Operations Analytics

Tutorial #3: Newsvendor model

Solutions

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[1]Please refer to "Recipe 3" for a review of the formulas used.

Exercise 1: Matt Herron

Matt Herron is the chief buyer at Investment Clothiers, a retail store known for excellence in apparel. It is time to order merchandise for the Christmas season. During a recent trip to Hong Kong, Matt spotted a particular men's overcoat that he expects will sell very well. Based on past experience, Matt expects demand to range from 100 to 400 overcoats, with probabilities that are as follows:

Estimated demand	Probability
100	0.1
200	0.4
300	0.4
400	0.1

The total cost to Investment Clothiers would be \$60 per overcoat, and the retail price is estimated at \$110 per overcoat. Any overcoat left over after the Christmas season is expected to be sold at \$40 each.

(1a) Calculate the expected profits for each of the following four cases: when Matt buys 100, 200, 300 or 400 overcoats. How many overcoats should Matt buy if he wishes to maximize expected profits over the coming Christmas season?

(1b) Now solve (1a) again using marginal analysis.

(1c) The assumption of a discrete demand distribution is a bit artificial. Solve the problem using a normal distribution for demand (with the same mean and standard deviation as the discrete distribution of the original problem description). Does this change make much difference?

Solution

(1a) $Q = 100$: He orders 100 overcoats, sells them all, as demand is always greater than or equal to his order. $E\left[\text{profit}\right] = E\left[\text{revenue}\right] - E\left[\text{cost}\right] = 100 \cdot 110 - 100 \cdot 60 = \boxed{\$5,000}$.

$Q = 200$: He orders 200, has a 0.10 probability of having an excess stock at the end of the Christmas season of 100 (when $D = 100$), and sells them at a discount; with probability 0.90 he sells them all. $E\left[\text{cost}\right] = 200 \cdot 60 = \$12,000$, $E\left[\text{revenue}\right] = 0.10 \cdot (100 \cdot 110 + 100 \cdot 40) + 0.90 \cdot (200 \cdot 110) = \$21,300$. Thus, $E\left[\text{profit}\right] = E\left[\text{revenue}\right] - E\left[\text{cost}\right] = \boxed{\$9,300}$.

$Q = 300$: He orders 300, has a 0.10 probability of having an excess of 200 (when $D = 100$), has a 0.40 probability of having an excess of 100 (when $D = 200$), and sells all ordered units with probability 0.50 (when $D = 300$ or 400). $E\left[\text{cost}\right] = 300 \cdot 60 = \$18,000$, $E\left[\text{revenue}\right] = 0.10 \cdot (100 \cdot 110 + 200 \cdot 40) + 0.40 \cdot (200 \cdot 110 + 100 \cdot 40) + 0.50 \cdot (300 \cdot 110) = \$28,800$. Thus, $E\left[\text{profit}\right] = E\left[\text{revenue}\right] - E\left[\text{cost}\right] = \boxed{\$10,800}$.

$Q = 400$: He orders 400, has a 0.10 probability of having an excess of 300 (when $D = 100$), has a 0.40 probability of having an excess of 200 (when $D = 200$), a 0.40 probability of having an excess of 100 (when $D = 300$), and sells all orders with probability 0.10 (when $D = 400$). $E\left[\text{cost}\right] = 400 \cdot 60 = \$24,000$, $E\left[\text{revenue}\right] = 0.10 \cdot (100 \cdot 110 + 300 \cdot 40) + 0.40 \cdot (200 \cdot 110 + 200 \cdot 40) + 0.40 \cdot (300 \cdot 110 + 100 \cdot 40) + 0.10 \cdot (400 \cdot 110) = \$33,500$. Thus, $E\left[\text{profit}\right] = E\left[\text{revenue}\right] - E\left[\text{cost}\right] = \boxed{\$9,500}$.

Thus, comparing the expected profits we find that the optimal (profit-maximizing) order quantity is $Q^* = 300$, with an expected profit equal to $\$10,800$.

