Lista de Exercícios sobre Funções Trigonométricas

(1) $A = \text{sen } 75^\circ + \text{sen } \frac{5\pi}{3} \cdot \cos \frac{13\pi}{6} + tg\, 225^\circ \cdot \cos 120^\circ$

	30	45	60
sen	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$
tg	$\sqrt{3}/3$	1	$\sqrt{3}$

$\text{sen}(75^\circ)$

$\text{sen}(45^\circ + 30^\circ) = \text{sen}45 \cdot \cos 30 - \text{sen}30 \cdot \cos 45$

$\sqrt{2}/2 \cdot \sqrt{3}/2 - 1/2 \cdot \sqrt{2}/2$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$ //

$\text{sen}(5\pi/3) = \text{sen}(300^\circ) = \text{sen}(-60) = -\text{sen}(60) = -\sqrt{3}/2$

$\cos(13\pi/6) = \cos(\pi/6) = \cos(30^\circ) = \sqrt{3}/2$ //

$\cos(120^\circ) = \cos(60 + 60)^\circ = \cos 60 \cdot \cos 60^\circ - \text{sen } 60^\circ \text{ sen } 60^\circ$

$1/2 \cdot 1/2 - \sqrt{3}/2 \cdot \sqrt{3}/2$

$1/4 - 3/4 = \frac{-2}{4} = -1/2$ //

$A = \frac{\sqrt{6}-\sqrt{2}}{4} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + 1 \Rightarrow \frac{\sqrt{6}-\sqrt{2}}{4} - \frac{3}{4} + 1$ (*)

$tg\, 225 = \frac{\text{sen } 225}{\cos 225}$

$\text{sen}(225) = \text{sen}(225 - 360)$

$\text{sen}(-135) = -\text{sen}(135)$

$= -\sqrt{2}/2$

$tag\, 225 = \frac{-\sqrt{2}/2}{-\sqrt{2}/2} \Rightarrow \frac{-\sqrt{2}}{2} \cdot \frac{2}{-\sqrt{2}}$

$\cos(225) = \cos(225 - 360)$

$\cos(-135) = \cos(135) = -\sqrt{2}/2$

$\Rightarrow \frac{-2\sqrt{2}}{-2\sqrt{2}} = 1$ //

(*) $\frac{\sqrt{6}-\sqrt{2}}{4} - \frac{3}{4} + \frac{4}{4} \Rightarrow \frac{\sqrt{6}-\sqrt{2}+1}{4}$ //

$$f(x)=\frac{1}{2}+\frac{1}{2}\cos(2x)$$

(2) $f(x) = \frac{1}{2} + \boxed{\frac{1}{2}\cos(2x)}$ $\quad p = \frac{2\pi}{|c|} \Rightarrow \frac{2\pi}{\cancel{2}} \quad \pi$

Imagem: $a = \frac{1}{2}$ $b = \frac{1}{2}$

amplitude $1/2$,

$[a-b, a+b]$

$[0, 1]$

x	cos x	cos 2x	½ cos 2x	½ + ½ cos (2x)
0	1	1	$1/2$	1
$\pi/4$	$\sqrt{2}/2$	0	0	$1/2$
$\pi/2$	0	-1	$-1/2$	0
$3\pi/4$	$\sqrt{2}/2$	0	0	$1/2$
π	-1	1	$1/2$	1

$\cos(135) = \sqrt{2}/2$

$\frac{3\pi}{4} \cdot \frac{2}{1} \quad \frac{6\pi}{4}$

$\frac{3\pi}{2}$

3. $f(x) = \text{sen } x$ $\qquad g(x) = 3\,\text{sen}\left(x + \frac{\pi}{2}\right)$

período $= \frac{2\pi}{|c|}$ $\quad$ período $= 2\pi$

$Im(g) = [a+b, a-b] = a = 0 \quad b = 3$

$[3, -3] \Rightarrow [-3, 3]$ //

x	sen x	sen (x + π/2)	3 sen (x + π/2)
0	0	1	3
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	$3\sqrt{2}/2$
$\pi/2$	1	0	0
$3\pi/4$	$\sqrt{2}/2$	$-\sqrt{2}/2$	$-3\sqrt{2}/2$
π	0	-1	-3

