Gabarito – Lógica de Predicados

Questão 2.

a)

Construct a proof for the argument: $\forall xPx \rightarrow Qa \therefore \exists x(Px \rightarrow Qa)$

1	$\forall xPx \rightarrow Qa$	
2	$\forall xPx$	
3	Pj	
4	Qa	→E 1, 2
5	$Pj \rightarrow Qa$	→I 3–4
6	$\exists x(Px \rightarrow Qa)$	∃I 5
7	$\neg\forall xPx$	
8	$\exists x\neg Px$	CQ 7
9	$\neg Pb$	
10	Pb	
11	$\bot$	¬E 9, 10
12	Qa	X 11
13	$Pb \rightarrow Qa$	→I 10–12
14	$\exists x(Px \rightarrow Qa)$	∃I 13
15	$\exists x(Px \rightarrow Qa)$	∃E 8, 9–14
16	$\exists x(Px \rightarrow Qa)$	LEM 2–6, 7–15

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

b)

Construct a proof for the argument: $\forall x(Px \vee Qx) \therefore \forall xPx \vee \exists xQx$

1	$\forall x(Px \vee Qx)$	
2	$\quad \forall xPx$	
3	$\quad \forall xPx \vee \exists xQx$	$\vee$I 2
4	$\quad \neg\forall xPx$	
5	$\quad \exists x\neg Px$	CQ 4
6	$\quad\quad \neg Pm$	
7	$\quad\quad Pm \vee Qm$	$\forall$E 1
8	$\quad\quad Qm$	DS 7, 6
9	$\quad\quad \exists xQx$	$\exists$I 8
10	$\quad \exists xQx$	$\exists$E 5, 6–9
11	$\quad \forall xPx \vee \exists xQx$	$\vee$I 10
12	$\forall xPx \vee \exists xQx$	LEM 2–3, 4–11

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

d)

Construct a proof for the argument: $\forall x(\neg Px \wedge Qx) \therefore \forall x(Px \rightarrow Qx)$

1	$\forall x(\neg Px \wedge Qx)$	
2	$\quad Pa$	
3	$\quad \neg Pa \wedge Qa$	$\forall$E 1
4	$\quad \neg Pa$	$\wedge$E 3
5	$\quad \bot$	$\neg$E 4, 2
6	$\quad Qa$	X 5
7	$Pa \rightarrow Qa$	$\rightarrow$I 2–6
8	$\forall x(Px \rightarrow Qx)$	$\forall$I 7

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

e)

Construct a proof for the argument: $\forall x(Px \wedge Qx) \therefore \forall x(Px \rightarrow Qx)$

1	$\forall x(Px \wedge Qx)$	
2	Pa	
3	$Pa \wedge Qa$	$\forall$E 1
4	Qa	$\wedge$E 3
5	$Pa \rightarrow Qa$	$\rightarrow$I 2–4
6	$\forall x(Px \rightarrow Qx)$	$\forall$I 5

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

f)

Construct a proof for the argument: $\exists x(\neg Px \wedge \neg Qx) \therefore \exists x\neg(Px \wedge Qx)$

1	$\exists x(\neg Px \wedge \neg Qx)$	
2	$\neg Pa \wedge \neg Qa$	
3	$Pa \wedge Qa$	
4	Pa	$\wedge$E 3
5	$\neg Pa$	$\wedge$E 2
6	$\perp$	$\neg$E 5, 4
7	$\neg(Pa \wedge Qa)$	$\neg$I 3–6
8	$\exists x\neg(Px \wedge Qx)$	$\exists$I 7
9	$\exists x\neg(Px \wedge Qx)$	$\exists$E 1, 2–8

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

h)

Construct a proof for the argument: $\forall x(Px \wedge Qx) \therefore \forall xPx \wedge \forall xQx$

1	$\forall x(Px \wedge Qx)$	
2	$Pa \wedge Qa$	∀E 1
3	Pa	∧E 2
4	$\forall xPx$	∀I 3
5	Qa	∧E 2
6	$\forall xQx$	∀I 5
7	$\forall xPx \wedge \forall xQx$	∧I 4, 6

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

i)

Construct a proof for the argument: $\forall xPx \vee \forall xQx \therefore \forall x(Px \vee Qx)$

1	$\forall xPx \vee \forall xQx$	
2	$\forall xPx$	
3	Pa	∀E 2
4	$Pa \vee Qa$	∨I 3
5	$\forall xQx$	
6	Qa	∀E 5
7	$Pa \vee Qa$	∨I 6
8	$Pa \vee Qa$	∨E 1, 2–4, 5–7
9	$\forall x(Px \vee Qx)$	∀I 8

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

j)

Construct a proof for the argument: $\exists x(Px \wedge Qx) \therefore \exists xPx \wedge \exists xQx$

1	$\exists x(Px \wedge Qx)$	
2	$Pa \wedge Qa$	
3	Pa	$\wedge$E 2
4	$\exists xPx$	$\exists$I 3
5	Qa	$\wedge$E 2
6	$\exists xQx$	$\exists$I 5
7	$\exists xPx \wedge \exists xQx$	$\wedge$I 4, 6
8	$\exists xPx \wedge \exists xQx$	$\exists$E 1, 2–7

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

l)

Construct a proof for the argument: $\forall x\forall y(Sy \rightarrow Fx) \therefore \exists ySy \rightarrow \forall xFx$

1	$\forall x\forall y(Sy \rightarrow Fx)$	
2	$\exists ySy$	
3	$\forall y(Sy \rightarrow Fa)$	$\forall$E 1
4	$Sb \rightarrow Fa$	$\forall$E 3
5	Sb	
6	Fa	$\rightarrow$E 4, 5
7	Fa	$\exists$E 2, 5–6
8	$\forall xFx$	$\forall$I 7
9	$\exists ySy \rightarrow \forall xFx$	$\rightarrow$I 2–8

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.

m)

Construct a proof for the argument: $Pb \therefore \forall x(x = b \rightarrow Px)$

1	Pb	
2	$\quad a = b$	
3	$\quad Pa$	=E 2, 1
4	$a = b \rightarrow Pa$	→I 2–3
5	$\forall x(x = b \rightarrow Px)$	∀I 4

NEW LINE NEW SUBPROOF

☺ Congratulations! This proof is correct.