Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

BU7150 — Business Decision Optimization

Nonlinear Programming

Dr. Isilay Talay
Assistant Professor in Operations and Supply Chain Management

Date 02/07/2022

Practice Problems from Last Week

Rolls Bakery

Discussion of the Solution

The values of the shadow prices

The allowable increase and decrease of the resource/requirement values

Practice Problems from Last Week

Political Advertisement Agency

Discussion of the Solution

The values of the shadow prices

The allowable increase and decrease of the resource/requirement values

Practice Problems from Last Week

Landhills Winery

Three stages of reading:

1) What is the business function and context of application? What is the role of the decision maker?

Seasonal blending plan for wine ingredients so that the required quality will be obtained with the minimum resource utilization, made by a vintner working for a premium wine producer

Summary of other contextual information

-Types of wine: varietal and blends

-Grape characteristics and government regulations

-The vintner assumes all bottles produced could be sold

Practice Problems from Last Week

Landhills Winery

Three stages of reading:

2) What is the 'best' we are trying to achieve? What are our limitations? What do we need to decide to create our blending plan?

Maximize the net profit, we need to consider: grape availability, acidity level upper bound, sugar upper bound, varietal wine inclusion lower bound, alcohol level lower and upper bounds, vintage wine year inclusion lower bound, vintage wine viticulture area inclusion lower bound

First, we need to determine which products we can produce, since they can be categorized based on whether the product is vintage, and if so, which viticulture and year it is labelled as

Practice Problems from Last Week

Landhills Winery

Three stages of reading:

2) What is the ‘best’ we are trying to achieve? What are our limitations? What do we need to decide to create our blending plan?

Based on the case information, we can produce:

-Vintage Cabernet Sauvignon-Santa Barbara-2011 (model product code: 1)

-Vintage Cabernet Sauvignon-San Luis Obispo-2010 (model product code: 2)

-Vintage Cabernet Sauvignon-San Luis Obispo-2011 (model product code: 3)

-Non-Vintage Cabernet Sauvignon (model product code: 4)

-Non-Vintage Merlot (model product code: 5)

Practice Problems from Last Week

Landhills Winery

Three stages of reading:

2) What is the ‘best’ we are trying to achieve? What are our limitations? What do we need to decide to create our blending plan?

To produce these products, I can use the grape types listed in the case:

-Cabernet Sauvignon-Santa Barbara-2011 (model ingredient code: 1)

-Cabernet Sauvignon-San Luis Obispo-2010 (model ingredient code: 2)

-Cabernet Sauvignon-San Luis Obispo-2011 (model ingredient code: 3)

-Merlot (model ingredient code: 4)

Practice Problems from Last Week

Landhills Winery

Three stages of reading:

2) What is the ‘best’ we are trying to achieve? What are our limitations? What do we need to decide to create our blending plan?

Units (?)	Cabernet Sauvignon-Santa Barbara-2011	Cabernet Sauvignon-San Luis Obispo-2010	Cabernet Sauvignon-San Luis Obispo-2011	Merlot
Vintage Cabernet Sauvignon-Santa Barbara-2011	1,1	1,2	1,3	1,4
Vintage Cabernet Sauvignon-San Luis Obispo-2010	2,1	2,2	2,3	2,4
Vintage Cabernet Sauvignon-San Luis Obispo-2011	3,1	3,2	3,3	3,4
Non-Vintage Cabernet Sauvignon	4,1	4,2	4,3	4,4
Non-Vintage Merlot	5,1	5,2	5,3	5,4

Practice Problems from Last Week

Landhills Winery

Three stages of reading:

2) What is the ‘best’ we are trying to achieve? What are our limitations? What do we need to decide to create our blending plan?

