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Operations Analytics

Tutorial #1

Processes

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Exercise 1: Instant Dolls Co.

Instant Dolls specializes in manufacturing dolls. Their business process consists on the following activities, which must be conducted in the sequence described below[1]:

Activity 1: Get the specifications of the doll from the customer, get the payment and submit the order to the production line. (8 minutes)

Activity 2: Assemble the main body of the doll (10 minutes)

Activity 3: Add hair and paint. (12 minutes)

Activity 4: Dress the doll (7 minutes)

Activity 5: Package the doll (3 minutes)

The whole process is conducted by three workers (W). The assignment of tasks to workers is the following: W1 does Activity 1; W2 does Activities 2 and 3; W3 does Activities 4 and 5.

1a) What is the bottleneck of the process?

1b) How much time will it take to produce 250 units? (assume that the process starts with an empty production line)

1c) What is the utilization of Worker 3?

1d) What is the average labor utilization of the workers? Assume the process operates at its capacity and there are no empty system effects.

1e) To increase the production rate, Instant Dolls is considering to hire a new worker to help any of the workers without changing the tasks performed by each worker. Where would you assign the new employee?

Solution

1a) We expect W2 to be the bottleneck. This is intuitive because the two activities this worker does, take much longer than activities done by W1 and W3. Recall that the step with the longest processing time or lowest capacity is the bottleneck.

1b) It is important to note that the process starts *empty*[2]. When the first order gets to the production, W1 spends 8 mins and passes it to W2 who spends 22 mins and passes it to W3 that spends 10 mins on it and the first product will be out. So the first product takes $8 + 22 + 10 = 40$ mins. Now take the second order. When W1 gets done with the first order and passes it to W2, she picks up the second order and works on it which takes only 8 mins. Therefore, when W2 is done with the first one, she will see an order already waiting for her and so she will immediately start working on it (so she does not need to wait to start working). She will take of course 22 mins to finish the work. W3 finishes the

[1]The time required for each activity is shown in parenthesis.

[2]If the process does not start empty, the analysis would be much simpler. The process flow time equals to the process time of the bottleneck. The time needed to do one order by the bottleneck is 22 min and thus the time for 250 orders would be $250 \cdot 22\text{min} = 5500$ min.

first order in 10 mins but then has to wait for W2 to finish the second order, i.e. she has to wait for $22 - 10 = 12$ mins to get the second order. As soon as she gets it, she will do the job in 10 mins and complete the order. Therefore, the production line spent 62 min to complete the first 2 orders, that is it spent $62 - 40 = 22$ mins to complete the second order. The same logic applies to the remaining orders. We conclude that the total time to be spent on 250 orders is: $40 + 22 \cdot 249 = 5,518$ min.

1c) Utilization (ρ) is equal to the ratio input rate/ capacity $\left(\frac{\lambda}{\mu}\right)$. We know that $\mu = \frac{1}{10}$ [dolls/min]. The input rate should be calculated though, and transformed into the same units that μ has (utilization is a unitless performance measure). If not, we usually assume that plants should be operated with the bottleneck utilized 100%. From 1b) we know that it takes 22min to produce one doll. That is:

$$22 \text{ min} \mapsto 1 \text{ doll}$$

$$1 \text{ min} \mapsto \lambda \text{ dolls}$$

$$(1 \text{ hour} \mapsto 60 \cdot \lambda \text{ dolls})$$

Thus, the process produces $\lambda = \frac{1}{22}$ [dolls/min] (or $\frac{60}{22}$ [dolls/ hour]). Overall, we conclude that the utilization of W3 is $\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{22}}{\frac{1}{10}} = 0.46$ (unitless quantity).

