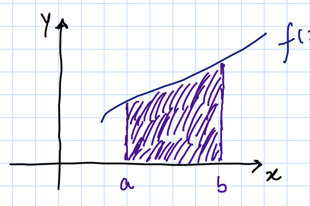
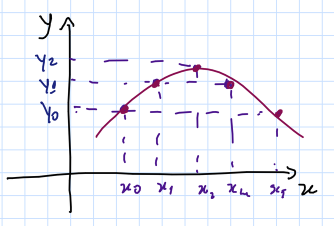
INTEGRAÇÃO NUMÉRICA

$$I = \int_a^b f(x)\,dx = F(b) - F(a)$$

e, se $f(x) > 0 \;\forall\, x \in [a,b]$, então a integral fornece a área entre o gráfico da função e o eixo X.





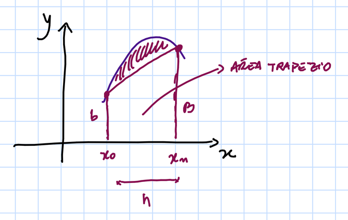
$\int_a^b f(x)\,dx = ?$

OBJEÇÕES:

i) A PRIMITIVA F(x) É MUITO COMPLEXA OU IMPOSSÍVEL DE SER DETERMINADA

ii) TEMOS SOMENTE A TABELA DA FUNÇÃO $f(x)$, O QUE COMPROMETE A DETERMINAÇÃO DE F(x)

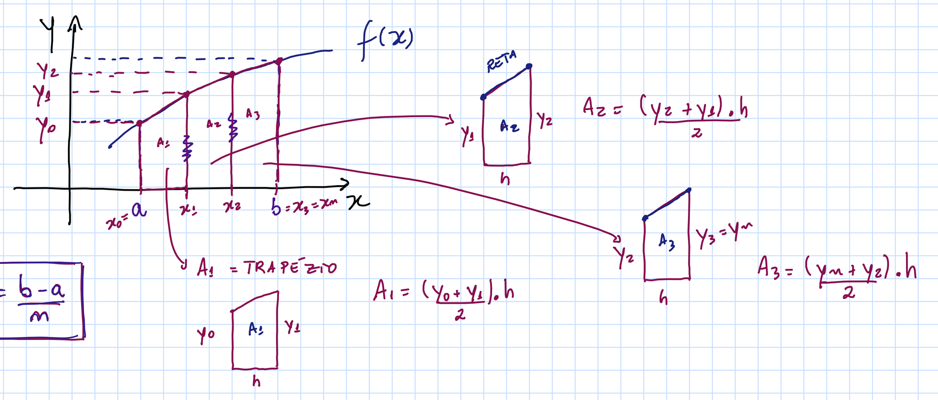
$\underbrace{\qquad\qquad\qquad}_{\text{INTEGRAÇÃO NUMÉRICA}}$



1. MÉTODO DOS TRAPÉZIOS

$$I = \int_a^b f(x)\,dx \approx \int_a^b \underbrace{p(x)}_{\text{POLINÔMIO INTERPOLADOR}}\,dx$$

NO MÉTODO DOS TRAPÉZIOS, ESSE POLINÔMIO INTERPOLADOR TEM GRAU 1 (UMA RETA)



$\boxed{h = \dfrac{b-a}{m}}$

$$\int_a^b f(x)\,dx \approx A_1 + A_2 + A_3$$

$$\approx (y_1 + y_0).\frac{h}{2} + (y_1 + y_2).\frac{h}{2} + (y_2 + y_3).\frac{h}{2}$$

$$\approx \frac{h}{2}.\left[y_0 + 2y_1 + 2y_2 + y_m\right]$$

$\vdots$

$$\boxed{I_m^T = \frac{h}{2}.\left[y_0 + 2y_1 + 2y_2 \ldots + 2y_{m-1} + y_m\right]}$$

MÉTODO DOS TRAPÉZIOS

EX01. DETERMINE $\int_{3,0}^{3,6} \frac{1}{x}\,dx$ USANDO O MÉTODO DO TRAPÉZIO COM $m = 3$