(1b) The critical ratio is

$$\frac{C_u}{C_u + C_o} = \frac{r - c}{r - s} = \frac{110 - 60}{110 - 40} = \frac{50}{70} = 0.714$$

We are in the Case 2 of the Recipe: we have a discrete distribution of demand. But we don't know it; we are given the probability *density* function (PDF)! Summing up step-by-step the probabilities we get the discrete *distribution* of demand (cummulative distribution function; CDF):

Quantity (Q)	$F(Q) = P\left[D \leq Q\right]$
100	0.1
200	0.5
300	0.9
400	1

Now, we need to find a value for Q such that $F(Q) = 0.714$. This value is not included in the above table, so using the *round-up rule* of the Recipe, we find that the optimal order quantity is $Q^* = 300$, which corresponds to an expected profit of $\$10,800$ (calculated above).

(1c) To use the Normal distribution we first need to know its parameters, i.e. the mean (μ) and the variance (σ^2). These would be identical to the mean and variance of the discrete distribution in (1a) and (1b).

Mean: $\mu = E\left[D\right] = 0.1 \cdot 100 + 0.40 \cdot 200 + 0.40 \cdot 300 + 0.10 \cdot 400 = 250$

Variance[2]: $\sigma^2 = E\left[D^2\right] - \left(E\left[D\right]\right)^2$. Therefore:

$$\begin{aligned}
\sigma^2 &= E\left[D^2\right] - \left(E\left[D\right]\right)^2 \\
&= \left(0.1 \cdot 100^2 + 0.40 \cdot 200^2 + 0.40 \cdot 300^2 + 0.10 \cdot 400^2\right) \\
&\qquad - \left(0.1 \cdot 100 + 0.40 \cdot 200 + 0.40 \cdot 300 + 0.10 \cdot 400\right)^2 \\
&= 69,000 - 250^2 = 6,500
\end{aligned}$$

[2]We may also calculate σ using its definition: $\sigma^2 = E\left[D - E\left[D\right]\right]^2 = 0.1 \cdot (100 - 250)^2 + 0.4 \cdot (200 - 250)^2 + 0.4 \cdot (300 - 250)^2 + 0.1 \cdot (400 - 250)^2$. This will give the exact same result.

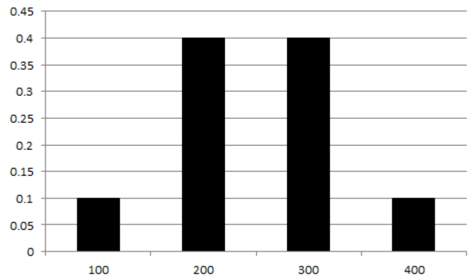

Fig. 1.1 The empirical density function of our estimates.

and the standard deviation equals:

$$\sigma = \sqrt{\sigma^2} = \sqrt{6,500} = 80.6$$

We therefore assume a normal (continuous) demand model, with the same mean and the same standard distribution as the above discrete distribution (which already was looking somewhat bell-shaped, see 1.1). The underage and overage costs have not changed and therefore, we need to find z^* corresponding to the critical ratio 0.714. From 1.1, this is $z^* = 0.57$. Hence, in the normal distribution case, we have that

$$Q^* = \mu + z^* \cdot \sigma = 250 + 0.57 \cdot 80.6 = 296 \text{ overcoats}$$

(round to the nearest integer number of overcoats). The continuous and the discrete models have the same mean and variance, so it is not surprising that the results for the optimal order quantities are close too.

Next, we calculate the expected profit.

Use the z-value you found before to look up in the Standard Normal Loss Function 1.2 the corresponding $L(z)$-value.

Expected lost sales $= \sigma \cdot L(0.57) = 80.6 \cdot 0.1771 = 14.2743$ overcoats

Expected sales $= \mu -$ Expected lost sales $= 250 - 14.2743 = 235.7257$ overcoats

Expected leftover inventory $= Q -$Expected sales $= 296 - 235.7257 = 60.2743$ overcoats

Expected profit $= [(r - c) \times$ Expected sales$] - [(c - s) \times$ Expected leftover inventory$] =$ $[(\$50) \times 235.7257] - [(\$20) \times 60.2743] = \boxed{\$10,581}$

The result for the continuous model ($10,581) is indeed close to that obtained for the discrete model (in part 1a): $10,800. This is what we expected given the above mentioned similarity between the two demand models. □

Exercise 2: Incentive Conflicts in a Supply Chain

Consider the situation by a manufacturer of PlayStations, SUNNY, who is facing Christmas demand. SUNNY sells the PlayStations through a large retail chain, TOYS-are-MINE, at a manufacturer (wholesale) price of 500€ per PlayStation. SUNNY's unit manufacturing cost is 350€ per PlayStation.