1d) We know that the flow rate[3] $\lambda = \frac{1}{22}$ [dolls/min]. So, we have:

$$\rho\,(\text{W1}) = \frac{\lambda_{\text{W1}}}{\mu_{\text{W1}}} = \frac{\frac{1}{22}}{\frac{1}{8}} \simeq 0.36$$

$$\rho\,(\text{W2}) = \frac{\lambda_{\text{W2}}}{\mu_{\text{W2}}} = \frac{\frac{1}{22}}{\frac{1}{22}} = 1$$

$$\rho\,(\text{W3}) = \frac{\lambda_{\text{W3}}}{\mu_{\text{W3}}} = \frac{\frac{1}{22}}{\frac{1}{10}} \simeq 0.45$$

Therefore, the utilization of the average worker is $\frac{1}{3} \cdot (0.36 + 1 + 0.45) = 60\%$.

1e) We already know the answer... Intuitively, it should make the life easier of the most busy (mostly utilized) resource, the bottleneck! □

Exercise 2: Improving operations at the Sugar plant

After a hectic but fun year at TCD, you ditched a career in consulting to become the MD of the Sugar plant. You are looking forward to apply your extensive knowledge of Operations principles to this plant.

[3]Unless stated, we always assume that the plant operates at the 100% utilization level of its bottleneck.

On your first visit to the sugar producing facility, you learn the following. Sugar production has basically three steps: Purification of Molasses, followed by Superheating of the purified molasses, and finally Crystallization. The facility has three extremely reliable continuous processing machines for the three process steps. To ensure the freshness of sugar, at no point in the process, can one build work in process inventories. The performance of the individual processing machines is described below.

Step 1, Purification: Capacity: 100 Tons/hour (T/hr); Yield: 90% (in other words, if 100 Tons of input are introduced in the machine, only 90 tons are available for the next step)

Step 2, Superheating: Capacity: 80 Tons/hour; Yield: 80%

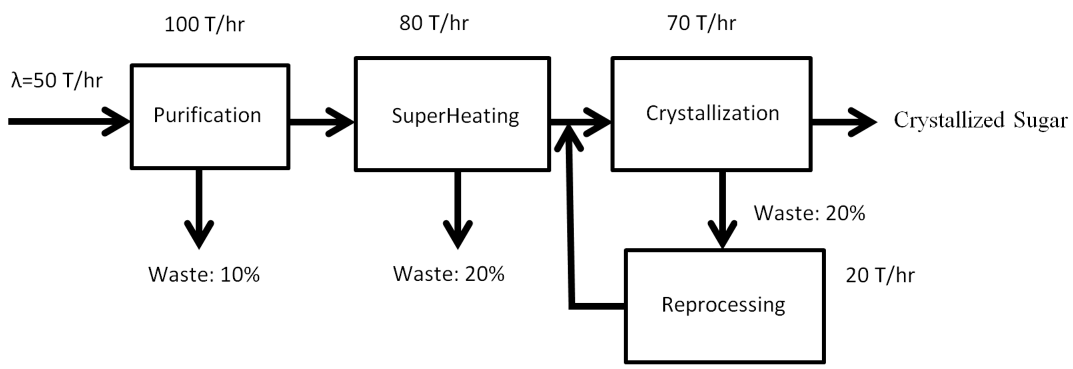
Step 3, Crystallization: Capacity: 70 Tons/hour; Yield: 80%

2a.i) Draw the process flow diagram assuming 50 T/hr of molasses are introduced into the Purification stage.

2a.ii) What would be the utilization of each of the stages? What stage is the bottleneck? What is the capacity of this plant (in Tons of crystallized sugar per hour)? What is the input required at the Purification stage to reach plant's capacity?

2b) After analyzing the plant, you feel a little frustrated by the high yield losses. You contact Tommy Tierman, a TCD Alum who is now the leading process improvement consultant for the sugar industry. He mentions a new technology available that can re-process the losses from the Crystallization stage. This could improve the efficiency of the plant, increasing its total capacity.

