Limites Notáveis ou Fundamentais

$$\lim_{u \to 0} \frac{\text{sen}(u)}{u} = 1$$

$$\lim_{u \to 0} \frac{1 - \cos(u)}{u} = 0$$

$$\lim_{u \to \infty} \left(1 + \frac{1}{u}\right)^u = e$$

$$\lim_{u \to 0} \frac{a^u - 1}{u} = \ln(a)$$

Exemplos:

Use os limites notáveis ou fundamentais, sempre que possível para calcular os limites dados a seguir.

a. $\lim\limits_{x \to 0} \frac{sen(3x)}{sen(5x)} = \frac{0}{0}$

$$\mathrm{L} = \lim_{x \to 0} \frac{sen(3x)}{sen(5x)} = \lim_{x \to 0} \frac{sen(3x)}{sen(5x)} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to 0} \frac{\frac{sen(3x)}{x}}{\frac{sen(5x)}{x}} = \lim_{x \to 0} \frac{\frac{3\ sen(3x)}{3x}}{\frac{5\ sen(5x)}{5x}}$$

$$\mathrm{L} = \frac{\lim\limits_{x \to 0} \frac{3\ sen(3x)}{3x}}{\lim\limits_{x \to 0} \frac{5\ sen(5x)}{5x}} = \frac{3 \ \cdot \ \lim\limits_{x \to 0} \frac{sen(3x)}{3x}}{5 \ \cdot \ \lim\limits_{x \to 0} \frac{sen(5x)}{5x}}$$

Definindo $u = 3x$ e $v = 5x$, temos que:

$$\mathrm{L} = \frac{3 \lim\limits_{u \to 0} \frac{sen(u)}{u}}{5 \ \lim\limits_{v \to 0} \frac{sen(v)}{v}} \Rightarrow \boxed{\mathrm{L} = \frac{3}{5}}$$

$$b.\ \lim_{x\to 4}\frac{sen(x)-sen(4)}{3^{2(x-4)}-1}=\frac{0}{0}$$

Definindo $u=x-4$, temos que:

$$L=\lim_{u\to 0}\frac{sen(u+4)-sen(4)}{3^{2u}-1}$$

$$L=\lim_{u\to 0}\frac{sen(u)\cos(4)+\cos(u)\,sen(4)-sen(4)}{3^{2u}-1}$$

$$L=\lim_{u\to 0}\frac{sen(u)\cos(4)-\mathrm{sen}(4)\,[1-\cos(u)]}{3^{2u}-1}$$

$$L=\lim_{u\to 0}\frac{\dfrac{sen(u)\cos(4)-\mathrm{sen}(4)\,[1-\cos(u)]}{u}}{\dfrac{3^{2u}-1}{u}}$$

$$L=\frac{\lim\limits_{u\to 0}\dfrac{sen(u)\cos(4)-\mathrm{sen}(4)\,[1-\cos(u)]}{u}}{\lim\limits_{u\to 0}\dfrac{3^{2u}-1}{u}}$$

$$L=\frac{\lim\limits_{u\to 0}\dfrac{sen(u)\cos(4)}{u}-\lim\limits_{u\to 0}\dfrac{\mathrm{sen}(4)\,[1-\cos(u)]}{u}}{\lim\limits_{u\to 0}\dfrac{3^{2u}-1}{u}}$$

$$L=\frac{\cos(4)\lim\limits_{u\to 0}\dfrac{sen(u)}{u}-\mathrm{sen}(4)\lim\limits_{u\to 0}\dfrac{1-\cos(u)}{u}}{\lim\limits_{u\to 0}\dfrac{3^{2u}-1}{u}}$$

$$L=\frac{\cos(4)\,.\,1-\mathrm{sen}(4)\,.\,0}{\lim\limits_{u\to 0}\dfrac{(3^2)^u-1}{u}}$$

$$L=\frac{\cos(4)}{\ln(3^2)}\ \Rightarrow\ \boxed{L=\frac{\cos(4)}{2\ln(3)}}$$

c. $\lim_{x \to +\infty} \left[x\left(ln(x+7) - ln(x) \right) \right]$

$$L = \lim_{x \to +\infty} \left(x \, ln \left(\frac{x+7}{x} \right) \right) = \lim_{x \to +\infty} \left(x \, ln \left(1 + \frac{7}{x} \right) \right) = \lim_{x \to +\infty} \left(ln \left(1 + \frac{7}{x} \right)^{x} \right) = ln \left(\underbrace{\lim_{x \to +\infty} \left(1 + \frac{7}{x} \right)^{x}}_{L_1} \right)$$

$$L_1 = \lim_{x \to +\infty} \left(1 + \frac{7}{x} \right)^{x}$$

Definindo $\frac{1}{u} = \frac{7}{x}$, temos que:

$$L_1 = \lim_{u \to +\infty} \left(1 + \frac{1}{u} \right)^{7u}$$

$$L_1 = \lim_{u \to +\infty} \left(\left(1 + \frac{1}{u} \right)^{u} \right)^{7}$$

$$L_1 = \left(\lim_{u \to +\infty} \left(1 + \frac{1}{u} \right)^{u} \right)^{7}$$

$$\boxed{L_1 = e^{7}}$$

$$\Rightarrow \boxed{L = \ln(e^{7}) = 7}$$

$$\lim_{u \to 0} \frac{sen(u)}{u} = 1$$

$$\lim_{u \to 0} \frac{1 - \cos(u)}{u} = 0$$

$$\lim_{u \to \infty} \left(1 + \frac{1}{u} \right)^{u} = e$$

$$\lim_{u \to 0} \frac{a^{u} - 1}{u} = \ln(a)$$

$d.\ \lim_{x \to 3} \frac{senh(x-3)}{x-3} = \frac{0}{0}$

Definindo $u = x - 3$, temos que:

$$L = \lim_{u \to 0} \frac{senh(u)}{u} = \lim_{u \to 0} \frac{e^{u} - e^{-u}}{2u} = \frac{1}{2} \lim_{u \to 0} \frac{e^{u} - 1 + 1 - e^{-u}}{u} = \frac{1}{2}\left(\lim_{u \to 0} \frac{e^{u} - 1}{u} - \lim_{u \to 0} \frac{e^{-u} - 1}{u} \right)$$

$$L = \frac{1}{2}\ln(e) - \frac{1}{2} \lim_{u \to 0} \frac{(e^{-1})^{u} - 1}{u} = \frac{1}{2} - \frac{1}{2}\ln(e^{-1}) \Rightarrow \boxed{L = 1}$$