It is possible to measure both the products and the ingredients in bottles, and the monetary values are given in terms of bottles as well as the others parameters could be transformed into bottles since standard bottle size is 750 ml, so it would be easier to decide on how many bottles of each ingredient to put into each product

Practice Problems from Last Week

Landhills Winery

Three stages of reading:

3) How can we express these descriptions in stage 2 mathematically so that we can enter them into an optimization software and solve our production planning problem?

Decisions:

Bottles	Cabernet Sauvignon-Santa Barbara-2011	Cabernet Sauvignon-San Luis Obispo-2010	Cabernet Sauvignon-San Luis Obispo-2011	Merlot
Vintage Cabernet Sauvignon-Santa Barbara-2011	X_11	X_12	X_13	X_14
Vintage Cabernet Sauvignon-San Luis Obispo-2010	X_21	X_22	X_23	X_24
Vintage Cabernet Sauvignon-San Luis Obispo-2011	X_31	X_32	X_33	X_34
Non-Vintage Cabernet Sauvignon	X_41	X_42	X_43	X_44
Non-Vintage Merlot	X_51	X_52	X_53	X_54

Practice Problems from Last Week

Landhills Winery

Three stages of reading:

3) How can we express these descriptions in stage 2 mathematically so that we can enter them into an optimization software and solve our production planning problem?

Decisions:

Bottles	Cabernet Sauvignon-Santa Barbara-2011	Cabernet Sauvignon-San Luis Obispo-2010	Cabernet Sauvignon-San Luis Obispo-2011	Merlot	Total
Vintage Cabernet Sauvignon-	X_11	X_12	X_13	X_14	X_11+X_12+X_13+X_14
Vintage Cabernet Sauvignon-San Luis Obispo-2010	X_21	X_22	X_23	X_24	X_21+X_22+X_23+X_24
Vintage Cabernet Sauvignon-San Luis Obispo-2011	X_31	X_32	X_33	X_34	X_31+X_32+X_33+X_34
Non-Vintage Cabernet	X_41	X_42	X_43	X_44	X_41+X_42+X_43+X_44
Non-Vintage Merlot	X_51	X_52	X_53	X_54	X_51+X_52+X_53+X_54
Total	$\Sigma_{\{i\}}$ X_i1	$\Sigma_{\{i\}}$ X_i2	$\Sigma_{\{i\}}$ X_i3	$\Sigma_{\{i\}}$ X_i4	

Practice Problems from Last Week

Landhills Winery

Three stages of reading:

3) How can we express these descriptions in stage 2 mathematically so that we can enter them into an optimization software and solve our production planning problem?

Objective:

Maximize (Revenue) – (Cost)

$$\left(\sum_{j=1}^{4} X_{1j}\right)\text{bottles}*9\,\frac{dollars}{bottle} + \left(\sum_{j=1}^{4} X_{2j}\right)\text{bottles}*9\,\frac{dollars}{bottle}$$

$$+\left(\sum_{j=1}^{4} X_{3j}\right)\text{bottles}*9\,\frac{dollars}{bottle} + \left(\sum_{j=1}^{4} X_{4j}\right)\text{bottles}*5.5\,\frac{dollars}{bottle}$$

$$+\left(\sum_{j=1}^{4} X_{5j}\right)\text{bottles}*2.95\,\frac{dollars}{bottle} - \left(\sum_{i=1}^{5} X_{i1}\right)\text{bottles}*2.35\,\frac{dollars}{bottle}$$

$$-\left(\sum_{i=1}^{5} X_{i2}\right)\text{bottles}*2.60\,\frac{dollars}{bottle} - \left(\sum_{i=1}^{5} X_{i3}\right)\text{bottles}*2.10\,\frac{dollars}{bottle}$$

$$-\left(\sum_{i=1}^{5} X_{i4}\right)\text{bottles}*1.55\,\frac{dollars}{bottle}$$

Practice Problems from Last Week

Landhills Winery

Three stages of reading:

3) How can we express these descriptions in stage 2 mathematically so that we can enter them into an optimization software and solve our production planning problem?