PASSO: $h = \frac{b-a}{m} = \frac{3,6 - 3,0}{3} = \frac{0,6}{3} = 0,2$

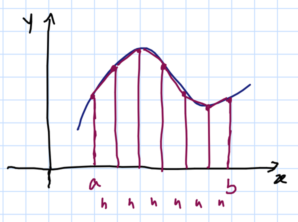
$f(x) = \frac{1}{x}$

i	x	y
0	3	0,33333
1	3,2	0,3125
2	3,4	0,29412
3	3,6	0,27778

$$I_3^T = \frac{h}{2}.\left[y_0 + 2y_1 + 2y_2 + y_3\right]$$

$$I_3^T = \frac{0,2}{2}.\left[0,33333 + 2.0,3125 + 2.0,29412 + 0,27778\right]$$

$$\boxed{I_3^T = 0,18243}$$



Sol. ANALÍTICA

$$\int_{3,0}^{3,6} \frac{1}{x}\,dx = \ln|x|\Big|_{3,0}^{3,6} = \ln(3,6) - \ln(3,0) = 0,18232$$

$$E_R = \frac{|0,18232 - 0,18243|}{0,18232} = 6\times10^{-4} = 6\times10^{-2} = 0,06\,\%$$

EX02 $\int_0^1 f(x)\,dx$?

x	0	0,25	0,50	0,75	1
f(x)	1	1,28	1,65	2,12	2,72
	y_0	y_1	y_2	y_3	y_4

$h = 0,25$

$$I_4^T = \frac{h}{2}.\left[y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right]$$

$$I_4^T = \frac{0,25}{2}.\left[1 + 2.1,28 + 2.1,65 + 2.2,12 + 2,72\right]$$

$$I_4^T = \frac{0,25}{2}.13,82$$

$$\boxed{I_4^T = 1,7275}$$

2. PRIMEIRA REGRA DE SIMPSON

NESSE CASO, O POLINÔMIO INTERPOLADOR TEM GRAU 2. $\Rightarrow P_2(x)$

LAGRANGE

$$P_2(x) = y_0.L_0 + y_1.L_1 + y_2.L_2$$

$$P_2(x) = y_0\left(\frac{(x-x_1).(x-x_2)}{(x_0-x_1).(x_0-x_2)}\right) + y_1\left(\frac{(x-x_0).(x-x_2)}{(x_1-x_0).(x_1-x_2)}\right) + y_2\left(\frac{(x-x_0).(x-x_1)}{(x_2-x_0).(x_2-x_1)}\right)$$

$$\int_{x_0}^{x_2} P_2(x)\,dx = y_0.\frac{h}{3} + y_1.\frac{4h}{3} + y_2.\frac{h}{3}$$

$$= \frac{h}{3}.(y_0 + 4y_1 + y_2)$$

DEMONSTRAÇÃO PARCIAL!

$$\int_{x_0}^{x_2} P_2(x)\,dx = \int_{x_0}^{x_2} y_0\left(\frac{(x-x_1).(x-x_2)}{(x_0-x_1).(x_0-x_2)}\right)dx$$

$x_0 - x_1 = -h$ e $x_0 - x_2 = -2h$

$$= y_0.\int_{x_0}^{x_2} \frac{(x-x_1).(x-x_2)}{(-h).(-2h)}\,dx$$

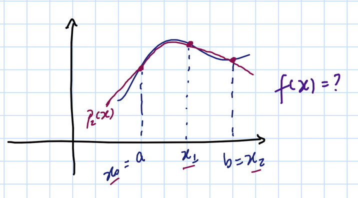
se $u = \frac{x - x_0}{h}$ $\Rightarrow uh + x_0 = x$, $h\,du = dx$; $u_0 = 0$, $u_2 = \frac{x_2 - x_0}{h} = \frac{2h}{h} = 2$

$$= \frac{y_0}{2h^2}\int_0^2 (uh + \underbrace{x_0 - x_1}_{-h}).(uh + \underbrace{x_0 - x_2}_{-2h}).h\,du$$

$$= \frac{y_0}{2h}\int_0^2 (uh - h).(uh - 2h)\,du$$

$$= \frac{y_0}{2h}.\int_0^2 (u^2h^2 - 3uh^2 + 2h^2)\,du$$

$$= \frac{y_0}{2h}\left(\frac{u^3}{3}h^2 - \frac{3u^2}{2}h^2 + 2h^2u\right)\Big|_0^2 \Rightarrow \frac{y_0}{2h}.\left(\frac{8h^2}{3} - 2h^2\right) \Rightarrow \frac{y_0}{2h}\left(\frac{2h^2}{3}\right) = y_0.\frac{h}{3}$$



