Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

**FACULTY OF ARTS, HUMANITIES & SOCIAL SCIENCES**

**Trinity Business School**

**MSc Business Analytics** **Semester 2**
**Annual Examination 2022-23**

# BU7148 Operations Analytics

**27/April/ 2023** **ONLINE** **9:30- 11:30**

**Shivam Gupta (External Examiner)**

**Dr. Konstantinos Stouras (Lecturer)**

**Time Allowed: 2 hours**

**Instructions for Candidates**

- This is an individual exam. You are responsible for the academic integrity of your submission.
- Distribution tables are available in the end of the exam paper.
- If anything is ambiguous to you, please state your assumption(s) and submit your answer according to your best interpretation of the question.
- Please answer all questions include all your answers in the separate dedicated Answer Sheet.
- Do not overthink!

**I. Business Intuition** (50 marks in total)

**(Q1)** Explain why we should always fully utilize the bottleneck of a process. Further, in order to increase the total output of a process would you recommend implementing yield improvements at steps before, at, or after the process' bottleneck resource? Why? You may use examples to illustrate your answers. (7 marks)

**(Q2)** Retail stores Yuppy and Snoopy both apply the newsvendor model to manage their inventories facing normally distributed demand with mean $\mu$=100. However, since Yuppy is a recently built store, its demand variability $\sigma$ is higher than Snoopy's. Compare the expected sales of Snoopy with those of Yuppy: are they higher, equal, or lower? Briefly justify your answer with reference to the definition of expected sales used in class. [Hint: "Variability is…"] (7 marks)

**(Q3) (a)** What is the most important pros and cons of short-term and long-term (business) relationships mentioned in class? Provide three assumptions that need to hold for long-term business relationships to work.
**(b)** Li & Fung is a B2B sourcing intermediary for buying from China. Companies such as Walmart or Target transact with a lot of Chinese factories directly, but for apparel or toys they only connect to suppliers abroad through the middleman Li & Fung. Conventional wisdom suggests that the advantage of sourcing through an intermediary lies mainly in any of the following 4 arguments: (i) intermediaries improve matching supply with demand for odd products, (ii) trust, (iii) price discovery, (iv) transaction costs. First, explain why none of the above traditional arguments provides a reason for why Walmart sources through Li & Fung for apparel. Then, consider a fifth advantage of intermediaries outlined in class and briefly elaborate on the key benefit for Walmart to source through Li & Fung for apparel. (10 marks)

**(Q4)** Which of the following levers *cannot* be used to reduce the amount of inventory for a firm?

a. Postpone product differentiation until closer to actual demand realization.
b. Reduce lead time.
c. Pool safety inventory for multiple locations.
d. Use common components.
e. Improve forecasting.
f. (d) and (e).
g. None of the above. (4 marks)

**(Q5)** Suppose the newsvendor model is used to manage inventories for a normally distributed demand N($\mu$=10, $\sigma$=2). Which of the following can happen when the order quantity is increased by one unit?

a. Expected sales increases by more than one unit.
b. Expected lost sales increases by more than one unit.
c. Expected leftover inventory increases by more than one unit.
d. Expected sales decreases by less than one unit.
e. Expected lost sales decreases by less than one unit.
f. Expected leftover inventory decreases by less than one unit.
g. None of the above. (7 marks)

**(Q6)** Fooslib is a start-up that rents out foosball tables by the hour to TCD students who seek to relax in between classes. Since TCD students are often overworked there is large variability in their demand for foosball table rentals. Fooslib's CEO was invited as a Guest Speaker to an Ops session and learnt there that revenue sharing contracts are a superior way to deal with this variability. As such, they consider ending their wholesale contract where they buy the foosball tables at a unit price of 1000 euros, and switching to a revenue sharing contract, where the upfront price is only 100 euros per table, but the supplier must also be paid 60% of Fooslib's revenues from every hour rented. What is definitely true if such a switch is implemented?

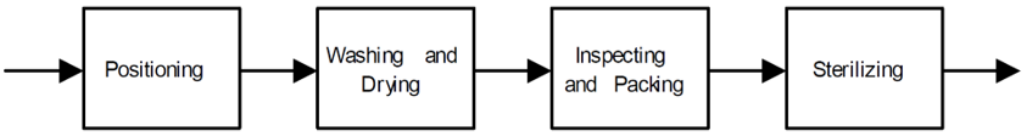
a. The cost of overage decreases, while the cost of underage increases.
b. The cost of overage increases, while the cost of underage decreases.
c. The cost of overage decreases, while the cost of underage decreases and Fooslib orders fewer tables.
d. The cost of overage decreases, while the cost of underage decreases and Fooslib orders more tables.
e. None of the above. (7 marks)

