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Operations Analytics

Tutorial #2: Queues

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Exercise 4: A Petroleum Refinery

An integrated petroleum company is considering expansion of its one unloading facility at its Australian refinery. Due to random variations in weather, loading delays, and other factors, ships arriving at the refinery to unload crude oil arrive according to the Poisson distribution ($CV_a = 1$) with an average arrival rate of 8 ships per week. Service time is exponential with an average service rate of 12 ships per week ($CV_s = 1$).

4a) Current performance measures:

4a.i) What is the average number of ships waiting to gain access to the single unloading facility?

4a.ii) What is the average time a ship must wait before beginning to deliver its cargo to the refinery?

4a.iii) What is the average total time that a ship spends at the refinery?

4b) The company has under consideration a second unloading berth that could be rented for $\$6,000$ per week. The service time for this berth is also assumed to be exponential with the same service rate of 12 ships per week as the company's own berth. The opportunity cost of a ship not being at sea is $\$30,000$ per week.

4b.i) If the second berth is rented, what will be the average number of ships waiting?

4b.ii) What would be the average total time a ship would spend at the refinery?

4b.iii) Is the benefit of reduced waiting time (in dollars) worth the rental cost for the second berth?

4c) An alternative way to improve unloading facilities is for the company to modify the current operations to speed up the unloading process. Specifically, the new service time would still be exponential, but with a mean service rate of 15 ships per week. The incremental cost of the new operations would be $\$4,000$ per week.

4c.i) Compute the (new) expected waiting time and total time for ships.

4c.ii) Is the benefit worth the incremental cost?

4c.iii) How would you answer (4c.i) and (4c.ii) above if, instead of speeding up the service rate of the facility, the new operations made the service time constant, but still with the old service rate of 12 ships per week?

Exercise 5: Voters' impatience to wait

During two former presidential elections in the United States, very long wait times have been witnessed at precincts (voting stations) in States that ultimately decided the election (Florida in 2000 and Ohio in 2004).

In Philadelphia as well, some voters complained about the long lines in some precincts, with most complaints coming from precinct A. In 2004, the average number of voters arriving at precinct A was of 35 per hour and the arrivals of voters was random with inter-arrival times that had a coefficient of variation of 1 ($CV_a = 1$).

Philadelphia had deployed 1 voting machine in precinct A. Suppose that each voter spent on average 100 seconds in the voting booth (this is the time needed to cast her/ his vote using a voting machine), with a standard deviation of 120 seconds.

5a) How long on average would a voter have to wait in line in precinct A in 2004 before entering in a booth to cast her/ his vote.

Given the long wait times for precinct A, the city of Philadelphia is thinking of alternative solutions to improve voting conditions. One of the proposed solutions is as follows.

Proposal 1: "Deploy three additional voting machines in precinct A. Assume that the voter turnout is expected to be a little more organized in 2008 as in the previous election and the time spent in the voting booth will now have a standard deviation of 75 seconds".

5b) Under Proposal 1, how long on average would a voter have to wait in line in precinct A in 2008 before casting her/ his vote.

Table: L_q values for the Multi-Server Queue

Values of L_q for s servers, with mean utilization rate ρ, assuming Poisson arrivals and Exponential service times.

Utilization rate (ρ)	Number of servers (s) 1	2	3	4	5
.10	.0111	.0020	.0004	.0001	.0000
.20	.0500	.0167	.0062	.0024	.0010
.30	.1286	.0593	.0300	.0159	.0086
.35	.1885	.0977	.0552	.0325	.0196
.40	.2667	.1524	.0941	.0605	.0398
.45	.3682	.2285	.1522	.1052	.0743
.50	.5000	.3333	.2368	.1739	.1304
.55	.6722	.4771	.3583	.2772	.2185
.60	.9000	.6750	.5321	.4306	.3542
.62	1.0116	.7743	.6213	.5109	.4269
.64	1.1378	.8880	.7246	.6051	.5130
.66	1.2812	1.0188	.8446	.7158	.6152
.68	1.4450	1.1698	.9847	.8461	.7367
.70	1.6333	1.3451	1.1488	1.0002	.8816
.72	1.8514	1.5500	1.3423	1.1834	1.0553
.74	2.1062	1.7914	1.5721	1.4025	1.2646
.76	2.4067	2.0785	1.8472	1.6668	1.5187
.78	2.7655	2.4237	2.1803	1.9887	1.8302
.80	3.2000	2.8444	2.5888	2.3857	2.2165
.82	3.7356	3.3661	3.0979	2.8832	2.7029
.84	4.4100	4.0265	3.7456	3.5190	3.3273
.86	5.2829	4.8852	4.5914	4.3526	4.1493
.88	6.4533	6.0414	5.7345	5.3834	5.2682
.90	8.1000	7.6737	7.3535	7.0898	6.8624
.92	10.5800	10.1392	9.8056	9.5290	9.2893
.94	14.7267	14.2712	13.9240	13.6344	13.3821
.96	23.0400	22.5698	22.2088	21.9060	21.6408
.98	48.0200	47.5350	47.1602	46.8439	46.5656
.99	98.0101	97.5176	97.1357	96.8127	96.4274