4) Simplifique as expressões

a) $$\frac{(\text{sen}(2x)\,\text{cossec}(x))\,(\text{sen}^2(x)+\cos^2(x)+tg^2(x))}{tg(x)\,.\,\sec^2(x)\,.\,\cos(x)}$$

$$\frac{(\text{sen}(2x).\text{cossec}(x))\,(1+tg^2(x))}{tg(x).\sec^2(x).\cos(x)} \Rightarrow \frac{(\text{sen}(2x).\ \text{cossec}\ x).\ \cancel{\sec^2 x}}{tg(x).\ \cancel{\sec^2(x)}.\cos(x)}$$

$$\frac{(\text{sen}(2x).\text{cossec}(x))}{tg(x).\cos(x)} \longrightarrow 2.\text{sen}\cancel{(x)}.\cos(x).\ \frac{1}{\text{sen}\cancel{(x)}} = 2\cos x$$

$$\frac{2\cos x}{tg(x).\cos(x)} \rightarrow \Rightarrow \frac{2\cos(x)}{\frac{\text{sen}(x)}{\cancel{\cos(x)}}.\ \cancel{\cos(x)}} \Rightarrow \frac{2\cos(x)}{\text{sen}(x)} = 2\,\text{cotg}(x)$$ //

b) $$\frac{\text{sen}\,x}{1+\cos x} + \frac{1+\cos x}{\text{sen}\,x} = \frac{(\text{sen}\,x)^2\ \ (1+\cos x).(1+\cos x)}{(1+\cos x).\ \text{sen}\,x}$$

$$\frac{\overbrace{(\text{sen}\,x)^2\ \ 1+\cos x+\cos x+(\cos x)^2}}{(1+\cos x).\ \text{sen}(x)} \Rightarrow \frac{2+\cos x+\cos x}{(1+\cos x).\text{sen}\,x}$$

$$\frac{2+2\cos x}{(1+\cos x).\text{sen}\,x} \Rightarrow \frac{2(1+\cancel{\cos x})}{(1\ \cancel{\cos x}).\ \text{sen}\,x} \Rightarrow \frac{2}{\text{sen}\,x} = 2.\frac{1}{\text{sen}\,x}$$

$2.\ \text{cossec}\ x$ //

2cossec (x)

5. Considerando as funções $f(x)=\sin x$ e $g(x)=\cos x$, relacione a segunda coluna de acordo com a primeira, estabelecendo identidades trigonométricas:

(1) $f(2x)$ (4) $\frac{1-g(2x)}{2}$

(2) $g(2x)$ (1) $2f(x)g(x)$

(3) $f^2(x)+g^2(x)$ (5) $\left(\frac{f(x)}{g(x)}\right)^2$

(4) $f(x)^2$ (2) $g^2(x)-f^2(x)$

(5) $\frac{1}{g^2(x)}-1$ (3) 1

① $f(x) = \text{sen}\,x \Rightarrow f(2x) \Rightarrow \text{sen}\,2x$

$2\,\text{sen}\,x\cos x \Rightarrow 2f(x)g(x)$

② $g(2x) = \cos 2x$

$2\cos x \cdot \text{sen}\,x$

$2\,g(x) - f(x)$

④ $\frac{1-g(2x)}{2} \Rightarrow \frac{1-\cos 2x}{2}$

$\Rightarrow \underline{1 - 2.\cos x.-\text{sen}\,x}$

③ $f^2(x) + g^2(x)$

$\cos^2(x) + \text{sen}^2(x) = 1$

⑥ $-1 \leq \frac{3x+2}{2} \leq 1 \Rightarrow -2 \leq 3x+2 \leq 2 \Rightarrow$ (-2) $\times 2$