TOYS-are-MINE offers the PlayStations for sale at its retail stores at the retail price of 800€ per PlayStation. Its demand forecast for the upcoming Christmas season is for 200,000 PlayStations, with a standard deviation of 50,000. The retail chain has to order ahead of the season.

Any PlayStations ordered will be delivered in time for the Christmas sales season, but no re-supply is possible during the sales season. Any PlayStation that will not be sold during the upcoming Christmas will be sold after the season at a discount, for a price of 300€ per PlayStation.

(2a.i) What is the optimal order quantity for TOYS-are-MINE, and what are its expected profits on this item?

(2a.ii) What is the expected profit of the manufacturer, SUNNY?

(2b) What is the optimal order quantity on this item, if the retailer and the manufacturer were integrated into a single company?

(2c) Compare the expected profits generated by the integrated supply chain (which you computed in your answer to (2b)), with that of the chain formed by the two independent firms, SUNNY and TOYS-are-MINE (in the latter case, we have assumed in (2a) that the order quantity was determined by the retailer). Which arrangement generates the highest expected profits for the supply chain? Why? What other major differences do you observe between these two inter-organisational arrangements?

Solution

(2a.i) The critical ratio is

$$\frac{C_u}{C_u + C_o} = \frac{r - c}{r - s} = \frac{800 - 500}{800 - 300} = \frac{300}{500} = 0.60$$

We are in the Case 1 of the Recipe: demand (D) follows the Normal distribution with $\mu = 200,000$ and $\sigma = 50,000$. We search for a z-value such that $\Phi(z) = 0.60$. Looking in 1.1 we find that the critical ratio value falls between two entries on the table $z_1 = 0.25$ and $z_2 = 0.26$. Applying the round-up rule, we choose the entry with the larger quantity, i.e. $z = 0.26$. Therefore, $Q = \mu + z \cdot \sigma = 200,000 + 0.26 \cdot 50,000 = 213,000$ PlayStations.

Next, we calculate the expected profit of the retailer TOYS-are-MINE (TaM).

Use the z-value you found before to look up in the Standard Normal Loss Function 1.2 the corresponding $L(z)$-value.

TaM's Expected lost sales $= \sigma \cdot L(z) = 50,000 \cdot 0.2824 = 14,120$ PlayStations

TaM's Expected sales $= \mu -$Expected lost sales $= 200,000 - 14,120 = 185,880$ PlayStations

TaM's Expected leftover inventory $= Q -$ Expected sales $= 213,000 - 185,880$
$= 27,120$ PlayStations

TaM's Expected profit $= [(r - c) \times$ Exp. sales$] - [(c - s) \times$ Exp. leftover inv.$]$
$= [(€300) \times 185,880] - [(€200) \times 27,120] = \boxed{€50,340,000}$

(2a.ii) SUNNY's profit from selling the PlayStation at TOYS-are-MINE store is $213,000 \times €150 = \boxed{€31,950,000}$, where $213,000$ is the number of PlayStations that TOYS-are-MINE purchases and €150 is SUNNY's gross margin $€500 - €350 = €150$.

While TOYS-are-MINE might be quite pleased with this situation (it does earn €50m relative to SUNNY's €32m approximately), it should not be. The total supply chain's profit is $€31,950,000 + €50,340,000 = \boxed{€82,290,000}$, but it could be higher as we explain next.

(2b) Here we need to find the *supply chain's optimal quantity*, since this is the quantity that maximizes the *integrated supply chain.* We can still use the newsvendor model to evaluate the supply chain's order quantity decision and performance measures. Each lost sale costs the supply chain (SC) the difference between the retail price and the manufacturing cost, $€800 - €350 = €450$; that is the SC's underage cost is $C_u = €450$. Each leftover PlayStation costs the SC the difference between the manufacturing cost and the salvage value, $€350 - €300 = €50$; that is, the SC's overage cost is $C_o = €50$. The SC's critical ratio is

$$\frac{C_u}{C_u + C_o} = \frac{450}{450 + 50} = 0.9$$

The appropriate z-statistic for that critical ratio is $z = 1.29$, because $\Phi(1.28) = 0.89973$ and $\Phi(1.29) = 0.90147$. The SC's expected profit-maximizing order quantity is then

$$Q = \mu + z \cdot \sigma = 200,000 + 1.29 \cdot 50,000 = 264,500$$

which is higher than TOYS-are-MINE's order of $213,000$ PlayStations.