**(Q7)** In your first consulting assignment at Dublin Consulting Group (DCG), you are asked to evaluate the post MSc retention of the Master students of TeaCiDee, a top business school. TeaCiDee provided data to DCG stating that even though 31 Master Students graduate every year, they spend approximately less than 3 years on average with their first employer. Recalling what you were taught by Konstantinos, you look at the data in detail and you find that the number of Master students staying with the first employer after their TeaCiDee Master has been: 180; 92; 150; 103; 125. Based on the data provided, what can you conclude regarding the average number of years TeaCiDee's Graduate Students spend with their first employer? (8 marks)

**II. Quantitative Questions** (50 marks in total)

**(Q8) Sounds Ops to me** (26 marks in total)

Surgical instruments, when they are not in use, are stored in "trays" that are wrapped and kept sterile until they are needed. Once a tray of instruments has been used in surgery and returned to the supplies area the instruments must be 1) positioned for washing, 2) washed and dried, 3) inspected for damage and packed back in the tray, and 4) sterilized. The four-step process is shown below:




Positioning is done by three people working in parallel and a person can do one tray in 8 minutes.

The washing and drying is done by 10 automated machines. Each washes and dries a tray of instruments in 60 minutes. No labor is required except to place the instruments in the machine and take them out, which we will ignore for this problem.

Inspecting and packing can be broken into two cases. Everything is fine (80% of the trays) and some instrument is damaged (20% of the trays). When everything is fine the person can work at a rate of 6.67 trays/hour (or 9 minutes/tray). If any instrument is damaged the service rate of one person to (1) repair or find a replacement and (2) inspect and pack is 2.73 trays/hour (or 22 minutes/tray). Two people work at this stage, and both people do both steps. Union contracts restrict us to use full time labor for this task and they cannot be employed for any other task.

The sterilizing step is like a baking or kiln operation that cannot be interrupted to add more trays. The sterilizing step takes 2.5 hours and one "oven" can handle up to 24 trays at once. There is one oven available for this step.

**(Q8a)** Given this configuration and inputs, what is the maximum output rate of this entire process in trays/ hour? (6 marks)

**(Q8b)** An expansion of the Surgical Suite will require that trays be handled at the rate of 12 trays/hour. What changes or improvements need to be made to meet this demand? [Hint: Think of which steps in the above process would need an improvement to meet this demand] (10 marks)

**(Q8c)** Given the increase in the required handling rate of trays to 12 trays/hour, the hospital's COO, Ranbir Kapoor, has asked the administration team to come up with cost savings measures. One suggestion to cut costs is to reduce the damage rate from 20% to something below 5% (assume the reduction in the damage rate is costless). A former Ops Consultant, Ms. Alia Bhatt has computed that the savings are minor 15% (22-9 minutes) equal to about 2 minutes/tray. She argues that even though nearly 100 trays per day will be handled (assuming roughly 8 hour workdays), this is only about 3 or so hours of one person's time multiplied by his/her wage rate. Do you **agree** or **disagree** with Alia's calculation and estimate of the cost savings? Why? **Briefly justify your answer.** (10 marks)

**(Q9) Lazy Profs** (24 marks in total)

The TCD library has started a program for faculty called "book-on-my-desk". Under this pro-gram, TCD's faculty can reserve a book through the intranet which will be delivered to their office using the campus mail distribution service. When a new book on Management Research is to be released the library must decide how many copies of the book to order and have in its library so as to provide good service to the faculty (assume only faculty read such books). From past experience, the library knows that for each day after the release of a research book, on average 0.2% of the faculty will request this new book. You may assume that the average daily rate of these requests does not differ over the planning horizon, and you may also assume that the CVa = 1, i.e., the coefficient of variation of time between requests for this book is 1. Further assume that TCD has 125 faculty members and that all faculty members are too lazy to physically go to the library.

The library knows for such new releases, on average a book spends the following amounts of time in transport and with the faculty.

| Step | Time | Mean |
|---|---|---|
| (1) | for mail service to deliver the book to faculty | 1.5 day |
| (2) | for faculty to keep the book | 5.5 days |
| (3) | for faculty's assistant to return the book to the library | 1 day |

Therefore, the total time the book spends out of the library for each usage cycle is 8 days on an average. Assume that the standard deviation of this time is 8 days.

The library is interested in understanding how long the faculty that request the book in this period will have to wait on an average before they receive such a book should they request one. Although there is a small amount of processing time to receive the book by the library, pick the book from the shelf and dispatch it, assume that this is negligible compared to the time the faculty held the book and the time that the mail service and assistants spent in transporting it. So, if faculty's request is in stock (in the library), then it is dispatched to the mail service immediately, which then delivers it to faculty. If there are none in stock, then the faculty's request must wait until the book is returned by someone else.

