LISTA SOBRE DEDUÇÃO NATURAL

Questão 1

a)

Construa uma prova para o argumento: $J \to \neg J \therefore \neg J$

1	$J \to \neg J$	
2	$\quad J$	
3	$\quad \neg J$	$\to$E 1, 2
4	$\quad \bot$	$\neg$E 3, 2
5	$\neg J$	$\neg$I 2–4

NOVA LINHA NOVA SUBPROVA

☺ Parabéns! Esta prova está correta.

b)

Construa uma prova para o argumento: $Q \to (Q \land \neg Q) \therefore \neg Q$

1	$Q \to (Q \land \neg Q)$	
2	$\quad Q$	
3	$\quad Q \land \neg Q$	$\to$E 1, 2
4	$\quad \neg Q$	$\land$E 3
5	$\quad \bot$	$\neg$E 4, 2
6	$\neg Q$	$\neg$I 2-5

NOVA LINHA NOVA SUBPROVA

☺ Parabéns! Esta prova está correta.

c)

Construct a proof for the argument: $A \to (B \to C) \therefore (A \land B) \to C$

1	$A \to (B \to C)$	
2	$\quad A \land B$	
3	$\quad A$	$\land$E 2
4	$\quad B \to C$	$\to$E 1, 3
5	$\quad B$	$\land$E 2
6	$\quad C$	$\to$E 4, 5
7	$(A \land B) \to C$	$\to$I 2–6

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

d)

Construct a proof for the argument: $K \wedge L \therefore K \leftrightarrow L$

1	$K \wedge L$	
2	K	∧E 1
3	L	∧E 1
4	$\quad K$	
5	$\quad L$	R 3
6	$\quad L$	
7	$\quad K$	R 2
8	$K \leftrightarrow L$	↔I 4–5, 6–7

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

e)

Construct a proof for the argument: $(C \wedge D) \vee E \therefore E \vee D$

1	$(C \wedge D) \vee E$	
2	$\quad C \wedge D$	
3	$\quad D$	∧E 2
4	$\quad E \vee D$	∨I 3
5	$\quad E$	
6	$\quad E \vee D$	∨I 5
7	$E \vee D$	∨E 1, 2–4, 5–6

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

f)

Construct a proof for the argument: $A \leftrightarrow B, B \leftrightarrow C \therefore A \leftrightarrow C$

1	$A \leftrightarrow B$	
2	$B \leftrightarrow C$	
3	$\quad A$	
4	$\quad B$	$\leftrightarrow$E 1, 3
5	$\quad C$	$\leftrightarrow$E 2, 4
6	$\quad C$	
7	$\quad B$	$\leftrightarrow$E 2, 6
8	$\quad A$	$\leftrightarrow$E 1, 7
9	$A \leftrightarrow C$	$\leftrightarrow$I 3–5, 6–8

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

g)

Construct a proof for the argument: $\neg F \rightarrow G, F \rightarrow H \therefore G \vee H$

1	$\neg F \rightarrow G$	
2	$F \rightarrow H$	
3	$\quad \neg(F \vee \neg F)$	
4	$\quad\quad F$	
5	$\quad\quad F \vee \neg F$	$\vee$I 4
6	$\quad\quad \bot$	$\neg$E 3, 5
7	$\quad \neg F$	$\neg$I 4–6
8	$\quad F \vee \neg F$	$\vee$I 7
9	$\quad \bot$	$\neg$E 3, 8
10	$F \vee \neg F$	IP 3–9
11	$\quad \neg F$	
12	$\quad G$	$\rightarrow$E 1, 11
13	$\quad G \vee H$	$\vee$I 12
14	$\quad F$	
15	$\quad H$	$\rightarrow$E 2, 14
16	$\quad G \vee H$	$\vee$I 15
17	$G \vee H$	$\vee$E 10, 11–13, 14–16

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

h)

Construct a proof for the argument: $(Z \land K) \lor (K \land M)$, $K \to D$ $\therefore$ D

1	$(Z \land K) \lor (K \land M)$	
2	$K \to D$	
3	$\quad Z \land K$	
4	$\quad K$	$\land$E 3
5	$\quad D$	$\to$E 2, 4
6	$\quad K \land M$	
7	$\quad K$	$\land$E 6
8	$\quad D$	$\to$E 2, 7
9	D	$\lor$E 1, 3–5, 6–8