(2c) The SC's performance measures can then be evaluated assuming the SC optimal order quantity, $264,500$ units:

Use the z-value you found before to look up in the Standard Normal Loss Function 1.2 the corresponding $L(z)$-value.

SC's Expected lost sales $= \sigma \cdot L(1.29) = 50,000 \cdot 0.0465 = 2,325$ PlayStations

SC's Expected sales $= \mu -$Expected lost sales $= 200,000 - 2,325 = 197,675$ PlayStations

SC's Expected leftover inventory $= Q -$ Expected sales $= 264,500 - 197,675$
$= 66,825$ PlayStations

SC's Expected profit $= [(r - c) \times$ Exp. sales$] - [(c - s) \times$ Exp. leftover inv.$]$

$= [(€450) \times 197,675] - [(€50) \times 66,825] = \boxed{€85,612,500}$

Thus, while SUNNY and TOYS-are-MINE currently earn an expected profit of €82, 290, 000, their SC could enjoy an expected profit increase €3, 420, 000 that is about 4% higher.

But, why the current supply chain perform worse than it could? The obvious answer is that TOYS-are-MINE does not order enough PlayStations: TOYS-are-MINE orders 213, 000 of them, but the SC's optimal order quantity is 264, 500 units. But why doesn't TOYS-are-MINE order enough? Because TOYS-are-MINE is acting in its own self interest to maximize its *own* profit.

To explain further, TOYS-are-MINE must pay SUNNY €500 per PlayStation and so TOYS-are-MINE acts as if the cost to produce each PlayStation is €500, not the actual €350. From TOYS-are-MINE's perspective, it does not matter if the actual manufacturing cost is €350, €400, or even €0; its "manufacturing cost" is €500.

This is related to an important phenomenon called *double marginalization*. In simple words, TOYS-are-MINE only earns a "piece of the pie" (€300) of the total "pie" benefit per sale (€450). Thus, it is logical that TOYS-are-MINE is not willing to purchase as much inventory as would be optimal for the supply chain.

Note also that under the integrated SC, the profit of each firm has changed, i.e. the pie is not cut differently. Specifically, under the integrated SC:

$$\text{SUNNY's expected profits} = (500 - 300) \times Q = 150 \times 264,500 = €39,675,000$$

$$\text{TaM's expected profits} = 85,612,500 - 39,675,000 = €45,937,500$$

The retailer's profit is actually smaller (by 9.6%) than the profit computed in part (a). This is because in part (a), the solution was pursuing the maximization of retail profits, which is no longer the case in part (b).

To sum up, this exercise illustrates an important finding:

> Even if every firm in a supply chain chooses actions to maximize its own expected profit, the total profit earned from all the firms in the supply chain may be less than the maximum possible profit of the integrated supply chain.

Aligning the incentives within the supply chain is key. □

Exercise 3: Banking Crisis

Nwankwo Kanu is Chief Investment Officer at a leading hedge fund in Africa. While his fund has been performing well, the contagion and fear in capital markets is getting to his investors. This is leading to huge redemptions from his fund. He must plan to meet these redemptions at the lowest costs. Kanu, a TCD Alum, recalls some quantitative methods he learned at Operations Analytics, his favorite TCD module, and suggests this method to his economic team to estimate these redemptions and predict that over the course of the next month, the expected redemptions will be US$ 300m. The coefficient of variation of this estimate is 0.333. Unfortunately, all Kanu's hedge fund assets are illiquid and cannot be disposed for the next year. He must thus go to the capital markets to raise additional funds. He has two sources of financing that he is planning to use simultaneously:

Source 1*:* Borrow from the government sponsored TARP (The-Augmentation-of-Rich-People) program. This facility charges a usurious 25% annual rate of interest. Further, it can be availed only if a request is made far in advance.

Source 2*:* Alternatively, when TARP funding comes short, you can borrow from the overnight credit markets. You can use this facility anytime and it provides funding immediately. Unfortunately, the rate of interest charged is 40% per annum.

(3a) How much money would you borrow from the TARP funds?

