MINI CASE: WORLD CUP TOP GOAL SCORER[1]

Betting on sporting events is big business in the UK and abroad. For instance, according to the UK Gambling Commission 2010 statistics, it is estimated that the amount legally bet on Football events alone in Britain was over a billion pounds. One of the popular bets is on who will score the most goals throughout the season of World Cup. The attached table shows the odds on the list of 180 players posted by one of the largest on-line betting shops in Britain just before the start of the 2018 World Cup.

The table is interpreted as follows: if you bet on Lionel Messi, for example, the betting shop pays you 10 times your bet and you get your bet back if he becomes the top scorer at the end of the 2018 World Cup, and you lose your bet if he does not. We say the *odds* are 1 to 10. The odds posted are for a player becoming the top goal scorer: you may not bet against a player using its posted odds (though, you could theoretically create a bet against a player by using bets for the other players). And we assume only one top scorer in the end.

If a bet on Lionel Messi at these odds were a *fair* bet, then on average you would neither win nor lose by making this bet. Let *p* denote the probability of winning implied by a fair bet; hence, *(1-p)* is the probability of losing. Then, *10*p* is the anticipated amount to be won, *1*(1-p)* is the anticipated amount to be lost, and, if the bet is fair, *10*p = 1*(1-p)*. Solving for *p* yields *p = 1/11*. More generally, a fair bet with odds of *1* to *b* implies the probability p of winning is *p=1/(b+1)*.

Data file in Excel form can be downloaded from module page.

1. Using the odds given in the table, compute the probability implied by a fair bet for each player. Do these probabilities satisfy the rules for probability (i.e. do the probabilities add up to 1)?

An *arbitrage* opportunity exists if there is a sequence of gambles at the posted odds that *never loses money* and wins a positive amount with positive probability. Betting houses try to arrange the bets they accept in order to create and exploit an arbitrage opportunity.

2. Consider several schemes the betting house could use:

(a) Accept exactly 180 bets, each of size £1 on each of the 180 listed players;
(b) Accept exactly 180 bets -- £180 on Lionel Messi, £179 on Neymar, …, £1 on Tom Rogic.

Does either of these schemes represent an arbitrage opportunity for the betting house? If not, does an arbitrage opportunity exist, and what might one look like? (Hint: consider a scheme that accepts exactly 180 bets, with the amount accepted on each player proportional to the implied fair-bet probability you computed above.)

[1] Based on Patrick Larkey, "Fair Bets on Winners in Professional Golf," *Chance*, V. 3, N. 4, 1990, 24-26. This case was developed from the original cases written by Prof. Steven Lippman at UCLA and Dr. Steve Yoo at UCL. The course instructor thanks Dr. Steve Yoo at UCL for sharing the case materials. The data is based on https://www.nicerodds.co.uk/top-scorer as on 05 May 2018.