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

i)

Construct a proof for the argument: $P \land (Q \lor R)$, $P \to \neg R$ $\therefore$ $Q \lor E$

1	$P \land (Q \lor R)$	
2	$P \to \neg R$	
3	$Q \lor R$	$\land$E 1
4	P	$\land$E 1
5	$\neg R$	$\to$E 2, 4
6	$\quad R$	
7	$\quad \bot$	$\neg$E 5, 6
8	$\quad Q$	X 7
9	$\quad Q$	
10	$\quad Q$	R 9
11	Q	$\lor$E 3, 6–8, 9–10
12	$Q \lor E$	$\lor$I 11

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

j)

Construct a proof for the argument: $S \leftrightarrow T \therefore S \leftrightarrow (T \vee S)$

1	$S \leftrightarrow T$	
2	│ S	
3	│ T	↔E 1, 2
4	│ $T \vee S$	∨I 3
5	│ $T \vee S$	
6	│ │ T	
7	│ │ S	↔E 1, 6
8	│ │ S	
9	│ │ S	R 8
10	│ S	∨E 5, 6–7, 8–9
11	$S \leftrightarrow (T \vee S)$	↔I 2–4, 5–10

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

k)

Construct a proof for the argument: $\neg(P \to Q) \therefore \neg Q$

1	$\neg(P \to Q)$	
2	│ Q	
3	│ │ P	
4	│ │ Q	R 2
5	│ $P \to Q$	→I 3–4
6	│ $\bot$	¬E 1, 5
7	$\neg Q$	¬I 2–6

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

l)

Construct a proof for the argument: $\neg(P \to Q) \therefore P$

1	$\neg(P \to Q)$	
2	| $\neg P$	
3	| | P	
4	| | $\bot$	¬E 2, 3
5	| | Q	X 4
6	| $P \to Q$	→I 3–5
7	| $\bot$	¬E 1, 6
8	P	IP 2–7

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

Questão 2

a)

Construct a proof for the argument: $A \to B, A \to C \therefore A \to (B \land C)$

1	$A \to B$	
2	$A \to C$	
3	| A	
4	| B	→E 1, 3
5	| C	→E 2, 3
6	| $B \land C$	∧I 4, 5
7	$A \to (B \land C)$	→I 3–6

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

b)

Construct a proof for the argument: $(A \wedge B) \rightarrow C \therefore A \rightarrow (B \rightarrow C)$

1	$(A \wedge B) \rightarrow C$	
2	$\quad A$	
3	$\quad\quad B$	
4	$\quad\quad A \wedge B$	$\wedge$I 2, 3
5	$\quad\quad C$	$\rightarrow$E 1, 4
6	$\quad B \rightarrow C$	$\rightarrow$I 3–5
7	$A \rightarrow (B \rightarrow C)$	$\rightarrow$I 2–6

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

c)

Construct a proof for the argument: $A \rightarrow (B \rightarrow C) \therefore (A \rightarrow B) \rightarrow (A \rightarrow C)$

1	$A \rightarrow (B \rightarrow C)$	
2	$\quad A \rightarrow B$	
3	$\quad\quad A$	
4	$\quad\quad B \rightarrow C$	$\rightarrow$E 1, 3
5	$\quad\quad B$	$\rightarrow$E 2, 3
6	$\quad\quad C$	$\rightarrow$E 4, 5
7	$\quad A \rightarrow C$	$\rightarrow$I 3–6
8	$(A \rightarrow B) \rightarrow (A \rightarrow C)$	$\rightarrow$I 2–7

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

d)

Construct a proof for the argument: $A \vee (B \wedge C) \therefore (A \vee B) \wedge (A \vee C)$

1	$A \vee (B \wedge C)$	
2	│ A	
3	│ $A \vee B$	∨I 2
4	│ $A \vee C$	∨I 2
5	│ $(A \vee B) \wedge (A \vee C)$	∧I 3, 4
6	│ $B \wedge C$	
7	│ B	∧E 6
8	│ $A \vee B$	∨I 7
9	│ C	∧E 6
10	│ $A \vee C$	∨I 9
11	│ $(A \vee B) \wedge (A \vee C)$	∧I 8, 10
12	$(A \vee B) \wedge (A \vee C)$	∨E 1, 2–5, 6–11

