

# Lógica de Predicados

1. a) Sócrates é um homem

s = Sócrates

$H(s)$  //

H = é um homem

b) Todo homem é mortal

M

$\forall x. (H(x) \rightarrow M(x))$

c) Jonas é um homem e todo homem é mortal

J

$H(J) \wedge \forall x. (H(x) \rightarrow M(x))$

d) Toda cobra é venenosa

C = é cobra.

V

$\forall x. (C(x) \rightarrow V(x))$

e) Não existe bêbado feliz.

B = bêbado

F = ser feliz

$\neg \exists x. (B(x) \wedge F(x))$  //

f) Alguns políticos não são honestos

P = políticos

H = honestos

$\exists x. (P(x) \wedge \neg H(x))$

g) Há aves que não voam  
 $A = \text{aves}$   $V = \text{voam}$

$$\exists x. (A(x) \wedge \neg V(x))$$

h) Todos Mentem  
 $M = \text{mentem}$

$$\forall x. (M(x))$$

i) Existe pôneis alienígenas  
 $P = \text{são pôneis}$   $A = \text{são alienígenas}$

j) Todo peixe nada  
 $P = \text{é peixe}$   $N = \text{Nada}$

$$\forall x. (P(x) \rightarrow N(x))$$

k) Algumas aves voam.  
 $A = \text{é ave}$   
 $V = \text{voam}$

$$\exists x. (A(x) \wedge V(x))$$

l) Nenhuma ave voa.  
 $A = \text{é ave}$   
 $V = \text{voa}$

$$\neg \exists x. (A(x) \wedge V(x))$$

$$\forall x. \neg (A(x) \wedge V(x))$$

m) Nem tudo que reluz é ouro  
 $R = \text{reluz}$   $O = \text{ouro}$

$$\neg \forall x. (R(x) \rightarrow O(x))$$

$$\neg \forall x. (R(x) \rightarrow x = O)$$

3. Considere a seguinte argumentação

a) o mais forte hebreu é Sansão.  
 $F = \text{mais forte} \quad H = \text{é hebreu} \quad s = \text{Sansão}$   
 $F(H(s))$ ,

b) Hércules é mais forte que Sansão  
 $h \quad s$   
 $F(h, s)$ ,

c) Se  $a$  é mais forte que  $b$ , então  $b$  não é mais forte que  $a$ .

$F(a, b) = a \text{ é mais forte que } b.$

$F(a, b) \rightarrow \neg F(b, a)$

d) Logo, Hércules não é Hebreu,

$\text{Par}^1 \{0, 2, 4, 6, \dots\}; \text{Mult} \rightarrow \times; \text{Some} \rightarrow +$

$\forall x. \forall y ( \text{Par}(x) \wedge \text{Par}(y) \rightarrow \text{Par}(\text{Mult}(x, y)) )$ ,

$\text{Impar}^1 = \{1, 3, 5, 7, \dots\}$

$\forall x \forall y ( \text{Impar}(x) \wedge \text{Impar}(y) \rightarrow \text{Impar}(\text{Some}(x, y)) )$

(a)  $\forall x. P(x) \rightarrow Q(a) \vdash \exists x. (P(x) \rightarrow Q(a))$

1	$\forall x. P(x) \rightarrow Q(a)$
2	$\neg(\forall x. P(x) \vee \neg \forall x. P(x)) \quad \text{h.}$
3	$\forall x. P(x) \quad \text{h.}$
4	$\forall x. P(x) \vee \neg \forall x. P(x)$
5	$\perp$
6	$\neg \forall x. P(x)$
7	$\forall x. P(x) \vee \neg \forall x. P(x)$
8	$\perp$
9	$\forall x. P(x) \vee \neg \forall x. P(x)$
10	$\forall x. P(x)$
11	$P(c)$
12	$Q(a)$
13	$P(c) \rightarrow Q(a)$
14	$\exists x. (P(x) \rightarrow Q(a))$
15	$\neg \forall x. P(x)$
16	$\exists x. \neg P(x)$
17	$\neg P(c)$
18	$\exists x. \neg P(x)$
19	$\perp$
20	$P(c)$
21	$\forall x. P(x)$
22	$\perp$
23	$\exists x. \neg P(x)$
24	$\neg P(b)$
25	$P(b)$
26	$\perp$
27	$Q(a)$
28	$P(b) \rightarrow Q(a)$
29	$\exists x. (P(x) \rightarrow Q(a))$
30	$\exists x. (P(x) \rightarrow Q(a))$
31	$\exists x. (P(x) \rightarrow Q(a))$