$\Rightarrow -4 \leq 3x \leq 0$ $(x\,1/3)$ $\Rightarrow \frac{-4}{3} \leq x \leq 0$

⑦ a) $r(x) = 2\sin(x/3)$ $a=0\,;\, b=2$

$Im = [0-2, 0+2] = [-2, 2]$

b) $q(x) = 2 + 5\cos x \qquad a = 2 \,;\, b = 5$

$Im[a-b, a+b] \Rightarrow [-3, 7]$

c) $p(x) = 5\cos(2x + \pi/4] \Rightarrow \quad a = 0 \,;\, b = 5$

$Im(p) = [-5, 5]$

$sen\, -\theta = -sen\,\theta$

$\cos -\theta = \cos\theta$

$tg\, -\theta = -tg\,\theta$

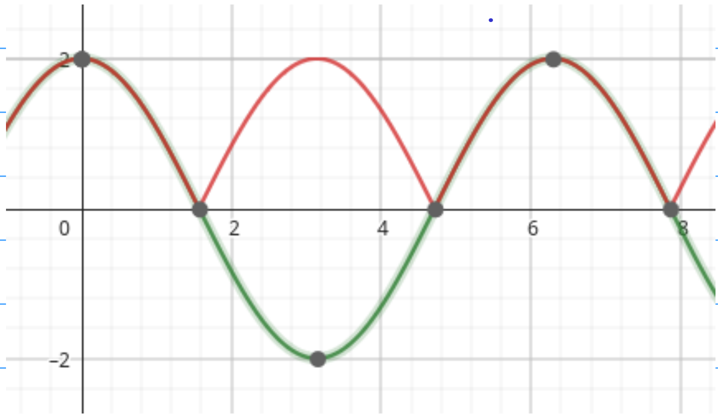
(8) a) $r(x) = \dfrac{2\sin x}{\cos(x) \cdot tg(x)}$

$r(-x) = \dfrac{2\,sen\,-x}{\cos(-x) \cdot tg(-x)} \Rightarrow \dfrac{2\,-sen\,x}{\cos(x) \cdot (-tg\,x)}$ $\ominus$ $\ominus$ função par!

b) $p(x) = \dfrac{\pi\, sen^3(x) \cdot \cos(x) \cdot tg(x)}{sen(x) + \sqrt{1-\cos^2(x)}} \Rightarrow$

$p(-x) = \dfrac{\pi\, sen^3(-x) \cdot \cos(-x) \cdot tg(-x)}{sen(-x) + \sqrt{1-\cos^2(-x)}}$

$\dfrac{-\pi \cdot sen^3(x) \cdot \cos(x) \cdot -tg(x)}{-sen(x) + \sqrt{1-\cos^2(x)}}$ $\quad \dfrac{+}{\pm}$ → como não temos uma multiplicação o valor pode dar os dois sendo assim nem par nem impar.



⑨ $f(x) = |2\cos x|$

b) $g(x) = \cos(x/2)$ periodo $= \dfrac{2\pi}{|c|} = \dfrac{2\pi}{1/2}$

$\dfrac{2\pi}{1} \cdot \dfrac{2}{1} = \dfrac{4\pi}{1} = 4\pi$ //

c) $h(x) = tg(x + \pi/2)$

⑩ a) $f(x) =$ cossec $x \Rightarrow \dfrac{1}{\text{sen}\,x}$

$\{x \in \mathbb{R} / x = k\pi,\ k \in \mathbb{Z}\}$ //

b) $f(x) = \dfrac{1 - \text{sen}^2 x}{1 + \text{sen}\,x}$ $\quad 1 + \text{sen}\,x \neq 0$