In this problem, we model service requests for a book title as a queueing system, whose "service time" consists of the three steps above (1), (2) and (3).

**(Q9a)** In words, describe when does an arrival occur to this service system. Also, what are the servers? (3 marks)

**(Q9b)** In words, describe a departure from this queuing system. (3 marks)

**(Q9c)** What are the items in the queue? (3 marks)

**(Q9d)** At least how many books should the library purchase of this new release to ensure that faculty wait no more than 2 days on average from when they request a book until it is received by them in the mail. Briefly explain the intuition behind your solution. (15 marks)

**~ End ~**

[oOo]

**Table A:** Standardised Normal Probability Table

| **Z** | **0.00** | **-0.01** | **-0.02** | **-0.03** | **-0.04** | **-0.05** | **-0.06** | **-0.07** | **-0.08** | **-0.09** |
|---|---|---|---|---|---|---|---|---|---|---|
| **-2.9** | 0.00187 | 0.00181 | 0.00175 | 0.00169 | 0.00164 | 0.00159 | 0.00154 | 0.00149 | 0.00144 | 0.00139 |
| **-2.8** | 0.00256 | 0.00248 | 0.00240 | 0.00233 | 0.00226 | 0.00219 | 0.00212 | 0.00205 | 0.00199 | 0.00193 |
| **-2.7** | 0.00347 | 0.00336 | 0.00326 | 0.00317 | 0.00307 | 0.00298 | 0.00289 | 0.00280 | 0.00272 | 0.00264 |
| **-2.6** | 0.00466 | 0.00453 | 0.00440 | 0.00427 | 0.00415 | 0.00402 | 0.00391 | 0.00379 | 0.00368 | 0.00357 |
| **-2.5** | 0.00621 | 0.00604 | 0.00587 | 0.00570 | 0.00554 | 0.00539 | 0.00523 | 0.00508 | 0.00494 | 0.00480 |
| **-2.4** | 0.00820 | 0.00798 | 0.00776 | 0.00755 | 0.00734 | 0.00714 | 0.00695 | 0.00676 | 0.00657 | 0.00639 |
| **-2.3** | 0.01072 | 0.01044 | 0.01017 | 0.00990 | 0.00964 | 0.00939 | 0.00914 | 0.00889 | 0.00866 | 0.00842 |
| **-2.2** | 0.01390 | 0.01355 | 0.01321 | 0.01287 | 0.01255 | 0.01222 | 0.01191 | 0.01160 | 0.01130 | 0.01101 |
| **-2.1** | 0.01786 | 0.01743 | 0.01700 | 0.01659 | 0.01618 | 0.01578 | 0.01539 | 0.01500 | 0.01463 | 0.01426 |
| **-2.0** | 0.02275 | 0.02222 | 0.02169 | 0.02118 | 0.02068 | 0.02018 | 0.01970 | 0.01923 | 0.01876 | 0.01831 |
| **-1.9** | 0.02872 | 0.02807 | 0.02743 | 0.02680 | 0.02619 | 0.02559 | 0.02500 | 0.02442 | 0.02385 | 0.02330 |
| **-1.8** | 0.03593 | 0.03515 | 0.03438 | 0.03362 | 0.03288 | 0.03216 | 0.03144 | 0.03074 | 0.03005 | 0.02938 |
| **-1.7** | 0.04457 | 0.04363 | 0.04272 | 0.04182 | 0.04093 | 0.04006 | 0.03920 | 0.03836 | 0.03754 | 0.03673 |
| **-1.6** | 0.05480 | 0.05370 | 0.05262 | 0.05155 | 0.05050 | 0.04947 | 0.04846 | 0.04746 | 0.04648 | 0.04551 |
| **-1.5** | 0.06681 | 0.06552 | 0.06426 | 0.06301 | 0.06178 | 0.06057 | 0.05938 | 0.05821 | 0.05705 | 0.05592 |