(b)  $\forall x. (P(x) \vee Q(x)) \vdash \forall x. P(x) \vee \exists x. Q(x)$

1	$\forall x. (P(x) \vee Q(x))$
2	$\neg(\forall x. P(x) \vee \neg \forall x. P(x))$
3	$\neg \forall x. P(x)$
4	$\forall x. P(x) \vee \neg \forall x. P(x)$
5	$\perp$
6	$\forall x. P(x)$
7	$\forall x. P(x) \vee \neg \forall x. P(x)$
8	$\perp$
9	$\forall x. P(x) \vee \neg \forall x. P(x)$
10	$\forall x. P(x)$
11	$\forall x. P(x) \vee \exists x. Q(x)$
12	$\neg \forall x. P(x)$
13	$\neg \exists x. \neg P(x)$
14	$\neg P(c)$
15	$\exists x. \neg P(x)$
16	$\perp$
17	$P(c)$
18	$\forall x. P(x)$
19	$\perp$
20	$\exists x. \neg P(x)$
21	$\neg P(m)$
22	$P(m) \vee Q(m)$
23	$P(m)$
24	$\perp$
25	$Q(m)$
26	$Q(m)$
27	$Q(m)$
28	$Q(m)$
29	$\exists x. Q(x)$
30	$\exists x. Q(x)$
31	$\forall x. P(x) \vee \exists x. Q(x)$
32	$\forall x. P(x) \vee \exists x. Q(x)$

(d)  $\forall x. (\neg P(x) \wedge Q(x)) \vdash \forall x. (P(x) \rightarrow Q(x))$

1	$\forall x. (\neg P(x) \wedge Q(x))$
2	$P(a)$
3	$\neg P(a) \wedge Q(a)$
4	$\neg P(a)$
5	$\perp$
6	$Q(a)$
7	$P(a) \rightarrow Q(a)$
8	$\forall x. (P(x) \rightarrow Q(x))$

(e)  $\forall x. (P(x) \wedge Q(x)) \vdash \forall x. P(x) \wedge \forall x. Q(x)$

1	$\forall x. (P(x) \wedge Q(x))$
2	$P(a) \wedge Q(a) \quad \forall E1$
3	$P(a) \quad \wedge E2$
4	$Q(a) \quad \wedge E2$
5	$\forall x. P(x) \quad \forall I3$
6	$\forall x. Q(x) \quad \forall I3$
7	$\forall x. P(x) \wedge \forall x. Q(x) \quad \wedge I5,6$

(f)  $\forall x. (P(x) \wedge Q(x)) \vdash \forall x. (P(x) \rightarrow Q(x))$

1	$\forall x. (P(x) \wedge Q(x))$
2	$P(a) \wedge Q(a)$
3	$P(a)$
4	$Q(a)$
5	$P(a)$
6	$Q(a)$
7	$P(a) \rightarrow Q(a)$
8	$\forall x. (P(x) \rightarrow Q(x))$

(g)  $\exists x. (P(x) \wedge Q(x)) \vdash \exists x. P(x) \wedge \exists x. Q(x)$

1	$\exists x. (P(x) \wedge Q(x))$
2	$P(a) \wedge Q(a)$
3	$P(a) \quad \wedge E2$
4	$\exists x. P(x) \quad \exists I3$
5	$\exists x. P(x) \quad \exists E1,2-4$
6	$P(a) \wedge Q(a)$
7	$Q(a) \quad \wedge E6$
8	$\exists x. Q(x) \quad \exists I7$
9	$\exists x. Q(x) \quad \exists E1,6-8$
10	$\exists x. P(x) \wedge \exists x. Q(x) \quad \wedge I5,9$