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

e)

Construct a proof for the argument: $(A \wedge B) \vee (A \wedge C) \therefore A \wedge (B \vee C)$

1	$(A \wedge B) \vee (A \wedge C)$	
2	│ $A \wedge B$	
3	│ A	∧E 2
4	│ B	∧E 2
5	│ $B \vee C$	∨I 4
6	│ $A \wedge (B \vee C)$	∧I 3, 5
7	│ $A \wedge C$	
8	│ A	∧E 7
9	│ C	∧E 7
10	│ $B \vee C$	∨I 9
11	│ $A \wedge (B \vee C)$	∧I 8, 10
12	$A \wedge (B \vee C)$	∨E 1, 2–6, 7–11

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

f)

Construct a proof for the argument: $A \vee B, A \rightarrow C, B \rightarrow D \therefore C \vee D$

1	$A \vee B$	
2	$A \rightarrow C$	
3	$\underline{B \rightarrow D}$	
4	$\quad \underline{A}$	
5	$\quad C$	$\rightarrow$E 2, 4
6	$\quad C \vee D$	$\vee$I 5
7	$\quad \underline{B}$	
8	$\quad D$	$\rightarrow$E 3, 7
9	$\quad C \vee D$	$\vee$I 8
10	$C \vee D$	$\vee$E 1, 4–6, 7–9

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

g)

Construct a proof for the argument: $\neg A \vee \neg B \therefore \neg(A \wedge B)$

1	$\underline{\neg A \vee \neg B}$	
2	$\quad \underline{A \wedge B}$	
3	$\quad A$	$\wedge$E 2
4	$\quad B$	$\wedge$E 2
5	$\quad\quad \underline{\neg A}$	
6	$\quad\quad \bot$	$\neg$E 5, 3
7	$\quad\quad \underline{\neg B}$	
8	$\quad\quad \bot$	$\neg$E 7, 4
9	$\quad \bot$	$\vee$E 1, 5–6, 7–8
10	$\neg(A \wedge B)$	$\neg$I 2–9

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

h)

Construct a proof for the argument: $A \wedge \neg B \therefore \neg(A \rightarrow B)$

```
1 │ A ∧ ¬B
  ├──────
2 │ │ A → B
  │ ├──────
3 │ │ A                 ∧E 1
4 │ │ ¬B                ∧E 1
5 │ │ B                 →E 2, 3
6 │ │ ⊥                 ¬E 4, 5
7 │ ¬(A → B)            ¬I 2–6
```

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

i)

Construct a proof for the argument: $\neg A \rightarrow \neg B \therefore B \rightarrow A$

```
1 │ ¬A → ¬B
  ├────────
2 │ │ B
  │ ├───
3 │ │ │ ¬A
  │ │ ├───
4 │ │ │ ¬B              →E 1, 3
5 │ │ │ ⊥               ¬E 4, 2
6 │ │ A                 IP 3–5
7 │ B → A               →I 2–6
```

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

j)

Construct a proof for the argument: $A \rightarrow B \therefore \neg A \vee B$

```
 1 │ A → B
   ├──────
 2 │ │ ¬(¬A ∨ B)
   │ ├──────────
 3 │ │ │ A
   │ │ ├───
 4 │ │ │ B             →E 1, 3
 5 │ │ │ ¬A ∨ B        ∨I 4
 6 │ │ │ ⊥             ¬E 5, 2
 7 │ │ ¬A              ¬I 3–6
 8 │ │ ¬A ∨ B          ∨I 7
 9 │ │ ⊥               ¬E 2, 8
10 │ ¬A ∨ B            IP 2–9
```

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

K)

Construct a proof for the argument: $A \to (B \vee C) \therefore (A \to B) \vee (A \to C)$

