss Function Table the corresponding $L(z)$-value. On MS Excel to find $L(z)$ for a given $z$: =z * normsdist(z) + normsdist(z, 0, 1, FALSE) - z. Then

$$\text{Expected lost sales} = \sigma \cdot L(z)$$

2. Expected sales $= \mu -$ Expected lost sales
3. Expected leftover inventory $= Q -$ Expected sales. It should not be surprising when both exp. leftover inventory and exp. lost sales are positive! (Why?)
4. Expected profit $= [(r - c) \times \text{Expected sales}] - [(c - s) \times \text{Expected leftover inventory}]$
5. In-stock probability: the probability the firm ends the season having satisfied all demand, or equival. the prob. the firm has stock available for every customer. This occurs if

demand is less than the order quantity

$$\text{In-stock probability} = F(Q)$$

Stock-out probability: the prob. the firm stocks out for some customer during the selling season (i.e. a lost sale occurs because demand exceeds the order quantity)

$$\text{Stockout probability} = 1 - F(Q) = 1 - \text{In-stock probability}$$

If you have computed the $z$-value above then look up on the Standard Normal Distribution Function Table to find $\Phi(z)$ which equals the in-stock probability. So, stock-out probability is $1 - \Phi(z)$. (For discrete distributions, the stock-out probability is $1 - F(Q)$, where $F(Q)$ is the probability that demand is $Q$ or lower).

- Note that we could use the steps in (5) above in reverse order to determine an order quantity $Q$ that satisfies a target in-stock probability: first, find the $\Phi(z)$ value that corresponds to $z$ you have (apply the "round-up rule" if needed). Then, calculate $Q = \mu + z \cdot \sigma$. (refer to the exercises we solved in class for an example)

# Notes and definitions

1. *In-stock probability*: the probability the firm ends the season having satisfied all demand, or equival. the prob. the firm has stock available for every customer. This occurs if demand is less than the order quantity

   $$\text{In-stock probability} = F\left(Q\right)$$

   *Stock-out probability*: the prob. the firm stocks out for some customer during the selling season (i.e. a lost sale occurs because demand exceeds the order quantity)

   $$\text{Stockout probability} = 1 - F\left(Q\right) = 1 - \text{In-stock probability}$$

   If you have computed the $z$-value above then look up on the Standard Normal Distribution Function Table to find $\Phi\left(z\right)$ which equals the in-stock probability. So, the stock-out probability is $1-\Phi\left(z\right)$. (For discrete distributions, the stock-out probability is $1-F\left(Q\right)$, where $F\left(Q\right)$ is the probability that demand is $Q$ or lower). We could use the previous steps in reverse order to determine an order quantity $Q$ that satisfies a target in-stock probability. First, find the $\Phi\left(z\right)$ value that corresponds to $z$ you have (apply the "round-up rule" if needed). Second, calculate $Q = \mu + z \cdot \sigma$.

2. *Fill rate*: the expected ratio of satisfied demand (i.e. sales) to total demand:

   $$\begin{aligned}\text{Fill rate} &= \frac{\text{Expected sales}}{\text{Expected demand}} = \frac{\text{Expected sales}}{\mu} = 1 - \frac{\text{Expected lost sales}}{\mu} \\ &= 1 - \frac{\text{Expected lost sales}}{\mu} = 1 - \frac{\sigma \cdot L\left(z\right)}{\mu}\end{aligned}$$

   In other words, the fill rate is the percentage of demand that is satisfied. The fill rate is a measure of customer service: The higher the fill rate, the more likely a customer will find a unit available to purchase. *Example*: the fill rate is 89.6 percent.

3. Terms used to describe our inventory:

   a) *On-order inventory*: The number of units that we have ordered in previous periods that we have not yet received. Our on-order inventory should never be negative, but it can be zero.

   b) *On-hand inventory*: The number of units of inventory we have on-hand, immediately available to serve demand.

c) *Pipeline inventory*: The expected on-order inventory, that is the average amount of inventory on order at any given time. Referring to Little's Law:

$$\text{Inventory} = \text{Flow rate} \times \text{Flow time}$$

Inventory is the number of units on order; flow rate is the expected demand (the expected order equals the expected demand, so on-order inventory is being created at a rate equal to expected demand); and flow time is the lead time, how much time does every unit spend on order. Therefore,

$$\text{Pipeline inventory} = \text{Expected demand} \times \text{Lead time}$$

*Note*: The above equation holds for any demand distribution (Normal, empirical, etc...), because Little's Law depends only on average rates, and *not* on the variability of those rates. *Example*: Patients are waiting in a radiology unit spend for their chest X-ray. Inventory reflects here the time a flow unit has to spend in the process in order to be transformed from input to output. Even with unlimited resources, patients would still need to spend some time in the interventional radiology unit; their flow time would be the length of the critical path. Some times, you might hear managers make statements of the type "we need to achieve zero inventory in our process". If we substitute Pipeline inventory=0 into Little's Law above, the immediate result is that a process with zero inventory is also a process with zero flow rate (unless we have zero flow time, which means that the process does not do anything to the flow unit). Thus, as long as it takes an operation even a minimum amount of time to work on a flow unit, the process will always exhibit pipeline inventory. There can be no hospitals without patients waiting and no factory can operate without some work in process! Little's Law also points us toward the best way to reduce pipeline inventory. As reducing flow rate (and with it demand and profit) is typically not a desirable option, the *only* other way to reduce pipeline inventory is by reducing flow time.