(f)  $\exists x. (\neg P(x) \wedge \neg Q(x)) \vdash \exists x. \neg(P(x) \wedge Q(x))$

1	$\exists x. (\neg P(x) \wedge \neg Q(x))$
2	$\neg P(a) \wedge \neg Q(a)$ h
3	$P(a) \wedge Q(a)$ h
4	$P(a)$ $\wedge E3$
5	$\neg P(a)$ $\wedge E2$
6	$\perp$ $\neg E4,5$
7	$\neg(P(a) \wedge Q(a))$
8	$\exists x. \neg(P(x) \wedge Q(x))$ $\exists I7$
9	$\exists x. \neg(P(x) \wedge Q(x))$ $\exists I, 2-8$

(i)  $\forall x. P(x) \vee \forall x. Q(x) \vdash \forall x. (P(x) \vee Q(x))$

1	$\forall x. P(x) \vee \forall x. Q(x)$
2	$\forall x. Px$
3	$Pa$ $\forall E2$
4	$Pa \vee Qa$ $\vee I3$
5	$\forall x. Qx$
6	$Qa$ $\forall E5$
7	$Pa \vee Qa$ $\vee I6$
8	$Pa \vee Qa$ $\vee E1,2-4,5-7$
9	$\forall x. (P(x) \vee Q(x))$ $\forall I8$

(l)  $\forall x. \forall y. (S(y) \rightarrow F(x)) \vdash \exists y. S(y) \rightarrow \forall x. F(x)$

1	$\forall x. \forall y. (S(y) \rightarrow F(x))$
2	$\exists y. Sy$
3	$\forall y. (S(y) \rightarrow F(a))$ $\forall E1$
4	$S(b) \rightarrow F(a)$ $\forall E3$
5	$S(b)$
6	$F(a)$ $\rightarrow E4,5$
7	$F(a)$ $\exists E2,5-6$
8	$\forall x. F(x)$
9	$\exists y. S(y) \rightarrow \forall x. F(x)$ $\rightarrow I2-8$

(m)  $P(b) \vdash \forall x. (x=b \rightarrow P(x))$

1	$P(b)$
2	$a=b$ h
3	$P(a)$ $=E2,1$
4	$a=b \rightarrow P(a)$ $\rightarrow I2-3$
5	$\forall x. (x=b \rightarrow P(x))$ $\forall I4$

(g)  $P(a) \rightarrow \forall x. Q(x) \vdash \forall x. (P(a) \rightarrow Q(x))$

1	$P(a) \rightarrow \forall x. Q(x)$
2	$P(a)$ h
3	$\forall x. Q(x)$ $\rightarrow E1,2$
4	$Q(c)$ $\forall E3$
5	$P(a) \rightarrow Q(c)$ $\rightarrow I2-4$
6	$\forall x. (P(a) \rightarrow Q(x))$ $\forall I5$

(n)  $P(b), \forall x. \forall y. (P(x) \wedge P(y) \rightarrow x=y) \vdash \forall x. ((P(x) \rightarrow x=b) \wedge (x=b \rightarrow P(x)))$

1	$P(b)$ premise
2	$\forall x. \forall y. (P(x) \wedge P(y) \rightarrow x=y)$ premise
3	$\forall y. (P(a) \wedge P(y) \rightarrow a=y)$ $\forall E2$
4	$P(a) \wedge P(b) \rightarrow a=b$ $\forall E3$
5	$P(a)$ h
6	$P(a) \wedge P(b)$ $\wedge I3,5$
7	$a=b$ $\rightarrow E4,6$
8	$P(a) \rightarrow a=b$
9	$a=b$
10	$P(a)$ $=E9,9$
11	$a=b \rightarrow P(a)$
12	$(P(a) \rightarrow a=b) \wedge (a=b \rightarrow P(a))$
13	$\forall x. ((P(x) \rightarrow x=b) \wedge (x=b \rightarrow P(x)))$

(o)  $\exists x. \exists y. (H(x,y) \vee H(y,x)), \neg \exists x. H(x,x) \vdash \exists x. \exists y. \neg(x=y)$

