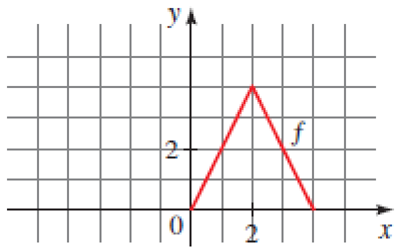
Lista de exercícios de MBA: Propriedades das funções Profª Graciela Moro

1. Dado o gráfico de uma função f, obtenha o gráfico das seguintes transformações de f:

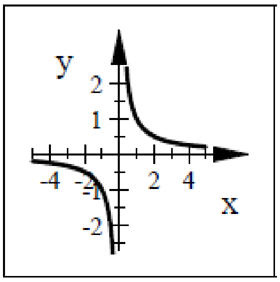
(a) $y = f(x-2)$ **(b)** $y = f(x) - 2$

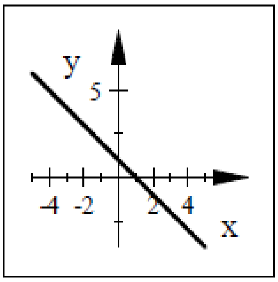
(c) $y = 2f(x)$ **(d)** $y = -f(x) + 3$

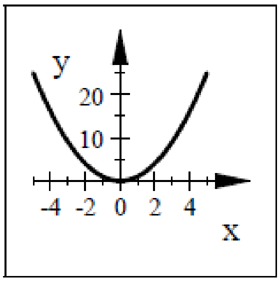
(e) $y = f(-x)$ **(f)** $y = \frac{1}{2}f(x-1)$

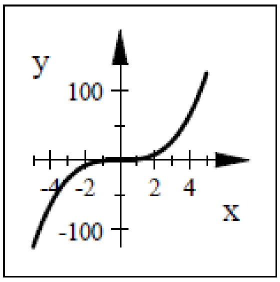


2. Classifique as funções abaixo como pares, ímpares ou nenhum dos casos:









3. Verifique se as funções abaixo são pares, ímpares ou nenhum destes casos. Justifique sua resposta:

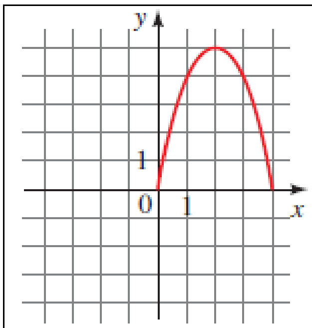
a) $f(x) = x^4$

b) $f(x) = x^3 - x$

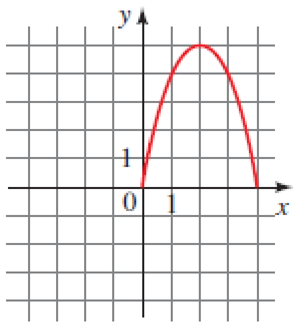
c) $f(x) = x + \frac{1}{x}$

d) $f(x) = 1 - \sqrt[3]{x}$

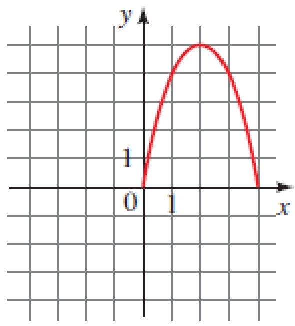
4. Em cada caso, complete o gráfico de f de modo que f seja uma (a) função par (b) ímpar