Objective:

Maximize (Revenue) – (Cost)

$$X_{11} * (9 - 2.35) + X_{12} * (9 - 2.60) + \ldots$$

	Grape 1	Grape 2	Grape 3	Grape 4	Total	Wholesale Price
Cost	2.35	2.6	2.1	1.55		
Unit Profit Wine 1	6.65	6.4	6.9	7.45		9
Unit Profit Wine 2	6.65	6.4	6.9	7.45		9
Unit Profit Wine 3	6.65	6.4	6.9	7.45		9
Unit Profit Wine 4	3.15	2.9	3.4	3.95		5.5
Unit Profit Wine 5	0.6	0.35	0.85	1.4		2.95

Practice Problems from Last Week

Landhills Winery

Three stages of reading:

3) How can we express these descriptions in stage 2 mathematically so that we can enter them into an optimization software and solve our production planning problem?

Constraints:

Acidity Level for Vintage Cabernet Sauvignon-Santa Barbara-2011 (standard wine bottle 750 ml):

$$\frac{X_{11} bottles * 7.5 \frac{100ml}{bottle} * 0.35 \frac{gm}{100ml} + \cdots}{X_{11} bottles * 7.5 \frac{100ml}{bottle} + \cdots} \leq 0.7 \frac{gm}{100ml}$$

$$\frac{X_{11} * {\color{red}7.5} * 0.35 + \cdots}{X_{11} * {\color{red}7.5} + \cdots} \leq 0.7$$

$$\frac{X_{11} * 0.35 + \cdots}{X_{11} + \cdots} \leq 0.7$$

$$X_{11} * 0.35 + \cdots \leq 0.7 * (X_{11} + \cdots)$$

$$X_{11} * (-0.35) + \cdots \leq 0$$

Constraints					LHS	RHS
Acidity Wine 1	-0.35	0.05	-0.15	-0.45	-17750	0
Acidity Wine 2	-0.35	0.05	-0.15	-0.45	0	0
Acidity Wine 3	-0.35	0.05	-0.15	-0.45	0	0
Acidity Wine 4	-0.35	0.05	-0.15	-0.45	-9897.45	0
Acidity Wine 5	0.05	0.45	0.25	-0.05	-0.05	0

14-8 Nonlinear Optimization Models

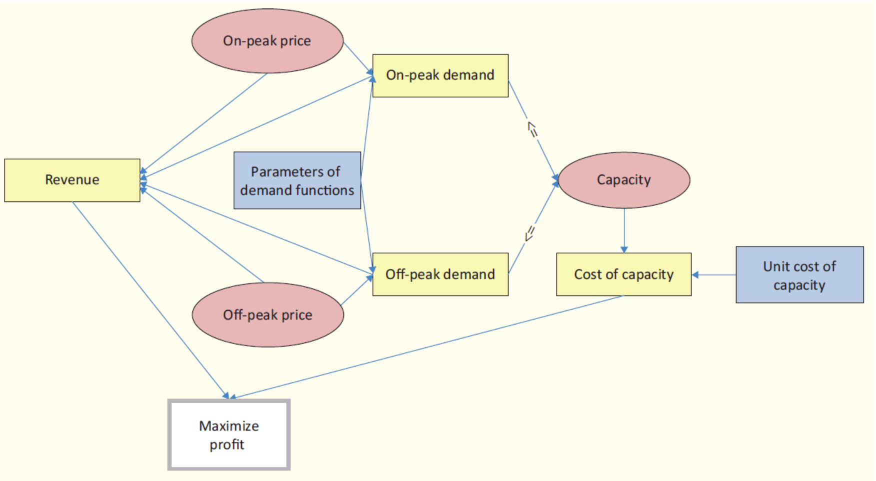
- In a **nonlinear programming (NLP) model**, the objective and/or the constraints are nonlinear functions of the decision variables.
- When you solve an NLP model, it is very possible that Solver will obtain a suboptimal solution.
  - This is because a nonlinear function can have local optimal solutions that are not the global optimal solution.
    - A **local optimal solution** is one that is better than all nearby points.
    - The **global optimum** is the one that beats all points in the entire feasible region.
- There are mathematical conditions that guarantee that the Solver solution is the global optimum, but these are difficult to check, and they aren't always satisfied.
  - A simpler way is to run Solver several times, each time with different starting values in the decision variable cells.