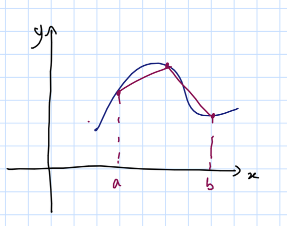
$$I = \int_{x_0}^{x_2} f(x)\,dx \approx \int_{x_0}^{x_2} P_2(x)\,dx \;\ldots = \frac{h}{3}\left(y_0 + 4y_1 + y_2\right)$$

generalizando p/ todos os pontos

$$\int_a^b f(x)\,dx \approx \int_{x_0}^{x_2} p_2(x)\,dx + \int_{x_2}^{x_4} p(x)\,dx \ldots \int_{x_{n-2}}^{x_n} p_2(x)\,dx$$

$$\approx \frac{h}{3}\left(y_0 + 4y_1 + y_2\right) + \frac{h}{3}\left(y_2 + 4y_3 + y_4\right) \ldots \frac{h}{3}.\left(y_{n-2} + 4y_{n-1} + y_n\right)$$

$$\approx \frac{h}{3}.\left(y_0 + 4y_1 + 2y_2 + 4y_3 \ldots 2y_{n-2} + 4y_{n-1} + y_n\right)$$



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$$\boxed{I_m^{1/3} = \frac{h}{3}.\left(y_0 + 4y_1 + 2y_2 \ldots 2y_{n-2} + 4y_{n-1} + y_n\right)}$$

EM QUE m É UM NÚMERO PAR DE SUBINTERVALOS

1º REGRA DE SIMPSON OU MÉTODO 1/3 DE SIMPSON

EX01. DETERMINE UMA APROXIMAÇÃO PARA O VALOR DA INTEGRAL $\int_1^3 \frac{\sqrt{x}}{1+x^2}\,dx$ USANDO A 1º REGRA DE SIMPSON E $m > 6$

USE 4 CASAS DECIMAIS APÓS A VÍRGULA EM SEUS CÁLCULOS

$f(x) = \frac{\sqrt{x}}{1+x^2}$

i	x	y	
0	1	0,5	
1	1,25	0,4363	4
2	1,50	0,3768	2
3	1,75	0,3256	4
4	2,0	0,2828	2
5	2,25	0,2474	4
6	2,50	0,2181	2
7	2,75	0,1937	4
8	3,0	0,1732	

$m = 8$ $\quad h = \frac{3-1}{8} = \frac{2}{8} = 0,25$

$$I_{1/3}^S = \frac{h}{3}.\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + 4y_5 \ldots \ldots + 2y_6 + 4y_7 + y_8\right]$$

$$I_{1/3}^S = \frac{0,25}{3}.\left[0,5 + 4.0,4368 + 2.0,3768 + 4.0,3256 \ldots \ldots + 2.0,2828 + 4.0,2474 + 2.0,2181 + \ldots \ldots + 4.0,1937 + 0,1732\right]$$

$$I_{1/3}^S = \frac{0,25}{3}.(7,2406)$$

$$\boxed{I_{1/3}^S = 0,6034}$$

EX02 - CALCULE $\int_2^4 \frac{\ln(x) + x^2}{(x+3)^2}\,dx$ COM $m = 10$

$h = \frac{4-2}{10} = \frac{2}{10} = 0,2$

$f(x) = \frac{\ln(x) + x^2}{(x+3)^2}$

i	x	y	
0	2	0,1877	
1	2,2	0,2082	4
2	2,4	0,2276	2
3	2,6	0,2460	4
4	2,8	0,2637	2
5	3,0	0,2805	4
6	3,2	0,2966	2
7	3,4	0,3121	4
8	3,6	0,3270	2
9	3,8	0,3412	4
10	4,0	0,3548	