| **-1.4** | 0.08076 | 0.07927 | 0.07780 | 0.07636 | 0.07493 | 0.07353 | 0.07215 | 0.07078 | 0.06944 | 0.06811 |
| **-1.3** | 0.09680 | 0.09510 | 0.09342 | 0.09176 | 0.09012 | 0.08851 | 0.08691 | 0.08534 | 0.08379 | 0.08226 |
| **-1.2** | 0.11507 | 0.11314 | 0.11123 | 0.10935 | 0.10749 | 0.10565 | 0.10383 | 0.10204 | 0.10027 | 0.09853 |
| **-1.1** | 0.13567 | 0.13350 | 0.13136 | 0.12924 | 0.12714 | 0.12507 | 0.12302 | 0.12100 | 0.11900 | 0.11702 |
| **-1.0** | 0.15866 | 0.15625 | 0.15386 | 0.15151 | 0.14917 | 0.14686 | 0.14457 | 0.14231 | 0.14007 | 0.13786 |
| **-0.9** | 0.18406 | 0.18141 | 0.17879 | 0.17619 | 0.17361 | 0.17106 | 0.16853 | 0.16602 | 0.16354 | 0.16109 |
| **-0.8** | 0.21186 | 0.20897 | 0.20611 | 0.20327 | 0.20045 | 0.19766 | 0.19489 | 0.19215 | 0.18943 | 0.18673 |
| **-0.7** | 0.24196 | 0.23885 | 0.23576 | 0.23270 | 0.22965 | 0.22663 | 0.22363 | 0.22065 | 0.21770 | 0.21476 |
| **-0.6** | 0.27425 | 0.27093 | 0.26763 | 0.26435 | 0.26109 | 0.25785 | 0.25463 | 0.25143 | 0.24825 | 0.24510 |
| **-0.5** | 0.30854 | 0.30503 | 0.30153 | 0.29806 | 0.29460 | 0.29116 | 0.28774 | 0.28434 | 0.28096 | 0.27760 |
| **-0.4** | 0.34458 | 0.34090 | 0.33724 | 0.33360 | 0.32997 | 0.32636 | 0.32276 | 0.31918 | 0.31561 | 0.31207 |
| **-0.3** | 0.38209 | 0.37828 | 0.37448 | 0.37070 | 0.36693 | 0.36317 | 0.35942 | 0.35569 | 0.35197 | 0.34827 |
| **-0.2** | 0.42074 | 0.41683 | 0.41294 | 0.40905 | 0.40517 | 0.40129 | 0.39743 | 0.39358 | 0.38974 | 0.38591 |
| **-0.1** | 0.46017 | 0.45620 | 0.45224 | 0.44828 | 0.44433 | 0.44038 | 0.43644 | 0.43251 | 0.42858 | 0.42465 |
| **0.0** | 0.50000 | 0.49601 | 0.49202 | 0.48803 | 0.48405 | 0.48006 | 0.47608 | 0.47210 | 0.46812 | 0.46414 |
| **z** | **0.00** | **0.01** | **0.02** | **0.03** | **0.04** | **0.05** | **0.06** | **0.07** | **0.08** | **0.09** |
| **0.0** | 0.50000 | 0.50399 | 0.50798 | 0.51197 | 0.51595 | 0.51994 | 0.52392 | 0.52790 | 0.53188 | 0.53586 |
| **0.1** | 0.53983 | 0.54380 | 0.54776 | 0.55172 | 0.55567 | 0.55962 | 0.56356 | 0.56749 | 0.57142 | 0.57535 |
| **0.2** | 0.57926 | 0.58317 | 0.58706 | 0.59095 | 0.59483 | 0.59871 | 0.60257 | 0.60642 | 0.61026 | 0.61409 |
| **0.3** | 0.61791 | 0.62172 | 0.62552 | 0.62930 | 0.63307 | 0.63683 | 0.64058 | 0.64431 | 0.64803 | 0.65173 |
| **0.4** | 0.65542 | 0.65910 | 0.66276 | 0.66640 | 0.67003 | 0.67364 | 0.67724 | 0.68082 | 0.68439 | 0.68793 |
| **0.5** | 0.69146 | 0.69497 | 0.69847 | 0.70194 | 0.70540 | 0.70884 | 0.71226 | 0.71566 | 0.71904 | 0.72240 |
| **0.6** | 0.72575 | 0.72907 | 0.73237 | 0.73565 | 0.73891 | 0.74215 | 0.74537 | 0.74857 | 0.75175 | 0.75490 |