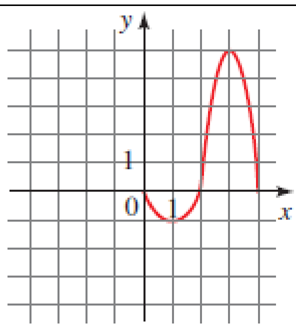


(a) Par

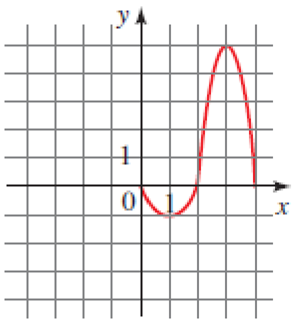


(b) ímpar

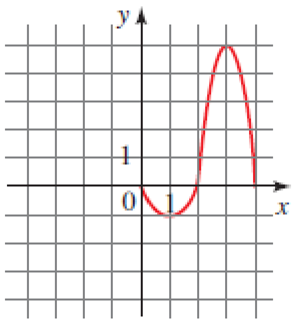




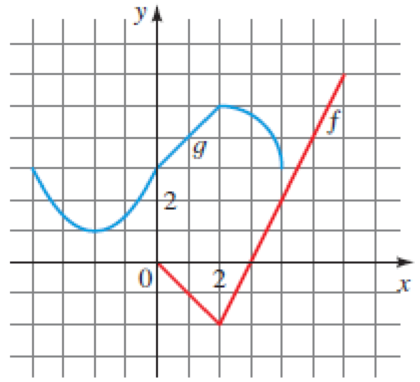
(a) Par



(b) ímpar



5. Use o gráfico de f e g para obter o valor de:



$f(g(2))$

$(g \circ f)(4)$

$(g \circ g)(-2)$

$g(f(0))$

$(f \circ g)(0)$

$(f \circ f)(4)$

6. Use a tabela para obter o valor de:

x	1	2	3	4	5	6
$f(x)$	2	3	5	1	6	3
$g(x)$	3	5	6	2	1	4

$f(g(2))$	$g(f(2))$
$f(f(1))$	$g(g(2))$
$(f \circ g)(6)$	$(g \circ f)(2)$
$(f \circ f)(5)$	$(g \circ g)(2)$

7. Encontre as funções $f \circ g$, $g \circ f$, $f \circ f$ e $g \circ g$, e seus domínios:

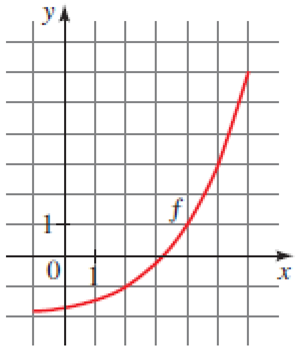
a) $f(x) = \dfrac{1}{x}, \quad g(x) = 2x + 4$

b) $f(x) = x^2, \quad g(x) = \sqrt{x - 3}$

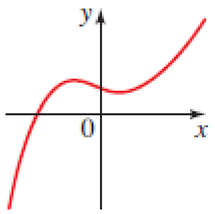
c) $f(x) = \dfrac{1}{\sqrt{x}}, \quad g(x) = x^2 - 4x$

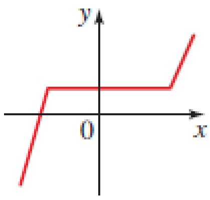
d) $f(x) = x - 4, \quad g(x) = |x + 4|$

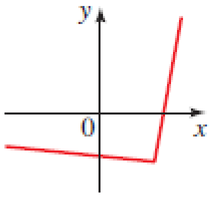
8. A função f, representada graficamente ao lado, admite inversa? Se admitir, determine $f^{-1}(1) =$ ______ e $f^{-1}(3) =$ ______

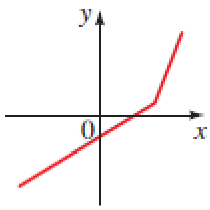


9. Verifique se f é injetora em cada um dos casos:







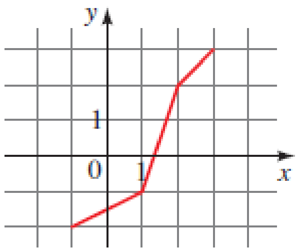
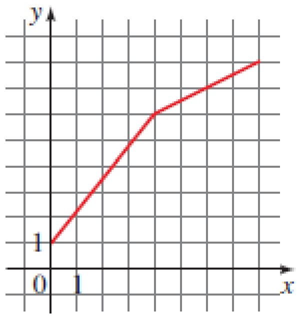


10. Encontre os valores indicados usando a tabela:

x	1	2	3	4	5	6
$f(x)$	4	6	2	5	0	1

$f^{-1}(5)$	$f^{-1}(0)$
$f^{-1}(f(1))$	$f(f^{-1}(6))$
$f^{-1}(f^{-1}(1))$	$f^{-1}(f^{-1}(0))$

11. Use o gráfico de f para traçar o gráfico de f^{-1}:

12. Determine a inversa das funções abaixo:

a) $f(x) = 5 - 4x^3$ c) $f(x) = \dfrac{x}{x+4}$ e) $f(x) = 4 - x^2, \quad x \geq 0$ g) $f(x) = \sqrt{4 - x^2}, \quad 0 \leq x \leq 2$

b) $f(x) = \dfrac{1}{x+2}$ d) $f(x) = \dfrac{2x+5}{x-7}$ f) $f(x) = \sqrt{5 + 8x}$

13.

Considere as funções $f(x) = x + 2$, $g(x) = x + 3$, $h(x) = |x|$ e $F(x) = f(x)\,g(x)$. Encon todos os valores de $x \in \Re$ que satisfazem a inequação

$$(h \circ F)(x) < (h(x))^2 - 6.$$

14.