1	$A \to (B \vee C)$	
2	$\quad \neg((A \to B) \vee (A \to C))$	
3	$\quad\quad A$	
4	$\quad\quad B \vee C$	→E 1, 3
5	$\quad\quad\quad B$	
6	$\quad\quad\quad\quad A$	
7	$\quad\quad\quad\quad B$	R 5
8	$\quad\quad\quad A \to B$	→I 6–7
9	$\quad\quad\quad (A \to B) \vee (A \to C)$	∨I 8
10	$\quad\quad\quad C$	
11	$\quad\quad\quad\quad A$	
12	$\quad\quad\quad\quad C$	R 10
13	$\quad\quad\quad A \to C$	→I 11–12
14	$\quad\quad\quad (A \to B) \vee (A \to C)$	∨I 13
15	$\quad\quad (A \to B) \vee (A \to C)$	∨E 4, 5–9, 10–14
16	$\quad\quad \bot$	¬E 2, 15
17	$\quad\quad C$	X 16
18	$\quad A \to C$	→I 3–17
19	$\quad (A \to B) \vee (A \to C)$	∨I 18
20	$\quad \bot$	¬E 2, 19
21	$(A \to B) \vee (A \to C)$	IP 2–20

NEW LINE | NEW SUBPROOF

☺ Congratulations! This proof is correct.

Questão 3

a)

Construct a proof for the argument: $\therefore \neg A \to (A \to \bot)$

1	$\quad \neg A$	
2	$\quad\quad A$	
3	$\quad\quad \bot$	¬E 1, 2
4	$\quad A \to \bot$	→I 2–3
5	$\neg A \to (A \to \bot)$	→I 1–4

NEW LINE | NEW SUBPROOF

☺ Congratulations! This proof is correct.

b)

Construct a proof for the argument: $\therefore \neg(A \wedge \neg A)$

1	$A \wedge \neg A$	
2	A	$\wedge$E 1
3	$\neg A$	$\wedge$E 1
4	$\bot$	$\neg$E 3, 2
5	$\neg(A \wedge \neg A)$	$\neg$I 1–4

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

c)

Construct a proof for the argument: $\therefore [(A \rightarrow C) \wedge (B \rightarrow C)] \rightarrow [(A \vee B) \rightarrow C]$

1	$(A \rightarrow C) \wedge (B \rightarrow C)$	
2	$(A \rightarrow C)$	$\wedge$E 1
3	$(B \rightarrow C)$	$\wedge$E 1
4	$A \vee B$	
5	A	
6	C	$\rightarrow$E 2, 5
7	B	
8	C	$\rightarrow$E 3, 7
9	C	$\vee$E 4, 5–6, 7–8
10	$(A \vee B) \rightarrow C$	$\rightarrow$I 4–9
11	$[(A \rightarrow C) \wedge (B \rightarrow C)] \rightarrow [(A \vee B) \rightarrow C]$	$\rightarrow$I 1–10

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

d)

Construct a proof for the argument: $\therefore \neg(A \to B) \to (A \wedge \neg B)$

1	$\neg(A \to B)$	
2	$\neg A$	
3	A	
4	$\bot$	¬E 2, 3
5	B	X 4
6	$A \to B$	→I 3–5
7	$\bot$	¬E 1, 6
8	A	IP 2–7
9	B	
10	A	
11	B	R 9
12	$A \to B$	→I 10–11
13	$\bot$	¬E 1, 12
14	$\neg B$	¬I 9–13
15	$A \wedge \neg B$	∧I 8, 14
16	$\neg(A \to B) \to (A \wedge \neg B)$	→I 1–15

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

e)

Construct a proof for the argument: $\therefore (\neg A \vee B) \to (A \to B)$

1	$\neg A \vee B$	
2	A	
3	$\neg A$	
4	$\bot$	¬E 3, 2
5	B	X 4
6	B	
7	B	R 6
8	B	∨E 1, 3–5, 6–7
9	$A \to B$	→I 2–8
10	$(\neg A \vee B) \to (A \to B)$	→I 1–9

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

f)

Construct a proof for the argument: $\therefore \neg\neg A \rightarrow A$

1	$\vert\ \vert\ \neg\neg A$	
2	$\vert\ \vert\ \vert\ \neg A$	
3	$\vert\ \vert\ \vert\ \bot$	¬E 1, 2
4	$\vert\ \vert\ A$	IP 2–3
5	$\vert\ \neg\neg A \rightarrow A$	→I 1–4

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

g)

Construct a proof for the argument: $\therefore \neg(A \wedge B) \rightarrow (\neg A \vee \neg B)$