| **0.7** | 0.75804 | 0.76115 | 0.76424 | 0.76730 | 0.77035 | 0.77337 | 0.77637 | 0.77935 | 0.78230 | 0.78524 |
| **0.8** | 0.78814 | 0.79103 | 0.79389 | 0.79673 | 0.79955 | 0.80234 | 0.80511 | 0.80785 | 0.81057 | 0.81327 |
| **0.9** | 0.81594 | 0.81859 | 0.82121 | 0.82381 | 0.82639 | 0.82894 | 0.83147 | 0.83398 | 0.83646 | 0.83891 |
| **1.0** | 0.84134 | 0.84375 | 0.84614 | 0.84849 | 0.85083 | 0.85314 | 0.85543 | 0.85769 | 0.85993 | 0.86214 |
| **1.1** | 0.86433 | 0.86650 | 0.86864 | 0.87076 | 0.87286 | 0.87493 | 0.87698 | 0.87900 | 0.88100 | 0.88298 |
| **1.2** | 0.88493 | 0.88686 | 0.88877 | 0.89065 | 0.89251 | 0.89435 | 0.89617 | 0.89796 | 0.89973 | 0.90147 |
| **1.3** | 0.90320 | 0.90490 | 0.90658 | 0.90824 | 0.90988 | 0.91149 | 0.91309 | 0.91466 | 0.91621 | 0.91774 |
| **1.4** | 0.91924 | 0.92073 | 0.92220 | 0.92364 | 0.92507 | 0.92647 | 0.92785 | 0.92922 | 0.93056 | 0.93189 |
| **1.5** | 0.93319 | 0.93448 | 0.93574 | 0.93699 | 0.93822 | 0.93943 | 0.94062 | 0.94179 | 0.94295 | 0.94408 |
| **1.6** | 0.94520 | 0.94630 | 0.94738 | 0.94845 | 0.94950 | 0.95053 | 0.95154 | 0.95254 | 0.95352 | 0.95449 |
| **1.7** | 0.95543 | 0.95637 | 0.95728 | 0.95818 | 0.95907 | 0.95994 | 0.96080 | 0.96164 | 0.96246 | 0.96327 |
| **1.8** | 0.96407 | 0.96485 | 0.96562 | 0.96638 | 0.96712 | 0.96784 | 0.96856 | 0.96926 | 0.96995 | 0.97062 |
| **1.9** | 0.97128 | 0.97193 | 0.97257 | 0.97320 | 0.97381 | 0.97441 | 0.97500 | 0.97558 | 0.97615 | 0.97670 |
| **2.0** | 0.97725 | 0.97778 | 0.97831 | 0.97882 | 0.97932 | 0.97982 | 0.98030 | 0.98077 | 0.98124 | 0.98169 |
| **2.1** | 0.98214 | 0.98257 | 0.98300 | 0.98341 | 0.98382 | 0.98422 | 0.98461 | 0.98500 | 0.98537 | 0.98574 |
| **2.2** | 0.98610 | 0.98645 | 0.98679 | 0.98713 | 0.98745 | 0.98778 | 0.98809 | 0.98840 | 0.98870 | 0.98899 |
| **2.3** | 0.98928 | 0.98956 | 0.98983 | 0.99010 | 0.99036 | 0.99061 | 0.99086 | 0.99111 | 0.99134 | 0.99158 |
| **2.4** | 0.99180 | 0.99202 | 0.99224 | 0.99245 | 0.99266 | 0.99286 | 0.99305 | 0.99324 | 0.99343 | 0.99361 |
| **2.5** | 0.99379 | 0.99396 | 0.99413 | 0.99430 | 0.99446 | 0.99461 | 0.99477 | 0.99492 | 0.99506 | 0.99520 |
| **2.6** | 0.99534 | 0.99547 | 0.99560 | 0.99573 | 0.99585 | 0.99598 | 0.99609 | 0.99621 | 0.99632 | 0.99643 |
| **2.7** | 0.99653 | 0.99664 | 0.99674 | 0.99683 | 0.99693 | 0.99702 | 0.99711 | 0.99720 | 0.99728 | 0.99736 |
| **2.8** | 0.99744 | 0.99752 | 0.99760 | 0.99767 | 0.99774 | 0.99781 | 0.99788 | 0.99795 | 0.99801 | 0.99807 |
| **2.9** | 0.99813 | 0.99819 | 0.99825 | 0.99831 | 0.99836 | 0.99841 | 0.99846 | 0.99851 | 0.99856 | 0.99861 |

