

GABARITO EXERCÍCIO 2

n)

1	Pb	
2	$\forall x \forall y [(Px \wedge Py) \rightarrow x = y]$	
3	$\forall y ((Pa \wedge Py) \rightarrow a = y)$	$\forall E\ 2$
4	$(Pa \wedge Pb) \rightarrow a = b$	$\forall E\ 3$
5	Pa	
6	$Pa \wedge Pb$	$\wedge I\ 1, 5$
7	$a = b$	$\rightarrow E\ 4, 6$
8	$Pa \rightarrow a = b$	$\rightarrow I\ 5-7$
9	$a = b$	
10	Pa	$=E\ 1, 9$
11	$(a = b) \rightarrow (Pa)$	$\rightarrow I\ 9-10$
12	$(Pa \rightarrow a = b) \wedge ((a = b) \rightarrow (Pa))$	$\wedge I\ 11, 8$
13	$\forall x ((Px \rightarrow x = b) \wedge ((x = b) \rightarrow (Px)))$	$\forall I\ 12$

o)

1	$\exists x \exists y (Hxy \vee Hyx)$	
2	$\neg \exists x Hxx$	
3	$\exists y (Hay \vee Hya)$	
4	$Hab \vee Hba$	
5	$a = b$	
6	Hab	
7	Haa	$=E\ 6, 5$
8	Hba	
9	Haa	$=E\ 5, 8$
10	Haa	$\vee E\ 4, 6-7, 8-9$
11	$\exists x (Hxx)$	$\exists I\ 10$
12	\perp	$\neg E\ 2, 11$
13	$\neg(a = b)$	$\neg I\ 5-12$
14	$\exists y (\neg(a = y))$	$\exists I\ 13$
15	$\exists y (\neg(a = y))$	$\exists E\ 3, 4-14$
16	$\exists x \exists y \neg(x = y)$	$\exists I\ 15$
17	$\exists x \exists y \neg(x = y)$	$\exists E\ 1, 3-16$

NEW LINE

NEW SUBPROOF

p)

1	$\forall x[(Px \rightarrow x = b) \wedge (x = b \rightarrow Px)]$	
2	$(Pb \rightarrow b = b) \wedge (b = b \rightarrow Pb)$	$\forall E 1$
3	$(b = b \rightarrow Pb)$	$\wedge E 2$
4	$b = b$	$=I$
5	Pb	$\rightarrow E 3, 4$
6	$Pa \wedge Pc$	
7	$(Pa \rightarrow a = b) \wedge (a = b \rightarrow Pa)$	$\forall E 1$
8	$(Pa \rightarrow a = b)$	$\wedge E 7$
9	Pa	$\wedge E 6$
10	$a = b$	$\rightarrow E 9, 8$
11	$(Pc \rightarrow c = b) \wedge (c = b \rightarrow Pc)$	$\forall E 1$
12	$(Pc \rightarrow c = b)$	$\wedge E 11$
13	Pc	$\wedge E 6$
14	$c = b$	$\rightarrow E 13, 12$
15	$a = c$	$=E 14, 10$
16	$(Pa \wedge Pc) \rightarrow (a = c)$	$\rightarrow I 6-15$
17	$\forall y((Pa \wedge Py) \rightarrow (a = y))$	$\forall I 16$
18	$\forall x\forall y((Px \wedge Py) \rightarrow (x = y))$	$\forall I 17$
19	$Pb \wedge \forall x\forall y((Px \wedge Py) \rightarrow (x = y))$	$\wedge I 18, 5$

q)

1	$Pa \rightarrow \forall xQx$	
2	Pa	
3	$\forall xQx$	$\rightarrow E 2, 1$
4	Qb	$\forall E 3$
5	$Pa \rightarrow Qb$	$\rightarrow I 2-4$
6	$\forall x(Pa \rightarrow Qx)$	$\forall I 5$

r)