Example 14.11: Electricity Pricing at Florida Power and Light (slide 1 of 2)

- **Objective:** To use a nonlinear model to determine prices and capacity when there are two different daily usage patterns, on-peak and off-peak.
- **Solution:** Florida Power and Light must determine the price per megawatt hour (mWh) to charge during both peak-load and off-peak periods.
- The monthly demand for power during each period (in mWh) is related to price as follows:

$$D_p = 2.253 - 0.013P_p + 0.003P_o$$
$$D_o = 1.142 - 0.015\,P_o + 0.005P_p$$

  - D_p and P_p are demand and price during on-peak times, and D_0 and P_0 are demand and price during off-peak times.



- Assume it costs FPL $75 per month to maintain one mWh of capacity.

Example 14.11: Electricity Pricing at Florida Power and Light (slide 2 of 2)

- The spreadsheet model is shown below.

	A	B	C	D	E	F	G	H
1	**Electricity pricing model**							
2								
3	**Input data**					**Range names used:**		
4	Coefficients of demand functions					Capacity	=Model!$B$15	
5		Constant	On-peak price	Off-peak price		Common_Capacity	=Model!$B$21:$C$21	
6	On-peak demand	2.253	−0.013	0.003		Demands	=Model!$B$19:$C$19	
7	Off-peak demand	1.142	0.005	−0.015		Prices	=Model!$B$13:$C$13	
8						Profit	=Model!$B$26	
9	Cost of capacity/mWh	$75						
10								
11	**Decisions**							
12		On-peak	Off-peak					
13	Price per mWh	$137.57	$75.85					
14								
15	Capacity (millions of mWh)	0.692						
16								
17	**Constraints on demand (in million of mWh)**							
18		On-peak	Off-peak					
19	Demand	0.692	0.692					
20		<=	<=					
21	Capacity	0.692	0.692					
22								
23	**Monetary summary ($ millions)**							
24	Revenue	147.712						
25	Cost of capacity	51.908						
26	Profit	95.804						

14-8c Portfolio Optimization Models

- **Portfolio optimization models** are used to determine the percentage of assets to invest in stocks, gold, and Treasury bills.
- To do any work with investments, the following formulas must be understood:

$$\text{Expected value of } R_p = \mu_1 x_1 + \mu_2 x_2 + \cdots + \mu_n x_n$$

$$\text{Variance of } R_p = \sigma_1^2 x_1^2 + \sigma_2^2 x_2^2 + \cdots + \sigma_n^2 x_n^2 + \Sigma_{ij} \rho_{ij} \sigma_i \sigma_j x_i x_j$$

  - All investors want to choose portfolios with high return (measured by the expected value in the first equation), but they also want portfolios with low risk (usually measured by the variance in the second equation).
  - The second equation can be rewritten slightly by using *covariances* instead of correlations, as shown below.

$$\text{Estimated variance of } R_p = \Sigma_{i,j} c_{ij} x_i x_j$$

    - This makes calculating the portfolio variance very easy with matrix function of Excel®.

Matrix Functions in Excel®

(slide 1 of 2)

- Two built-in Excel matrix functions, MMULT and TRANSPOSE, simplify the calculation for the variance of portfolio return.
- The *matrix* is a rectangular array of numbers.
  - A matrix is an *i* x *j* matrix if it consists of *i* rows and *j* columns.
    - An example of a 2 x 3 matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$$

Matrix Functions in Excel®
(slide 2 of 2)

- If matrix A has the same number of columns as matrix B has rows, it is possible to calculate the *matrix product* of A and B, denoted AB.