**Table B:** Standardised Normal Loss Table

| z | 0.00 | -0.01 | -0.02 | -0.03 | -0.04 | -0.05 | -0.06 | -0.07 | -0.08 | -0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **-2.9** | 2.90054 | 2.91052 | 2.92051 | 2.93049 | 2.94047 | 2.95046 | 2.96044 | 2.97042 | 2.98041 | 2.99040 |
| **-2.8** | 2.80076 | 2.81074 | 2.82071 | 2.83069 | 2.84066 | 2.85064 | 2.86062 | 2.87060 | 2.88058 | 2.89056 |
| **-2.7** | 2.70106 | 2.71103 | 2.72099 | 2.73096 | 2.74093 | 2.75090 | 2.76087 | 2.77084 | 2.78081 | 2.79079 |
| **-2.6** | 2.60146 | 2.61142 | 2.62137 | 2.63133 | 2.64129 | 2.65125 | 2.66121 | 2.67117 | 2.68113 | 2.69110 |
| **-2.5** | 2.50200 | 2.51194 | 2.52188 | 2.53183 | 2.54177 | 2.55171 | 2.56166 | 2.57161 | 2.58156 | 2.59151 |
| **-2.4** | 2.40272 | 2.41264 | 2.42256 | 2.43248 | 2.44241 | 2.45234 | 2.46227 | 2.47220 | 2.48213 | 2.49207 |
| **-2.3** | 2.30366 | 2.31356 | 2.32345 | 2.33335 | 2.34325 | 2.35316 | 2.36307 | 2.37298 | 2.38289 | 2.39280 |
| **-2.2** | 2.20489 | 2.21475 | 2.22462 | 2.23449 | 2.24436 | 2.25423 | 2.26411 | 2.27400 | 2.28388 | 2.29377 |
| **-2.1** | 2.10647 | 2.11629 | 2.12612 | 2.13595 | 2.14579 | 2.15563 | 2.16547 | 2.17532 | 2.18517 | 2.19503 |
| **-2.0** | 2.00849 | 2.01827 | 2.02805 | 2.03783 | 2.04762 | 2.05742 | 2.06722 | 2.07702 | 2.08683 | 2.09665 |
| **-1.9** | 1.91105 | 1.92077 | 1.93049 | 1.94022 | 1.94996 | 1.95970 | 1.96945 | 1.97920 | 1.98896 | 1.99872 |
| **-1.8** | 1.81428 | 1.82392 | 1.83357 | 1.84323 | 1.85290 | 1.86257 | 1.87226 | 1.88195 | 1.89164 | 1.90134 |
| **-1.7** | 1.71829 | 1.72785 | 1.73742 | 1.74699 | 1.75658 | 1.76617 | 1.77578 | 1.78539 | 1.79501 | 1.80464 |
| **-1.6** | 1.62324 | 1.63270 | 1.64217 | 1.65165 | 1.66114 | 1.67064 | 1.68015 | 1.68967 | 1.69920 | 1.70874 |
| **-1.5** | 1.52931 | 1.53865 | 1.54800 | 1.55736 | 1.56674 | 1.57612 | 1.58552 | 1.59494 | 1.60436 | 1.61380 |
| **-1.4** | 1.43667 | 1.44587 | 1.45508 | 1.46431 | 1.47356 | 1.48281 | 1.49208 | 1.50137 | 1.51067 | 1.51998 |
| **-1.3** | 1.34553 | 1.35457 | 1.36363 | 1.37270 | 1.38179 | 1.39090 | 1.40002 | 1.40916 | 1.41831 | 1.42748 |
| **-1.2** | 1.25610 | 1.26496 | 1.27384 | 1.28274 | 1.29165 | 1.30059 | 1.30954 | 1.31851 | 1.32750 | 1.33650 |
| **-1.1** | 1.16862 | 1.17727 | 1.18595 | 1.19465 | 1.20336 | 1.21210 | 1.22086 | 1.22964 | 1.23844 | 1.24726 |
| **-1.0** | 1.08332 | 1.09174 | 1.10019 | 1.10866 | 1.11716 | 1.12568 | 1.13422 | 1.14279 | 1.15138 | 1.15999 |
| **-0.9** | 1.00043 | 1.00860 | 1.01680 | 1.02503 | 1.03328 | 1.04156 | 1.04986 | 1.05819 | 1.06654 | 1.07491 |
| **-0.8** | 0.92021 | 0.92810 | 0.93603 | 0.94398 | 0.95196 | 0.95997 | 0.96801 | 0.97607 | 0.98417 | 0.99229 |
| **-0.7** | 0.84288 | 0.85048 | 0.85810 | 0.86576 | 0.87345 | 0.88117 | 0.88892 | 0.89669 | 0.90450 | 0.91234 |
| **-0.6** | 0.76867 | 0.77595 | 0.78325 | 0.79059 | 0.79797 | 0.80537 | 0.81281 | 0.82028 | 0.82778 | 0.83531 |
| **-0.5** | 0.69780 | 0.70473 | 0.71170 | 0.71870 | 0.72573 | 0.73281 | 0.73991 | 0.74705 | 0.75422 | 0.76143 |
| **-0.4** | 0.63044 | 0.63701 | 0.64362 | 0.65027 | 0.65695 | 0.66367 | 0.67042 | 0.67721 | 0.68404 | 0.69090 |
| **-0.3** | 0.56676 | 0.57296 | 0.57920 | 0.58547 | 0.59178 | 0.59813 | 0.60452 | 0.61094 | 0.61740 | 0.62390 |
| **-0.2** | 0.50689 | 0.51271 | 0.51856 | 0.52445 | 0.53038 | 0.53634 | 0.54235 | 0.54840 | 0.55448 | 0.56060 |
| **-0.1** | 0.45094 | 0.45635 | 0.46181 | 0.46731 | 0.47285 | 0.47842 | 0.48404 | 0.48969 | 0.49539 | 0.50112 |
| **0.0** | 0.39894 | 0.40396 | 0.40902 | 0.41412 | 0.41926 | 0.42444 | 0.42966 | 0.43492 | 0.44022 | 0.44556 |