(3b) What are your annualized capital costs if you employ the solution prescribed in (a)?

(3c) Abou Abed, one of Kanu's classmates at TCD is in exactly the same financial position with Kanu. While he runs an identical fund, most of his clients are located in the beautiful downtown Beirut. He calls Kanu up and proposes merging the two funds. He somewhat remembers the Ops class and suggests that merging would reduce the above mentioned capital costs for both funds. Do you agree? Justify your answer in no more than 100 words.

(3d) The crisis is getting worse. The overnight credit markets have now completely frozen up. Kanu no longer cares about minimizing the capital costs. All he cares about is the odds of survival, or the probability that the fund will not be able to meet redemptions. What is the minimum amount of money Kanu needs to borrow from TARP funds to ensure that he will be able to meet all redemption requests with a 95% probability?

(3e) What is the average utilization of Kanu's hedge fund in Africa?

Solution

(3a) This is a classic newsvendor setup with reactive capacity: The cost of borrowing too little is $C_u = 40 - 25 = 15$, whereas the cost of borrowing too much is $C_o = 25$. Note that units do not matter here. The critical ratio is $\frac{15}{15+25} = 0.375$. From 1.1, the correct z-value is -0.31. So, the amount borrowed is $Q^* = \mu + z^* \cdot \sigma = 300 + (-0.31) \cdot 0.333 \cdot 300 =$ US$ 269m, which can also be referred to as the *total capacity* of the fund.

(3b) For US$ 269m you got to pay 25% of it, that is $269 \cdot 0.25 =$ US$ 67.25m. On the other hand, because you need to satisfy whatever demand there is, you will have to borrow from the overnight market if there is demand and you do not have the money available. In other words, if anything above US$ 269m comes, you would have to borrow. This means that you have to borrow the *expected lost sales* from the overnight market. For z of -0.31 lost sales, from 1.2 you find: $L(-0.31) = 0.5730$. Expected lost sales are then: $\sigma \cdot L(-0.31) = 100 \cdot 0.5730 =$ US$ 57.3m. For this amount you need to pay 40%, or US$ 23.92m.

So, the total costs are $67.25 + 23.92 =$ US$ 90.17m.

Note: It is not enough just to write down the expected lost sales! It must be clear from the logic of the solution why we need to compute this, i.e. this is the money that you need to borrow from the overnight market.

(3c) Yes, merging the two funds will decrease capital costs for both as Kanu and Abou will now aggregate demand for redemptions from both markets. This will have lower uncertainty and this will lead to the capital costs being lower. This is the same principle of aggregating demand from multiple markets that we have seen multiple times in class.

(3d) This translates to $P[Q \geq D] = 0.95$. Such a demand is Normal with mean 300 and standard deviation 100, but we can not find such a Q from the 1.1! In fact, we need to convert it to the Standard Normal case: $P\left[\frac{Q-300}{100} \geq \frac{D-300}{100}\right] = 0.95$. So the above probability is equal to: $P\left[Z \leq \frac{Q-300}{100}\right] = 0.95$. From 1.1 the value of z corresponding to 0.95 is 1.65 and therefore: $\frac{Q-300}{100} = 1.65$ or $Q =$ US$ 465m.

(3e) The average utilization is the fraction of the available capacity that gets used on average. "Capacity" in our hedge fund context is simply the money available, which is calculated as $Q^* =$ US$ 269m in (a). To compute this number, we need the installed capacity and the average capacity used. We already know the total capacity from (a). Next, we need to compute the average capacity used. Average capacity used corresponds to the average sales in a standard newsvendor setup. So we need to compute the expected sales:

$$\begin{aligned} E[\text{sales}] &= E[\text{demand}] - E[\text{lost sales}] \\ &= 300 - 100 \cdot L(z^* = -0.31) \\ &= 300 - 100 \cdot 0.5730 \\ &= \text{US\$ } 242.7\text{m} \end{aligned}$$

And the average utilization of the fund is $\frac{242.7}{269} = 0.902$, or the money of the fund is being used (hedged) at a percentage of 90% on average. □

z	0.00	-0.01	-0.02	-0.03	-0.04	-0.05	-0.06	-0.07	-0.08	-0.09
-2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
-2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
-2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
-2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
-2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
-2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
-2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
-2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
-2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
-2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
-1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
-1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
-1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
-1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
-1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
-1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
-1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
-1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
-1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
-1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
-0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
-0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
-0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
-0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
-0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
-0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
-0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
-0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
-0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861