1	$\vert\ \vert\ \neg(A \wedge B)$	
2	$\vert\ \vert\ \vert\ \neg(A \vee \neg A)$	
3	$\vert\ \vert\ \vert\ \vert\ A$	
4	$\vert\ \vert\ \vert\ \vert\ A \vee \neg A$	∨I 3
5	$\vert\ \vert\ \vert\ \vert\ \bot$	¬E 2, 4
6	$\vert\ \vert\ \vert\ \neg A$	¬I 3–5
7	$\vert\ \vert\ \vert\ A \vee \neg A$	∨I 6
8	$\vert\ \vert\ \vert\ \bot$	¬E 2, 7
9	$\vert\ \vert\ A \vee \neg A$	IP 2–8
10	$\vert\ \vert\ \vert\ A$	
11	$\vert\ \vert\ \vert\ \vert\ B$	
12	$\vert\ \vert\ \vert\ \vert\ A \wedge B$	∧I 10, 11
13	$\vert\ \vert\ \vert\ \vert\ \bot$	¬E 1, 12
14	$\vert\ \vert\ \vert\ \neg B$	¬I 11–13
15	$\vert\ \vert\ \vert\ \neg A \vee \neg B$	∨I 14
16	$\vert\ \vert\ \vert\ \neg A$	
17	$\vert\ \vert\ \vert\ \neg A \vee \neg B$	∨I 16
18	$\vert\ \vert\ \neg A \vee \neg B$	∨E 9, 10–15, 16–17
19	$\vert\ \neg(A \wedge B) \rightarrow (\neg A \vee \neg B)$	→I 1–18

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

h)

Construct a proof for the argument: $\therefore (A \to B) \vee (B \to A)$

1	$\neg((A \to B) \vee (B \to A))$	
2	$\neg(A \to B)$	
3	$\neg(B \to A)$	
4	A	
5	B	
6	A	R 4
7	$B \to A$	$\to$I 5–6
8	$\bot$	$\neg$E 3, 7
9	B	X 8
10	$A \to B$	$\to$I 4–9
11	$\bot$	$\neg$E 2, 10
12	$B \to A$	IP 3–11
13	$(A \to B) \vee (B \to A)$	$\vee$I 12
14	$\bot$	$\neg$E 1, 13
15	$A \to B$	IP 2–14
16	$(A \to B) \vee (B \to A)$	$\vee$I 15
17	$\bot$	$\neg$E 1, 16
18	$(A \to B) \vee (B \to A)$	IP 1–17

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

i)

Construct a proof for the argument: $\therefore [(A \to B) \to A] \to A$

1	$(A \to B) \to A$	
2	$\neg A$	
3	A	
4	$\bot$	$\neg$E 2, 3
5	B	X 4
6	$A \to B$	$\to$I 3–5
7	A	$\to$E 1, 6
8	$\bot$	$\neg$E 2, 7
9	A	IP 2–8
10	$((A \to B) \to A) \to A$	$\to$I 1–9

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

Questão 4

a)

Construct a proof for the argument: $A \lor B, \neg A \therefore B$

1	$A \lor B$	
2	$\neg A$	
3	$\quad\vert\ A$	
4	$\quad\vert\ \bot$	$\neg$E 2, 3
5	$\quad\vert\ B$	X 4
6	$\quad\vert\ B$	
7	$\quad\vert\ B$	R 6
8	B	$\lor$E 1, 3–5, 6–7

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

b)

Construct a proof for the argument: $A \lor B, \neg B \therefore A$

1	$A \lor B$	
2	$\neg B$	
3	$\quad\vert\ A$	
4	$\quad\vert\ A$	R 3
5	$\quad\vert\ B$	
6	$\quad\vert\ \bot$	$\neg$E 2, 5
7	$\quad\vert\ A$	X 6
8	A	$\lor$E 1, 3–4, 5–7

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

c)

Construct a proof for the argument: $A \to B, \neg B \therefore \neg A$

1	$A \to B$	
2	$\neg B$	
3	$\quad A$	
4	$\quad B$	$\to$E 1, 3
5	$\quad \bot$	$\neg$E 2, 4
6	$\neg A$	$\neg$I 3–5

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

d)

Construct a proof for the argument: $\therefore \neg\neg A \to A$

1	$\quad \neg\neg A$	
2	$\quad\quad \neg A$	
3	$\quad\quad \bot$	$\neg$E 1, 2
4	$\quad A$	IP 2–3
5	$\neg\neg A \to A$	$\to$I 1–4

