Test 2: Money Market modeling with a random-coefficient linear model

Time Series Analysis
Olivier Wintenberger (olivier.wintenberger@upmc.fr)

You have to deliver your Test by email in a pdf file accompanied with the R codes before the 10th of December, midnight. The statistical treatment of the discrete time process must be done in the software R. Commands can be described in the pdf only if necessary.

Let us now consider the following model for the USA Money Supply

$$DM1_t = \beta_{0,t} + \beta_{1,t} OBL_{t-1} + \beta_{2,t} INF_{t-1} + \beta_{3,t} SUR_{t-1} + \beta_{4,t} DM1_{t-1} + \zeta_t. \quad (1)$$

where $1 \leq t \leq T$ refers to the quarterly scale (with no seasonality adjustment) and

- $DM1_t$ refers to the log-ratio $100 \log(M1_t/M1_{t-1})$ with $M1_t$ the Money Supply ("M1 Money Supply").
- OBL_t refers to the increment $\mathcal{R}_t - \mathcal{R}_{t-1}$ of the short term rates $\mathcal{R}_t$ ("Three Month Treasury Bill: Secondary Market Rate").
- INF_t refers to the inflation rate $100 \log(CPI_t/CPI_{t-1})$ where CPI is the american consumer price index ("Consumer Price Index: Total All Items for the United States").
- SUR_t refers to the surplus (or deficit) of the federal government in a hundred billions of dollars ("Federal government budget surplus or deficit").
- ζ_t is a gaussian white noise: ζ_t are iid $\mathcal{N}(0, \sigma_\zeta^2)$.

The equation (1) models the following phenomena: The growth of the USA monetary supply ($DM1_t$) is a combination of short term needs and a major political objective: the control of the inflation (INF_{t-1}). The monetary politic is mainly based on the adjustment of the short term rates (OBL_{t-1}) with respect to the needs of the federal government (SUR_{t-1}) and the monetary growth of the last period ($DM1_{t-1}$). For more details we refer to the article of Kim and Nelson (1989)

The Time-Varying-Parameter Model for Modeling Changing Conditional Variance: The Case of the Lucas Hypothesis.

The coefficients $\beta_{i,t}$, $i = 1, \dots, 4$ quantify the influence of the different macro economical variables in the linear regression (1). The intercept $\beta_{0,t}$ can be viewed as a deterministic trend. All these coefficients can vary due to the economic situation of the USA at time t. We assume in the following that these coefficients $\beta_{i,t}$ and the intercept $\beta_{0,t}$ are random walks themselves:

$$\beta_{i,t} = \beta_{i,t-1} + \epsilon_{i,t} \text{ with independent } \epsilon_{i,t} \rightsquigarrow \mathcal{N}(0, \sigma_i^2) \text{ for } i = 0, \dots, 4. \quad (2)$$

Denoting $\beta_t = (\beta_{0,t}, \dots, \beta_{4,t})'$ the time series $(DM1_t)$ admits a state space model representation $1 \le t \le T$:

$$\begin{cases} \text{Space equation: } DM1_t = B_t \beta_t + \zeta_t, \\ \text{State equation: } \beta_t = A_t \beta_{t-1} + \epsilon_t, \end{cases}$$

with $A_t = I_5$, $B_t = (1, OBL_{t-1}, INF_{t-1}, SUR_{t-1}, DM1_{t-1})$ and

$$\epsilon_t = (\epsilon_{0,t}, \dots, \epsilon_{4,t})'.$$

The parameters of the model are merged into the vector

$$\theta = (\sigma_\zeta^2, \sigma_0^2, \dots, \sigma_4^2)' \in (0, \infty) \times [0, \infty)^5.$$

Denotes $\mathcal{F}_{t-1} = \sigma(DM1_0, B_1, \dots, DM1_{t-1}, B_t)$ the past observations. Fix the initial law of β_0 to $\mathcal{N}(0, 50 * I_5)$ and assume it is independent of the ϵ_t and ζ_t. We assume in Questions 1-6 that θ is deterministic and fixed.

1. Upload in R the data from `https://research.stlouisfed.org` (or Data.csv where the last 2 years are missing).

2. Give the expression of the density of the vectors ϵ_t.

3. Describe the recursive steps of the Kalman prediction algorithm that provide the prediction distribution $\mathcal{L}(\beta_t | \mathcal{F}_{t-1})$ that is assumed to be gaussian (see lecture notes).

4. Express the likelihood contrast $L_T(\theta)$ of $(DM1_1, \dots, DM1_T)$ (conditional on $DM1_0$) with respect to $V_{t/t-1}$, $\beta_{t/t-1}$ and B_t, $1 \le t \le T$.

5. Implement the state-space model in R thanks to the command SSM-regression of the package KFAS.

6. Using the command fitSSM of the package KFAS, compute the maximum likelihood estimator $\hat{\theta}$ from the initial value in the optimization routine
$$(0.5, 0.1, 0.1, 0.1, 0.1, 0.1)'.$$

7. Use the Kalman filter via the command KFS to compute $\beta_{t/t-1}$ and $\Sigma_{t/t-1}$ for $\theta = \hat{\theta}$.

8. Draw the curves of the coefficients $\beta_{t/t-1}$ and their variances (the diagonal terms in $\Sigma_{t/t}$) in a separate graph. Also draw the Monetary Supply $DM1_t$, their predictions at time $t-1$ and their conditional confidence intervals at level 95% given the past $\mathcal{F}_{t-1}$ for all t.