For example, if

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

then AB is the following 2×2 matrix:

$$AB = \begin{pmatrix} 1(1) + 2(3) + 3(5) & 1(2) + 2(4) + 3(6) \\ 2(1) + 4(3) + 5(5) & 2(2) + 4(4) + 5(6) \end{pmatrix} = \begin{pmatrix} 22 & 28 \\ 39 & 50 \end{pmatrix}$$

- The Excel® MMULT function performs matrix multiplications in a single step.

The Portfolio Selection Model

- Most investors have two objectives in forming portfolios: to obtain a large expected return and to obtain a small variance (to minimize risk).
  - The problem is inherently nonlinear because the portfolio variance is nonlinear in the investment amounts.
  - The most common way of handling this two-objective problem is to specify a minimal required expected return and then minimize the variance subject to the constraint on the expected return.

Example 14.12: Portfolio Optimization at Perlman & Brothers (slide 1 of 3)

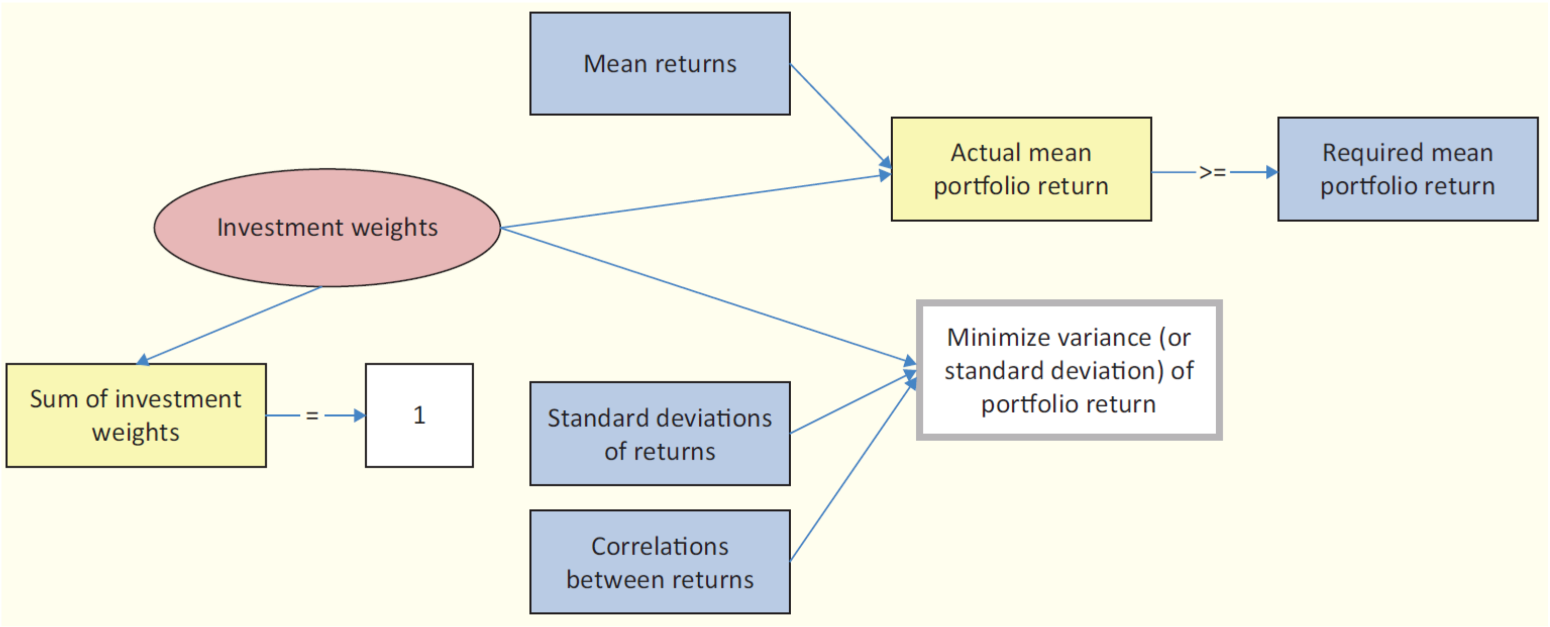
- **Objective:** To use NLP to find the portfolio that minimizes the risk, measured by portfolio variance, subject to achieving an expected return of at least 12%.
- **Solution:** Perlman & Brothers intends to invest a given amount of money in three stocks.
- The means and standard deviations of annual returns have been estimated in the table on the left.
- The correlations between the annual returns on the stocks are listed in the table on the right.