$\text{sen}\,x \neq 1$

$\frac{\pi}{2}$ ou $\frac{3\pi}{2}$

$\{x \in \mathbb{R} / x \neq 3\pi/2 + 2k\pi,\ k \in \mathbb{Z}\}$ //

(11)

a) $Arcsen(-\sqrt{3}/2) = \pi/4$ //

b) $arctan(1) = 45^\circ$ $\pi/4$

c) $arccos(1/2) = \pi/3$

d) $arcsec(-2) = \dfrac{1}{\cos(x)} = -2$

$\cos(x) \cdot -2 = 1$

$-\cos(x) = \dfrac{1}{2}$ $\cos(x) = -1/2$ // $= 120$

ou $2\pi/3$

e) $arcsen(sen\, \pi/7) = \pi/2$

f) $arctan(tan\, \dfrac{13\pi}{6}) = 13\pi/6$

(12) $arctan(4/3)$ $tan\, x = 4/3$

- $tan\,\theta = 4/3$; - $cotg\,\theta = \dfrac{1}{tg\,\theta} \Rightarrow \dfrac{1}{4/3} = \dfrac{3}{4}$ //

- $1 + tg^2\theta = sec^2\theta$

$1 + (4/3)^2 = \dfrac{1}{\cos^2\theta}$

$1 + 16/9 = \dfrac{1}{\cos^2\theta}$

- $25/9 = 1/\cos^2\theta$

$\cos^2\theta = 9/25$

$\cos\theta = \sqrt{9/25}$

- $\cos\theta = \pm 3/5$ //

- $sen^2\theta + \cos^2\theta = 1$

$sen^2\theta = 1 - 9/25$

$sen^2\theta = 16/25$

- $sen\,\theta = \pm\sqrt{16/25}$

$cossec\,\theta = \dfrac{1}{sen\,\theta}$ $\therefore$ $5/4$ //

13) $f(x) = \arccos(12 - x)$

(-12)

$(x \cdot -1)$

$-1 \leq 12 - x \leq 1$

$-13 \leq -x \leq -11$

$13 \geq x \geq 11$

$x \geq 11$

$x \leq 13$ $\therefore [11, 13]$

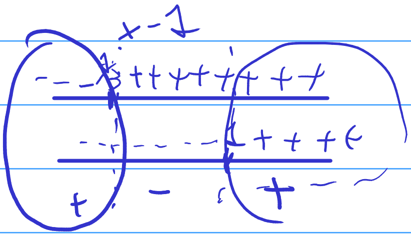
14) $f(x) = \operatorname{arcsen}\left(\dfrac{2x}{x-1}\right)$

$\boxed{-1 \leq \dfrac{2x}{x-1} \leq 1}$ $\Rightarrow \dfrac{2x}{x-1} \geq -1 \Rightarrow \dfrac{2x}{x-1} + 1 \geq 0$

$\dfrac{2x + x - 1}{x-1} \geq 0 \Rightarrow \dfrac{3x-1}{x-1} \Rightarrow x = 1/3$

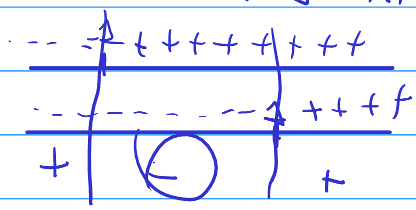
$\Rightarrow x = 1$

$x \neq 1$



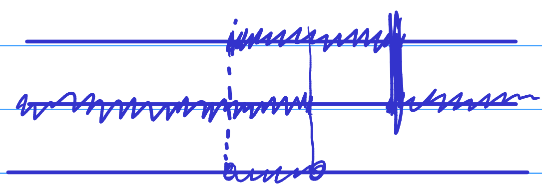
$(-\infty, 1/3] \cup (1, +\infty)$

$\dfrac{2x}{x-1} \leq 1 \Rightarrow \dfrac{2x}{x-1} - 1 \leq 0$ $\quad \dfrac{x+1}{x-1} \leq 0$ $\quad x \neq 1$ $\quad x = -1$, $x = 1$



$[-1, 1)$

$Df = [-1, 1] \cap ((-\infty, 1/3] \cup (1, +\infty))$



$[-1, 1/3]$ //