. Considere as funções $g(x) = x^2 - 1 + |x - 1|$ e $h(x) = \frac{1}{x-2}$.

(a) A função g é uma função par, ímpar ou nem par nem ímpar? Justifique com argumentos consistentes.

(b) Determine o conjunto solução da inequação $g(x-1)\,h(x) < h^{-1}\left(\frac{1}{3}\right)$.

Algumas respostas:

1.

2. ímpar; nem par, nem ímpar; par; ímpar.

3. par ; ímpar; ímpar; nem par nem ímpar

4.

5. a) f(g(2))=4 b) 5 c)4 d)3 e)0 f) -2

6.

$f(g(2))$ $f(f(1))$ $(f \circ g)(6)$ $(f \circ f)(5)$	$g(f(2))$ $g(g(2))$ $(g \circ f)(2)$ $(g \circ g)(2)$
Respostas da coluna acima: a) 6 b) 3 c) 1	Respostas da coluna acima: a) 6 b) 1 c) 6

d) 3	d) 1

7.

a) $f(g(x)) = \frac{1}{2x+4}$; Dom: $\mathbb{R} - \{-2\}$ $g(f(x)) = \frac{2}{x} + 4$; Dom: $\mathbb{R}^*$ $f(f(x)) = x$, Dom: $\mathbb{R}^*$ $g(g(x)) = 4x + 12$; Dom: $\mathbb{R}$	b) $(g(x)) = x - 3; Dom: [3, +\infty)$ $g(f(x)) = \sqrt{x^2 - 3}; (-\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty)$ $f(f(x)) = x^4$, Dom: $\mathbb{R}$ $g(g(x)) = \sqrt{\sqrt{x-3} - 3}; Dom: [12, +\infty)$
c) $f(g(x)) = \frac{1}{\sqrt{x^2-4x}}$; Dom: $(-\infty, 0) \cup (4, +\infty)$ $g(f(x)) = \frac{1}{x} - \frac{4}{\sqrt{x}}$; Dom: $(0, +\infty)$ $f(f(x)) = \sqrt[4]{x}$, Dom: $(0, +\infty)$ $g(g(x)) = x^4 - 8x^3 + 12x^2 + 16x$; Dom: $\mathbb{R}$	d) $f(g(x)) = \|x + 4\| - 4$; Dom: $\mathbb{R}$ $g(f(x)) = \|x\|$; Dom: $\mathbb{R}$ $f(f(x)) = x - 8$, Dom: $\mathbb{R}$ $g(g(x)) = \|x + 8\|$; Dom: $\mathbb{R}$

8. $f^{-1}(1) = 4$ e $f^{-1}(3) = 5$

9. Injetora no 1º e 4º gráficos

10.

$f^{-1}(5)$ $f^{-1}(f(1))$ $f^{-1}(f^{-1}(1))$	=4 =1 =2	$f^{-1}(0)$ $f(f^{-1}(6))$ $f^{-1}(f^{-1}(0))$	=5 =6 =4

11.

12.

a) $f^{-1}: \mathbb{R} \to \mathbb{R}$ $tal\ que\ f^{-1}(x) = \sqrt[5]{\frac{5-x}{4}}$
b) $f^{-1}: \mathbb{R} - \{0\} \to \mathbb{R} - \{-2\}$ $tal\ que\ f^{-1}(x) = \frac{1-2x}{x}$
c) $f^{-1}: \mathbb{R} - \{1\} \to \mathbb{R} - \{-4\}$ $tal\ que\ f^{-1}(x) = \frac{-4x}{x-1}$
d) $f^{-1}: \mathbb{R} - \{2\} \to \mathbb{R} - \{7\}$ $tal\ que\ f^{-1}(x) = \frac{7x+5}{x-2}$
e) $f^{-1}: (-\infty, 4] \to \mathbb{R} - \{0\}$ $tal\ que\ f^{-1}(x) = \sqrt{4-x}$
f) $f^{-1}: [0, +\infty) \to (-\frac{5}{8}, +\infty]$ $tal\ que\ f^{-1}(x) = \frac{x^2-5}{8}$
g) $f^{-1}: [0,2] \to [0,2]$ $tal\ que\ f^{-1}(x) = \sqrt{4-x^2}$

13. A desigualdade acima fica reescrita na forma: $|x^2 + 5x + 6| < x^2 - 6$

$$S = (-\infty, -\frac{5}{2})$$

14. a) Nem par, nem ímpar b) $S = (-\infty, 4) - \{2\}$