Stock	Mean	Standard Deviation
1	0.14	0.20
2	0.11	0.15
3	0.10	0.08

Combination	Correlation
Stocks 1 and 2	0.6
Stocks 1 and 3	0.4
Stocks 2 and 3	0.7

Example 14.12: Portfolio Optimization at Perlman & Brothers (slide 2 of 3)

- The big picture model is shown below.
- One interesting aspect of this model is that it is not necessary to specify the amount of money invested.



Example 14.12: Portfolio Optimization at Perlman & Brothers (slide 3 of 3)

- The portfolio optimization model is shown below.

	A	B	C	D	E	F	G	H	I
1	**Portfolio selection model**								
2									
3	**Stock input data**								
4		Stock 1	Stock 2	Stock 3					
5	Mean return	0.14	0.11	0.1					
6	StDev of return	0.2	0.15	0.08					
7									
8	Correlations	Stock 1	Stock 2	Stock 3		Covariances	Stock 1	Stock 2	Stock 3
9	Stock 1	1	0.6	0.4		Stock 1	0.04	0.018	0.0064
10	Stock 2	0.6	1	0.7		Stock 2	0.018	0.0225	0.0084
11	Stock 3	0.4	0.7	1		Stock 3	0.0064	0.0084	0.0064
12									
13	**Investment decisions**								
14		Stock 1	Stock 2	Stock 3					
15	Investment weights	0.500	0.000	0.500					
16									
17	**Constraint on investing everything**								
18		Total weights		Required value					
19		1.00	=	1					
20									
21	**Constraint on expected portfolio return**								
22		Mean portfolio return		Required mean return					
23		0.120	>=	0.120					
24									
25	**Portfolio variance**	0.0148							
26	Portfolio stdev	0.1217							
27									
28	**Range names used:**								
29	Investment_weights	=Model!$B$15:$D$15							
30	Mean_portfolio_return	=Model!$B$23							
31	Portfolio_stdev	=Model!$B$26							
32	Portfolio_variance	=Model!$B$25							
33	Required_mean_return	=Model!$D$23							
34	Total_weights	=Model!$B$19							

Modeling Issues for Portfolio Selection Problems

- Typical real-world portfolio selection problems involve a large number of potential investments. However, the basic model does not change at all.
- If a company is allowed to short a stock, the fraction invested in that stock is allowed to be negative.
  - To implement this, eliminate the nonnegativity constraints on the decision variable cells.
- An alternative objective might be to minimize the probability that the portfolio loses money.

Keys to Solving Most Optimization Problems

- Determine the decision variable cells.
- Set up the model so that you can easily calculate what you wish to maximize or minimize.
- Set up the model so that the relationships between the cells in the spreadsheet and the constraints of the problem are readily apparent.
- Optimization models do not always fall into ready-made categories.
  - A model might involve a combination of the ideas discussed in the production scheduling, blending, and aggregate planning examples.

Post-Lecture Practice

1) Asllani, A. (2014). Business Analytics with Management Science Models and Methods. Ch. 4 – World Class Furniture

Canning Consultants: The OPAC Assessment 905E06-PDF-ENG
https://hbsp.harvard.edu/tu/fd052871