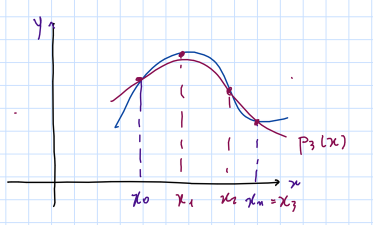
$$I_{1/3}^S = \frac{0,2}{3}.\left[0,1877 + 4.0,2082 + 2.0,2276 \ldots \ldots + 4.0,2460 + 2.0,2637 + 4.0,2805 \ldots + 2.0,2966 + 4.0,3121 + 2.0,3270 \ldots 4.0,3412 + 0,3548\right]$$

$$I_{1/3}^S = \frac{0,2}{3}.(8,3241)$$

$$\boxed{I_{1/3}^S = 0,5549}$$

3. SEGUNDA REGRA DE SIMPSON

NESSE CASO, O POLINÔMIO INTERPOLADOR TEM GRAU 3



$$I = \int_{x_0}^{x_3} f(x)\,dx \approx \int_{x_0}^{x_3} P_3(x) = \ldots \frac{3h}{8}.\left(y_0 + 3y_1 + 3y_2 + y_3\right)$$

generalizando p/ todos os pontos

$$I = \int_a^b f(x)\,dx \approx \int_{x_0}^{x_3} p_3(x)\,dx + \int_{x_3}^{x_6} p_3(x)\,dx \ldots \int_{x_{m-3}}^{x_m} p_3(x)\,dx$$

$$\approx \frac{3h}{8}\left(y_0 + 3y_1 + 3y_2 + y_3\right) + \frac{3h}{8}\left(y_3 + 3y_4 + 3y_5 + y_6\right) + \frac{3h}{8}\left(y_{m-3} + 3y_{m-2} + 3y_{m-1} + y_m\right)$$

$$\approx \frac{3h}{8}\left(y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 + 2y_6 \ldots 2y_{m-3} + 3y_{m-2} + 3y_{m-1} + y_m\right)$$

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$$\boxed{I_{3/8}^T = \frac{3h}{8}.\left(y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 \ldots 2y_{n-3} + 3y_{n-2} + 3y_{n-1} + y_n\right)}$$

c/ n MÚLTIPLO DE 3 e $h = \frac{x_m - x_0}{m}$

EX01 - CALCULAR UMA APROXIMAÇÃO PARA $I = \int_1^4 \ln(x^3 + \sqrt{e^x + 1})\,dx$ USANDO A SEGUNDA REGRA DE SIMPSON COM $m = 6$. CALCULE O ERRO RELATIVO SABENDO QUE O VALOR EXATO DA INTEGRAL É 8,56161.

$f(x) = \ln(x^3 + \sqrt{e^x + 1})$

$h = \frac{4-1}{6} = \frac{3}{6} = 0,5$

i	x	y	
0	1	1,07442	
1	1,5	1,74332	3
2	2,0	2,38843	3
3	2,5	2,95781	2
4	3,0	3,45290	3
5	3,5	3,88600	3
6	4,0	4,26908	

$$I_{3/8}^S = \frac{3h}{8}.\left[y_0 + 3y_1 + 3y_2 + 2y_3 + 3y_4 + 3y_5 + y_6\right]$$

$$I_{3/8}^S = \frac{3.0,5}{8}.\left[1,07442 + 3.1,74332 + 3.2,38843 + 2.2,95781 \ldots \ldots + 3.3,45290 + 3.3,88600 + 4,26908\right]$$

$$I_{3/8}^S = \frac{3.0,5}{8}.(45,67107)$$

$$\boxed{I_{3/8}^S = 8,56332}$$

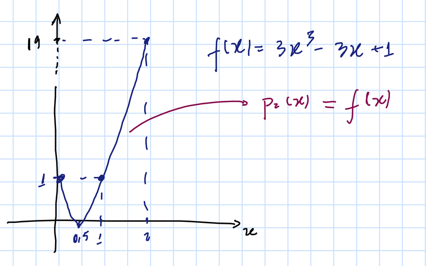
$$E_R = \frac{|8,56161 - 8,56332|}{8,56161} = 1,99\times10^{-4} = 1,99\times10^{-2} \approx 0,02\,\%$$

EX02 $\int_0^2 (3x^3 - 3x + 1)\,dx$

$h = \frac{2}{2} = 1$

$$I_{1/3}^S = \frac{h}{3}\left(1 + 4.1 + 19\right) \Rightarrow \frac{1}{3}.24 = 8$$

x	y
0	1
1	1
2	19



RESPOSTA ANALÍTICA $\int_0^2 3x^3 - 3x + 1 = \frac{3x^4}{4} - \frac{3x^2}{2} + x\Big|_0^2 = \frac{3.2^4}{4} - \frac{3.2^2}{2} + 2 = 12 - 6 + 2 = 8$