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

e)

Construct a proof for the argument: $A \to B$, $\neg A \to B \therefore B$

1	$A \to B$	
2	$\neg A \to B$	
3	│ $\neg B$	
4	│ │ A	
5	│ │ B	$\to$E 1, 4
6	│ │ $\bot$	$\neg$E 3, 5
7	│ $\neg A$	$\neg$I 4–6
8	│ B	$\to$E 2, 7
9	│ $\bot$	$\neg$E 3, 8
10	B	IP 3–9

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

f)

Construct a proof for the argument: $\neg(A \wedge B) \therefore \neg A \vee \neg B$

1	$\neg(A \wedge B)$	
2	│ A	
3	│ │ B	
4	│ │ $A \wedge B$	$\wedge$I 2, 3
5	│ │ $\bot$	$\neg$E 1, 4
6	│ $\neg B$	$\neg$I 3–5
7	│ $\neg A \vee \neg B$	$\vee$I 6
8	│ $\neg A$	
9	│ $\neg A \vee \neg B$	$\vee$I 8
10	$\neg A \vee \neg B$	LEM 2–7, 8–9

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

g)

Construct a proof for the argument: $\neg A \vee \neg B \therefore \neg(A \wedge B)$

1	$\neg A \vee \neg B$	
2	$\quad A \wedge B$	
3	$\quad A$	$\wedge$E 2
4	$\quad B$	$\wedge$E 2
5	$\quad\quad \neg A$	
6	$\quad\quad \bot$	$\neg$E 5, 3
7	$\quad\quad \neg B$	
8	$\quad\quad \bot$	$\neg$E 7, 4
9	$\quad \bot$	$\vee$E 1, 5–6, 7–8
10	$\neg(A \wedge B)$	$\neg$I 2–9

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

h)

Construct a proof for the argument: $\neg(A \vee B) \therefore \neg A \wedge \neg B$

1	$\neg(A \vee B)$	
2	$\quad A$	
3	$\quad A \vee B$	$\vee$I 2
4	$\quad \bot$	$\neg$E 1, 3
5	$\neg A$	$\neg$I 2–4
6	$\quad B$	
7	$\quad A \vee B$	$\vee$I 6
8	$\quad \bot$	$\neg$E 1, 7
9	$\neg B$	$\neg$I 6–8
10	$\neg A \wedge \neg B$	$\wedge$I 5, 9

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

i)

Construct a proof for the argument: $\neg A \land \neg B \therefore \neg(A \lor B)$

1	$\neg A \land \neg B$	
2	$\neg A$	$\land$E 1
3	$\neg B$	$\land$E 1
4	│ $A \lor B$	
5	│ │ A	
6	│ │ $\neg A$	R 2
7	│ │ $\bot$	$\neg$E 6, 5
8	│ │ B	
9	│ │ $\neg B$	R 3
10	│ │ $\bot$	$\neg$E 9, 8
11	│ $\bot$	$\lor$E 4, 5–7, 8–10
12	$\neg(A \lor B)$	$\neg$I 4–11

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

Questão 5

a) Usando as regras derivadas

Construct a proof for the argument: $E \lor F, F \lor G, \neg F \therefore E \land G$

1	$E \lor F$	
2	$F \lor G$	
3	$\neg F$	
4	E	DS 1, 3
5	G	DS 2, 3
6	$E \land G$	$\land$I 4, 5

NEW LINE NEW SUBPROOF

Congratulations! This proof is correct.

a) Sem usar regras derivadas

Construct a proof for the argument: $E \vee F, F \vee G, \neg F \therefore E \wedge G$

1	$E \vee F$	
2	$F \vee G$	
3	$\neg F$	
4	$\quad F$	
5	$\quad \bot$	¬E 4, 3
6	$\quad G$	X 5
7	$\quad G$	
8	$\quad G$	R 7
9	G	∨E 2, 4–6, 7–8
10	$\quad E$	
11	$\quad E$	R 10
12	$\quad F$	
13	$\quad \bot$	¬E 3, 12
14	$\quad E$	X 13
15	E	∨E 1, 10–11, 12–14
16	$E \wedge G$	∧I 9, 15