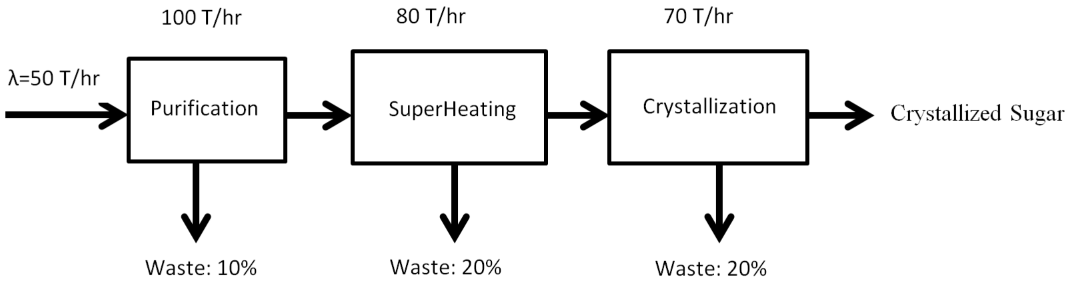
Assume we take the losses from the crystallization stage and convert them to super-heated liquid, which must be again re-introduced into the crystallization stage. We introduce a "Re-processing machine" with a capacity of 20 T/Hr. The process flow diagram is illustrated below.



What would be the utilization of each of the stages? What stage is the bottleneck here? What is the new capacity of the plant? What is the input required at the Purification stage to reach plant's capacity?

Solution

2a.i)



2a.ii) It is useful to create the following table.

Stage	Purification	Superheating	Crystallization
input (λ)	50	45	36
yield	0.9	0.8	0.8
output	$50 \cdot 0.9 = 45$	$45 \cdot 0.8 = 36$	$36 \cdot 0.8 = 28.8$
capacity (μ)	100	80	70
utilization (ρ)	$\rho = \frac{\lambda}{\mu} = \frac{50}{100} = 50\%$	$\rho = \frac{45}{80} = 56\%$	$\rho = \frac{36}{70} = 51\%$

Superheating is the bottleneck stage, since it has the highest utilization at the sustainable production rate. To compute the capacity of the plant, the bottleneck should be fully utilized. So the input to the Superheating stage must be 80 T/hr. Because Superheating produces only 80% good output, then crystallization receives 80 [T/hr] $\cdot$ 0.8 = 64 [T/hr] input. Then, the final production would be 64 [T/hr] $\cdot$ 0.8 = 51.2 [T/hr], which is the production line capacity. Note that to reach the capacity of the plant an input of 80/0.9 = 88.89 [T/hr] is required at the Purification stage.

2b) This is a case with a "feedback loop". We have that

$$0.2 \cdot (x + y) = y \Longleftrightarrow y = \frac{1}{4}x$$

where x: input rate to Crystallization from Superheating and y: input rate to Reprocessing from Crystallization. Crystallization has two inputs, from Superheating and Reprocessing, or the total flow into Crystallization is now $\frac{5}{4}$ the output of Superheating. It is useful to create the following table.

Stage	Purification	Superheating	Crystallization	Re-processing
input (λ)	50	45	$36+\frac{1}{4}\cdot 36 = 45$	$\frac{1}{4}\cdot 36 = 9$
yield	0.9	0.8	0.8	1
output	$50 \cdot 0.9 = 45$	$45 \cdot 0.8 = 36$	$45 \cdot 0.8 = 36$	9
capacity (μ)	100	80	70	20
utilization (ρ)	$\rho = \frac{50}{100} = 50\%$	$\rho = \frac{45}{80} = 56\%$	$\rho = \frac{45}{70} = 64\%$	$\rho = \frac{9}{20} = 45\%$

Crystallization is the bottleneck, as it has the highest utilization. The plant currently produces output at a rate of $45 \cdot 0.8 = 36$ [T/Hr]. However, the bottleneck should be 100% utilized for the plant to reach its capacity. That is, we increase the input rate to the Crystallization stage so that it is equal its capacity (70 T/Hr). Then, the "good" output from the Crystallization stage would give us an output of the total plant of $70 \cdot 0.8 = 56$ [T/Hr]. Note that to reach the capacity of the plant an initial input of 77.8 T/Hr is required at the Purification stage, which is the solution to the equation $\lambda \cdot 0.9 \cdot 0.8 \cdot \frac{5}{4} = 70$. □