| **z** | **0.00** | **0.01** | **0.02** | **0.03** | **0.04** | **0.05** | **0.06** | **0.07** | **0.08** | **0.09** |
| **0.0** | 0.39894 | 0.39396 | 0.38902 | 0.38412 | 0.37926 | 0.37444 | 0.36966 | 0.36492 | 0.36022 | 0.35556 |
| **0.1** | 0.35094 | 0.34635 | 0.34181 | 0.33731 | 0.33285 | 0.32842 | 0.32404 | 0.31969 | 0.31539 | 0.31112 |
| **0.2** | 0.30689 | 0.30271 | 0.29856 | 0.29445 | 0.29038 | 0.28634 | 0.28235 | 0.27840 | 0.27448 | 0.27060 |
| **0.3** | 0.26676 | 0.26296 | 0.25920 | 0.25547 | 0.25178 | 0.24813 | 0.24452 | 0.24094 | 0.23740 | 0.23390 |
| **0.4** | 0.23044 | 0.22701 | 0.22362 | 0.22027 | 0.21695 | 0.21367 | 0.21042 | 0.20721 | 0.20404 | 0.20090 |
| **0.5** | 0.19780 | 0.19473 | 0.19170 | 0.18870 | 0.18573 | 0.18281 | 0.17991 | 0.17705 | 0.17422 | 0.17143 |
| **0.6** | 0.16867 | 0.16595 | 0.16325 | 0.16059 | 0.15797 | 0.15537 | 0.15281 | 0.15028 | 0.14778 | 0.14531 |
| **0.7** | 0.14288 | 0.14048 | 0.13810 | 0.13576 | 0.13345 | 0.13117 | 0.12892 | 0.12669 | 0.12450 | 0.12234 |
| **0.8** | 0.12021 | 0.11810 | 0.11603 | 0.11398 | 0.11196 | 0.10997 | 0.10801 | 0.10607 | 0.10417 | 0.10229 |
| **0.9** | 0.10043 | 0.09860 | 0.09680 | 0.09503 | 0.09328 | 0.09156 | 0.08986 | 0.08819 | 0.08654 | 0.08491 |
| **1.0** | 0.08332 | 0.08174 | 0.08019 | 0.07866 | 0.07716 | 0.07568 | 0.07422 | 0.07279 | 0.07138 | 0.06999 |
| **1.1** | 0.06862 | 0.06727 | 0.06595 | 0.06465 | 0.06336 | 0.06210 | 0.06086 | 0.05964 | 0.05844 | 0.05726 |
| **1.2** | 0.05610 | 0.05496 | 0.05384 | 0.05274 | 0.05165 | 0.05059 | 0.04954 | 0.04851 | 0.04750 | 0.04650 |
| **1.3** | 0.04553 | 0.04457 | 0.04363 | 0.04270 | 0.04179 | 0.04090 | 0.04002 | 0.03916 | 0.03831 | 0.03748 |
| **1.4** | 0.03667 | 0.03587 | 0.03508 | 0.03431 | 0.03356 | 0.03281 | 0.03208 | 0.03137 | 0.03067 | 0.02998 |
| **1.5** | 0.02931 | 0.02865 | 0.02800 | 0.02736 | 0.02674 | 0.02612 | 0.02552 | 0.02494 | 0.02436 | 0.02380 |
| **1.6** | 0.02324 | 0.02270 | 0.02217 | 0.02165 | 0.02114 | 0.02064 | 0.02015 | 0.01967 | 0.01920 | 0.01874 |
| **1.7** | 0.01829 | 0.01785 | 0.01742 | 0.01699 | 0.01658 | 0.01617 | 0.01578 | 0.01539 | 0.01501 | 0.01464 |
| **1.8** | 0.01428 | 0.01392 | 0.01357 | 0.01323 | 0.01290 | 0.01257 | 0.01226 | 0.01195 | 0.01164 | 0.01134 |
| **1.9** | 0.01105 | 0.01077 | 0.01049 | 0.01022 | 0.00996 | 0.00970 | 0.00945 | 0.00920 | 0.00896 | 0.00872 |
| **2.0** | 0.00849 | 0.00827 | 0.00805 | 0.00783 | 0.00762 | 0.00742 | 0.00722 | 0.00702 | 0.00683 | 0.00665 |
| **2.1** | 0.00647 | 0.00629 | 0.00612 | 0.00595 | 0.00579 | 0.00563 | 0.00547 | 0.00532 | 0.00517 | 0.00503 |
| **2.2** | 0.00489 | 0.00475 | 0.00462 | 0.00449 | 0.00436 | 0.00423 | 0.00411 | 0.00400 | 0.00388 | 0.00377 |
| **2.3** | 0.00366 | 0.00356 | 0.00345 | 0.00335 | 0.00325 | 0.00316 | 0.00307 | 0.00298 | 0.00289 | 0.00280 |
| **2.4** | 0.00272 | 0.00264 | 0.00256 | 0.00248 | 0.00241 | 0.00234 | 0.00227 | 0.00220 | 0.00213 | 0.00207 |
| **2.5** | 0.00200 | 0.00194 | 0.00188 | 0.00183 | 0.00177 | 0.00171 | 0.00166 | 0.00161 | 0.00156 | 0.00151 |
| **2.6** | 0.00146 | 0.00142 | 0.00137 | 0.00133 | 0.00129 | 0.00125 | 0.00121 | 0.00117 | 0.00113 | 0.00110 |
| **2.7** | 0.00106 | 0.00103 | 0.00099 | 0.00096 | 0.00093 | 0.00090 | 0.00087 | 0.00084 | 0.00081 | 0.00079 |
| **2.8** | 0.00076 | 0.00074 | 0.00071 | 0.00069 | 0.00066 | 0.00064 | 0.00062 | 0.00060 | 0.00058 | 0.00056 |
| **2.9** | 0.00054 | 0.00052 | 0.00051 | 0.00049 | 0.00047 | 0.00046 | 0.00044 | 0.00042 | 0.00041 | 0.00040 |