1	$\exists x. \exists y. (H(x,y) \vee H(y,x))$ p
2	$\neg \exists x. H(x,x)$ p
3	$\exists y. (H(a,y) \vee H(y,a))$ h
4	$H(a,b) \vee H(b,a)$ h
5	$a=b$ h
6	$H(a,b)$ h
7	$H(a,a)$ $=E5,6$
8	$H(b,a)$ h
9	$H(a,a)$ $=E8,8$
10	$H(a,a)$ $\vee E4,6,7,8-9$
11	$\exists x. (H(x,x))$ $\exists I10$
12	$\perp$ $\neg E2,11$
13	$\neg(a=b)$ $\neg I5-12$
14	$\exists y. (\neg(a=y))$ $\exists I13$
15	$\exists y. (\neg(a=y))$ $\exists E3,4-14$
16	$\exists x. \exists y. (\neg(x=y))$ $\exists I15$
17	$\exists x. \exists y. (\neg(x=y))$ $\exists E1,3-16$

(p)  $\forall x. ((P(x) \rightarrow x=b) \wedge (x=b \rightarrow P(x))) \vdash P(b) \wedge \forall x. \forall y. (P(x) \wedge P(y) \rightarrow x=y)$

1	$\forall x. [(P(x) \rightarrow x=b) \wedge (x=b \rightarrow P(x))]$
2	$(P(b) \rightarrow b=b) \wedge (b=b \rightarrow P(b))$ $\forall E1$
3	$b=b \rightarrow P(b)$ $\wedge E2$
4	$b=b$ $=I$
5	$P(b)$ $\rightarrow E3,4$
6	$P(a) \wedge P(c)$
7	$(P(a) \rightarrow a=b) \wedge (a=b \rightarrow P(a))$ $\forall E1$
8	$P(a) \rightarrow a=b$ $\wedge E7$
9	$P(a)$ $\wedge E6$
10	$a=b$ $\rightarrow E8,9$
11	$(P(c) \rightarrow c=b) \wedge (c=b \rightarrow P(c))$ $\forall E1$
12	$P(c) \rightarrow c=b$ $\wedge E11$
13	$P(c)$ $\wedge E6$
14	$c=b$ $\rightarrow E12,13$
15	$a=c$ $=E10,14$
16	$P(a) \wedge P(c) \rightarrow a=c$ $\rightarrow I6-15$
17	$\forall y. (P(a) \wedge P(y) \rightarrow a=y)$ $\forall I16$
18	$\forall x. \forall y. (P(x) \wedge P(y) \rightarrow x=y)$ $\forall I17$
19	$P(b) \wedge \forall x. \forall y. (P(x) \wedge P(y) \rightarrow x=y)$ $\wedge I5,18$

(r)  $\forall x. \forall y. \forall z. (S(x, y) \wedge S(y, z) \rightarrow S(x, z)), \forall x. \neg S(x, x) \vdash \forall x. \forall y. (S(x, y) \rightarrow \neg S(y, x))$

1	$\forall x. \forall y. \forall z. (S(x, y) \wedge S(y, z) \rightarrow S(x, z))$
2	$\forall x. \neg S(x, x)$
3	$\forall y. \forall z. (S(a, y) \wedge S(y, z) \rightarrow S(a, z)) \quad \forall E 1$
4	$\forall z. (S(a, b) \wedge S(b, z) \rightarrow S(a, z)) \quad \forall E 3$
5	$(S(a, b) \wedge S(b, a) \rightarrow S(a, a)) \quad \forall E 4$
6	$\neg S(a, a) \quad \forall E 2$
7	$S(a, b) \quad h$
8	$S(b, a) \quad h$
9	$S(a, b) \wedge S(b, a) \quad \wedge I 7, 8$
10	$S(a, a) \quad \rightarrow E$
11	$\perp \quad \neg E 6, 10$
12	$\neg S(b, a) \quad \neg I 8-11$
13	$S(a, b) \rightarrow \neg S(b, a) \quad \rightarrow I 7-12$
14	$\forall y (S(a, y) \rightarrow \neg S(y, a)) \quad \forall I 13$
15	$\forall x. \forall y (S(x, y) \rightarrow \neg S(y, x)) \quad \forall I 14, //$