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

b)

Construct a proof for the argument: $M \vee (N \to M) \therefore \neg M \to \neg N$

1	$M \vee (N \to M)$	
2	$\quad \neg M$	
3	$\quad N \to M$	DS 1, 2
4	$\quad \neg N$	MT 3, 2
5	$\neg M \to \neg N$	→I 2–4

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

c)

Construct a proof for the argument: $(M \vee N) \wedge (O \vee P)$, $N \to P$, $\neg P \therefore M \wedge O$

1	$(M \vee N) \wedge (O \vee P)$	
2	$N \to P$	
3	$\neg P$	
4	$\neg N$	MT 2, 3
5	$M \vee N$	∧E 1
6	M	DS 5, 4
7	$O \vee P$	∧E 1
8	O	DS 7, 3
9	$M \wedge O$	∧I 6, 8

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

d)

Construct a proof for the argument: $(X \wedge Y) \vee (X \wedge Z)$, $\neg(X \wedge D)$, $D \vee M \therefore M$

1	$(X \wedge Y) \vee (X \wedge Z)$	
2	$\neg(X \wedge D)$	
3	$D \vee M$	
4	$\quad X \wedge Y$	
5	$\quad X$	∧E 4
6	$\quad X \wedge Z$	
7	$\quad X$	∧E 6
8	X	∨E 1, 4–5, 6–7
9	$\neg X \vee \neg D$	DeM 2
10	$\quad \neg X$	
11	$\quad \bot$	¬E 10, 8
12	$\quad M$	X 11
13	$\quad \neg D$	
14	$\quad M$	DS 3, 13
15	M	∨E 9, 10–12, 13–14

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

e)

Construct a proof for the argument: $C \to (E \wedge G), \neg C \to G \therefore G$

1	$C \to (E \wedge G)$	
2	$\neg C \to G$	
3	$\quad C$	
4	$\quad E \wedge G$	$\to$E 1, 3
5	$\quad G$	$\wedge$E 4
6	$\quad \neg C$	
7	$\quad G$	$\to$E 2, 6
8	G	LEM 3–5, 6–7

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

f)

Construct a proof for the argument: $M \wedge (\neg N \to \neg M) \therefore (N \wedge M) \vee \neg M$

1	$M \wedge (\neg N \to \neg M)$	
2	M	$\wedge$E 1
3	$(\neg N \to \neg M)$	$\wedge$E 1
4	$\quad \neg N$	
5	$\quad \neg M$	$\to$E 3, 4
6	$\quad \bot$	$\neg$E 5, 2
7	N	IP 4–6
8	$N \wedge M$	$\wedge$I 7, 2
9	$(N \wedge M) \vee \neg M$	$\vee$I 8

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

g)

Construct a proof for the argument: $(Z \wedge K) \leftrightarrow (Y \wedge M), D \wedge (D \rightarrow M) \therefore Y \rightarrow Z$

1	$(Z \wedge K) \leftrightarrow (Y \wedge M)$	
2	$D \wedge (D \rightarrow M)$	
3	Y	
4	$D \rightarrow M$	$\wedge$E 2
5	D	$\wedge$E 2
6	M	$\rightarrow$E 4, 5
7	$Y \wedge M$	$\wedge$I 3, 6
8	$Z \wedge K$	$\leftrightarrow$E 1, 7
9	Z	$\wedge$E 8
10	$Y \rightarrow Z$	$\rightarrow$I 3–9

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

h)

Construct a proof for the argument: $(W \vee X) \vee (Y \vee Z), X \rightarrow Y, \neg Z \therefore W \vee Y$

1	$(W \vee X) \vee (Y \vee Z)$	
2	$X \rightarrow Y$	
3	$\neg Z$	
4	$W \vee X$	
5	W	
6	$W \vee Y$	$\vee$I 5
7	X	
8	Y	$\rightarrow$E 2, 7
9	$W \vee Y$	$\vee$I 8
10	$W \vee Y$	$\vee$E 4, 5–6, 7–9
11	$Y \vee Z$	
12	$\neg Z$	R 3
13	Y	DS 11, 12
14	$W \vee Y$	$\vee$I 13
15	$W \vee Y$	$\vee$E 1, 4–10, 11–14

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.