1	$\forall x \forall y \forall z [(Sxy \wedge Syz) \rightarrow Sxz]$	
2	$\forall x \neg Sxx$	
3	$\forall y \forall z ((Say \wedge Syz) \rightarrow Saz)$	$\forall E 1$
4	$\forall z ((Sab \wedge Sbz) \rightarrow Saz)$	$\forall E 3$
5	$((Sab \wedge Sba) \rightarrow Saa)$	$\forall E 4$
6	$\neg Saa$	$\forall E 2$
7	Sab	
8	Sba	
9	$Sab \wedge Sba$	$\wedge I 7, 8$
10	Saa	$\rightarrow E 5, 9$
11	\perp	$\neg E 10, 6$
12	$\neg Sba$	$\neg I 8-11$
13	$Sab \rightarrow \neg Sba$	$\rightarrow I 7-12$
14	$\forall y (Say \rightarrow \neg Sya)$	$\forall I 13$
15	$\forall x \forall y (Sxy \rightarrow \neg Syx)$	$\forall I 14$

s)

1	$\forall x (Px \vee Qx)$	
2	$\exists x \neg Qx$	
3	$\forall x (Rx \rightarrow \neg Px)$	
4	$Pa \vee Qa$	$\forall E 1$
5	$Ra \rightarrow \neg Pa$	$\forall E 3$
6	$\neg Qa$	
7	Ra	
8	$\neg Pa$	$\rightarrow E 5, 7$
9	Pa	
10	\perp	$\neg E 9, 8$
11	Qa	
12	\perp	$\neg E 11, 6$
13	\perp	$\vee E 4, 9-10, 11-12$
14	$\neg Ra$	$\neg I 7-13$
15	$\exists x \neg Rx$	$\exists I 14$
16	$\exists x \neg Rx$	$\exists E 2, 6-15$

t)

1	$\forall x [Px \rightarrow (Qx \vee Rx)]$	
2	$\neg \exists x (Px \wedge Rx)$	
3	$Pa \rightarrow (Qa \vee Ra)$	$\forall E 1$
4	Pa	
5	$Qa \vee Ra$	$\rightarrow E 3, 4$
6	Ra	
7	$Pa \wedge Ra$	$\wedge I 4, 6$
8	$\exists x (Px \wedge Rx)$	$\exists I 7$
9	\perp	$\neg E 2, 8$
10	Qa	$\vee E 5, 9$
11	Qa	
12	Qa	$\vee E 5, 11-11, 6-10$
13	$Pa \rightarrow Qa$	$\rightarrow I 4-12$
14	$\forall x (Px \rightarrow Qx)$	$\forall I 13$

u

1	$\exists x \exists y (Sxy \vee Syx)$	
2	$\exists y (Say \vee Sya)$	
3	$(Sab \vee Sba)$	
4	Sab	
5	$\exists y (Say)$	$\exists I$ 4
6	$\exists x \exists y (Sxy)$	$\exists I$ 5
7	Sba	
8	$\exists y (Sby)$	$\exists I$ 7
9	$\exists x \exists y (Sxy)$	$\exists I$ 8
10	$\exists x \exists y (Sxy)$	$\vee E$ 3, 4-6, 7-9
11	$\exists x \exists y (Sxy)$	$\exists E$ 2, 3-10
12	$\exists x \exists y (Sxy)$	$\exists E$ 1, 2-11

v)

1	$\exists x (Px \wedge Qx)$	
2	$\forall x (Px \rightarrow Rx)$	
3	$Pa \rightarrow Ra$	$\forall E$ 2
4	$Pa \wedge Qa$	
5	Pa	$\wedge E$ 4
6	Qa	$\wedge E$ 4
7	Ra	$\rightarrow E$ 3, 5
8	$Ra \wedge Qa$	$\wedge I$ 6, 7
9	$\exists x (Rx \wedge Qx)$	$\exists I$ 8
10	$\exists x (Rx \wedge Qx)$	$\exists E$ 1, 4-9