(u)  $\exists x. \exists y. (S(x, y) \vee S(y, x)) \vdash \exists x. \exists y. S(x, y)$

1	$\exists x. \exists y. (S(x, y) \vee S(y, x))$
2	$\exists y (S(a, y) \vee S(y, a)) \quad h$
3	$S(a, b) \vee S(b, a) \quad h$
4	$S(a, b) \quad h$
5	$\exists y (S(a, y)) \quad \exists I 4$
6	$\exists x \exists y (S(x, y)) \quad \exists I 5$
7	$S(b, a) \quad h$
8	$\exists y (S(y, a)) \quad \exists I 7$
9	$\exists x \exists y (S(x, y)) \quad \exists I 8$
10	$\exists x \exists y (S(x, y)) \quad \vee E 3, 4-6, 7-9$
11	$\exists x. \exists y. (S(x, y)) \quad \exists E 2, 3-10$
12	$\exists x. \exists y. (S(x, y)) \quad \exists E 1, 2, 12$

(s)  $\forall x. (P(x) \vee Q(x)), \exists x. \neg Q(x), \forall x. (R(x) \rightarrow \neg P(x)) \vdash \exists x. \neg R(x)$

1	$\forall x. (P(x) \vee Q(x))$
2	$\exists x. \neg Q(x)$
3	$\forall x. (R(x) \rightarrow \neg P(x))$
4	$P(a) \vee Q(a) \quad \forall E 1$
5	$R(a) \rightarrow \neg P(a) \quad \forall E 3$
6	$\neg Q(a) \quad h$
7	$R(a) \quad h$
8	$\neg P(a) \quad \rightarrow E 5, 7$
9	$P(a) \quad h$
10	$\perp \quad \neg E 8, 9$
11	$Q(a) \quad h$
12	$\perp \quad \neg E 6, 11$
13	$\perp \quad \vee E 4, 9-10, 11-12$
14	$\neg R(a) \quad \neg I 7-13$
15	$\exists x. \neg R(x) \quad \exists I 14$
16	$\exists x. \neg R(x) \quad \exists E 2, 6-15$

(v)  $\exists x. (P(x) \wedge Q(x)), \forall x. (P(x) \rightarrow R(x)) \vdash \exists x. (R(x) \wedge Q(x))$

1	$\exists x. (P(x) \wedge Q(x))$
2	$\forall x. (P(x) \rightarrow R(x))$
3	$P(a) \rightarrow R(a) \quad \forall E 2$
4	$P(a) \wedge Q(a) \quad h$
5	$P(a) \quad \wedge E 4$
6	$Q(a) \quad \wedge E 4$
7	$R(a) \quad \rightarrow E 3, 5$
8	$R(a) \wedge Q(a) \quad \wedge I 6, 7$
9	$\exists x. (R(x) \wedge Q(x)) \quad \exists I 8$
10	$\exists x. (R(x) \wedge Q(x)) \quad \exists E 1, 4-9$

(t)  $\forall x. (P(x) \rightarrow (Q(x) \vee R(x))), \neg \exists x. (P(x) \wedge R(x)) \vdash \forall x. (P(x) \rightarrow Q(x))$

1	$\forall x. (P(x) \rightarrow (Q(x) \vee R(x)))$
2	$\neg \exists x. (P(x) \wedge R(x))$
3	$P(a) \rightarrow (Q(a) \vee R(a)) \quad \forall E 1$
4	$P(a) \quad h$
5	$(Q(a) \vee R(a)) \quad \rightarrow E 3, 4$
6	$R(a) \quad h$
7	$P(a) \wedge R(a) \quad \wedge I 4, 6$
8	$\exists x. (P(x) \wedge R(x)) \quad \exists I 7$
9	$\perp \quad \neg E 2, 8$
10	$Q(a) \quad \vee E 5, 9$
11	$Q(a) \quad h$
12	$Q(a) \quad \vee E 5, 6-10, 11-12$
13	$P(a) \rightarrow Q(a) \quad \rightarrow I 4-12$
14	$\forall x. (P(x) \rightarrow Q(x)) \quad \forall I 13, //$