Table 1.1 Standard Normal Distribution, $\Phi(z)$.

z	0.00	-0.01	-0.02	-0.03	-0.04	-0.05	-0.06	-0.07	-0.08	-0.09
-2.9	2.90054	2.91052	2.92051	2.93049	2.94047	2.95046	2.96044	2.97042	2.98041	2.99040
-2.8	2.80076	2.81074	2.82071	2.83069	2.84066	2.85064	2.86062	2.87060	2.88058	2.89056
-2.7	2.70106	2.71103	2.72099	2.73096	2.74093	2.75090	2.76087	2.77084	2.78081	2.79079
-2.6	2.60146	2.61142	2.62137	2.63133	2.64129	2.65125	2.66121	2.67117	2.68113	2.69110
-2.5	2.50200	2.51194	2.52188	2.53183	2.54177	2.55171	2.56166	2.57161	2.58156	2.59151
-2.4	2.40272	2.41264	2.42256	2.43248	2.44241	2.45234	2.46227	2.47220	2.48213	2.49207
-2.3	2.30366	2.31356	2.32345	2.33335	2.34325	2.35316	2.36307	2.37298	2.38289	2.39280
-2.2	2.20489	2.21475	2.22462	2.23449	2.24436	2.25423	2.26411	2.27400	2.28388	2.29377
-2.1	2.10647	2.11629	2.12612	2.13595	2.14579	2.15563	2.16547	2.17532	2.18517	2.19503
-2.0	2.00849	2.01827	2.02805	2.03783	2.04762	2.05742	2.06722	2.07702	2.08683	2.09665
-1.9	1.91105	1.92077	1.93049	1.94022	1.94996	1.95970	1.96945	1.97920	1.98896	1.99872
-1.8	1.81428	1.82392	1.83357	1.84323	1.85290	1.86257	1.87226	1.88195	1.89164	1.90134
-1.7	1.71829	1.72785	1.73742	1.74699	1.75658	1.76617	1.77578	1.78539	1.79501	1.80464
-1.6	1.62324	1.63270	1.64217	1.65165	1.66114	1.67064	1.68015	1.68967	1.69920	1.70874
-1.5	1.52931	1.53865	1.54800	1.55736	1.56674	1.57612	1.58552	1.59494	1.60436	1.61380
-1.4	1.43667	1.44587	1.45508	1.46431	1.47356	1.48281	1.49208	1.50137	1.51067	1.51998
-1.3	1.34553	1.35457	1.36363	1.37270	1.38179	1.39090	1.40002	1.40916	1.41831	1.42748
-1.2	1.25610	1.26496	1.27384	1.28274	1.29165	1.30059	1.30954	1.31851	1.32750	1.33650
-1.1	1.16862	1.17727	1.18595	1.19465	1.20336	1.21210	1.22086	1.22964	1.23844	1.24726
-1.0	1.08332	1.09174	1.10019	1.10866	1.11716	1.12568	1.13422	1.14279	1.15138	1.15999
-0.9	1.00043	1.00860	1.01680	1.02503	1.03328	1.04156	1.04986	1.05819	1.06654	1.07491
-0.8	0.92021	0.92810	0.93603	0.94398	0.95196	0.95997	0.96801	0.97607	0.98417	0.99229
-0.7	0.84288	0.85048	0.85810	0.86576	0.87345	0.88117	0.88892	0.89669	0.90450	0.91234
-0.6	0.76867	0.77595	0.78325	0.79059	0.79797	0.80537	0.81281	0.82028	0.82778	0.83531
-0.5	0.69780	0.70473	0.71170	0.71870	0.72573	0.73281	0.73991	0.74705	0.75422	0.76143
-0.4	0.63044	0.63701	0.64362	0.65027	0.65695	0.66367	0.67042	0.67721	0.68404	0.69090
-0.3	0.56676	0.57296	0.57920	0.58547	0.59178	0.59813	0.60452	0.61094	0.61740	0.62390
-0.2	0.50689	0.51271	0.51856	0.52445	0.53038	0.53634	0.54235	0.54840	0.55448	0.56060
-0.1	0.45094	0.45635	0.46181	0.46731	0.47285	0.47842	0.48404	0.48969	0.49539	0.50112
0.0	0.39894	0.40396	0.40902	0.41412	0.41926	0.42444	0.42966	0.43492	0.44022	0.44556