**Table C:** $L_q$ Values for the Multi-server Queue

Values of $L_q$ for *s* servers, with mean utilization rate $\rho$, assuming Poisson arrivals and exponential service times.

| Utilization rate | Number of servers (s) | | | | |
|---|---|---|---|---|---|
| ($\rho$) | 1 | 2 | 3 | 4 | 5 |
| .10 | .0111 | .0020 | .0004 | .0001 | .0000 |
| .20 | .0500 | .0167 | .0062 | .0024 | .0010 |
| .30 | .1286 | .0593 | .0300 | .0159 | .0086 |
| .35 | .1885 | .0977 | .0552 | .0325 | .0196 |
| .40 | .2667 | .1524 | .0941 | .0605 | .0398 |
| .45 | .3682 | .2285 | .1522 | .1052 | .0743 |
| .50 | .5000 | .3333 | .2368 | .1739 | .1304 |
| .55 | .6722 | .4771 | .3583 | .2772 | .2185 |
| .60 | .9000 | .6750 | .5321 | .4306 | .3542 |
| .62 | 1.0116 | .7743 | .6213 | .5109 | .4269 |
| .64 | 1.1378 | .8880 | .7246 | .6051 | .5130 |
| .66 | 1.2812 | 1.0188 | .8446 | .7158 | .6152 |
| .68 | 1.4450 | 1.1698 | .9847 | .8461 | .7367 |
| .70 | 1.6333 | 1.3451 | 1.1488 | 1.0002 | .8816 |
| .72 | 1.8514 | 1.5500 | 1.3423 | 1.1834 | 1.0553 |
| .74 | 2.1062 | 1.7914 | 1.5721 | 1.4025 | 1.2646 |
| .76 | 2.4067 | 2.0785 | 1.8472 | 1.6668 | 1.5187 |
| .78 | 2.7655 | 2.4237 | 2.1803 | 1.9887 | 1.8302 |
| .80 | 3.2000 | 2.8444 | 2.5888 | 2.3857 | 2.2165 |
| .82 | 3.7356 | 3.3661 | 3.0979 | 2.8832 | 2.7029 |
| .84 | 4.4100 | 4.0265 | 3.7456 | 3.5190 | 3.3273 |
| .86 | 5.2829 | 4.8852 | 4.5914 | 4.3526 | 4.1493 |
| .88 | 6.4533 | 6.0414 | 5.7345 | 5.3834 | 5.2682 |
| .90 | 8.1000 | 7.6737 | 7.3535 | 7.0898 | 6.8624 |
| .92 | 10.5800 | 10.1392 | 9.8056 | 9.5290 | 9.2893 |
| .94 | 14.7267 | 14.2712 | 13.9240 | 13.6344 | 13.3821 |
| .96 | 23.0400 | 22.5698 | 22.2088 | 21.9060 | 21.6408 |
| .98 | 48.0200 | 47.5350 | 47.1602 | 46.8439 | 46.5656 |
| .99 | 98.0101 | 97.5176 | 97.1357 | 96.8127 | 96.4274 |