z	**0.00**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**
0.0	0.39894	0.39396	0.38902	0.38412	0.37926	0.37444	0.36966	0.36492	0.36022	0.35556
0.1	0.35094	0.34635	0.34181	0.33731	0.33285	0.32842	0.32404	0.31969	0.31539	0.31112
0.2	0.30689	0.30271	0.29856	0.29445	0.29038	0.28634	0.28235	0.27840	0.27448	0.27060
0.3	0.26676	0.26296	0.25920	0.25547	0.25178	0.24813	0.24452	0.24094	0.23740	0.23390
0.4	0.23044	0.22701	0.22362	0.22027	0.21695	0.21367	0.21042	0.20721	0.20404	0.20090
0.5	0.19780	0.19473	0.19170	0.18870	0.18573	0.18281	0.17991	0.17705	0.17422	0.17143
0.6	0.16867	0.16595	0.16325	0.16059	0.15797	0.15537	0.15281	0.15028	0.14778	0.14531
0.7	0.14288	0.14048	0.13810	0.13576	0.13345	0.13117	0.12892	0.12669	0.12450	0.12234
0.8	0.12021	0.11810	0.11603	0.11398	0.11196	0.10997	0.10801	0.10607	0.10417	0.10229
0.9	0.10043	0.09860	0.09680	0.09503	0.09328	0.09156	0.08986	0.08819	0.08654	0.08491
1.0	0.08332	0.08174	0.08019	0.07866	0.07716	0.07568	0.07422	0.07279	0.07138	0.06999
1.1	0.06862	0.06727	0.06595	0.06465	0.06336	0.06210	0.06086	0.05964	0.05844	0.05726
1.2	0.05610	0.05496	0.05384	0.05274	0.05165	0.05059	0.04954	0.04851	0.04750	0.04650
1.3	0.04553	0.04457	0.04363	0.04270	0.04179	0.04090	0.04002	0.03916	0.03831	0.03748
1.4	0.03667	0.03587	0.03508	0.03431	0.03356	0.03281	0.03208	0.03137	0.03067	0.02998
1.5	0.02931	0.02865	0.02800	0.02736	0.02674	0.02612	0.02552	0.02494	0.02436	0.02380
1.6	0.02324	0.02270	0.02217	0.02165	0.02114	0.02064	0.02015	0.01967	0.01920	0.01874
1.7	0.01829	0.01785	0.01742	0.01699	0.01658	0.01617	0.01578	0.01539	0.01501	0.01464
1.8	0.01428	0.01392	0.01357	0.01323	0.01290	0.01257	0.01226	0.01195	0.01164	0.01134
1.9	0.01105	0.01077	0.01049	0.01022	0.00996	0.00970	0.00945	0.00920	0.00896	0.00872
2.0	0.00849	0.00827	0.00805	0.00783	0.00762	0.00742	0.00722	0.00702	0.00683	0.00665
2.1	0.00647	0.00629	0.00612	0.00595	0.00579	0.00563	0.00547	0.00532	0.00517	0.00503
2.2	0.00489	0.00475	0.00462	0.00449	0.00436	0.00423	0.00411	0.00400	0.00388	0.00377
2.3	0.00366	0.00356	0.00345	0.00335	0.00325	0.00316	0.00307	0.00298	0.00289	0.00280
2.4	0.00272	0.00264	0.00256	0.00248	0.00241	0.00234	0.00227	0.00220	0.00213	0.00207
2.5	0.00200	0.00194	0.00188	0.00183	0.00177	0.00171	0.00166	0.00161	0.00156	0.00151
2.6	0.00146	0.00142	0.00137	0.00133	0.00129	0.00125	0.00121	0.00117	0.00113	0.00110
2.7	0.00106	0.00103	0.00099	0.00096	0.00093	0.00090	0.00087	0.00084	0.00081	0.00079
2.8	0.00076	0.00074	0.00071	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056
2.9	0.00054	0.00052	0.00051	0.00049	0.00047	0.00046	0.00044	0.00042	0.00041	0.00040

Table 1.2 Standard Normal Loss Function, $L(z)$.